

1. (c) For solid sphere

$$I = \frac{2}{5} M R^2 = \frac{2}{5} \left(\frac{4}{3} \pi R^3 \rho \right) R^2$$

$$\rho = \frac{176}{105} R^5 \rho$$

2. (c) An object is said to be moving with a uniform acceleration, if its velocity changes by equal amount in equal intervals of time. The velocity-time graph of uniformly accelerated motion is a straight line inclined to time axis. Acceleration of an object in a uniformly accelerated motion in one dimension is equal to the slope of the velocity-time graph with time axis.

3. (a)

$$t_1 = \frac{2u \sin \theta}{g} \text{ and}$$

$$t_2 = \frac{2u \sin(90 - \theta)}{g} = \frac{2u \cos \theta}{g}$$

$$\therefore t_1 t_2 = \frac{4u^2 \cos \theta \sin \theta}{g^2} = \frac{2}{g} \left[\frac{u^2 \sin 2\theta}{g} \right]$$

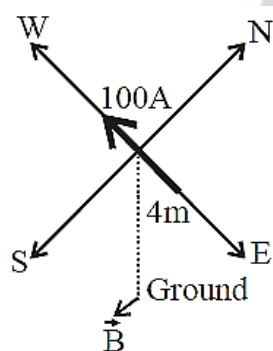
$$= \frac{2}{g} R,$$

where R is the range.

Hence $t_1 t_2 \propto R$

4. (c) The magnetic field is

$$B = \frac{\mu_0}{4\pi} \frac{2I}{r} = 10^{-7} \times \frac{2 \times 100}{4} = 5 \times 10^{-6} \text{ T}$$



According to right hand palm rule, the magnetic field is directed towards south.

5. (b)

6. (a) Since unit of energy = (unit of force).(unit of length) so if we increase unit of length and force, each by four times, then unit of energy will increase by sixteen times.
7. (d) Power is defined as the rate of doing work. For the automobile, the power output is the amount of work done (overcoming friction) divided by the length of time in which the work was done.
8. (b) Dimension of at = Dimension of F

$$[at] = [F] \Rightarrow [a] = \left[\frac{F}{t} \right]$$

$$[b] = \left[\frac{MLT^{-2}}{T} \right] \Rightarrow [a] = [MLT^{-3}]$$

$$\text{Dimension of } bt^2 = \text{Dimension of } F [bt^2] = [F] \Rightarrow [b] = \left[\frac{F}{t^2} \right]$$

$$[b] = \left[\frac{MLT^{-4}}{T^2} \right] \Rightarrow [b] = [MLT^{-4}]$$

9. (b) As $a_{CM} = 0$ [$v_{CM} = \text{constant}$], Tangential acceleration of each point $|\vec{a}_{AB}| = \frac{2v^2}{R}$

10. (b)

$$g = \frac{GM}{R^2} \Rightarrow \frac{dg}{g} = -2 \frac{dR}{R}$$

$$\frac{dR}{R} = -1\% \Rightarrow \frac{dg}{g} = 2\%$$

11. (c) For a perfectly rigid body strain produced is zero for the given force applied, so $Y = \text{stress} / \text{strain} = \infty$

12. (b) Ice is lighter than water. When ice melts, the volume occupied by water is less than that of ice. Due to which the level of water goes down.

13. (b)

14. (d) The entropy change of the body in the two cases is same as entropy is a state function.

15. (c)

16. (b) Differentiate $PV = \text{constant}$ w.r.t V

$$\Rightarrow P\Delta V + V\Delta P = 0 \Rightarrow \frac{\Delta P}{P} = -\frac{\Delta V}{V}$$

17. (a) $\Delta W = \text{area under the } p - V \text{ curve}$
 $= \frac{1}{2} \times 3p \times 2V = 3pV$

18. (c)

19. (b)

$$\gamma = 1 + \frac{2}{f}, \Rightarrow \gamma - 1 = \frac{2}{f}$$

$$\Rightarrow \frac{f}{2} = \frac{1}{\gamma - 1} \Rightarrow f = \frac{2}{\gamma - 1}$$

20. (b)

21. (a) $y(x, t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}xt)} = e^{-(\sqrt{ax} + \sqrt{bt})^2}$

It is a function of type $y = f(\omega t + kx)$

$\therefore y(x, t)$ represents wave travelling along

$-x$ direction.

$$\text{Speed of wave} = \frac{\omega}{k} = \frac{\sqrt{b}}{\sqrt{a}} = \sqrt{\frac{b}{a}}$$

22. (b)

$$v_s = \frac{v}{10} \quad n' = n \frac{v}{v-v_s}$$

$$\frac{n'}{n} = \frac{v}{\left(v - \frac{v}{10}\right)} = \frac{10}{9}$$

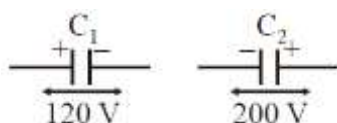
23. (c) $\phi = E(ds)\cos\theta = E(2\pi r^2)\cos 0^\circ = 2\pi r^2 E.$

24. (a)

$$-eE = mg$$

$$\vec{E} = -\frac{9.1 \times 10^{-31} \times 10}{1.6 \times 10^{-19}} = -5.6 \times 10^{-11} \text{ N/C}$$

25. (b)



For potential to be made zero, after connection

$$120C_1 = 200C_2 \quad \left[\because C = \frac{q}{v} \right]$$

$$\Rightarrow 3C_1 = 5C_2$$

26. (d)

$$V_A = IR$$

$$V_B = \left(\frac{2I}{3}\right) 1.5R = IR \quad V_C = \left(\frac{I}{3}\right) 3R = IR$$

$$\therefore V_A = V_B = V_C$$

27. (b) $1.5 = \frac{u^2 \sin 30}{g}; R = \frac{u^2 \sin 90}{g} = 3 \text{ km}$

28. (c) $\vec{\tau} = \vec{m} \times \vec{B}$

29. (d) Work done, $W = MB(1 - \cos\theta)$

$$\theta = 90^\circ$$

$$W = MB$$

30. (d)

