1. **(a)** As we know,

Gravitational potential energy $= \frac{-GMm}{r}$ and orbital velocity, $v_0 = \sqrt{GM/(R+h)}$

$$= \sqrt{\frac{GM}{(R+2R)}} = \sqrt{\frac{GM}{3R}}$$

$$E_f = \frac{1}{2}mv_0^2 - \frac{GMm}{3R} = \frac{1}{2}m\frac{GM}{3R} - \frac{GMm}{3R}$$

$$= \frac{GMm}{3R}\left(\frac{1}{2} - 1\right) = \frac{-GMm}{6R}$$

$$E_i = \frac{-GMm}{R} + K$$

$$E_i = E_f$$

Therefore, minimum required energy,

$$K = \frac{5GMm}{6R}$$

2. **(a)**

$$W = T\Delta A = 4\pi R^2\, T\left(n^{1/3} - 1\right)$$

$$= 4 \times 3.14 \times (10^{-2})^2 \times 460 \times 10^{-3}\left[(10^6)^{1/3} - 1\right]$$

$$= 0.057$$

3. **(c)**

$$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{\infty} - \frac{1}{25}\right) = -\frac{1}{50},$$

$$\frac{1}{f_2} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{25} + \frac{1}{20}\right) = \frac{3}{100}$$

$$\text{and } \frac{1}{f_3} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{-20} - \frac{1}{\infty}\right) = -\frac{1}{40}$$

$$\text{Now } \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

$$= -\frac{1}{50} + \frac{3}{100} - \frac{1}{40}$$

$$\therefore\ f = -66.6\text{ cm}$$

4. **(b)** When a charged particle enters a magnetic field at a direction perpendicular to the direction of motion, the path of the motion is circular. In circular motion the direction of velocity changes at every point (the magnitude remains constant). Therefore, the tangential momentum will change at every point. But kinetic energy will remain constant as it is given by $\frac{1}{2}mv^2$ and v^2 is the square of the magnitude of velocity which does not change.

5. **(b)**

$$N = N_0\left(\frac{1}{2}\right)^n$$

or $$\frac{N_0}{16} = N_0\left(\frac{1}{2}\right)^n$$

or $$n = 4$$

Halflife $$t_{1/2} = \frac{t}{n} = \frac{2}{4} = \frac{1}{2}h$$

6. (a) The charging of inductance given by,

$$i = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$\frac{i_0}{2} = i_0\left(1 - e^{-\frac{Rt}{L}}\right) \Rightarrow e^{-\frac{Rt}{L}} = \frac{1}{2}$$

Taking log on both the sides,

$$-\frac{Rt}{L} = \log 1 - \log 2$$

$$\Rightarrow \mathrm{t} = \frac{\mathrm{L}}{\mathrm{R}}\log 2 = \frac{300 \times 10^{-3}}{2} \times 0.69$$

$$\Rightarrow \mathrm{t} = 0.1\mathrm{sec.}$$

7. (a) For, $\mathrm{r} < \mathrm{r_A}, \mathrm{E} = 0$

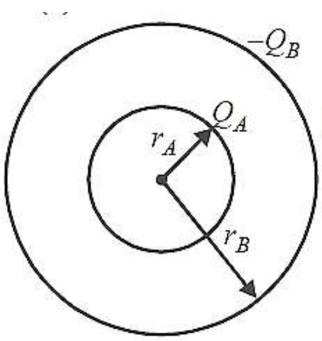
$\mathrm{r} = \mathrm{r_A}$, $$\mathrm{E} = \frac{1}{4\pi\epsilon_0}\frac{\mathrm{Q_A}}{\mathrm{r_A^2}}.$$

$\mathrm{r_A} < \mathrm{r} < \mathrm{r_B}$, $$\mathrm{E} = \frac{1}{4\pi\epsilon_0}\frac{\mathrm{Q_A}}{\mathrm{r^2}}$$

$\mathrm{r} = \mathrm{r_B}$, $$\mathrm{E} = \frac{1}{4\pi\epsilon_0}\left(\frac{\mathrm{Q_A} - \mathrm{Q_B}}{\mathrm{r_B^2}}\right).$$

$$= -\frac{1}{4\pi\epsilon_0}\left(\frac{\mathrm{Q_B} - \mathrm{Q_A}}{\mathrm{r_B^2}}\right)$$

These values are correctly represent in option (a).



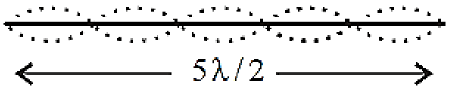
8. (b)

$$M = (\pi r^2 h)\rho = \pi r^2\left(\frac{2T\cos\theta}{\rho r g}\right)\rho$$

and $$M' = \pi(2r)^2\left(\frac{2T\cos\theta}{\rho \times 2r \times g}\right)\rho$$

$$= 2M.$$

9. **(a)** $f = \frac{5}{2\ell}\sqrt{\frac{F}{\mu}} = \frac{5}{2\ell}\sqrt{\frac{9g}{\mu}}$



and $f = \frac{3}{2\ell}\sqrt{\frac{Mg}{\mu}}$ ....(ii)

From above equations, we get M = 25 kg.

10. **(a)** We have, $E = W_0 + K$

or $\frac{hc}{400\times10^{-9}} = W_0 + \frac{1}{2}mv^2$

and $\frac{hc}{250\times10^{-9}} = W_0 + \frac{1}{2}m(2v)^2$

On simplifying above equations, we get

$W_0 = 2hc \times 10^6$ J.

11. **(d)** $V = 0$, and so $C = \frac{q}{V} \to \infty$.

