1. **(b)** The escape velocity from earth is given by

$v_e = \sqrt{2\,gR_e}$ ...

The orbital velocity of a satellite revolving around earth is given by

$$v_0 = \frac{\sqrt{GM_e}}{(R_e + h)}$$

where, M_e = mass of earth, R_e = radius of earth, h = height of satellite from surface of earth.
By the relation $GM_e = gR_e^2$

So, $v_0 = \frac{\sqrt{gR_e^2}}{(R_e+h)}$

Dividing equation (i) by (ii), we get

$$\frac{v_e}{v_0} = \frac{\sqrt{2(R_e + h)}}{(R_e)}$$

Given, $v_0 = \frac{v_e}{2}$

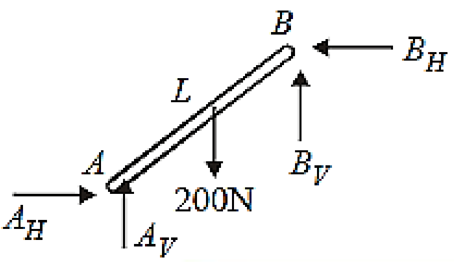
$$\frac{2v_e}{v_e} = \frac{\sqrt{2(R_e + h)}}{R_e}$$

Squaring on both side, we get

$$4 = \frac{2(R_e + h)}{R_e}$$

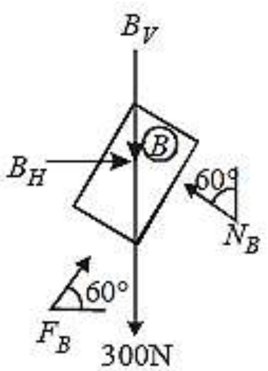
or $R_e + h = 2R_e$ i.e., $h = R_e$

2. **(a)** Consider FBD of structure.



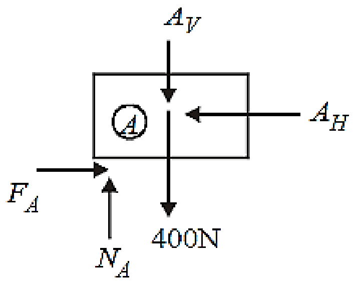
Applying equilibrium equations, $Av + Bv = 200$ N ... (i)

$A_H = B_H$ ... (ii)
From FBD of block B,



$$B_H + F_B\cos 60^\circ - N_B\sin 60^\circ = 0$$
$$N_B\cos 60^\circ - B_{V'} - 300 + F_B\sin 60^\circ = 0$$
$$F_B = 0.25N_B$$
$$B_H - 0.74N_B = 0 \ \ldots \text{(iii)}$$
$$-B_V + 0.71N_B = 300 \ \ldots \text{(iv)}$$

FBD of block A



$$F_A - A_H = 0$$
$$N_A - A_V = 400 \ \ldots \text{(v)}$$
$$F_A = \mu_A N_A$$
$$\therefore \mu_A N_A - A_H = 0 \ \ldots \text{(vi)}$$

On solving above equations, we get

$$N_A = 650\ \text{N}, F_A = 260\ \text{N}, F_A = \mu_A N_A$$
$$\therefore \mu_A = \frac{260}{250} = 0.4$$

3. **(a)** Let v = speed of sound and v_S = speed of tuning forks. Apparent frequency of fork moving towards the observer is

$$n_1 = \left(\frac{v}{v - v_S}\right) n$$

Apparent frequency of the fork moving away from the observer is

$$n_2 = \left(\frac{v}{v + v_S}\right) n$$

If f is the number of beats heard per second. then $f = n_1 - n_2$

$$\Rightarrow f = \left(\frac{v}{v - v_s}\right) n - \left(\frac{v}{v + v_s}\right) n$$
$$\Rightarrow f = \frac{v(v + v_s) - v(v - v_s)}{v^2 - v_s^2}(n)$$
$$\Rightarrow \frac{2v_s n}{v^2 - v_s^2} = f \Rightarrow 2\left(\frac{v_s}{v}\right) = n = f\{if v_s \ll v\}$$
$$\Rightarrow v_s = \frac{fv}{2n}$$

putting $v = 340\ \text{m/s}, f = 3, n = 340\ \text{Hz}$ we get,

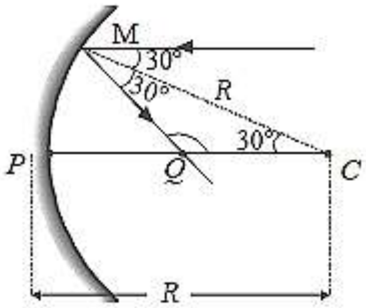
$$v_s = \frac{340 \times 3}{3 \times 340} = 1.5\ \text{m/s}$$

4. (a) The displacement of the particle is given by:

$$
\begin{aligned}
x &= A\sin(-2\omega t) + B\sin^2 \omega t \\
&= -A\sin 2\omega t + \frac{B}{2}(1 - \cos 2\omega t) \\
&= -\left(A\sin 2\omega t + \frac{B}{2}\cos 2\omega t\right) + \frac{B}{2}
\end{aligned}
$$

This motion represents SHM with an amplitude: $\sqrt{A^2 + \frac{B^2}{4}}$, and mean position $\frac{B}{2}$.

5. (d) From similar triangles,



$$
\frac{QC}{\sin 30^\circ} = \frac{R}{\sin 120^\circ}
$$

$$
\text{or } QC = R \times \frac{\sin 30^\circ}{\sin 120^\circ} = \frac{R}{\sqrt{3}}
$$

Thus

$$
PQ = PC - QC = R - \frac{R}{\sqrt{3}} = R\left(1 - \frac{1}{\sqrt{3}}\right)
$$

6. (c) Using Gauss's law, we have

$$
\oint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0}\int (\rho dv) = \frac{1}{\epsilon_0}\int_0^R kr^a \times 4\pi r^2 dr
$$

$$
\text{or } E \times 4\pi R^2 = \left(\frac{4\pi k}{\epsilon_0}\right)\frac{R^{(a+3)}}{(a+3)}
$$

$$
\therefore\ E_1 = \frac{kR^{(a+1)}}{\epsilon_0(a+3)}
$$

For $r = \frac{R}{2} \cdot E_2 = \frac{k\left(\frac{R}{2}\right)^{a+1}}{\epsilon_0(a+3)}$

Given, $E_2 = \frac{E_1}{8}$

$$
\text{or } \frac{k\left(\frac{R}{2}\right)^{a+1}}{\epsilon_0(a+3)} = \frac{1}{8}\frac{kR^{(a+1)}}{\epsilon_0(a+3)}
$$

$$
\therefore\ 2^{\frac{1}{a+1}} = \frac{1}{8}
$$

$$
\text{or } a = 2.
$$

7. **(a)** As we know $F = qvB = \frac{mv^2}{r}$

$$\therefore r = \frac{mv}{Bq}$$

And $KE = k = \frac{1}{2}mv^2$

$$\therefore mv = \sqrt{2\,km}$$
$$\therefore r = \frac{mv}{qB} = \frac{\sqrt{2\,km}}{qB}$$

$\Rightarrow r \propto \sqrt{k}$ or $r = c^{1/2}$ (c is a constant)

$$\frac{dr}{dr} = c\frac{dk^{1/2}}{dr} \text{ or } \frac{c\Delta k}{\Delta r} = 2\sqrt{k}$$
$$\text{or } \frac{\Delta r}{r} = \frac{c\Delta k}{2\sqrt{k}c\sqrt{k}} = \frac{\Delta k}{2k}$$

Therefore, percentage changes in radius of path,

$$\frac{\Delta r}{r} \times 100 = \frac{\Delta k}{2k} \times 100 = 2\%$$

8. **(d)** Given: fringe with $\beta = 0.03$ cm, $D = 1$ m $= 100$ cm
Distance between images of the source $= 0.8$ cm.
Image distance $v = 80$ cm
Object distance $= u$
Using mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{60} + \frac{1}{u} = \frac{1}{16}$$
$$\Rightarrow u = 20 \text{ cm}$$

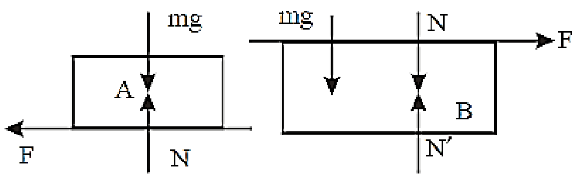
Magnification, $m = \frac{v}{u} = \frac{80}{20} = 4$

$$\text{Magnification} = \frac{\text{distances between images of slits}}{\text{distance between slits}}$$
$$= \frac{0.8}{d} = \frac{0.8}{d} = 4 \Rightarrow d = 0.2 \text{ cm}$$

Fringe width $\beta = \frac{D\lambda}{d}$

or, $\beta = \frac{100\lambda}{0.2} = 0.03 \times 10^{-2}$
Therefore, wavelength of light used $\lambda = 6000$ Å

9. **(d)** For the blocks A and B FBD as shown below



Equations of motion

$$a_A = \frac{F}{M}(\text{ in } -x \text{ direction })$$
$$a_B = \frac{F}{M}(\text{ in } +x \text{ direction })$$

Relative acceleration, of A w.r.t. B,

$$\begin{aligned} a_{A,B} &= a_A - a_B = -\frac{F}{m} - \frac{F}{M} \\ &= -F\left(\frac{M+m}{Mm}\right)(\text{ along } -x \text{ direction }) \end{aligned}$$

Initial relative velocity of A w.r.t. B,

$u_{AB} = v_0$

using equation $v^2 = u^2 + 2$ as

$$0 = v_0^2 - \frac{2F(m+M)S}{Mm} \Rightarrow S = \frac{Mmv_0^2}{2F(m+M)}$$

i.e., Distance moved by A relative to B

$$S_{AB} = \frac{Mmv_0^2}{2F(m+M)}$$

10. (b) In the string elastic force is conservative in nature.