31. (b) $\tan\phi = \frac{X}{R} \times \frac{4}{3}$

$$\text{Power factor} = \cos\phi = \frac{3}{5} = 0.6$$

32. (b) $V_{\text{rms}} = \sqrt{\frac{(T/2)V_0^2 + 0}{T}} = \frac{V_0}{\sqrt{2}}$

33. (b) $\mu_g \sin i = \mu_{\text{air}} \sin 90^\circ \Rightarrow \mu_g = \frac{1}{\sin i}$

34. (b)

35. (a) The angular fringe width is given by $\alpha = \frac{\lambda}{d}$

where λ is wavelength and d is the distance between two coherent sources. Thus

$$d = \frac{\lambda}{\alpha}$$

Given, $\lambda = 6280 \text{ \AA}, \alpha = 1^\circ = \frac{\pi}{180} \text{ radian.}$

$$\text{Thus } d = \frac{6280 \times 10^{-10}}{3.14} \times 180$$

$$= 3.6 \times 10^{-5} \text{ m} = 0.036 \text{ mm}$$

36. (c)

37. (a)

$$B. E_H = \frac{2.22}{2} = 1.11$$

$$B. E_{He} = \frac{28.3}{4} = 7.08$$

$$B. E_{Fe} = \frac{492}{56} = 8.78 = \text{maximum}$$

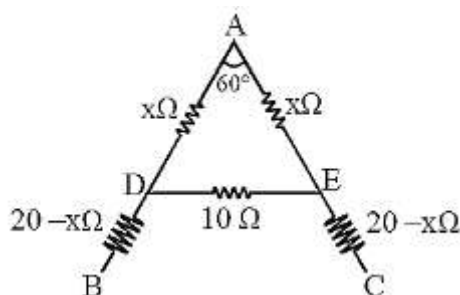
$$B. E_U = \frac{1786}{235} = 7.6$$

$^{56}_{26}\text{Fe}$ is most stable as it has maximum binding energy per nucleon.

38. (a) A positive feedback from output to input in an amplifier provides oscillations of constant amplitude.

39. (c)

40. (d)



41. (a) $100 \text{ amu of He} = \frac{100}{4} \text{ atoms of He} = 25 \text{ atoms.}$

[1 a.m.u. = mass of one proton (approx.)]

42. (a)

$$\text{Radius of orbit} = \frac{n^2 a_0}{z} \quad (a_0 = 0.529 \text{ \AA})$$

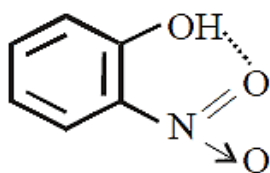
$$\text{Radius of H} = \frac{(1)^2 \times 0.529 \text{ \AA}}{1} = 0.53 \text{ \AA}$$

Thus, the radius of ${}_3\text{Li}^{2+}$ will be:

$$= \frac{(1)^2 \times 0.529}{3} = 0.17 \text{ \AA}$$

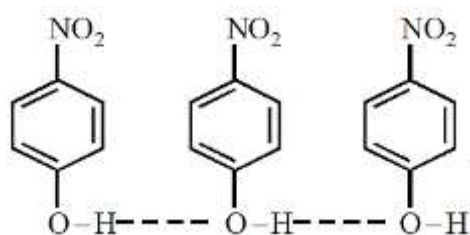
43. (d) P (At no. 15) has electronic configuration $1s^2, 2s^2p^6, 3s^2p^3$, hence no electron in d -subshell.

44. (c) Ortho-nitrophenol has intramolecular



H-bonding

and paranitrophenol has intermolecular H-bonding.

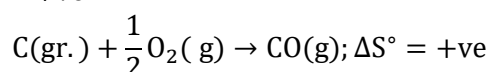


Hence former is more volatile than latter.

45. (d) In an ideal gas, the intermolecular forces of attraction are negligible and hence it cannot be liquefied.

46. (a) Since, in the first reaction gaseous products are forming from solid carbon hence entropy will increase i.e.

$\Delta S = +ve$.



Since, $\Delta G^\circ = \Delta H^\circ - T\Delta S$ hence the value of ΔG decrease on increasing temperature.

47. (a) $\frac{1}{2}H_2 + \frac{1}{2}X_2 \rightarrow HX$ Let the bond enthalpy of $X - X$ bond be x .

$$\Delta H_f(HX) = -50$$

$$= \frac{1}{2}\Delta H_{H-H} + \frac{1}{2}\Delta H_{X-X} - \Delta H_{H-X}$$

$$= \frac{1}{2}2x + \frac{1}{2}x - 2x = \frac{-x}{2}$$

$$\therefore x = 50 \times 2 = 100 \text{ kJ mol}^{-1}$$

48. (a)

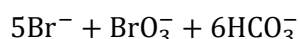
$$-\log(OH) = pOH; -\log 6.2 \times 10^{-9} = pOH;$$

$$\therefore pOH = 8.21$$

49. (d) Combination of $NaOH$ and CH_3COOH is the mixture of alkali and acetic acid. Therefore, this combination can not be buffer forming solution.

50. (c) By addition of SO_2 , equilibrium will shift to RHS which is exothermic. Hence temp, will increase.

51. (d) $3Br_2 + 6CO_3^{2-} + 3H_2O \rightarrow$



O.N. of Br_2 changes from 0 to -1 and +5, hence it is reduced as well as oxidised.

52. (c) The high boiling point of water is due to H-bonding.