12. **(d)** Mass of uranium changed into energy

$$= \frac{0.1}{100} \times 1$$
$$= 10^{-3} \text{ kg.}$$

The energy released $= mC^2$

$$= 10^{-3} \times (3 \times 10^8)^2$$
$$= 9 \times 10^{13} \text{ J.}$$

13. **(d)** The change in internal energy ΔU is same in all process.

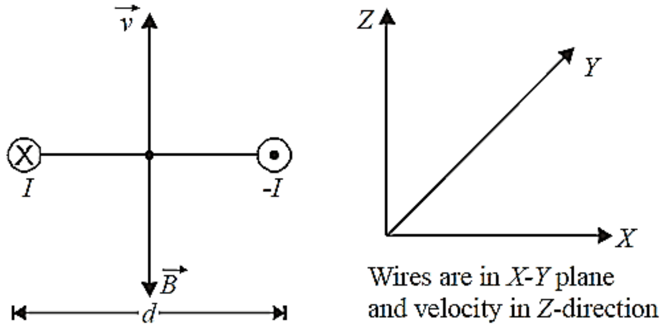
$Q_{ACB} = \Delta U + W_{ACB}$,
$Q_{ADB} = \Delta U$,
$Q_{AEB} = \Delta U + W_{AEB}$

Here W_{ACB} is positive and W_{AEB} is negative. Hence $Q_{ACB} > Q_{ADB} > Q_{AEB}$.

14. **(c)** Dimensions of

$$Y = \frac{\text{dimensions of X}}{\text{dimensions of Z}^2}$$
$$= \frac{M^{-1}L^{-2}T^4A^2}{(MT^{-2}A^{-1})^2}$$
$$= [M^{-3}L^{-2}T^8A^4]$$

15. **(d)** Net magnetic field due to the wires will be downward as shown below in the figure. Since angle between $\vec{v}$ and $\vec{B}$ is 180°,



Wires are in X-Y plane and velocity in Z-direction

Therefore, magnetic force $\vec{F}_m = q(\vec{v} \times \vec{B}) = 0$

16. (c) For projectile A

Maximum height, $H_A = \frac{u_A^2 \sin^2 45^\circ}{2g}$

For projectile B

Maximum height, $H_B = \frac{u_B^2 \sin^2 \theta}{2g}$

As we know, $H_A = H_B$

$$\frac{u_A^2 \sin^2 45^\circ}{2g} = \frac{u_B^2 \sin^2 \theta}{2g}$$

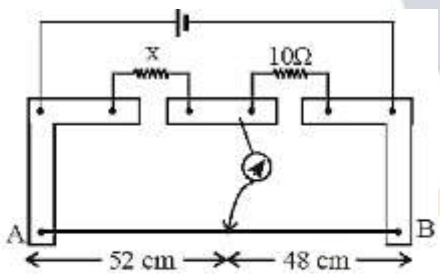
$$\frac{\sin^2 \theta}{\sin^2 45^\circ} = \frac{u_A^2}{u_B^2}$$

$$\sin^2 \theta = \left(\frac{u_A}{u_B}\right)^2 \sin^2 45^\circ$$

$$\sin^2 \theta = \left(\frac{1}{\sqrt{2}}\right)^2 \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{4}$$

$$\sin \theta = \frac{1}{2} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

17. (b) At Null point



$$\frac{X}{\ell_1} = \frac{10}{\ell_2}$$

Here $\ell_1 = 52 + \text{End correction}$

$= 52 + 1 = 53$ cm

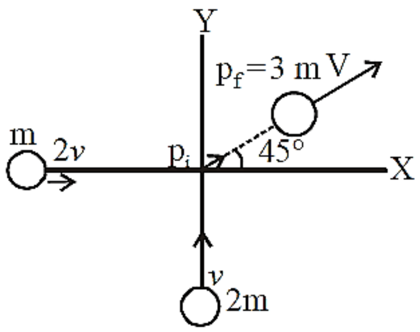
$\ell_2 = 48 + \text{End correction} = 48 + 2 = 50$ cm

$$\therefore \frac{X}{53} = \frac{10}{50} \quad \therefore X = \frac{53}{5} = 10.6\Omega$$

18. (a) Total flux through the cubical surface,

$$\phi = \frac{q_{in}}{\epsilon_0}$$
$$= \left[\frac{3+2+(-7)}{\epsilon_0}\right] C = -\frac{2C}{\epsilon_0}$$

19. (a)



Initial momentum of the system

$$p_i = m \times 2v\hat{i} + 2m \times v\hat{j}$$
$$= \sqrt{(m \times 2v)^2 + (2m \times v)^2} \text{ (magnitude)}$$
$$= 2\sqrt{2}mv$$

Final momentum of the system $= 3mV$
By the law of conservation of momentum

$$2\sqrt{2}\ mv = 3mV$$
$$\Rightarrow \frac{2\sqrt{2}v}{3} = V_{\text{combined}}$$

Loss in energy

$$\Delta E = \frac{1}{2}\ m_1\ V_1^2 + \frac{1}{2}\ m_2\ V_2^2 - \frac{1}{2}(\ m_1 + m_2)V_{\text{combined}}^2$$
$$\Delta E = 3mv^2 - \frac{4}{3}mv^2 = \frac{5}{3}mv^2 = 55.55\%$$

20. (d) Because of the Lenz's law of conservation of energy.