$\therefore W = -\Delta U$

Work done by elastic force of string,

$$\begin{aligned} W &= -(U_F - U_i) = U_i - U_F. \\ W &= \frac{1}{2}kx^2 - \frac{k}{2}(x+y)^2 \\ &= \frac{1}{2}kx^2 - \frac{1}{2}k(x^2 + y^2 + 2xy) \\ &= \frac{1}{2}kx^2 - \frac{1}{2}ky^2 - \frac{1}{2}kx^2 - \frac{1}{2}k(2xy) \\ &= -kxy - \frac{1}{2}ky^2 \end{aligned}$$

Therefore, the work done against elastic force

$$W_{\text{external}} = -W = \frac{ky}{2}(2x+y)$$

11. (c) Let the body is projected vertically upwards from A with a speed u_0.

Using equation, $s = ut + \frac{1}{2}at^2$

For case (1) $-h = u_0t_1 - \frac{1}{2}g_1^2$

For case (2) $-h = u_0t_2 - \frac{1}{2}gt_1^2$

Subtracting eq (2) from (1), we get

$0 = u_0(t_2 + t_1) + \frac{1}{2}g(t_2^2 - t_1^2)$

$\Rightarrow u_0 = \frac{1}{2}\, g(t_1 - t_2)$

Putting the value of u_0 in eq (2), we get

$-h = -\left(\frac{1}{2}\right)g(t_1 - t_2)t_2 - \left(\frac{1}{2}\right)g\,{}_2^2$

$\Rightarrow h = \frac{1}{2}\, g(t_1t_2)$

For case 3, $u_0 = 0, t = ?$

$-h = 0 \times t - \left(\frac{1}{2}\right)gt^2$

$h = \left(\frac{1}{2}\right)gt^2$

Comparing eq. (4) and (5), we get

$\frac{1}{2}gt^2 = \frac{1}{2}gt_1t_2 \;\therefore t = \sqrt{t_1t_2}$

12. (c)

$\frac{\Delta Q}{\Delta t} = \frac{\Delta W}{\Delta t} =$ work done per unit time $= \frac{ka\theta}{L}$

$\frac{dW}{dt} = P\frac{dv}{dt} = k\frac{a\theta}{L}, P = \frac{nRT}{V}$

$\Rightarrow \quad \frac{0.5R(300)}{V}A \cdot \frac{d\ell}{dt} = \frac{ka\theta}{L}$

$\Rightarrow \quad \frac{0.5R(300)}{A \cdot \frac{L}{2}}A \cdot v = \frac{ka\theta}{L}$

$\Rightarrow \quad v = \frac{ka}{R}\left(\frac{27}{300}\right) = \frac{k}{100R}$

13. (a) At any time t, the side of the square $a = (a_0 - \alpha t)$, where $a_0 =$ side at $t = 0$.

At this instant, flux through the square:

$\phi = BA\cos 0^\circ = B(a_0 - \alpha t)^2$

$\therefore$ emf induced $E = -\frac{d\phi}{dt}$

$\Rightarrow E = -B \cdot 2(a_0 - \alpha t)(0 - \alpha) = +2\alpha aB$

14. (b) As we know, threshold wavelength

$(\lambda_0) = \frac{hc}{\phi}$

$\Rightarrow \lambda_0 \quad = \frac{(6.63 \times 10^{-34}) \times 3 \times 10^8}{2.3 \times (1.6 \times 10^{-19})} = 5.404 \times 10^{-7}\text{ m}$

$\Rightarrow \quad \lambda_0 = 5404\text{Å}$

Hence, wavelength 4144Å and 4972Å will emit electron from the metal surface.
For each wavelength energy incident on the surface per unit time = intensity of each × area of the surface wavelength

$$= \frac{3.6 \times 10^{-3}}{3} \times (1\text{ cm})^2 = 1.2 \times 10^{-7} \text{ joule}$$

Therefore, energy incident on the surface for each wavelength in 2 s

$$E = (1.2 \times 10^{-7}) \times 2 = 2.4 \times 10^{-7}\text{ J}$$

Number of photons n_1 due to wavelength 4144Å

$$n_1 = \frac{(2.4 \times 10^{-7})(4144 \times 10^{-10})}{(6.63 \times 10^{-34})(3 \times 10^8)} = 0.5 \times 10^{12}$$

Number of photons n_2 due to the wavelength 4972Å

$$n_2 = \frac{(2.4 \times 10^{-7})(4972 \times 10^{-10})}{(6.63 \times 10^{-34})(3 \times 10^8)} = 0.572 \times 10^{12}$$

Therefore, total number of photoelectrons liberated in 2 s,

$$\begin{aligned} N &= n_1 + n_2 \\ &= 0.5 \times 10^{12} + 0.575 \times 10^{12} \\ &= 1.075 \times 10^{12} \end{aligned}$$

15. (c) The net force acting on the gate element of width dy at a depth y from the surface of the fluid, is

$$\begin{aligned} dy &= \left(p_0 + \rho_g y - p_0\right) \times 1dy \\ &= \rho gydy \end{aligned}$$

Torque about the hinge is $d\tau = pgydy \times \left(\frac{1}{2} - y\right)$

Net torque experienced by the gate is $\tau_{net} = \int d\tau + F \times \frac{1}{2}$

$$= \int_0^1 \rho gydy\left(\frac{1}{2} - y\right) + F \times \frac{1}{2} = 0$$

$$\Rightarrow F = \frac{\rho g}{6}$$

i.e., The force F required to hold the gate stationary is $\frac{pg}{6}$

16. (b) As we know,

$$F = qvB = m\frac{v^2}{R} \Rightarrow v = \frac{q}{m}BR$$

The kinetic energy of the photoelectron

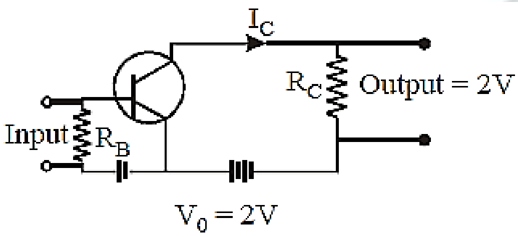
$$= \frac{1}{2}mv^2 = \frac{1}{2}\frac{e^2B^2R^2}{m}$$
$$= \frac{1}{2}\frac{(1.6\times10^{-19})^2(2\times10^{-2})^2(23\times10^{-3})^2}{(9.1\times10^{-31})}$$
$$= 2.97\times10^{-15}\ \text{J}$$
$$= \frac{2.97\times10^{-15}}{1.6\times10^{-19}} = 18.36\text{keV}$$

Energy of the incident photon $= \frac{hc}{\lambda}$

$$= \frac{12.4}{0.50} = 24.8\text{keV}$$

Therefore, Binding energy $= 24.8 - 18.6 = 6.2$keV

17. **(c)** Given: Voltage across the collector $V_0 = 2$ V; collector resistance, $R_c = 2\times10^3\,\Omega$; Base resistance $R_B = 1\times10^3\,\Omega$; Input signal voltage, $V_i = ?$



$$V_0 = I_C R_C = 2$$
$$\Rightarrow I_C = \frac{2}{2\times10^3} = 10^{-3}\ \text{A}$$
$$\text{Current gain } \alpha = \frac{I_C}{I_B} = 100$$
$$\Rightarrow I_B = \frac{I_C}{100} = \frac{10^{-3}}{100} = 10^{-5}\ \text{A}$$
$$V_i = R_B I_B \Rightarrow V_i = 1\times10^3\times10^{-5}$$
$$\Rightarrow V_i = 10^{-2}\ \text{V} \Rightarrow V_i = 10\text{mV}$$

18. **(a)** Initial potential energy of the system

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{a}+\frac{q^2}{a}+\frac{q^2}{a}\right] = \frac{1}{4\pi\varepsilon_0}\left(\frac{3q^2}{a}\right)$$
$$= 9\times10^9\left(3\times\frac{(0.1)^2}{1}\right) = 27\times10^7\ \text{J}$$

Let charge at A is moved to mid-point O, Then final potential energy of the system

$$U_f = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q^2}{(a/2)}+\frac{q^2}{a}\right]$$
$$= 5\times\frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{a^2}\right) = 45\times10^7\ \text{J}$$

Work done $= U_f - U_i = 18\times10^7$ J
Also, energy supplied per sec $= 1000$ J (given)
Time required to move one of the mid-point of the line joining the other two

$$t = \frac{18 \times 10^7}{1000} = 18 \times 10^4 \text{ s} = 50 \text{ h}$$

19. **(d)** Let there are n_1 moles of hydrogen and n_2 moles of helium in the given mixture. As $Pv = nRT$
Then the pressure of the mixture

$$P = \frac{n_1 RT}{V} + \frac{n_2 RT}{V} = (n_1 + n_2)\frac{RT}{V}$$
$$\Rightarrow \quad 2 \times 101.3 \times 10^3 = (n_1 + n_2) \times \frac{(8.3 \times 300)}{20 \times 10^{-3}}$$

or,

$$(n_1 + n_2) = \frac{2 \times 101.3 \times 10^3 \times 20 \times 10^{-3}}{(8.3)(300)}$$

or, $n_1 + n_2 = 1.62$
The mass of the mixture is (in grams)

$$n_1 \times 2 + n_2 \times 4 = 5$$
$$\Rightarrow \quad (n_1 + 2n_2) = 2.5$$

Solving the eqns. (1) and (2), we get $n_1 = 0.74$ and $n_2 = 0.88$
Hence, $\frac{m_H}{m_{He}} = \frac{0.74 \times 2}{0.88 \times 4} = \frac{1.48}{3.52} = \frac{2}{5}$

20. **(c)** Resistance of wire $(R) = \rho \frac{l}{A}$ If wire is bent in the middle then $l' = \frac{l}{2}, A' = 2A$

$\therefore$ New resistance, $R' = \rho \frac{l'}{A'} = \frac{\rho \frac{l}{2}}{2A} = \frac{\rho l}{4A} = \frac{R}{4}$.