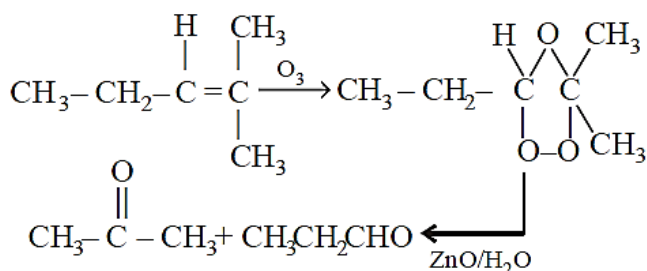
53. (b) The basic character of oxides increases down the group.

54. (a) The given two structures are optical isomers but as these are mirror image of each other, hence they represent enantiomers of each other.

55. (c) Paper chromatography is a special case of partition chromatography where the special quality paper containing water trapped in it acts as a stationary phase and solvent as a mobile phase. Thus, both phases are liquids.

56. (a) $-CCl_3$, $-NO_2$ and $-NH_3$ are m-directing in nature.

57. (a)



58. (b) In presence of U.V. rays O_2 is converted into O_3 .

59. (b) $\text{CO}_2 + \text{H}_2\text{O} \rightleftharpoons \text{H}_2\text{CO}_3 \rightarrow \text{H}^+ + \text{HCO}_3^-$

Here $[\text{H}^+]$ increases hence, pH decreases due to which soil fertility will also decrease.

60. (d) When an electrolyte dissociates van't Hoff factor i is greater than 1 and when it associates the i is less than 1.

61. (b)

$$\begin{aligned}
 m &= \frac{1000 \times k_b \times w}{W \times \Delta T_b} \\
 \text{or } k_b &= \frac{m \times W \times \Delta T_b}{1000 \times w} = \frac{100 \times 100 \times \Delta T_b}{1000 \times 10} \\
 &= \Delta T_b
 \end{aligned}$$

62. (c)

$$\begin{aligned}
 \lambda_m^\circ \text{ for BaCl}_2 &= \lambda_m^\circ \text{Ba}^{2+} + \lambda_m^\circ \text{Cl}^- \\
 &= \frac{1}{2} \times 127 + 76 = 139.5 \Omega^{-1} \text{ cm}^2
 \end{aligned}$$

63. (d) The reaction is of first and for a first order reaction, rate, $R = k[\text{N}_2\text{O}_5]$

$$\begin{aligned}
 2.4 \times 10^{-5} &= 3 \times 10^{-5} \times [\text{N}_2\text{O}_5] \\
 [\text{N}_2\text{O}_5] &= \frac{2.4 \times 10^{-5}}{3 \times 10^{-5}} = 0.8 \text{ mol L}^{-1}
 \end{aligned}$$

64. (c) The more the liquifiable nature of a gas, the more is the enthalpy of adsorption. Water is more liquifiable.

65. (a) According to Hardy-Schulze rule, coagulation power of ions is directly proportional to charge on ion.

$\therefore \text{Fe}(\text{OH})_3$ is positively charged colloid.

$\therefore$ It will be coagulated by anion.

(a) $\text{Mg}_3(\text{PO}_4)_2 \rightleftharpoons 3\text{Mg}^{2+} + 2\text{PO}_4^{3-}$

(b) $\text{BaCl}_2 \rightleftharpoons \text{Ba}^{2+} + 2\text{Cl}^-$

(c) $\text{NaCl} \rightleftharpoons \text{Na}^+ + \text{Cl}^-$

(d) $\text{KCN} \rightleftharpoons \text{K}^+ + \text{CN}^-$

Because PO_4^{3-} has highest charge among the given anions, therefore, $\text{Mg}_3(\text{PO}_4)_2$ is the most effective in the coagulation of $\text{Fe}(\text{OH})_3$ solution.

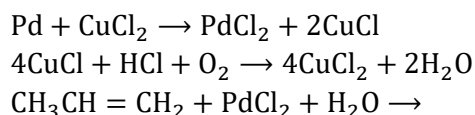
66. (c) $\text{Cl}_2 + \text{H}_2\text{O} \rightarrow 2\text{HCl} + \frac{1}{2}\text{O}_2$ Hydrogen has more affinity for chlorine.

67. (b) $2\text{Cu} + \text{H}_2\text{O} \xrightarrow[\text{Steam}]{\text{Hot}} \text{Cu}_2\text{O} + \text{H}_2 \uparrow$

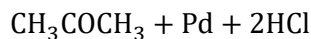
68. (b)

69. (b) At $120 - 140^\circ\text{C}$ temperature and 1.5 atm pressure, sodium phenoxide reacts with CO_2 to yield sodium salicylate which on further hydrolysis give to salicylic acid. This reaction is known as Kolbe's reaction.

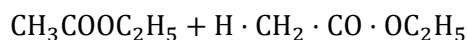
70. (b) In Wacker process, when mixture of propene and air is passed through mixture of Pd and CuCl_2 at high pressure acetone is formed.



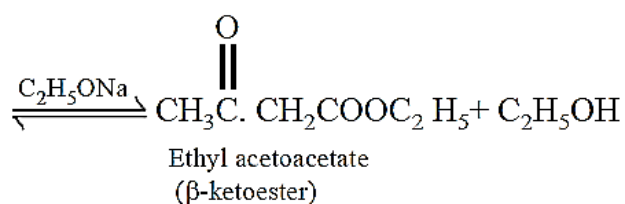
Propane



71. (d) In Claisen condensation intermolecular condensation of esters containing α -hydrogen atom in presence of strong base produce β -keto ester.