21. (c) We know that

$$Y = \frac{mg/A}{\Delta\ell/\ell} = \frac{mg\ell}{A\Delta\ell}$$

Also $\Delta\ell = \ell \propto \Delta T$
From (1) and (2)

$$Y = \frac{mg\ell}{A\ell\alpha\Delta T} = \frac{mg}{A\alpha\Delta T}$$
$$\therefore m = \frac{YA\alpha\Delta T}{g} = \frac{10^{11} \times \pi(10^{-3})^2 \times 10^{-5} \times 10}{10} = \pi \approx 3$$

22. (a)

$$C_{max} = 60^\circ$$

$$\therefore \quad r_{\mu_d} = \frac{1}{\sin 60^\circ}$$

$$\text{or} \quad \frac{\mu_g}{\mu_\ell} = \frac{2}{\sqrt{3}}$$

$$\therefore \quad \mu_\ell = \frac{\sqrt{3}}{2}\mu_g = \frac{\sqrt{3}}{2} \times 1.5$$

$$= 1.3$$

23. **(c)** The frequency of tuning fork, f = 392 Hz.

Also $392 = \frac{1}{2 \times 50}\sqrt{F/\mu}$

After decreasing the length by 2%, we have

$$f' = \frac{1}{2(49)}\sqrt{F/\mu}$$

From above equations,

f′ = 400 Hz.

∴ Beats frequency = 8 Hz.

24. **(a)**

$Z_1 = 1, Z_2 = 1, Z_3 = 2$ and $Z_4 = 3$.

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

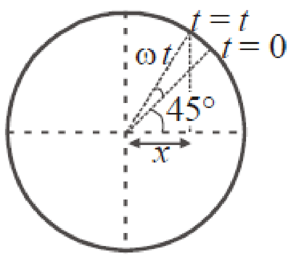
$$\text{or} \quad \lambda = \frac{4}{3RZ^2}$$

or $\lambda Z^2 =$ constant

So $\lambda_1(1)^2 = \lambda_2(1)^2 = \lambda_3(2)^2 = \lambda a(3^2)$

or $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$.

25. **(a)**



$$x = a\cos\left(\omega t + \frac{\pi}{4}\right)$$

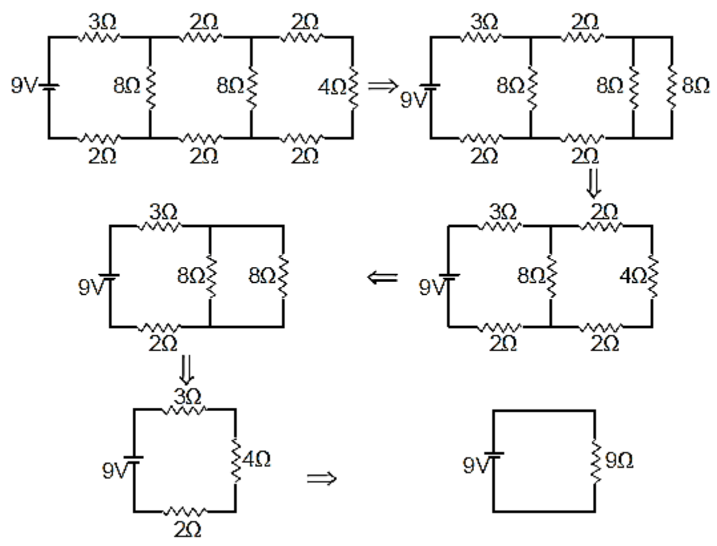
$$\text{or } x = a\cos\left(\frac{2\pi t}{4} + \frac{\pi}{4}\right)$$

26. **(b)** $\Delta x_{max} = 0$ and $\Delta x_{max} = 2\lambda$

Theortical maximas are $= 2n + 1 = 2 \times 2 + 1 = 5$

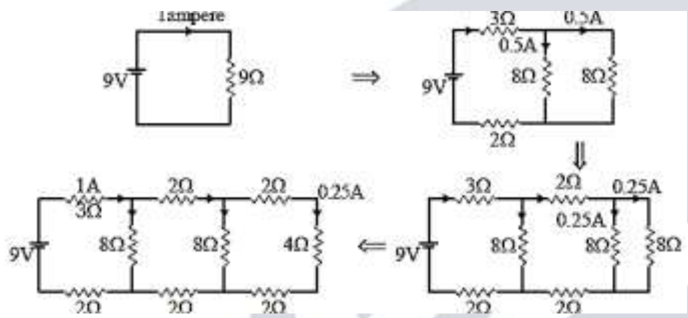
But on the screen there will be three maximas.

27. **(d)** The net resistance of the circuit is 9Ω as shown in the following figures.



$$I = \frac{V}{R} = \frac{9V}{9\Omega} = 1.0\ A$$

The flow of current in the circuit is as follows.



The current divides into two equal parts if passes through two equal resistances in parallel. Thus current through 4Ω resistor is 0.25 A.

28. (d) Here $\frac{x}{1000} = \frac{1.22\lambda}{D}$

or $x = \frac{1.22 \times 5 \times 10^{3} \times 10^{-10} \times 10^{3}}{10 \times 10^{-2}}$ or $x = 1.22 \times 5 \times 10^{-3}$ m $= 6.1$ m x is of the order of 5 mm.

29. (d) The change of state from liquid to vapour (for gas) is called vapourisation. It is observed that when liquid is heated, the temperature remains constant untill the entire amount of the liquid is converted into vapour. The temperature at which the liquid and the vapour states of the substance coexists is called its boiling point.

30. (d) When the wire is bent in the form of a square and connected between M and N as shown in fig. (2), the effective resistance between M and N decreases to one fourth of the value in fig. (1). The current increases fourtimes the initial value according to the relation $V = IR$. Since $H = I^2Rt$, the decrease in the value of resistance is more than compensated by the increases in the value of current. Hence heat produced increases.