21. **(b)** The path difference when transparent sheet is introduced $\Delta x = (\mu - 1)t$ If the central maxima occupies position of nth fringe, then $(\mu - 1)t = n\lambda = d\sin\theta$

$$\Rightarrow \sin\theta = \frac{(\mu - 1)t}{d} = \frac{(1.17 - 1) \times 1.5 \times 10^{-7}}{3 \times 10^{-7}} = 0.085$$

Therefore, angular position of central maxima

$\theta = \sin^{-1}(0.085) = 4.88° \approx 4.9$

For small angles, $\sin\theta \approx \theta \approx \tan\theta$

$$\Rightarrow \tan\theta = \frac{y}{D}$$
$$\therefore \frac{y}{D} = \frac{(\mu - 1)t}{d} \Rightarrow y = \frac{D(\mu - 1)t}{d}$$

22. **(c)** From question,
Horizontal velocity (initial),

$$u_x = \frac{40}{2} = 20 \text{ m/s}$$

Vertical velocity (initial), $50 = u_y t + \frac{1}{2}gt^2$

$\Rightarrow u_y \times 2 + \frac{1}{2}(-10) \times 4$

or, $50 = 2u_y - 20$
or, $u_y = \frac{70}{2} = 35 \text{ m/s}$
$\therefore \tan\theta = \frac{u_y}{u_x} = \frac{35}{20} = \frac{7}{4}$
$\Rightarrow$ Angle $\theta = \tan^{-1}\frac{7}{4}$

23. (a) Newton's law of viscosity, $F = \eta A \frac{dv}{dy}$

Stress $= \frac{F}{A} = \eta\left(\frac{dv}{dy}\right) = \eta k\left(\frac{4y}{a^2} - \frac{3y^2}{a^3}\right)$

At $y = a$, stress $= \eta k\left(\frac{4}{a} - \frac{3}{a}\right) = \frac{\eta k}{a}$

24. (b)

According to energy conservation principle,
If, x_1 is maximum elongation in the spring when the particle is in its lowest extreme position. Then,

$$\text{mgh} = \frac{1}{2}\text{kx}_1^2 - \text{mgx}_1$$

$$\Rightarrow \frac{1}{2}\text{kx}_1^2 - \text{mgx}_1 - \text{mgh} = 0$$

or, $x_1^2 - \frac{2mg}{k}x_1 - \frac{2mg}{k}\cdot h = 0$

$$\therefore x_1 = \frac{\frac{2\text{mg}}{\text{k}} \pm \sqrt{\left[\left(\frac{2\text{mg}}{\text{k}}\right)^2 + 4 \times \frac{2\text{mg}}{\text{k}}\text{h}\right]}}{2}$$

Amplitude $A = X_1 - X_0$ (elongation in spring for equilibrium position)

$$A = \frac{\text{mg}}{\text{k}}\sqrt{\left(1 + \frac{2\text{hk}}{\text{mg}}\right)}$$

25. (c) Let the initial mass of uranium be M_0 Final mass of uranium after time t,

$$M = \frac{3}{4}M_0$$

According to the law of radioactive disintegration.

$$\frac{M}{M_0} = \left(\frac{1}{2}\right)^{t/T} \Rightarrow \frac{M_0}{M} = (2)^{t/T}$$

$$\therefore \log_{10}\left(\frac{M_0}{M}\right) = \frac{t}{T}\log_{10}(2)$$

$$t = T\frac{\log_{10}\left(\frac{M_0}{M}\right)}{\log_{10}(2)} = \frac{T\log_{10}\left(\frac{4}{3}\right)}{\log_{10}(2)}$$

$$= \frac{T\log_{10}(1.333)}{\log_{10}(2)} = 4.5 \times 10^9\left(\frac{0.1249}{0.3010}\right)$$

$$\Rightarrow t = 1.867 \times 10^9 yr.$$

26. (b) Total carrent, $1 = (5 + 10\sin\omega t)$

$$\Rightarrow I_{eff} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T (5 + 10\sin\omega t)^2 dt\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T (25 + 100\sin\omega t + 100\sin^2\omega t)\right]^{1/2}$$

But, $\frac{1}{T}\int_0^T \sin\omega t.\,dt = 0$ and

$$\frac{1}{T}\int_0^T \sin^2\omega t.\,dt = \frac{1}{2}$$

So, $I_{eff} = \left[25 + \frac{1}{2} \times 100\right]^{1/2} = 5\sqrt{3}$ A

27. (a) If F be the equivalent focal length, then

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}\frac{1}{F} = (\mu_1 - 1)\left(\frac{1}{\infty} + \frac{1}{R}\right)$$

$$+(\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = \frac{\mu_1 - \mu_2}{R}$$

$$F = \frac{R}{\mu_1 - \mu_2}$$

28. (b) Total moment of inertia

$$= I_1 + I_2 + I_3 + I_4 = 2I_1 + 2I_2$$

$$= 2(I_1 + I_2)[I_3 = I_1, I_1 = I_4]$$

Now, $I_2 = I_3 = \frac{MI^2}{3}$

Using parallel axes theorem, we have

$I = I_{CM} + Mx^2$ and $x = \sqrt{l^2 + \frac{l^2}{4}}$

$$I_1 = I_4 = \frac{Ml^2}{12} + M\left[\sqrt{l^2 + \left(\frac{l}{2}\right)^2}\right]^2$$

Putting all values we get

Moment of inertia, $I = 10\left(\frac{Ml^2}{3}\right)$

29. (b) For 2^{nd} line of Balmer series in hydrogen spectrum

$$\frac{1}{\lambda} = R\,(1)\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3}{16}R$$

For Li^{2+} $\left[\frac{1}{\lambda} = R \times 9\left(\frac{1}{x^2} - \frac{1}{12^2}\right) = \frac{3R}{16}\right]$

which is satisfied by $n = 12 \rightarrow n = 6$.

30. (c) When a charge particle is allowed to move in a uniform magnetic field, then it describes spiral or circular path

Centripetal force, $\frac{mv^2}{R} = qvB \;\therefore v = \left(\frac{qB}{m}R\right)$

Hence, $\sqrt{\frac{2qV}{m}} = \left(\frac{qB}{m}\right)R \left[\because V = \sqrt{\frac{2qV}{m}}\right]$

$$\Rightarrow R = \left(\frac{2mV}{q}\right)^{1/2} \times \frac{1}{B}$$

or, $m \propto R^2$

$[\because V, q$ and B are constant $]$

or, $\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$

31. (b) Water wets glass and so the angle of contact is zero.

For full rise, neglecting the small mass in the meniscus

$$2\pi rT = \pi r^2 h\rho g \Rightarrow h = \frac{2T}{r\rho g}[\because \text{ water wets glass, } \theta = 0°]$$

$$= \frac{2 \times 0.07}{0.25 \times 10^{-3} \times 1000 \times 9.8}$$

As the tube is only 2 cm above the water and so, water will rise by 2 cm and meet the tube at an angle such that,

$$2\pi r\cos\theta = \pi r^2 h'\rho g$$

$$\Rightarrow 2\,T\cos\theta = h'r\rho g$$

$$\Rightarrow \cos\theta = \frac{h'r\rho g}{2T}$$

$$= \frac{2 \times 10^{-2} \times 0.25 \times 10^{-3} \times 1000 \times 9.8}{2 \times 0.07}$$

The liquid will meet the tube at an angle, $\theta \cong 70°$

32. (d) Given: Initial velocity $u_0 = 20\sqrt{2}$ m/s; angle of projection $\theta = 45°$

Therefore, horizontal and vertical components of initial velocity are

$$u_x = 20\sqrt{2}\cos 45° = 20 \text{ m/s}$$

and $u_y = 20\sqrt{2}\sin 45° = 20 \text{ m/s}$
After 1 s, horizontal component remains unchanged while the vertical component becomes

$$v_y = u_y - gt$$

Due to explosion, one part comes to rest. Hence, from the conservation of linear momentum, vertical component of second part will become $v_y' = 20 \text{ m/s}$.
Therefore, maximum height attained by the second part will be

$$H = h_1 + h_2$$

$$\text{Using, } h = ut + \frac{1}{2}at^2$$

$$\Rightarrow h_1 = (20 \times 1) - \frac{1}{2} \times 10 \times (1)^2 = 15 \text{ m}$$

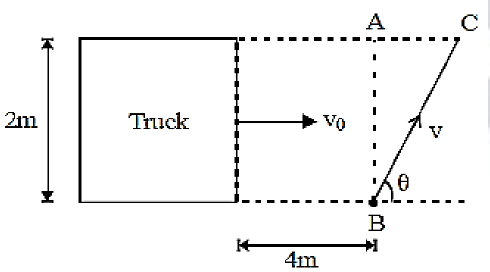
$$a = g = 10 \text{ m/s}^2$$

$$h_2 = \frac{v_y'^2}{2g} = \frac{(20)^2}{2 \times 10} = 20 \text{ m}$$

$$H = 20 + 15 = 35 \text{ m}$$

33. (c) Let the man starts crossing the road at an angle θ as shown in figure. For safe crossing the condition is that the man must cross the road by the time the truck describes the distance $4 + AC$ or $4 + 2\cot\theta$.

$$\therefore \frac{4 + 2\cot\theta}{8} = \frac{2/\sin\theta}{v} \text{ or } v = \frac{8}{2\sin\theta + \cos\theta}$$



For minimum v, $\frac{dv}{d\theta} = 0$
or $\frac{-8(2\cos\theta - \sin\theta)}{(2\sin\theta + \cos\theta)^2} = 0$ or $2\cos\theta - \sin\theta = 0$
or $\tan\theta = 2$

From equation (i),

$$v_{min} = \frac{8}{2\left(\frac{2}{\sqrt{5}}\right) + \frac{1}{\sqrt{5}}} = \frac{8}{\sqrt{5}} = 3.57 \text{ m/s}$$