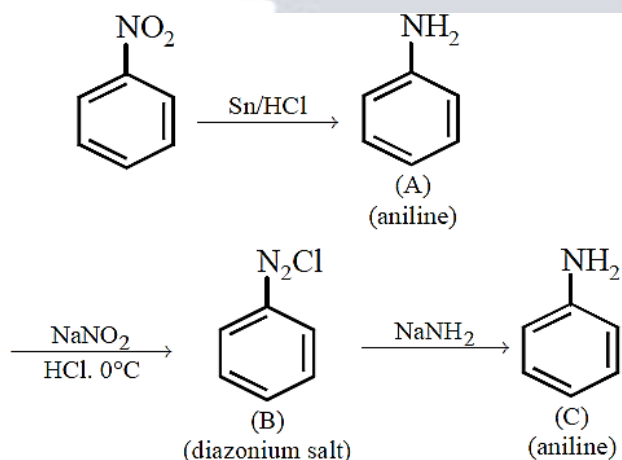


Ethyl acetate



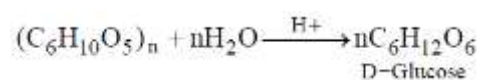
72. (b) Like clemmensen reduction, Wolf-Kishner reduction involves reduction of $> \text{C}=\text{O}$ to $> \text{CH}_2$, of course by different reagent.

73. (d)

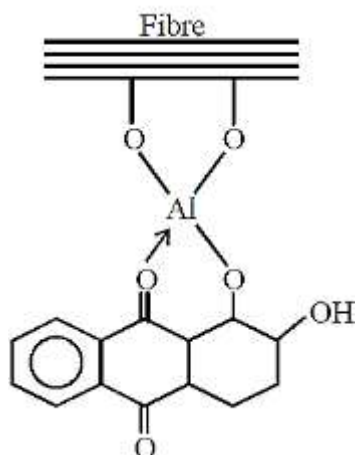


74. (c) Fibrous proteins have thread like molecules which lie side by side to form fibres. The various molecules are held together by hydrogen bonds.

75. (b)



76. (d) Alizarin is an anthraquinone dye. It gives a bright red colour with aluminium and a blue colour with barium.



77. (c) 2,4-dichlorophenoxyacetic acid is used as a herbicide.

78. (b)

Eq. of acid = Eq. of base,

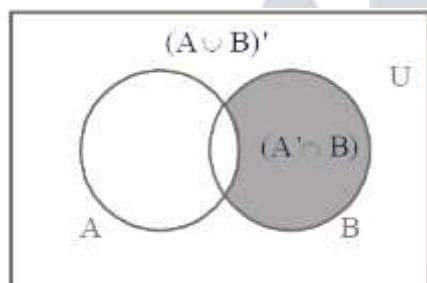
$$\therefore \frac{0.45}{\text{E. wt}} = \frac{20 \times 0.5}{1000} = \text{E. wt} = 45$$

$$\text{Basicity} = \frac{\text{M. wt}}{\text{E. wt}} = \frac{90}{45} = 2$$

79. (b)

80. (c) $\text{SnS} + (\text{NH}_4)_2\text{S}_2 \rightarrow (\text{NH}_4)_2\text{SnS}_3$ soluble

81. (a) From Venn-Euler's Diagram.



$$\therefore (A \cup B)' \cup (A' \cap B) = A'$$

82. (a) $xy - 12x - 12y = 0 \Rightarrow (x - 12)(y - 12) = 144$ Now 144 can be factorised into two factors x and y where $x \leq y$ and the factors are (1,144), (2,72), (3,48), (4,36), (6,24), (8,18), (9,16), (12,12).

Thus there are eight solutions.

83. (b) Using cosine formula

$$2\sin(\theta + \phi)\cos(\theta - \phi) = 1/2$$

$$2\cos(\theta + \phi)\cos(\theta - \phi) = 3/2$$

Squaring (1) and (2) and then adding

$$4\cos^2(\theta - \phi) = \frac{1}{4} + \frac{9}{4} = \frac{5}{2} \Rightarrow \cos^2(\theta - \phi) = \frac{5}{8}$$

84. (b) When $k = 1$, LHS = 1 but RHS = $1 + 10 = 11$

$\therefore T(1)$ is not true

Let $T(k)$ is true. That is

$$1 + 3 + 5 + \dots + (2k - 1) = k^2 + 10$$

Now, $1 + 3 + 5 + \dots + (2k - 1) + (2k + 1)$

$$= k^2 + 10 + 2k + 1 = (k + 1)^2 + 10$$

$\therefore T(k + 1)$ is true.

That is $T(k)$ is true $\Rightarrow T(k + 1)$ is true.

But $T(n)$ is not true for all $n \in \mathbb{N}$, as $T(1)$ is not true.

85. (c)

$$\sin \frac{\pi}{5} + i \left(1 - \cos \frac{\pi}{5} \right)$$

$$= 2 \sin \frac{\pi}{10} \cos \frac{\pi}{10} + i 2 \sin^2 \frac{\pi}{10}$$

$$= 2 \sin \frac{\pi}{10} \left(\cos \frac{\pi}{10} + i \sin \frac{\pi}{10} \right)$$

$$\text{For amplitude, } \tan \theta = \frac{\sin \frac{\pi}{10}}{\cos \frac{\pi}{10}} = \tan \frac{\pi}{10}$$

$$\Rightarrow \theta = \frac{\pi}{10}$$

86. (a) We have, $x = \omega - \omega^2 - 2$ or $x + 2 = \omega - \omega^2$

$$\text{Squaring, } x^2 + 4x + 4 = \omega^2 + \omega^4 - 2\omega^3$$

$$= \omega^2 + \omega^3 \omega - 2\omega^3 = \omega^2 + \omega - 2 \quad [\omega^3 = 1]$$

$$= -1 - 2 = -3 \Rightarrow x^2 + 4x + 7 = 0$$

Dividing $x^4 + 3x^3 + 2x^2 - 11x - 6$ by $x^2 + 4x + 7$, we get

$$\begin{aligned} x^4 + 3x^3 + 2x^2 - 11x - 6 &= (x^2 + 4x + 7)(x^2 - x - 1) + 1 \\ &= (0)(x^2 - x - 1) + 1 = 0 + 1 = 1 \end{aligned}$$

87. (a) First prize may be given to any one of the 4 boys; hence first prize can be distributed in 4 ways. similarly every one of second, third, fourth and fifth prizes can also be given in 4 ways.

$$\therefore \text{ the number of ways of their distribution} = 4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 1024$$

88. (b) We have: $30 = 2 \times 3 \times 5$. So, 2 can be assigned to either a or b or c i.e. 2 can be assigned in 3 ways.