Percentage loss in energy during the collision $\simeq 56\%$

31. (a) $U = mV = kmr$.

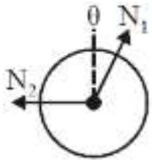
Force, $F = -\frac{dU}{dr} = -km$

Now, $\frac{mv^2}{r} = km \Rightarrow v \propto r^{1/2}$

$$\therefore T = \frac{2\pi r}{v} = \frac{2\pi r}{cr^{1/2}} \Rightarrow T \propto r^{1/2}$$

32. (b)

$\sin\theta = \frac{1}{2}$



Thus, $N_1 \sin\theta = N_2$

$\therefore \frac{N_1}{N_2} = \frac{1}{\sin\theta} = 2.$

33. **(a)** When charge is given to inner cylinder, an electric field will be in between the cylinders. So there is potential difference between the cylinders.

34. **(a)**

$$\Rightarrow \frac{x}{\sin 2\theta} = \frac{r}{\sin(90-\theta)}$$
$$\Rightarrow x = 2r\sin\theta$$
$$\therefore \frac{dx}{dt} = 2r\cos\theta \times \frac{d\theta}{dt}$$
$$\frac{d\theta}{dt} = \frac{dx/dt}{2r\cos\theta} = \frac{v_0}{2r\cos 60^\circ} = \frac{v_0}{r}$$

35. **(c)**

$$R^2 = [A^2 + B^2 + 2AB\cos\theta]$$
$$R^2 = R^2 + R^2 + 2R^2\cos\theta$$
$$-R^2 = 2R^2\cos\theta \text{ or } \cos\theta = -1/2$$
$$\text{or } \theta = 2\pi/3$$

36. **(b)** $B = \mu_0\mu_r H \Rightarrow \mu_r \propto \frac{B}{H} =$ slope of $B - H$ curve

According to the given graph, slope of the graph is highest at point Q.

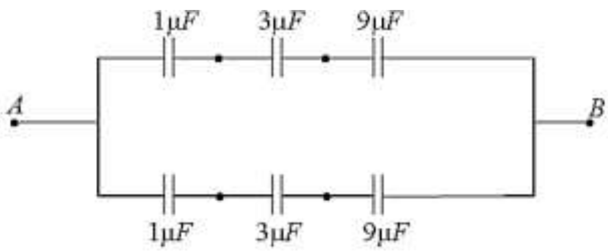
37. **(b)** $v_{av} = \left[\frac{500+600+700+800+900}{5}\right] = 700\ \mathrm{m/s}$

and

$$= \sqrt{\frac{500^2 + 600^2 + 700^2 + 800^2 + 900^2}{5}} = 714\ \mathrm{m/s}$$

Thus v_{rms} is greater than average speed by 14 m/s.

38. **(b)** The effective circuit is shown in figure.



The capacitance of upper series,

$$\frac{1}{C}=\frac{1}{1}+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+---\infty$$
$$\therefore C=\frac{2}{3}\mu C$$

Now $C_{AB}=2C=\frac{4}{3}\mu F$

39. **(a)** Process AB is isobasic and BC is isothermal, CD isochoric and DA isothermic compression.

40. **(d)** During the operation, either of D_1 and D_2 be in forward bias. Also R_1 and R_2 are different, so output across R will have different peaks.

41. **(b)** Polystyrene and polyethylene belong to the category of thermoplastic polymers which are capable of repeatedly softening on heating and harden on cooling.

42. **(d)** Hybridisation:

$$\begin{array}{ll} [Fe(CN)_6]^{4-}, & [Mn(CN)_6]^{4-}, \\ d^2sp^3 & d^2sp^3 \\ [Co(NH_3)_6]^{3+} & [Ni(NH_3)_6]^{2+} \\ d^2sp^3 & sp^3d^2 \end{array}$$

Hence $[Ni(NH_3)_6]^{2+}$ is outer orbital complex.

43. **(a)** The order of reaction is $\frac{3}{2}$ and molecularity is 2

44. **(d)** $CaSO_4$

45. **(a)** More is E°_{RP}, more is the tendency to get itself reduced or more is oxidising power.

46. **(c)**

$$\begin{aligned} \Delta G &= -2.303RT\log K \\ -nFE &= -2.303RT\log K \\ \log K &= \frac{nFE}{2.303RT} \\ &= 0.4342\frac{nFE}{RT} \\ \ln K &= \frac{nFE^\circ}{RT} \\ K &= e^{\frac{nFE^\circ}{RT}} \end{aligned}$$

47. **(c)**

Compound	O.S. of N
N_2O	+1
NO	+2
NO_2	+4
NO_3^-	+5
NH_4^+	-3

Therefore, increasing order of oxidation state of N is:

$NH_4^+ < N_2O < NO < NO_2 < NO_3^-$.

48. (a) Raoult's law becomes special case of Henry's law when K_H become equal to p_1°.