34. (b) Let speed of neutron before collision $= V$
Speed of neutron after collision $= V_1$
Speed of proton or hydrogen atom after collision $= V_2$
Energy of excitation $= \Delta E$
From the law of conservation of linear momentum,

$$mv = mv_1 + mv_2$$

And for law of conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

From squaring eq. (i), we get

$$v^2 = v_1^2 + v_2^2 + 2v_1v_2$$

From squaring eq. (ii), we get

$$v^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m}$$

From eqn (3) & (4)

$$\therefore 2v_1v_2 = \frac{2\Delta E}{m}$$

$$\therefore (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = v^2 - \frac{4\Delta E}{m}$$

As, $v_1 - v_2$ must be real, $v^2 - \frac{4\Delta E}{m} \geq 0$

$$\Rightarrow \frac{1}{2}mv^2 \geq 2\Delta E$$

The minimum energy that can be absorbed by the hydrogen atom in the ground state to go into the excited state is 10.2eV. Therefore, the maximum kinetic energy needed is

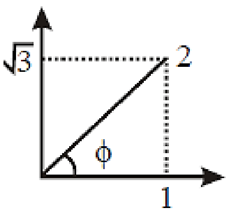
$$\frac{1}{2}mv_{min}^2 = 2 \times 10.2 = 20.4\text{eV}$$

35. **(a)** From, figure,

$$A_R = \sqrt{(\sqrt{3})^2 + (1)^2} = 2$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$$

$$\therefore \quad y = 2\sin\left(\omega t + \frac{\pi}{3}\right)$$



$$\frac{d^2y}{dt^2} = a = -2\omega^2\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$a_{max} = -2\omega^2 = g$$

For which mass just breaks off the plank

$$\omega = \sqrt{g/2}$$

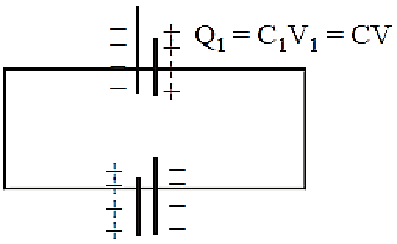
This will be happen for the first time when

$$\omega t + \frac{\pi}{3} + \frac{\pi}{2} \text{ or } \omega t = \frac{\pi}{6}$$

$$\therefore\ t = \frac{\pi}{6\omega} = \frac{\pi}{6}\sqrt{\frac{2}{g}}$$

36. (b) From the figure.

The net charge shared between the two capacitors



The two capacitors will have some potential, say V'.
The net capacitance of the parallel combination of the two capacitors

$$C' = C_1 + C_2 = C + 2C + 3C$$

The potential of the capacitors

$$V' = \frac{Q'}{C'} = \frac{3CV}{3C} = V$$

The electrostatic energy of the capacitors

$$E' = \frac{1}{2}C'V'^2 = \frac{1}{2}(3C)V^2 = \frac{3}{2}C\,V^2$$

37. (a)

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{10^2 + \left(2000 \times 5 \times 10^{-3} - \frac{1}{2000 \times 50 \times 10^{-6}}\right)^2}$$

$$= 10\Omega$$

$$i = \frac{V_0}{Z} = \frac{20}{10} = 2\text{ A.}$$

38. (b) The heat produced is given by

$$H = \frac{V^2}{R} \text{ and } R = \frac{\rho\ell}{\pi r^2}$$

$$\therefore H = V^2\left(\frac{\pi r^2}{\rho\ell}\right)$$

$$\text{or } H = \left(\frac{\pi V^2}{\rho}\right)\frac{r^2}{\ell}$$

Thus heat (H) is doubled if both length (ℓ) and radius (r) are doubled.

39. **(b)** As we know, frequency

$f \propto \sqrt{mg}$ or $f \propto \sqrt{g}$

In water, $f_w = 0.8 f_{air}$

$$\frac{g'}{g}(0.8)^2 = 0.64$$
$$\Rightarrow 1 - \frac{\rho_w}{\rho_m} = 0.64$$
$$\Rightarrow \frac{\rho_w}{\rho_m} = 0.36$$

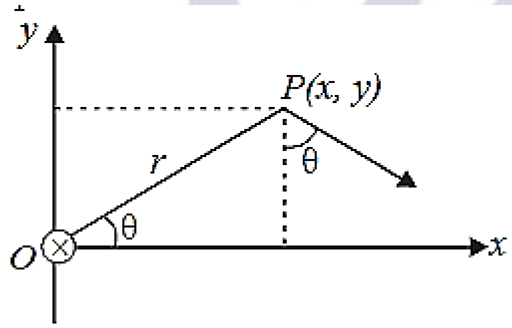
In liquid, $\frac{g'}{g} = (0.6)^2 = 0.36$

$1 - \frac{\rho_1}{\rho_m} = 0.36 \frac{\rho_1}{\rho_m} = 0.64$

From eq. (1) and (2)

$\frac{\rho_l}{\rho_n} = \frac{0.64}{0.36} \therefore \rho_l = 1.77$

40. **(a)** The wire carries a current I in the negative z-direction. We have to consider the magnetic vector field $\vec{B}$ at (x, y) in the $z = 0$ plane.
Magnetic field $\vec{B}$ is perpendicular to OP.



$$\therefore \vec{B} = B\sin\theta\hat{\imath} - B\cos\theta\hat{\jmath}$$
$$\sin\theta = \frac{y}{r}, \cos\theta = \frac{x}{r} B = \frac{\mu_0 I}{2\pi r}$$
$$\therefore \vec{B} = \frac{\mu_0 I}{2\pi r^2}(y\hat{\imath} - x\hat{\jmath})$$
$$\text{or } \vec{B} = \frac{\mu_0 I(y\hat{\imath} - x\hat{\jmath})}{2\pi(x^2 + y^2)}.$$

41. **(c)** NO pollutant is the main product of automobiles exhaust.
42. **(c)** The high concentration of hydrocarbon pollutants in atmosphere causes cancer.
43. **(b)** Electronic configuration of element with atomic number 118 will be $[Rn]5f^{14}6\,d^{10}\,7\,s^2 7p^6$. Since its elctronic configuration in the outer most orbit (ns^2np^6) resemble with that of inert or noble gases, therefore it will be noble gas element.

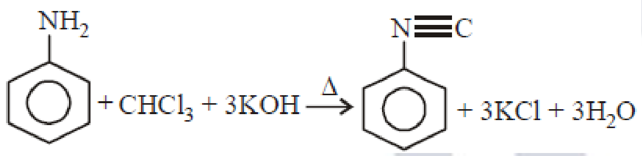
44. (c) The third law helps to calculate the absolute entropies of pure substances at different temperature.
The entropy of the substance at different temperature. T may be calculated by the measurement of heat capacity change

$$S_T - S_0 = \Delta S = \int_0^T \frac{C_p \cdot dt}{T}$$

Where S_T = Entropy at TK

$$\begin{aligned} S_0 &= \text{Entropy at } 0\text{ K} \\ &= C_p \cdot \log_e T \\ &= 2.303 C_p \cdot \log T \end{aligned}$$

45. (d) $CoCl_3 \cdot 5NH_3 \cdot H_2O$ is pink in colour
46. (b) Cobalt is present in vitamin B_{12}.
47. (d) Cobalt (60) isotope is used in the treatment of cancer.
48. (b) PMMA is used in bullet proof glass
49. (a) $ClO_4^- < ClO_3^- < ClO_2^- < ClO^-$ is the correct increasing order of Bronsted base. With increase in the number of oxygen atoms in the conjugate bases, the delocalisation of the π bond becomes more and more extended. This results in decrease in the electron density. Consequently basicity also decreases.
50. (a) Due to dipole-dipole interaction the boiling point of alkyl halide is higher as compared to corresponding alkanes.
51. (c) Complex compounds contains two different metallic elements but give test only for one of them. Because complex ions such as $[Fe\ (CN)_6]^4$ of $K_4[Fe(CN)_6]$, do not dissociate into Fe^{2+} and CN^- ions.
52. (b) Primary amines (aromatic or aliphatic) on warming with chloroform and alcoholic KOH, gives carbylamine having offensive smell. This reaction is called carbylamine reaction.



53. (b) Nitrous oxide (i.e., N_2O) is the laughing gas.
54. (d) Anthracene is purified by sublimation. In sublimation, a solid is converted directly into gaseous state on heating without passing through liquid phase.
55. (b) Zeise's salt is common name of

$K[PtCl_3(\eta^2 = C_2H_4)]$

56. (c) $CaCl_2$ is produced as a by product in solvay ammonia process.

(i) $NaCl + CO_2 + NH_3 + H_2O \longrightarrow NaHCO_3 + NH_4Cl$

(iii) $2NH_4Cl + CaO \longrightarrow 2NH_3 + \underset{\text{By product}}{CaCl_2} + H_2O$

57. (b) Semiconductor materials like Si and Ge are usually purified by zone refining. Zone refining is based on the principle of fractional crystallisation i.e. difference in solubilities of impurities in solid and molten states of metal, so that the zones of impurities are formed and finally removed.

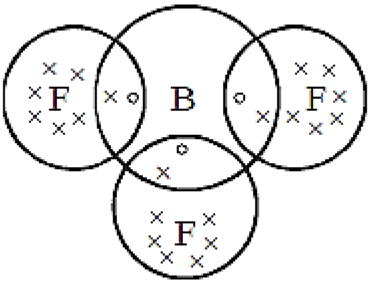
58. (c) Order of basic character is $NH_3 > PH_3 > AsH_3 > SbH_3$. Basic-character decreases down the group from N to Bi due to increase in atomic size.

59. (c) Normal glass is calcium alkali silicate glass made by fusing the alkali metal carbonate, $CaCO_3$ and SiO_2.

60. (c) Exa $= 10^{18}$

61. (b) More the oxidation state of the central atom (metal) more is its acidity. Hence SeO_2 (O. S. of Se $= +4$) is acidic. Further for a given O.S., the basic character of the oxides increases with the increasing size of the central atom. Thus Al_2O_3 and Sb_2O_3 are amphoteric and Bi_2O_3 is basic.