Similarly, each of 3 and 5 can be assigned in 3 ways. Thus, the number of solutions is $3 \times 3 \times 3 = 27$.

89. (b)

$$\text{Expression} = (1 + x^2)^{40} \cdot \left(x + \frac{1}{x} \right)^{-10}$$

$$= (1 + x^2)^{30} \cdot x^{10}$$

$$\text{The coefficient of } x^{20} \text{ in } x^{10}(1 + x^2)^{30}$$

$$= \text{the coefficient of } x^{10} \text{ in } (1 + x^2)^{30}$$

$$= {}^{30}C_5 = {}^{30}C_{30-5} = {}^{30}C_{25}$$

90. (a) The series is a G.P. with common ratio $= \left(\frac{1-3x}{1+3x}\right)$ and $|r| = \left|\frac{1-3x}{1+3x}\right|$ is less than 1 since x is positive $S_{\infty} = \frac{a}{1-r} =$

$$\frac{\frac{1}{1+3x}}{1 - \left(-\frac{1-3x}{1+3x}\right)} = \frac{1}{2}$$

91. (a) If 'D' be the foot of altitude, drawn from origin to the given line, then 'D' is the required point.

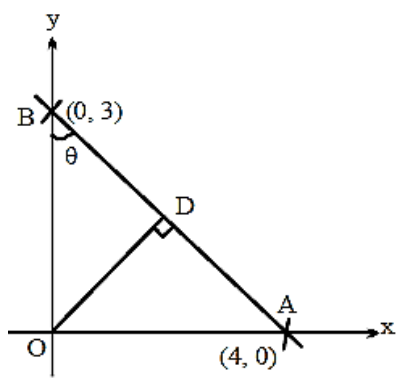
Let $\angle OBA = \theta$

$$\Rightarrow \tan \theta = 4/3$$

$$\Rightarrow \angle DOA = \theta$$

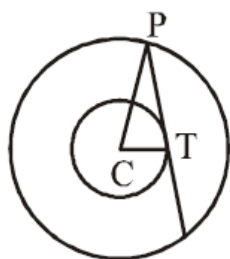
we have $OD = 12/5$.

If D is (h, k) then $h = OD \cos \theta, k = OD \sin \theta \Rightarrow h = 36/25, k = 48/25$



92. (a) Let the radius of the first circle be $CT = r_1$. Also, let the radius of the second circle be $CP = r_2$.

In the triangle PCT, T is a right angle



$$\begin{aligned} \text{So, } PT &= \sqrt{PC^2 - CT^2} = \sqrt{r_2^2 - r_1^2} \\ &= \sqrt{(f^2 - \lambda) - (f^2 - \mu)} = \sqrt{\mu - \lambda} \end{aligned}$$

93. (b) We have $x^2 - y^2 - 4x + 4y + 16 = 0$

$$\Rightarrow (x^2 - 4x) - (y^2 - 4y) = 16$$

$$\Rightarrow (x^2 - 4x + 4) - (y^2 - 4y + 4) = -16$$

$$\Rightarrow (x - 2)^2 - (y - 2)^2 = -16$$

$$\Rightarrow \frac{(x - 2)^2}{4^2} - \frac{(y - 2)^2}{4^2} = 1$$

This is rectangular hyperbola, whose eccentricity is always $\sqrt{2}$.

94. (c) Put $x = \frac{\pi}{2} - h$ as $x \rightarrow \frac{\pi}{2}, h \rightarrow 0$

$\therefore$ Given limit

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{1 - \tan\left(\frac{\pi}{4} - \frac{h}{2}\right)}{1 + \tan\left(\frac{\pi}{4} - \frac{h}{2}\right)} \cdot \frac{(1 - \cosh)}{(2h)^3} \\
 &= \lim_{h \rightarrow 0} \tan \frac{h}{2} \cdot \frac{2 \sin^2 \frac{h}{2}}{8h^3} \\
 &= \lim_{h \rightarrow 0} \frac{1}{4} \cdot \frac{\tan \frac{h}{2}}{\frac{h}{2} \times 2} \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}} \right)^2 \times \frac{1}{4} \\
 &= \lim_{h \rightarrow 0} \frac{1}{32} \cdot \left(\frac{\tan \frac{h}{2}}{\frac{h}{2} \times 2} \right) \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}} \right)^2 = \frac{1}{32}
 \end{aligned}$$

95. (a)

$$\begin{aligned}
 f'(5) &= \lim_{h \rightarrow 0} \frac{f(5+h) - f(5)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{f(5+h) - f(5+0)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{f(5) \cdot f(h) - f(5) + f(0)}{h} \\
 &(\because f(x+y) = f(x) \cdot f(y) \text{ for all } x, y) \\
 &= \left(\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} \right) \cdot f(5) = f'(0) \cdot f(5) \\
 &= 3 \times 2 = 6
 \end{aligned}$$

96. (a) $\because \delta_x^2 = \frac{\sum d^2 i}{n}$

But both A and B have 100 observations, then both the sample A and B have same standard deviation and the same variance.