49. (b)

$$E_{cell}^o = \frac{0.059}{2}\log K_c \text{ or } \frac{1.10 \times 2}{0.059} = \log K_c$$
$$\therefore K_c = 1.9 \times 10^{37}$$

50. (d)

$$\frac{P}{T} = \text{ constant (Gay Lussac's law)}$$
$$\Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow P_1\, T_2 = P_2\, T_1$$
$$PV = \text{ constant}$$
$$P_1\, V_1 = P_2\, V_2 \;\; \text{[Boyle's law]}$$

51. (b) $\Delta n = -\frac{1}{2}; \Delta H = \Delta E - \frac{1}{2}RT; \Rightarrow \Delta E > \Delta H$

52. (d) For H atom, $E_n = -\frac{13.6Z^2}{n^2}$ eV

For second orbit, $n = 2$

$$Z = \text{ At. no. } = 1 \text{ (for hydrogen)}$$
$$\therefore E_2 = -\frac{13.6 \times (1)^2}{(2)^2} = \frac{-13.6}{4}\text{eV}$$
$$= \frac{-13.6 \times 1.6 \times 10^{-19}}{4}\text{ J}$$
$$= -5.44 \times 10^{-19}\text{ J}$$

53. (b) The right sequence of $I.E_1$ of Li < B < Be < C.

54. (c) Photochemical smog does not involve SO_2.

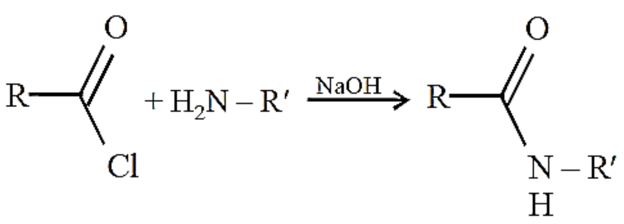
55. (c) There are four chief minerals present in a Portland cement tricalcium silicate (Ca_3SiO_5), dicalcium silicate (Ca_2SiO_4), tricalcium aluminate ($Ca_3Al_2O_6$) and calcium aluminoferrite ($Ca_4Al_nFe_{2-n}O_7$).

56. (a) Ammonia is a weak base and a salt containing its conjugate acid, the ammonium cation, such as NH_4OH functions as a buffer solution when they are present together in a solution.

57. (c) Among these ligands, ' F ' is a weak field ligand, makes only high spin complexes which has sp^3d^2 hybridization.

58. (d) Glycosidic linkage is a type of covalent bond that joins either two carbohydrate (sugar) molecule or one carbohydrate to another group. All molecules show such type of linkages.

59. (a) Schotten-Baumann Conditions



The use of added base to drive the equilibrium in the formation of amides from amines and acid chlorides.

60. (c) Note that in structures 1 and 2, every two adjacent hydrogen atoms are at maximum possible distance from each other (staggered conformation).

61. (b) Bond length decreases with an increase in bond order. Therefore, the order of bond length in these species is $O_2^+ < O_2^- > O_2 < O_2{}^{2-}$ (bond order - $O_2^+ = 2.5, O_2 = 2, O_2^- = 1.5, O_2^{2-} = 1$)

62. (a) For a given orbital with principal quantum number (n) and azimuthal quantum number (l) number of radial nodes $= (n - l - 1)$ for $3s$ orbital: $n = 3$ and $l = 0$ therefore, number of radial nodes $= 3 - 0 - 1 = 2$
for $2p$ orbital: $n = 2$ and $l = 1$ therefore, number of radial nodes $= 2 - 1 - 1 = 0$

63. (c)

$$M_1 V_1 = M_2 V_2$$
$$(0.025M)(0.050\ L) = (M_2)(0.025\ L)$$
$$M_2 = 0.05M$$
$$\text{but, there are } 2H\text{'s per } H_2SO_4 \text{ so } [H_2SO_4]$$
$$= 0.025M$$

64. (a) Given, mass ratio is $C : H : O (6 : 1 : 24)$ so, molar ratio will be $6/12 : 1/1 : 24/16 = 1 : 2 : 3$ therefore, $HO - (C = O) - OH$ has molar ratio $1 : 2 : 3$

65. (b) In bcc structure,
no. of atoms at corner $= 1/8 \times 8 = 1$
no. of atom at body centre $= 1$
therefore, total no of atom per unit cell $= 2$.

66. (b) PH_5 does not exist because d-orbital of ' P ' interacts with s-orbital of H. Bond formed is not stable and not energetically favorable. It depends on size and orientation of interaction.

67. (b) Ionic bonding is non directional, whereas covalent bonding is directional. So, CO_2 is directional.

68. (a) Given $\Delta H 35.5\ kJ\ mol^{-1}$
$\Delta S = 83.6 JK^{-1}\ mol^{-1}$

$\because \Delta G = \Delta H - T\Delta S$

For a reaction to be spontaneous, $\Delta G = -ve$ i.e., $\Delta H < T\Delta S$

$$\therefore T > \frac{\Delta H}{\Delta S} = \frac{35.5 \times 10^3 Jmol^{-1}}{83.6 JK^{-1}}$$

So, the given reaction will be spontaneous at $T > 425\ K$

69. (c)

$$\lambda_m = \frac{1000k}{0.1} = \frac{1000 \times 3.75 \times 10^{-4}}{0.1} = 3.75;$$
$$\alpha = \frac{\lambda_m}{\lambda_m^\infty} = \frac{3.75}{250} = 1.5 \times 10^{-2}$$
$$K_a = C^2 = 0.1 \times (1.5 \times 10^{-2})^2 = 2.25 \times 10^{-5}$$

70. (b)

$\text{Rate}_1 = k[A]^x[B]^y$

$\frac{\text{Rate}_1}{4} = k[A]^x[2B]^y$

or $\text{Rate}_1 = 4k[A]^x[2B]^y$

From (1) and (2) we get

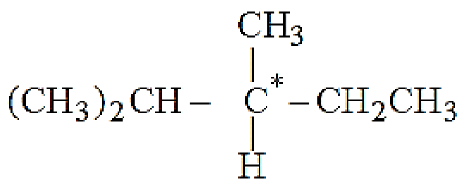
$\frac{k[A]^x[B]^y}{4} = k[A]^x[2B]^y$

$\frac{[B]^y}{4} = [2B]^y$

or $\frac{1}{4} = \left(\frac{2B}{B}\right)^y \Rightarrow \frac{1}{4} = 2^y$ or $(2)^{-2} = 2^y$