62. (b) BF_3 does not follow octate rule because central atom, boron lacks an electron pair. Thus, it also acts as Lewis acid.



63. (a) According to Le-Chatelier's principle increase in temperature favours the endothermic reaction while decrease in temperature favour the exothermic reaction. Increase in pressure shifts the equilibrium in that side in which number of gaseous moles decreases.

64. (a) Efficiency of fuel cell is:

$$\eta = \frac{-nFE_{cell}}{\Delta H} \times 100$$

65. (b) The mass of the substance deposited when one Faraday of charge is passed through its solution is equal to gram equivalent weight.

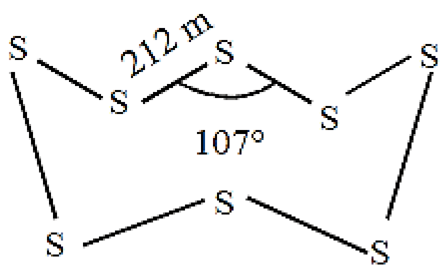
66. (a) Unit of rate constant for second order reaction is $Lmol^{-1}sec^{-1}$.

67. (b) For first order reaction $[A] = [A_0]e^{-kt}$

$\therefore$ The concentration of reactants will exponentially decreases with time.

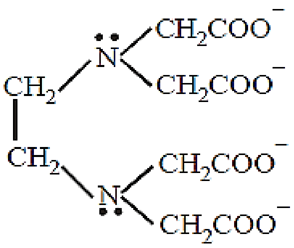
68. (a) In P_4 molecule, the four sp^3-hybridised phosphorous atoms lie at the corners of a regular tetrahedron with $\angle PPP = 60°$.

In S_8 molecule $S - S - S$ angle is $107°$ rings.



69. (a) 40 elements are present in d-block.

70. (a) EDTA is hexadentate ligand



71. (a) Am shows maximum number of oxidation states, $+3, +4, +5, +6$

72. (a)

Fe^{3+} ion can be detected by $K_4[Fe(CN)_6]$

$4Fe^{3+} + 3\ K_4[Fe(CN)_6]$

$\longrightarrow Fe_4[Fe(CN)_6]_3 + 12\ K^+$

73. (d) $\Delta G = \Delta H - T\Delta S$; ΔG is positive for a reaction to be non-spontaneous when ΔH is positive and ΔS is negative.

74. (a) This method is known as Clark's process. In this method temporary hardness is removed byadding lime water or milk of lime.

$$Ca(OH)_2 + Ca(HCO_3)_2 \longrightarrow \underset{\text{ppt.}}{2CaCO_3 \downarrow} + 2H_2O$$

75. (b) Graphite is covalent solid.

76. (c)

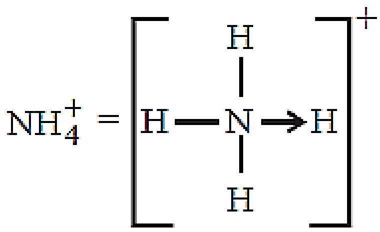
77. (b)

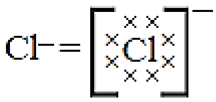
78. (c) Calcium silicophosphate (a mixture of $Ca_3(PO_4)_2$&Ca_2SiO_4) is called Thomas slag.

79. (a) The sequence of bases in mRNA are read in a serial order in groups of three at a time. Each triplet of nucleotides (having a specific sequence of bases) in known as codon. Each codon specifies one amino acid. Further since, there are four bases. therefore, $4^3 = 64$ triplets or codons are possible.

80. (b) Bond structure of molecules are:

$HCl = H^+ - Cl^-$





$H_2O_2 = H - O - O - H$

hence, clearly NH_4^+ion contains all three types of bonds.

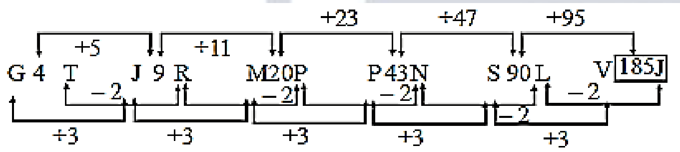
81. (c) 'Immutable' means 'unchangeable'. So, option (c) is correct choice.

82. (a) 'ignominious' means 'shameful'. So, option (a) is correct choice.

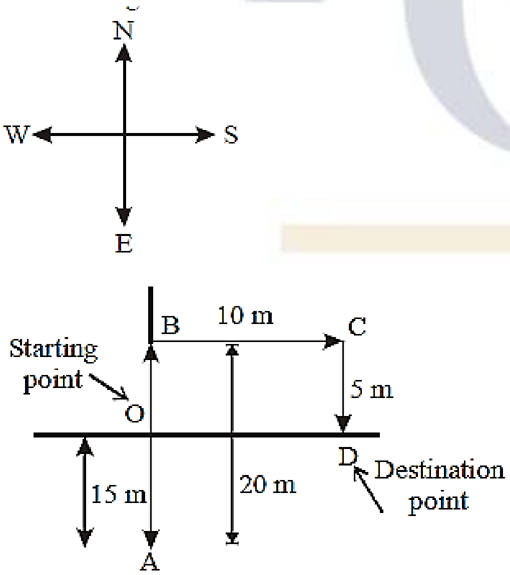
83. (c) 'callous' means 'showing or having an insensitive and cruel disregard for others'. So, option (c) is correct choice.

84. (a) Option (d) institutional as the word means relating to principles esp. of law, so legally also every human has rights of freedom and equality.

85. (a)
86. (c)
87. (b) Immortal means living forever, never dying or decaying. So, perishable is the correct opposite to it.
88. (c) Opposed is the correct answer of this. To patronise means favour or pat on the back.
89. (a) Tactful means having or showing skill and senstivity in dealing with others or with defficult essues. So, in cautious is the correct opposite of factful.
90. (c) Atheist is the best alternative.
91. (c) 'Epicure' is the best alternative.
92. (d)
93. (c)
94. (d)
95. (c)
96. (c)
97. (c)
98. (b) The contents of the third figure in each row (and column) are determined by the contents of the first two figures. Lines are carried forward from the first two figures to the third one, except where two lines appear in the same position, in which they are cancelled out.
99. (b) From figure, (i), (iii) and (iv), we have concluded that 2, 6,1 and 5 appear adjacent so 3 . clearly, 4 will appear opposite to 3
100. (c) In figure (A), the dot is placed in the region which is common to the circle and triangle. Now, we have to find similar common region in all the four options. Only in figure (c), we find such a region which is common to the circle and triangle.
101. (b)



102. (a) According to the given information, the direction of Neeraj is as following.

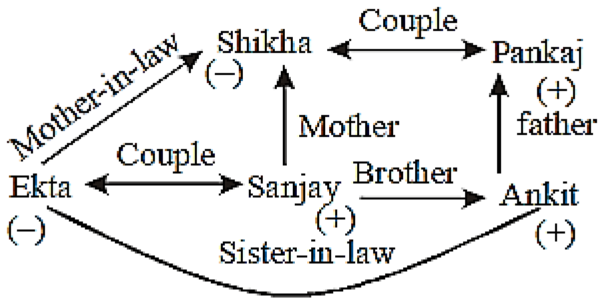


103. (b)

Let the average age of 8 men $= x$ yr
Total age of 8 men $= 8x$ yr
Now, new average age $= (x+2)$yr
Total age $= 8(x+2)$yr
Difference of ages $= 8(x+2) - 8x$
$= 8x + 16 - 8x = 16$yr

$\therefore$ Age of new man $= 20 + 16 = 36$yr
So, the new man is 16yr older to the man by whom the new man is replaced.

104. **(d)** The relation is as following:



It is clearly shown that Shikha is the mother of Ankit.

105. **(b)** Since Arun and Suresh interchange places, so Arun's new position (13th from left) is the same as Suresh's earlier position (6th from right).
So, number of children in the queue $= (12 + 1 + 5) = 18$.
Now, Suresh's new position is the same as Arun's earlier position fifth from left.
Therefore Suresh's position from the right $= (18 - 4) = 14$ th.

106. **(d)**

$$\text{Consider } \lim_{x\to\infty} \frac{\int_0^{2x} xe^{x^2} dx}{e^{4x^2}}$$
$$= \lim_{x\to\infty} \frac{2\int_0^{2x} xe^{x^2} dx}{2e^{4x^2}}$$
$$= \lim_{x\to\infty} \frac{2\int_0^{2x} e^{x^2} d(x^2)}{2e^{4x^2}}$$
$$= \lim_{x\to\infty} \frac{\left[e^{x^2}\right]_0^{2x}}{e^{4x^2}} = \lim_{x\to\infty} \frac{e^{4x^2} - 1}{2e^{4x^2}}$$
$$= \lim_{x\to\infty} \left(\frac{1}{2} - \frac{1}{2e^{4x^2}}\right) = \frac{1}{2}$$

107. **(a)** Consider $\frac{1}{2} + \frac{3}{8} + \frac{9}{32} + \frac{27}{128} + \cdots$.
Which can be written as

$$\frac{3^0}{2^1} + \frac{3^1}{2^3} + \frac{3^2}{2^5} + \frac{3^3}{2^7} + \cdots$$

$$= \frac{1}{2}\left[1 + \frac{3}{2^2} + \frac{3^2}{2^4} + \frac{3^3}{2^6} + \cdots.\right]$$

Since $\left(1+\frac{3}{2^2}+\frac{3^2}{2^4}+---\right)$ is a G.P. therfore by sum of infinite G.P, we have

$$=\frac{1}{2}\left[\frac{1}{1-\frac{3}{2^2}}\right]=\frac{1}{2}\left[\frac{1}{1-\frac{3}{4}}\right]=2$$

$\therefore$ Given expression

$$= -1$$
$$[\because 1+\omega+\omega^2=0]$$

108. (a)

$$\text{Roots } = \frac{4(2-i)\pm\sqrt{16(2-i)^2+8(1+i)(5+3i)}}{4(1+i)}$$
$$=\frac{4-i}{1+i} \text{ or } \frac{-i}{1+i}=\frac{3-5i}{2} \text{ or } \frac{-1-i}{2}$$

109. (b)

$$\text{Consider } \frac{3}{4}+\frac{15}{16}+\frac{63}{64}+\cdots \text{ upto n terms}$$
$$=\frac{2^2-1}{2^2}+\frac{2^4-1}{2^4}+\frac{2^6-1}{2^6} \text{ upto n terms}$$
$$=\left(1-\frac{1}{2^2}\right)+\left(1-\frac{1}{2^4}\right)+\left(1-\frac{1}{2^6}\right)+\cdots \text{ upto n terms}$$
$$=(1+1+1+\cdots \text{ upto n terms })$$
$$-\left(\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}+\cdots \text{ upto n terms}\right)$$
$$=n-\frac{1}{2^2}\left[\frac{1-\left(\frac{1}{2^2}\right)^n}{1-\frac{1}{2^2}}\right]=n-\frac{1}{3}(1-4^{-n})$$
$$=n+\frac{4^{-n}}{3}-\frac{1}{3}$$

110. (d) $\tan\theta$ is of period π so that $\tan 3\theta$ is of period $\pi/3$.