Hence, $\frac{V_A}{V_B} = 1$

97. (a)

Since, $P(\text{exactly one of } A, B \text{ occurs}) = q$.

$$\begin{aligned}
 \therefore P(A \cup B) - P(A \cap B) &= q \\
 \Rightarrow p - P(A \cap B) &= q \Rightarrow P(A \cap B) = p - q \\
 \Rightarrow 1 - P(A' \cup B') &= p - q \Rightarrow P(A' \cup B') = 1 - p + q \\
 \Rightarrow P(A') + P(B') - P(A' \cap B') &= 1 - p + q \\
 \Rightarrow P(A') + P(B') &= (1 - p + q) + [1 - P(A \cup B)] \\
 &= (1 - p + q) + (1 - p) = 2 - 2p + q
 \end{aligned}$$

98. (a) We have, $\text{fog}(-x) = f[g(-x)] = f[-g(x)]$

($\because g$ is odd)

$$= f[g(x)] \quad (\because f \text{ is even})$$

$$= \text{fog}(x) \quad \forall x \in \mathbb{R}.$$

$\therefore \text{fog}$ is an even function.

99. (d)

$$\begin{aligned} & \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) \\ &= \tan^{-1}\left[\frac{\frac{1}{4} + \frac{2}{9}}{1 - \frac{1}{4} \times \frac{2}{9}}\right] = \tan^{-1}\left[\frac{1}{2}\right] \end{aligned}$$

100. (d) $\sin^{-1} x + \cos^{-1} x + \tan^{-1} x = \frac{\pi}{2} + \tan^{-1} x$

Since domain of the function $x \in [-1, 1] \therefore -\frac{\pi}{4} \leq \tan^{-1} x \leq \frac{\pi}{4}$.

Hence, $k = \frac{\pi}{4}$ and $K = \frac{3\pi}{4}$

101. (a) Consider first two equations:

$$2x + 3y = -4 \text{ and } 3x + 4y = -6$$

$$\text{We have } \Delta = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = -1 \neq 0$$

$$\Delta_x = \begin{vmatrix} -4 & 3 \\ -6 & 4 \end{vmatrix} = 2 \text{ and } \Delta_y = \begin{vmatrix} 2 & -4 \\ 3 & -6 \end{vmatrix} = 0$$

$$\therefore x = -2 \text{ and } y = 0$$

Now this solution satisfies the third, so the equations are consistent with unique solution.

102. (d) Applying $C_1 - C_2$ and $C_2 - C_3$, we get

$$\begin{aligned} \text{Det.} &= \begin{vmatrix} 25 & 21 & 219 \\ 15 & 27 & 198 \\ 21 & 17 & 181 \end{vmatrix} = \begin{vmatrix} 4 & 21 & 9 \\ -12 & 27 & -72 \\ 4 & 17 & 11 \end{vmatrix} \\ &[\text{by } C_1 - C_2, C_3 - 10C_2] \\ &= \begin{vmatrix} 4 & 21 & 9 \\ 0 & 90 & -45 \\ 0 & -4 & 2 \end{vmatrix} [\text{By } R_2 + 3R_1, R_3 - R_1] \\ &= 4(180 - 180) = 0 \end{aligned}$$

103. (b)

$$\text{Given } x = a \sin \theta \text{ and } y = b \cos \theta$$

$$\Rightarrow \frac{dx}{d\theta} = a \cos \theta \text{ and } \frac{dy}{d\theta} = -b \sin \theta$$

$$\therefore \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = -\frac{b}{a} \tan \theta \Rightarrow \frac{d^2y}{dx^2} = \frac{-b}{a} \sec^2 \theta$$

104. (d) For Rolle's theorem in $[a, b]$, $f(a) = f(b)$,

$$\text{In } [0, 1] \Rightarrow f(0) = f(1) = 0$$

$\therefore$ the function has to be continuous in $[0, 1]$

$$\Rightarrow f(0) = \lim_{x \rightarrow 0^+} f(x) = 0$$

$$\Rightarrow \lim_{x \rightarrow 0} x^\alpha \log x = 0 \Rightarrow \lim_{x \rightarrow 0} \frac{\log x}{x^{-\alpha}} = 0$$

$$\text{Applying L.H. Rule } \lim_{x \rightarrow 0} \frac{1/x}{-\alpha x^{-\alpha-1}} = 0 \Rightarrow \lim_{x \rightarrow 0} \frac{-x^\alpha}{\alpha} = 0 \Rightarrow \alpha > 0$$

105. (b) Since $f(x)$ is continuous at $x = 2$

$$\therefore f(2) = \lim_{x \rightarrow 2^+} f(x) \Rightarrow 1 = \lim_{x \rightarrow 2^+} (ax + b)$$

$$\therefore 1 = 2a + b \dots (1)$$

Again $f(x)$ is continuous at $x = 4$,

$$\therefore f(4) = \lim_{x \rightarrow 4^-} f(x) \Rightarrow 7 = \lim_{x \rightarrow 4^-} (ax + b)$$

$$\therefore 7 = 4a + b \dots (2)$$

Solving (1) and (2), we get $a = 3, b = -5$

106. (c) $f'(x) = 3 \left(\frac{a^2-1}{a^2+1} \right) x^2 - 3$

$$f'(x) < 0 \text{ for all } x \text{ if } a^2 - 1 \leq 0 \Rightarrow -1 \leq a \leq 1$$

107. (b) Diagonal $D = \sqrt{2} \cdot a$

$$\text{Differentiating w.r.t. } t: \frac{dD}{dt} = \sqrt{2} \frac{da}{dt} \text{ or } \frac{da}{dt} = \frac{1}{\sqrt{2}} \frac{dD}{dt} = \frac{1}{\sqrt{2}} \times 0.5 \text{ cm/s}$$

Let Area is denoted by A

$$\frac{dA}{dt} = 2a \frac{da}{dt}$$

when area A is 400 cm^2 then $a = 20$

$$\therefore \frac{dA}{dt} = 2 \times 20 \times \frac{0.5}{\sqrt{2}} = 10\sqrt{2} \text{ cm}^2/\text{sec}$$