$y = -2$

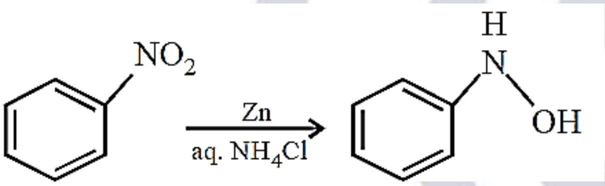
71. (a) A compound is said to exhibit optical isomerism if it atleast contains one chiral carbon atom, which is an atom bonded to 4 different atoms or groups.



72. (c) Meso compounds are characterized by an internal plane of symmetry that renders them achiral.

73. (a) Control rods slowdown the motion of neutrons and help in controlling the rate of fission. Cadmium is efficient for this purpose

74. (c) Reducing reagent is needed, as shown in given reaction.

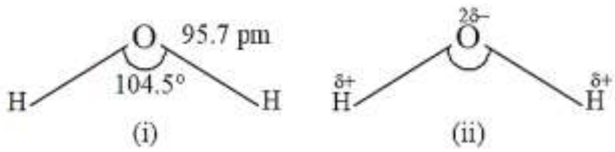


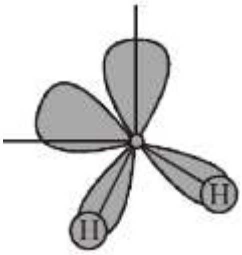
75. (c)

$$H_2CO_3 \rightleftharpoons H^+ + HCO_3^- \rightleftharpoons H^+ +$$

$CO_3^- \cdot HCO_3^-$ can donate and accept H^+.

76. (c)

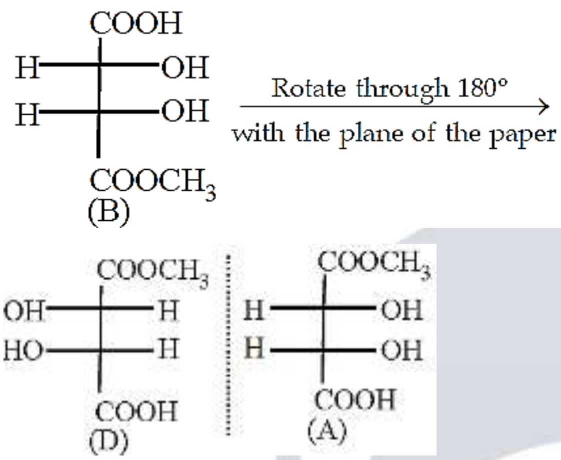




(iii)

77. **(c)** Because Na is very reactive and cannot be extracted by means of the reduction by C, CO etc. So it is extracted by electrolysis.

78. **(d)** Rotation of B through 180° within the plane of the paper gives D which is an enantiomer of A, hence A and B are enantiomers



79. **(b)**

80. **(a)** Thyroxine is an amine hormone.

81. **(a)** The word Loquacious (Adjective) means: talking a lot; talkative. Option (a) is the right synonym while others have different meanings.

82. **(c)** The word Meticulous (Adjective) means: paying careful attention to every detail; fastidious; thorough. Careless in option (c) is the correct antonomy.

83. **(c)**

84. **(b)**

85. **(d)**

86. **(b)**

87. **(b)** China is a big country. In area it is bigger than any other country except Russia. [except means other than, accept means consent, expect means to anticipate and access means entrance].

88. **(d)** The treasure was hidden off the shore. When something is hidden "off the shore," it just means that it's hidden somewhere near it.

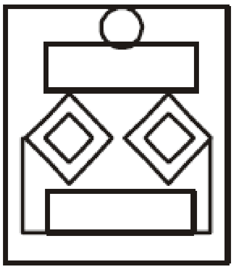
89. **(b)** Delete 'pair of' before binocular because the word 'binocular' itself suggests a pair.

90. **(b)** Delete 'all' before 'left'. Here the usage of 'all' is superfluous as 'the teacher as well as his students' itself signifies everyone.

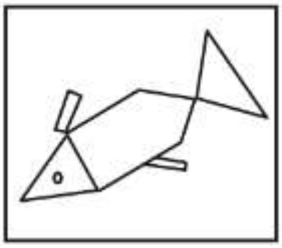
91. **(b)**

92. **(d)** Option (d) will complete the question figure.

93. **(a)**



94. (c) All the components of question figure are present in Answer Figure (c)



95. (b) In each row, the second figure is obtained from the first figure by adding two mutually perpendicular line segments at the centre and the third figure is obtained from the first figure by adding four circles outside the main figure.

96. (d) In each row, the second figure is obtained by rotating the first figure through 90°CW or 90°ACW and adding a circle to it. Also, the third figure is obtained by adding two circles to the first figure (without rotating the figure).

97. (a) O is the husband of P. M is the son of P. Therefore, M is the son of O.

98. (a) Wife of Vinod's father means the mother of Vinod.
Only brother of Vinod's mother means maternal uncle of Vinod.
Therefore, Vinod is cousin of Vishal.