111. (b) Let

$$f(x)=\frac{x}{1+x}+\frac{x}{(x+1)(2x+1)}+\frac{x}{(2x+1)(3x+1)}+\cdots\infty$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{x}{[(r-1)x+1](rx+1)}$$
$$=\lim_{x\to\infty}\sum_{r=1}^{n}\left[\frac{x}{[(r-1)x+1]}-\frac{1}{rx+1}\right]$$
$$=\lim_{n\to\infty}\left[1-\frac{1}{nx+1}\right]=1$$

For $x = 0$, we have $f(x) = 0$

Thus, we have $f(x) = \begin{cases} 1, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Clearly, $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) \neq f(0)$

So, $f(x)$ is not continuous at $x = 0$.

112. **(c)** Since, g is the inverse of function f. Therefore,

$g(x) = f^{-1}(x)$

$\Rightarrow f[g(x)] = x$

$\Rightarrow fog(x) = x$, for all x

Differentiate both side,w.r.tx

$\Rightarrow \frac{d}{dx}\{fog(x)\} = \frac{d}{dx}(x)$, for all x

$\Rightarrow f'[g(x)]g'(x) = 1$, for all x

$\Rightarrow \sin\{g(x)\}g'(x) = 1$, for all x

(By defn of $f'(x)$)

$$\Rightarrow g'(x) = \frac{1}{\sin\{g(x)\}}$$

113. **(a)** Total number of coins $= 2n + 1$

Consider the following events:

E_1 = Getting a coin having head on both sides from the bag.

E_2 = Getting a fair coin from the bag

A = Toss results in a head

Given: $P(A) = \frac{31}{42}, P(E_1) = \frac{n}{2n+1}$

and $P(E_2) = \frac{n+1}{2n+1}$

Then,

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$$

$$\Rightarrow \frac{31}{42} = \frac{n}{2n+1} \times 1 + \frac{n+1}{2n+1} \times \frac{1}{2}$$

$$\frac{31}{42} = \frac{n}{2n+1} + \frac{n+1}{2(2n+1)}$$

$$\Rightarrow \frac{31}{42} = \frac{3n+1}{2(2n+1)}$$

$$\Rightarrow \frac{31}{21} = \frac{3n+1}{2n+1}$$

$n = 10$

114. **(a)** Given differential equation is

$$dy + \{y\phi'(x) - \phi(x)\phi'(x)\}dx = 0$$

$$\Rightarrow \frac{dy}{dx} + \phi'(x)y = \phi(x)\phi'(x)$$

which is a linear differential equation with $P = \phi'(x), Q = \phi(x) \cdot \phi'(x)$ and

$I.F = e^{\int \phi'(x)dx} = e^{\phi(x)}$

Solution is $y. e^{\phi(x)} = \int \phi(x) \cdot \phi'(x)e^{\phi(x)}dx + C$

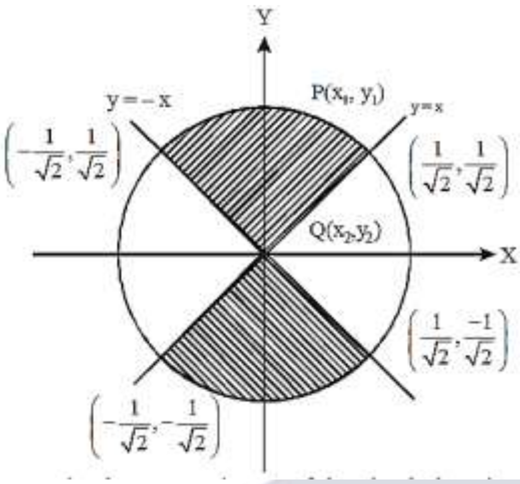
$$\Rightarrow y \cdot e^{\phi(x)} = \int \phi(x) \cdot e^{\phi(x)} \phi'(x) dx + C$$

$$\Rightarrow y \cdot e^{\phi(x)} = \phi(x) e^{\phi(x)} - \int \phi'(x) e^{\phi(x)} dx + C$$

$$\Rightarrow y \cdot e^{\phi(x)} = \phi(x) e^{\phi(x)} - e^{\phi(x)} + C$$

$$\Rightarrow y = [\phi(x) - 1] + Ce - \phi(x)$$

115. (c)



Required area = 4 (Area of the shaded region in first quadrant)

$$= 4 \int_0^{1\sqrt{2}} (y_1 - y_2) dx = 4 \int_0^{1\sqrt{2}} \left(\sqrt{1 - x^2} - x\right) dx$$

$$= 4 \left[\frac{x}{2}\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1} x - \frac{x^2}{2}\right]_0^{1/\sqrt{2}}$$

$$= 4 \left[\frac{1}{2\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{\pi}{4} - \frac{1}{4}\right]$$

$$= 4 \left[\frac{1}{4} + \frac{\pi}{8} - \frac{1}{4}\right] = \frac{4\pi}{8} = \frac{\pi}{2} \text{ sq units}$$

116. (c) $U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$

Solving for values of x, we get

$U = \{0,1,2,3\}$

$A = \{x : x^2 - 5x + 6 = 0\}$

Solving for values of x, we get

$A = \{2,3\}$

and $B = \{x : x^2 - 3x + 2 = 0\}$

Solving for values of x, we get

$B = \{2,1\}$

$A \cap B = \{2\}$

$\therefore (A \cap B)' = U - (A \cap B)$

$= \{0,1,2,3\} - \{2\} = \{0,1,3\}$

117. **(c)**

$$\cos^{-1}x-\cos-1\frac{y}{2}=\alpha$$
$$\Rightarrow \cos^{-1}\left(\frac{xy}{2}+\sqrt{(1-x^2)\left(1-\frac{y^2}{4}\right)}\right)=\alpha$$
$$\Rightarrow \cos^{-1}\left(\frac{xy+\sqrt{4-y^2-4x^2+x^2y^2}}{2}\right)=\alpha$$
$$\Rightarrow 4-y^2-4x^2+x^2y^2=$$
$$\Rightarrow 4x^2+y^2-4xy\cos\alpha=4\sin^2\alpha.$$

118. **(d)** Let

$$\frac{e^x+e^{5x}}{e^{3x}}=a_0+a_1x+a_2x^2+a_3a^3+\cdots$$
$$=\frac{e^x}{e^{3x}}+\frac{e^{5x}}{e^{3x}}=a_0+a_1x+a_2x^2+\cdots.$$
$$=e^{-2x}+e^{2x}=a_0+a_1x+a_2x^2+a_3x^3+\cdots.$$

By using

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+-----\text{ and}$$
$$e^{-x}=1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+--$$
$$e^{-2x}+e^{2x}=2\left[1+\frac{(2x)^2}{2!}+\frac{(2x)^4}{4!}+\cdots.\right]$$

$$=a_0+a_1x+a_2x^2+a_3a^3+\cdots$$
$$=a_1=a_3=a_5=\cdots=0$$

Hence, $2a_1+2^3a_3+2^5a_5+\cdots.=0$

119. **(d)** Let θ be the angle between **b** and **c**.

$$\text{Given, } |b\times c|=\sqrt{15}$$
$$\Rightarrow |b||c|\sin\theta=\sqrt{15}$$
$$\Rightarrow \sin\theta=\frac{\sqrt{15}}{|\,b|c\,|}$$
$$\Rightarrow \sin\theta=\sqrt{\frac{15}{4\times 1}}=\frac{\sqrt{15}}{4}$$
$$\therefore \cos\theta=\sqrt{1-\frac{15}{16}}=\frac{1}{\sqrt{16}}=\frac{1}{4}$$

Now given, $b-2c=\lambda a\Rightarrow |b-2c^2|=|\lambda a^2|$

$\Rightarrow |b|^2 + 4|c| - 4(b \cdot c) = \lambda^2 |a|^2$
$\Rightarrow 16 + 4 - 4|b||c| \cos\theta = \lambda^2$
$\Rightarrow 20 - 16\cos\theta = \lambda^2$
$\Rightarrow 20 - 4 = \lambda^2 \Rightarrow \lambda^2 = 16 \left(\because \cos\theta = \frac{1}{4}\right)$
$\Rightarrow \lambda = \pm 4$

120. **(b)** Total number of arrangements of 10 digits 0,1,2, ….,9 bytaking 4 at atime $= {}^{10}C_4 \times 4$! We observe that in every arrangement of 4 selected digits there is just one arrangement in which the digits are in descending order. ∴ Required number of 4-digit numbers.