108. (d)

Slope of normal to $y = f(x)$ at $(3,4)$ is $\frac{-1}{f'(3)}$.

$$\begin{aligned} \text{Thus, } \frac{-1}{f'(3)} &= \tan\left(\frac{3\pi}{4}\right) = \tan\left(\frac{\pi}{2} + \frac{\pi}{4}\right) \\ &= -\cot\frac{\pi}{4} = -1 \Rightarrow f'(3) = 1 \end{aligned}$$

109. (a) $I = \int \frac{\sqrt{x}}{4-x^3} dx = \int \frac{\sqrt{x} dx}{\sqrt{4-x^3}}$

Here integral of $\sqrt{x} = \frac{2}{3} x^{3/2}$ and $4 - x^3 = 4 - (x^{3/2})^2$

$$\text{Put } x^{3/2} = t \Rightarrow \sqrt{x} dx = \frac{2}{3} dt$$

$$\text{So } I = \frac{2}{3} \int \frac{dt}{\sqrt{4-t^2}} = \frac{2}{3} \sin^{-1}\left(\frac{x^{3/2}}{2}\right) + c$$

110. (c) $I = \int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx$

$$\begin{aligned} I &= \int_0^{\pi/2} \frac{2^{\sin(\pi/2-x)}}{2^{\sin(\pi/2-x)} + 2^{\cos(\pi/2-x)}} dx \\ &= \int \frac{2^{\cos x}}{2^{\cos x} + 2^{\sin x}} dx \Rightarrow 2I = \int_0^{\pi/2} \frac{2^{\cos x} + 2^{\sin x}}{2^{\cos x} + 2^{\sin x}} dx = \int_0^{\pi/2} 1 dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4} \end{aligned}$$

111. (c) Area $= \int_0^{\pi/2} y dx = \int_0^{\pi/2} \sin x dx = [-\cos x]_0^{\pi/2} = 1$

112. (d)

$$Ax^2 + By^2 = 1$$

$$Ax + By \frac{dy}{dx} = 0$$

$$A + By \frac{d^2y}{dx^2} + B \left(\frac{dy}{dx} \right)^2 = 0$$

From (2) and (3)

$$x \left\{ -By \frac{d^2y}{dx^2} - B \left(\frac{dy}{dx} \right)^2 \right\} + By \frac{dy}{dx} = 0$$

Dividing both sides by $-B$, we get

$$xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx} \right)^2 - y \frac{dy}{dx} = 0$$

Which is a DE of order 2 and degree 1

113. (c) Unit vector perpendicular to both the given vectors is,

$$\frac{(6\hat{i} + 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 6\hat{j} - 2\hat{k})}{|(6\hat{i} + 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 6\hat{j} - 2\hat{k})|} = \frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7}$$

114. (c)

$$a \cdot b = a \cdot c \Rightarrow a \cdot (b - c) = 0$$

$$\Rightarrow a = 0 \text{ or } b - c = 0 \text{ or } a \perp (b - c)$$

$$\Rightarrow a = 0 \text{ or } b = c \text{ or } a \perp (b - c)$$

$$\text{Also } a \times b = a \times c \Rightarrow a \times (b - c) = 0$$

$$\Rightarrow a = 0 \text{ or } b - c = 0 \text{ or } a \parallel (b - c)$$

$$\Rightarrow a = 0 \text{ or } b = c \text{ or } a \parallel (b - c)$$

Observing to (1) and (2) we find that

$$a = 0 \text{ or } b = c$$

115. (b) If (l_1, m_1, n_1) and (l_2, m_2, n_2) are the direction ratios then angle between the lines is

$$\cos \theta = \frac{l_1 l_2 + m_1 m_2 + n_1 n_2}{\sqrt{l_1^2 + m_1^2 + n_1^2} \sqrt{l_2^2 + m_2^2 + n_2^2}}$$

Here $l_1 = 1, m_1 = 1, n_1 = 1$ and

$$l_2 = 1, m_2 = -1, n_2 = n \text{ and } \theta = 60^\circ$$

$$\therefore \cos 60^\circ = \frac{1 \times 1 + 1 \times (-1) + 1 \times n}{\sqrt{1^2 + 1^2 + 1^2} \times \sqrt{1^2 + 1^2 + n^2}}$$

$$\Rightarrow \frac{1}{2} = \frac{\sqrt{3}\sqrt{2+n^2}}{\sqrt{3}(2+n^2)} = \frac{1}{\sqrt{2+n^2}}$$

$$\Rightarrow n^2 = 6 \Rightarrow n = \pm\sqrt{6}$$

116. (a) We know that the length of the perpendicular from the point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$ is

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

and the co-ordinate (α, β, γ) of the foot of the $\perp$ are given by

$$\begin{aligned} \frac{\alpha - x_1}{a} &= \frac{\beta - y_1}{b} = \frac{\gamma - z_1}{c} \\ &= -\left(\frac{ax_1 + by_1 + cz_1 + d}{a^2 + b^2 + c^2}\right) \end{aligned}$$

In the given ques., $x_1 = 7, y_1 = 14, z_1 = 5,$

$$a = 2, b = 4, c = -1, d = -2$$

By putting these values in (1), we get

$$\begin{aligned} \frac{\alpha - 7}{2} &= \frac{\beta - 14}{4} = \frac{\gamma - 5}{-1} = -\frac{63}{21} \\ \Rightarrow \alpha &= 1, \beta = 2 \text{ and } \gamma = 8 \end{aligned}$$

Hence, foot of $\perp$ is $(1, 2, 8)$

117. (b) The direction ratios of the line are

$$3 - 2, -4 - (-3), -5 - 1 \text{ i.e. } 1, -1, -6$$

Hence equation of the line joining the given points is $\frac{x-2}{1} = \frac{y+3}{-1} = \frac{z-1}{-6} = r$ (say)