99. (a) The pattern is as follows:

$$A \xrightarrow{+2} C \xrightarrow{+2} E \xrightarrow{+2} \boxed{G} \xrightarrow{+2} I \xrightarrow{+2} K$$

$$G \xrightarrow{+2} I \xrightarrow{+2} K \xrightarrow{+2} \boxed{M} \xrightarrow{+2} O \xrightarrow{+2} Q$$

$$M \xrightarrow{+2} O \xrightarrow{+2} Q \xrightarrow{+2} \boxed{S} \xrightarrow{+2} U \xrightarrow{+2} W$$

$$S \xrightarrow{+2} U \xrightarrow{+2} W \xrightarrow{+2} \boxed{Y} \xrightarrow{+2} A \xrightarrow{+2} C$$

$$Y \xrightarrow{+2} A \xrightarrow{+2} C \xrightarrow{+2} \boxed{E} \xrightarrow{+2} G \xrightarrow{+2} I$$

100. **(b)** The pattern is as follows:

$$P \xrightarrow{+3} S \xrightarrow{+3} V \xrightarrow{+3} Y \xrightarrow{+3} B$$

$$E \xrightarrow{+3} H \xrightarrow{+3} K \xrightarrow{+3} N \xrightarrow{+3} Q$$

$$T \xrightarrow{+3} W \xrightarrow{+3} Z \xrightarrow{+3} C \xrightarrow{+3} F$$

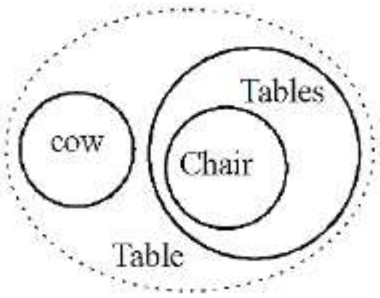
$$I \xrightarrow{+3} L \xrightarrow{+3} O \xrightarrow{+3} R \xrightarrow{+3} U$$

Therefore, the first term should be

$$A \xrightarrow{+3} D \xrightarrow{+3} G \xrightarrow{+3} J \xrightarrow{+3} M$$

101. **(d)** The statement implies that politicians win elections by the votes of people. Therefore, neither of the assumptions is implicit in the statement.

102. **(d)**



Conc I: True
Conc II: False
Conc III: False
Conc IV: False
or

103. **(a)** Temple and Church are places of worship. It does not imply that Hindus and Christians use the same place for worship. Church is different temple. Therefore, neither Conclusion I nor II follows.

104. **(a)** Growth and development of human organism is a continuous process. Some changes take place in human body now and then. Therefore, neither Conclusion I nor II follows.

105. **(c)**

106. **(b)**

We have $g(-3) = 0$

$\Rightarrow\ f(g(-3)) = f(0) = 7(0)^2 + 0 - 8 = -8$

$\therefore\quad fog(-3) = -8$

$g(9) = 9^2 + 4 = 85 \Rightarrow f(g(9)) = f(85) = 8(85)$

$+3 = 683$

$\therefore\quad fog(9) = 683$

$f(0) = 7.0^2 + 0 - 8 = -8 \Rightarrow g(f(0)) = g(-8)$

$= |-8| = 8$

$\therefore\quad gof(0) = 8$

$f(6) = 4(6) + 5 = 29 \Rightarrow g(f(6)) = g(29) = (29)^2$

$+4 = 845$

$\therefore\quad gof(6) = 845$

107. **(c)** $X - X - X - X - X$. The four digits 3,3,5,5 can be arranged at $(-)$ places in $\frac{4!}{2!2!} = 6$ ways. The five digits 2,2,8,8,8 can be arranged at (X) places in $\frac{5!}{2!3!}$ ways $= 10$ ways Total no. of arrangements $= 6 \times 10 = 60$ ways

108. **(b)**

$$\sum_{k=1}^{n}(k)(k+1)(k-1)=\sum_{k=1}^{n}k(k^2-1)=\sum_{k=1}^{n}(k^3-k)$$

$$=\left(\frac{n(n+1)}{2}\right)^2-\frac{n(n+1)}{2}$$

$$=\frac{n(n+1)}{2}\left(\frac{n(n+1)}{2}-1\right)$$

$$\frac{n^2+n}{2}\left(\frac{n^2+n-2}{2}\right)=\frac{n^4+n^3-2n^2+n^3+n^2-2n}{4}$$

$$=\frac{n^4}{4}+\frac{n^3}{2}-\frac{n^2}{4}-\frac{n}{2}\Rightarrow s=-\frac{1}{2}$$

109. **(c)** Let eq. of ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, length of semi-latus rectum

$$=\frac{b^2}{a}=\frac{a^2(1-e^2)}{a}=a(1-e^2)$$

Given $a(1-e^2)=\frac{1}{3}(2a)$

$$\Rightarrow 1-e^2=\frac{2}{3}\Rightarrow e^2=1-\frac{2}{3}=\frac{1}{3}\Rightarrow e=\frac{1}{\sqrt{3}}$$

110. **(c)** Given quadratic eqn. is $x^2+px+\frac{3p}{4}=0$

So, $\alpha+\beta=-p, \alpha\beta=\frac{3p}{4}$

Now, given $|\alpha-\beta|=\sqrt{10}\Rightarrow\alpha-\beta$

$=\pm\sqrt{10}$

$\Rightarrow(\alpha-\beta)^2=10\Rightarrow\alpha^2+\beta^2-2\alpha\beta=10$

$\Rightarrow(\alpha+\beta)^2-4\alpha\beta=10$

$$\Rightarrow p^2-4\times\frac{3p}{4}=10\Rightarrow p^2-3p-10=0$$

$\Rightarrow p=-2,5\Rightarrow p\in\{-2,5\}$

111. **(d)** The given system of lines passes through the point of intersection of the straight lines $2x+y-3=0$ and $3x+2y-5=0$ [$L_1+\lambda L_2=0$ form], which is (1,1). The required line will also pass through this point. Further, the line will be farthest from point $(4,-3)$ if it is in direction perpendicular to line joining (1,1) and $(4,-3)$.