$$= \frac{{}^{10}C_4 \times 4!}{4!} = {}^{10}C_4$$

121. **(c)** Given equation of a line parallel to X-axis is $y = k$.
Given equation of the curve is $y = \sqrt{x}$,
On solving equation of line with the equation of curve, we get $x = k^2$
Thus the intersecting point is (k^2, k) It is given that the line $y = k$ intersect the curve $y = \sqrt{x}$ at an angle of $\pi/4$.
This means that the slope of the tangent to $y = \sqrt{x}$ at (k^2, k) is $\tan\left(\pm\frac{\pi}{4}\right) = \pm 1$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(k^2,k)} = \pm 1 \Rightarrow \left(\frac{1}{2\sqrt{x}}\right)_{(k^2,k)} = \pm 1$$
$$\Rightarrow k = \pm\frac{1}{2}$$

122. **(a)** Let A, B and C be the three angles of $\triangle$ ABC and
Let $a = 10$ and $b = 9$
It is given that the angles are in AP.
∴ $2B = A + C$ on adding B both the sides, we get

$3B = A + B + C$
$\Rightarrow 3B = 180^\circ \Rightarrow B = 60^\circ$

Now, we know $\cos B = \frac{a^2+c^2-b^2}{2ac}$

$$\Rightarrow \cos 60^\circ = \frac{10^2 + c^2 - 9^2}{2 \times 10 \times c}$$
$$\Rightarrow \frac{1}{2} = \frac{100 + c^2 - 81}{20c}$$
$$\Rightarrow c^2 - 10c + 19 = 0 \Rightarrow c = 5 \pm \sqrt{6}$$

123. **(d)**

124. **(a)** We have $\frac{1}{n}\sum_{i=1}^{n}(x_i + 2)^2 = 18$ and

$$\frac{1}{n}\sum_{i=1}^{n}(x_i - 2)^2 = 10$$
$$\Rightarrow \sum_{i=1}^{n}(x_i + 2)^2 = 18n \text{ and}$$

$$\sum_{i=1}^{n}(x_i-2)^2=10n$$

$$\Rightarrow \sum_{i=1}^{n}(x_i+2)^2+\sum_{i=1}^{n}(x_i-2)^2=28n$$

$$\text{and } \sum_{i=1}^{n}(x_i+2)^2-\sum_{i=1}^{n}(x_i-2)^2=8n$$

$$\Rightarrow 2\sum_{i=1}^{n}(x_i+4)^2=28n \text{ and } 2\sum_{i=1}^{n}4x_i=8n$$

$$\Rightarrow \sum_{i=1}^{n}x_i^2+4n=14n \text{ and } \sum_{i=1}^{n}x_i=n$$

$$\Rightarrow \sum_{i=1}^{n}x_i^2=10n \text{ and } \sum_{i=1}^{n}x_i=n$$

$$\therefore \sigma=\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2-\left(\frac{1}{n}\sum_{i=1'}^{n}x_i\right)^2}=\sqrt{\frac{10n}{n}-\left(\frac{n}{n}\right)^2}=3$$

125. **(a)** The parametric equations of the parabola $y^2=8x$ are $x=2t^2$ and $y=4t$. and the given equation of circle is $x^2+y^2-2x-4y=0$

On putting $x=2t^2$ and $y=4t$ in circle we get

$4t^4+16t^2-4t^2-16t=0$
$\Rightarrow 4t^2+12t^2-16t=0$
$\Rightarrow 4t(t^3+3t-4)=0$
$\Rightarrow t(t-1)(t^2+t+4)=0$
$\Rightarrow t=0, t=1$
$[\because t^2+t+4\neq 0]$

Thus the coordinates of points of intersection of the circle and the parabola are Q(0,0) and P(2,4). Clearly these are diametrically opposite points on the circle.
The coordinates of the focus S of the parabola are (2,0) which lies on the circle.

$\therefore$ Area of $\triangle$ PQS $=\frac{1}{2}\times QS\times SP=\frac{1}{2}\times 2\times 4$
$= 4$ sq. units.

126. **(a)** Let $f(x)=e^{x-1}+x-2$ check for $x=1$

Then, $f(1)=e^0+1-2=0$
So, $x=1$ is a real root of the equation $f(x)=0$
Let $x=\alpha$ be the other root such that $\alpha>1$ or $\alpha<1$. Consider the interval $[1,\alpha]$ or $[\alpha,1]$.
Clearly $f(1)=f(\alpha)=0$
By Rolle's theorem $f'(x)=0$ has a root in $(1,\alpha)$ or in $(\alpha,1)$.
But $f'(x)=e^{x-1}+1>0$, for all x. Thus, $f'(x)\neq 0$, for any $x\in(1,\alpha)$ or $x\in(\alpha,1)$, which is a contradiction.
Hence, $f(x)=0$ has no real root other than 1 .

127. **(c)** Constraints will be

$x_{11}+x_{21}+\cdots.+x_{m1}=b_1$
$x_{12}+x_{22}+\cdots..x_{m2}=b_2$
$x_{1n}+x_{2n}+\cdots.+x_{mn}=b_n$
$x_{11}+x_{12}+\cdots+x_{1n}=b_1$
$x_{21}+x_{22}+\cdots+x_{2n}=b_2$
$x_{m1}+x_{m2}+\cdots+x_{mm}=b_n$

So, total number of constraints $= m+n$

128. **(c)** Let A ≡ event that drawn ball is red B ≡ event that drawn ball is white Then AB and BA are two disjoint cases of the given event.

$$\therefore P(AB+BA)=P(AB)+P(BA)$$
$$=P(A)\,P\left(\frac{B}{A}\right)+P(B)P\left(\frac{A}{B}\right)$$
$$=\frac{3}{6}\cdot\frac{3}{5}+\frac{3}{6}\cdot\frac{3}{5}=\frac{3}{5}$$

129. **(b)** We know that, $M(\text{adj } M)=|M|I$
Replacing M by adj M, we get
adj $\mathbf{M}[\text{adj}(\text{adj } M)=\det(\text{adj } \mathbf{M})I$
$=\det(M)M^{-1}[\text{adj}(\text{adj } M)=\alpha^2 1$
$\left[\because M^{-1}=\frac{1}{|M|}\text{adj}(M)\right]$
$\Rightarrow \alpha M^{-1}[\text{adj}(\text{adj } M)]=\alpha^2 I$
$\Rightarrow M^{-1}[\text{adj}(\text{adj } M)]=\alpha I$
But $M^{-1}[\text{adj}(\text{adj } M)]=KI$
Hence, $K=\alpha$

130. **(c)** Let (x_1,y_1) be one of the points of contact. Given curve is $y=\cos x$

$$\Rightarrow \frac{dy}{dx}=-\sin x$$
$$\Rightarrow \left.\frac{dy}{dx}\right|_{(x_1,y_1)}=-\sin x_1$$

Now the equation of the tangent at (x_1,y_1) is

$$y-y_1\left(\frac{dy}{dx}\right)_{(x_1,y_1)}(x-x_1)$$
$$\Rightarrow \quad y-y_1=-\sin x_1(0-x_1)$$

Since, it is given that equation of tangent passes through origin.

$$\therefore\ 0-y_1=-\sin x_1(0-x_1)$$
$$\Rightarrow\ y_1=-x_1\sin x_1$$

Also, point (x_1,y_1) lies on $y=\cos x$.

$\therefore\ y_1=\cos x_1$

From Eqs. (i), (ii), we get

$$\sin^2 x_1 + \cos^2 x_1 = \frac{y_1^2}{x_1^2} + y_1^2 = 1$$

$\Rightarrow \quad x_1^2 = y_1^2 + y_1^2 x_1^2$

Hence, the locus of (x_1, y_1) is

$x^2 = y^2 + y^2x^2 \Rightarrow x^2y^2 = x^2 - y^2$

131. **(b)**

132. **(a)** The equations of given lines can be written as

$$L_1: x - 5 = \frac{y}{3-\alpha} = \frac{z}{-2}$$

$$L_2: x - \alpha = \frac{y}{-1} = \frac{z}{2-\alpha}$$

Since, these lines are coplanar.

Therefore, $\begin{vmatrix} 5-\alpha & 0-0 & 0-0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$

$\Rightarrow (5-\alpha)(3-\alpha)(2-\alpha) - 2 = 0$

$\Rightarrow (5-\alpha)(6 - 3\alpha - 2\alpha + \alpha^2 - 2] = 0$

$\Rightarrow (5-\alpha)(\alpha^2 - 5\alpha + 4] = 0$

$\Rightarrow (5-\alpha)(\alpha-1)(\alpha-4) = 0$

$\Rightarrow \alpha = 1,4,5$

133. **(b)**

$e = \frac{1}{2}$. Directrix, $x = \frac{a}{e} = 4$

$\therefore a = 4 \times \frac{1}{2} = 2 \;\; \therefore b = 2\sqrt{1 - \frac{1}{4}} = \sqrt{3}$

Equation of ellispe is

$$\frac{x^2}{4} + \frac{y^2}{3} = 1 \Rightarrow 3x^2 + 4y^2 = 12$$

134. **(d)** $\frac{x}{2} + \frac{2}{x}$ is of the form $x + \frac{1}{x} \geq 2$ and equality holds for $x = 1$

135. **(a)**

$$y = \left(x + \sqrt{1+x^2}\right)^n$$

$$\frac{dy}{dx} = n\left(x + \sqrt{1+x^2}\right)^{n-1}\left(1 + \frac{1}{2}(1+x^2)^{-1/2} \cdot 2x\right)$$

$$\frac{dy}{dx} = n\left(x + \sqrt{1+x^2}\right)^{n-1}\frac{\left(\sqrt{1+x^2} + x\right)}{\sqrt{1+x^2}}$$

$$= \frac{n\left(\sqrt{1+x^2} + x\right)^n}{\sqrt{1+x^2}}$$

or $\sqrt{1+x^2}\frac{dy}{dx} = ny$ or $\sqrt{1+x^2}y_1 = ny$

$\left(y_1 = \frac{dy}{dx}\right)$

Squaring, $(1+x^2)y_1^2 = n^2y^2$
Differentiating,

$$(1+x^2)2y_1y_2 + y_1^2 \cdot 2x = n^2 \cdot 2yy_1$$
$$\text{or } (1+x^2)y_2 + xy_1 = n^2\text{y}$$