Coordinates of any point on this line are $(r + 2, -r - 3, -6r + 1)$

If this point lies on the given plane $2x + y + z = 7$, then $2(r + 2) + (-r - 3) + (-6r + 1) = 7 \Rightarrow r = -1$

Coordinates of any point on this line are $(-1 + 2, -(-1) - 3, -6(-1) + 1)$ i.e. $(1, -2, 7)$

118. (b) The probability of hitting the target 5th time at the 10th throw = $P(\text{the probability of hitting the target 4 times in the first 9 throws}) \times P(\text{the probability of hitting the target at the 10th throw}) =$

$$\left[{}^9C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^5 \right] \left(\frac{1}{2}\right) = \frac{9!}{4!5!} \times \left(\frac{1}{2}\right)^{10} = \frac{63}{2^9}$$

119. (c) The probability of showing same number by both dice $p = \frac{6}{36} = \frac{1}{6}$ In binomial distribution here $n = 4, r = 2, p =$

$$\frac{1}{6}, q = \frac{5}{6}$$

$$\begin{aligned} \therefore \text{req. probability} &= {}^nC_r q^{n-r} p^r = {}^4C_2 \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right)^2 \\ &= 6 \left(\frac{25}{36}\right) \left(\frac{1}{36}\right) = \frac{25}{216} \end{aligned}$$

120. (b)

$$A = 180^\circ - 60^\circ - 75^\circ = 180^\circ - 135^\circ = 45^\circ$$

$$\text{Now, } \frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\Rightarrow \frac{2}{\sin 45^\circ} = \frac{b}{\sin 60^\circ} \Rightarrow b = \frac{2 \cdot (\sqrt{3}/2)}{1/\sqrt{2}} = \sqrt{6}$$

121. (a) The function is given by profit function $= x \cdot \frac{8}{100} + y \times \frac{10}{100} = 0.08 + 0.10y$.

122. (c) Given,

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \dots + \frac{x^2}{2} + x + 1$$

$$\Rightarrow f'(x) = \frac{100x^{99}}{100} + \frac{99x^{98}}{99} + \dots + \frac{2x}{2} + 1 + 0$$

$$[\because f(x) = x^n \Rightarrow f'(x) = nx^{n-1}]$$

$$\Rightarrow f'(x) = x^{99} + x^{98} + \dots + x + 1$$

Putting $x = 1$, we get

$$f'(1) = \underbrace{(1)^{99} + 1^{98} + \dots + 1 + 1}_{100 \text{ times}} = \underbrace{1 + 1 + 1 \dots + 1 + 1}_{100 \text{ times}}$$

$$\Rightarrow f'(1) = 100$$

Again, putting $x = 0$, we get

$$f'(0) = 0 + 0 + \dots + 0 + 1 \Rightarrow f'(0) = 1$$

From eqs. (ii) and (iii), we get; $f'(1) = 100f'(0)$

Hence, $m = 100$

123. (a)

$$As^2 = 0, A^k = 0 \forall k \geq 2.$$

$$\text{Thus, } (A + I)^{50} = I + 50A \Rightarrow (A + I)^{50} - 50A = I$$

$$\therefore a = 1, b = 0, c = 0, d = 1$$

$$abc + abd + bcd + acd = 0$$

124. (c)

125. (d)

$$\frac{p}{2} [2a_1 + (p-1)d] = \frac{p^2}{2}$$

$$\frac{q}{2} [2a_1 + (q-1)d] = \frac{q^2}{2}$$

$$\Rightarrow \frac{2a_1 + (p-1)d}{2a_1 + (q-1)d} = \frac{p}{q}$$

$$\frac{a_1 + \left(\frac{p-1}{2}\right)d}{a_1 + \left(\frac{q-1}{2}\right)d} = \frac{p}{q} \text{ For } \frac{a_6}{a_{21}}, p = 11, q = 41$$

$$\Rightarrow \frac{a_6}{a_{21}} = \frac{11}{41}$$

126. (b) The word Florid (Adjective) means : rosy; gaudy; ornated; red; having too much decoration or detail. The word Pale (Adjective) means : light in colour; not strong or bright; having skin that is almost white because of illness. Hence, the words florid and pale are antonymous.
127. (c) The word Verity (Noun) means: a belief or principle about life that is accepted as true; truth. Hence, the words verity and falsehood are antonymous.
128. (a) The word Perspicuity (Noun) means: clarity. The word vagueness (Noun) means: no clarity in a person's mind Hence, the words perspicuity and Vagueness are antonymous.
129. (d) Disgrace means a state of shame.
130. (a) Striking means extraordinary, attractive.
131. (b) Fiasco means a complete failure.
132. (d)
133. (a) Word for word means: in exactly the same words or when translated exactly equivalent words.
134. (b) Huddle: come close in a group
135. (b) Right use of as - as comparison
136. (a)
137. (a)
138. (a)
139. (a)
140. (c)
141. (a)
142. (a)

$$(1 \times 2 \times 3 \times 5) + (1 + 2 + 3 + 5) = 41$$

$$(3 \times 4 \times 2 \times 6) + (3 + 4 + 2 + 6) = 159$$

$$(9 \times 8 \times 3 \times 4) + (9 + 8 + 3 + 4) = 888$$

143. (b) Each day of the week is repeated after 7 days.
So, after 63 days, it will be Monday. After 61 days, it will be Saturday.
144. (d)
145. (b) The number should be 404.

$$\times 1 + 100, \times 2 + 100, \times 3 + 100 \dots \dots$$

146. (b) After putting sign

$$24 = 4 \times 5 + 4$$

$$24 = 24$$

Hence, (b) is correct choice.

147. (a)
148. (d)
149. (d)
150. (c)