136. **(a)** As given,

$$A = \lim_{x\to\infty} x\sin\left(\frac{1}{x}\right) = \lim_{x\to\infty}\frac{\sin\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}$$

Let $t = \frac{1}{x}$ when $x \to \alpha, t \to 0$

$$\Rightarrow A \quad = \lim_{t\to\infty}\frac{\sin t}{t} = 1$$
$$\left[\because \lim_{t\to 0}\frac{\sin x}{x} = 1\right]$$

and $B = \lim_{x\to 0} x\sin\left(\frac{1}{x}\right)$

$\Rightarrow B = \lim_{x\to 0} x \cdot \lim_{x\to 0}\sin\left(\frac{1}{x}\right)$
$\Rightarrow B = 0$
$\therefore$ A = 1 and B = 0 is correct

137. **(b)** As given a and b are the roots of the equation

$$x^2 + ax + b = 0$$
$$\Rightarrow \text{ sum of roots, } a + b = -a$$
$$\Rightarrow b = -2a \ \ldots(1)$$

and product of roots, ab = b

$$\Rightarrow ab - b = 0$$
$$\Rightarrow b(a-1) = 0$$
$$\text{if } b = 0 \text{ then } a = 0$$
$$\text{if } b \neq 0 \text{ then } a = 1 \text{ and } b = -2$$

so, the expression will be,

$$f(x) = x^2 + x - 2$$
$$= x^2 + 2\cdot\frac{1}{2}x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 2$$

$$\Rightarrow f(x) = \left(x + \frac{1}{2}\right)^2 - \frac{9}{4}$$

So, f(x) will be minimum, if $\left(x + \frac{1}{2}\right)^2 = 0$
i.e. when $x = -\frac{1}{2}$
$\Rightarrow$ minimum value of function $= -\frac{9}{4}$

138. **(d)**

Let us assume the functions f(x) and g(x) given by

$f(x) = \tan x - x$ and $g(x) = x - \sin x$, for

$0 < x < \frac{\pi}{2}$

Now, $f'(x) = \sec^2 x - 1$ and

$g'(x) = 1 - \cos x$

$\Rightarrow f'(x) > 0$ and $g'(x) > 0, \forall x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow f(x) > f(0)$ and $g(x) > g(0) \forall x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow \tan x - x > 0$ and $x - \sin x > 0$

$\forall x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow \tan x > x$ and $x > \sin x, \forall x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow \sin x < x < \tan x, \forall x \in \left(0, \frac{\pi}{2}\right)$

139. **(a)** Put $x = \sin\theta$ and $y = \sin\phi$

$\Rightarrow \cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$

$\Rightarrow 2\cos \frac{\theta+\phi}{2} \cos\frac{\theta-\phi}{2} = 2a\cos\frac{\theta+\phi}{2} \sin\frac{\theta-\phi}{2}$

$\Rightarrow \cot\frac{\theta-\phi}{2} = a \Rightarrow \theta - \phi = 2\cot^{-1} a$

$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$

Differentiate $\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$

so the degree is one

140. **(b)** Let $f(x) = ax^3 + bx^2 + cx + d$

Put $x = 0$ and $x = 1$

Then, we get $f(0) = -1$ and $f(1) = 0$

$\Rightarrow d = -1$ and $a + b + c + d = 0$

$\Rightarrow a + b + c = 1$

It is given that $x = 0$ is a stationary point of f(x), but it is not a point of extremum.
Therefore, $f'(0) = 0 = f''(0)$ and $f'''(0) = 0$

Now, $f(x) = ax + bx + cx + d$

$\Rightarrow f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2\,b$ and $f'''(x) = 6a$

$f' = 0, f''(0) = 0$ and $f'''(0) = 0 \neq 0$

$\Rightarrow c = 0, b = 0$ and $a \neq 0$

From Eqs. (i) and (ii), we get

$a = 1, b = c = 0$ and $d = -1$

Put these values in f(x)
we get $f(x) = x^3 - 1$
Hence, $\int \frac{f(x)}{x^3-1} dx = \int \frac{x^3-1}{x^3-1} dx = \int 1dx = x + C$

141. **(b)** $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is defined if (i) $-1 \leq x - 3 \leq 1 \Rightarrow 2 \leq x \leq 4$ and
(ii) $9 - x^2 > 0 \Rightarrow -3 < x < 3$
Taking common solution of (i) and (ii), we get $2 \leq x < 3$
$\therefore$ Domain $= [2,3)$

142. **(a)** The equations of the lines are

$$p_1x + q_1y - 1 = 0$$
$$p_2x + q_2y - 1 = 0$$

and $p_3x + q_3y - 1 = 0$

As they are concurrent,

$$\begin{vmatrix} p_1 & q_1 & -1 \\ p_2 & q_2 & -1 \\ p_3 & q_3 & -1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} p_1 & q_1 & 1 \\ p_2 & q_2 & 1 \\ p_3 & q_3 & 1 \end{vmatrix} = 0$$

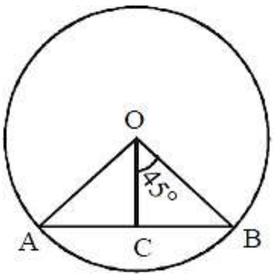
This is also the condition for the points $(p_1, q_1), (p_2, q_2)$ and (p_3, q_3) to be collinear.

143. **(c)** Let AB be the chord of length $\sqrt{2}$. Let O be the centre of the circle and let OC be the perpendicular from O on AB.

Then, $AC = BC = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$ In $\triangle$ OBC, we have

$$OB = BC \operatorname{cosec} 45°$$
$$= \frac{1}{\sqrt{2}} \times \sqrt{2} = 1$$

$\therefore$ Area of the circle $= \pi(OB)^2 = \pi$sq units



144. **(b)** $\cos A = n\cos B$ and $\sin A = m\sin B$
Squaring and adding, we get

$$1 = n^2\cos^2 B + m^2\sin^2 B$$
$$\Rightarrow 1 = n^2(1 - \sin^2 B) + m^2\sin^2 B$$
$$\therefore (m^2 - n^2)\sin^2 B = 1 - n^2$$

145. **(a)** $z_1, z_2, 0$ are vertices of an equilateral triangle, so we have $z_1^2 + z_2^2 + 0^2 = z_1z_2 + z_2 \cdot 0 + 0.z_1 \Rightarrow z_1^2 + z_2^2 = z_1z_2 \Rightarrow z_1^2 + z_2^2 - z_1z_2 = 0$

146. **(b)** Obviously, the relation is not reflexive and transitive, but it is symmetric, because

$x^2+x^2=2x^2\neq 1$
and $x^2+y^2=1, y^2+z^2=1$
$\Rightarrow x^2+z^2=1$

But $x^2+y^2=1 \Rightarrow y^2+x^2=1$

147. **(c)** Let l, m and n be the direction cosines.
Then, $l=\cos\theta, m=\cos\beta, n=\cos\theta$
we have $l^2+m^2+n^2=1$

$\Rightarrow \cos^2\theta+\cos^2\beta+\cos^2\theta=1$
$\Rightarrow 2\cos^2\theta+1-\sin^2\beta=1$
$\Rightarrow 2\cos^2\theta-\sin^2\beta=0$
$\Rightarrow 2\cos^2\theta-3\sin^2\beta=0$
$\Rightarrow$ $[\because \sin^2\beta=3\sin^2\theta$ (given) $]$
$\Rightarrow \tan^2\theta=2/3$
$\therefore \cos^2\theta=\dfrac{1}{1+\tan^2\theta}=\dfrac{1}{1+2/3}=\dfrac{3}{5}$

148. **(b)** Given $n=4$ and $P(X=0)=\frac{16}{81}$
Let p be the probability of success and q that of failure in a trial.
Then, $P(X=0)={}^4C_0p^0q^4=\frac{16}{81}$

$\Rightarrow q^4=\left(\dfrac{2}{3}\right)^4$
$(\because {}^nC_0=1)$
$\Rightarrow q=\dfrac{2}{3}\Rightarrow p=\dfrac{1}{3}$

$\therefore P(X=4)={}^4C_4p^4q^0=p^4=\left(\dfrac{1}{3}\right)^4=\dfrac{1}{81}$

149. **(d)** Let $f(x+y)=f(x)+f(y), \forall x,y\in R$
Put $x=0=y$

$\Rightarrow f(0)=f(0)+f(0)$
$\Rightarrow f(0)=0$

Now, $f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$f''(0)=\lim\limits_{h\to 0}\dfrac{f(h)}{h}$

Now,

$$f(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{f(x)+f(h)-f(x)}{h}$$

$\Rightarrow f'(x)=\lim\limits_{h\to 0}\dfrac{f(h)}{h}=f'(0)$
$\Rightarrow f(x)=x'(0)+C$
But $f(0)=0$
$\therefore C=0$

Hence, $f(x) = xf'(0), \forall x \in R$
Clearly, f(x) is everywhere continuous and differentiable and $f'(x)$ is constant.

$\forall x \in R$

150. **(d)** Let the coefficients of rth, (r + 1) th, and (r + 2)th term se in HP.

Then, $\frac{2}{^nC_r} = \frac{1}{^nC_{r-1}} + \frac{1}{^nC_{r+1}}$

$\Rightarrow 2 = \frac{^nC_r}{^nC_{r-1}} = \frac{^nC_r}{^nC_{r+1}}$

$\Rightarrow 2 = \frac{n-r+1}{r} + \frac{r+1}{n-r}$

$\Rightarrow n^2 - 4nr + 4r^2 + n = 0$

$\Rightarrow (n-2r)^2 + n = 0$

which is not possible for any value for n.