$\therefore$ The equation of the required line is

$$y-1=\frac{-1}{\frac{-3-1}{4-1}}(x-1)\Rightarrow 3x-4y+1=0$$

112. **(b)** $\frac{n!}{n^n}=\frac{3}{32}\Rightarrow\frac{n!}{n^n}=\frac{8\times3}{8\times32}=\frac{4!}{4^4}$

$\therefore n=4$

113. **(b)**

$R=(3+\sqrt{5})^{2n}, G=(3-\sqrt{5})^{2n}$

Let $[R]+1=I$ ($\because$ [.] greatest integer function)

$\Rightarrow$ $R+G=I(\because 0<G<1)$

$\Rightarrow$ $(3+\sqrt{5})^{2n}+(3-\sqrt{5})^{2n}=I$

seeing the option put $n=1$

$I=28$ isdivisibleby 4 i.e. 2^{n+1}

114. **(c)** For f(x) to be defined, we must have

$$-1 \leq \log_2\left(\frac{1}{2}x^2\right) \leq 1$$

$\Rightarrow 2^{-1} \leq \frac{1}{2}x^2 \leq 2^1 [\because$ the base $=2>1]$

$\Rightarrow 1 \leq x^2 \leq 4$

Now, $1 \leq x^2$

$\Rightarrow x^2-1 \geq 0$ i.e. $(x-1)(x+1) \geq 0$

$\Rightarrow x \leq -1$ or $x \geq 1$

Also, $x^2 \leq 4$

$\Rightarrow x^2-4 \leq 0$ i.e. $(x-2)(x+2) \leq 0$

$\Rightarrow -2 \leq x \leq 2.$

Form (2) and (3), we get the domain of

$f \quad = \{(-\infty,-1] \cup [1,\infty)\} \cap [-2,2]$

$\quad = [-2,-1] \cup [1,2]$

115. **(a)** We construct the following table taking assumed mean $a=55$ (step deviation method).

Class	x_i	f_i	c.f.	$u_i=\frac{x_i-a}{10}$	f_iu_i
10 – 20	15	2	2	-4	-8
20 – 30	25	3	5	-3	-9
30 – 40	35	4	9	-2	-8
40 – 50	45	5	14	-1	-5
50 – 60	55	6	20	0	0
60 – 70	65	12	32	1	12
70 – 80	75	14	46	2	28
80 – 90	85	10	56	3	30
90 – 100	95	4	60	4	16
Total		60			56

$\therefore$ The mean $= a + \dfrac{\sum f_i u_i}{\sum f_i} \times h$

$$= 55 + \frac{56}{60} \times 10 = 55 + \frac{56}{6} = 64.333$$

Here $n = 60 \Rightarrow \frac{n}{2} = 30$, therefore, $60 - 70$ is the median class Using the formula:

$$M = l + \frac{\frac{n}{2} - C}{f} \times c = 60 + \frac{30 - 20}{12} \times 10$$

$$= 60 + \frac{100}{12} = 60 + 8.333 = 68.333$$

116. (c)

$$2e^{iB} = e^{iA} + e^{iC}$$

$\Rightarrow$ $2\cos B = \cos A + \cos C$

& $2\sin B = \sin A + \sin C$

Squaring and adding we get

$\cos(A - C) = 1 \Rightarrow A - C = 0$

$\therefore$ $A = C$, From (i) and (ii) $\cos B = \cos A$

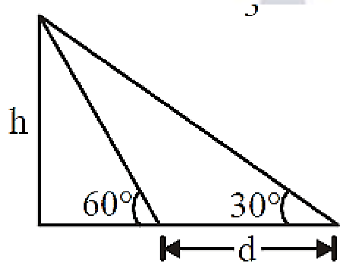
and $\sin B = \sin A$

So, $A = B. \Rightarrow A = B = C$

117. (c)

$d = h\cot 30^\circ - h\cot 60^\circ$ and time $= 3$ min.

$\therefore$ Speed $= \dfrac{h(\cot 30^\circ - \cot 60^\circ)}{3}$ per minute



It will travel distance h cot 60° in

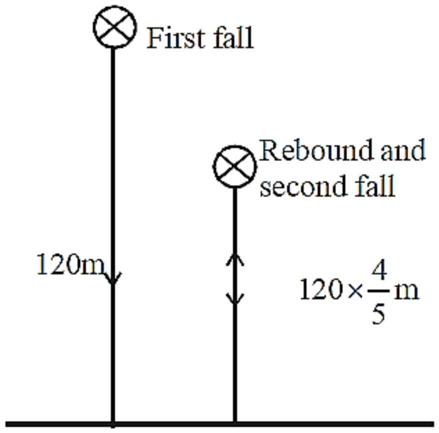
$$\frac{h\cot 60^\circ \times 3}{h(\cot 30^\circ - \cot 60^\circ)} = 1.5 \text{ minute}$$

118. (c) Clearly, the total distance described

$$= 120 + 2\left[120 \times \frac{4}{5} + 120 \times \frac{4}{5} \times \frac{4}{5} + 120 \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} + \cdots \text{ to } \infty\right]$$

Except in the first fall the same ball will travel twice in each step the same distance one upward and second downward travel.

$\therefore$ Distance travelled



$$= 120 + 2 \times 120\left[\frac{4}{5} + \left(\frac{4}{5}\right)^2 + \cdots \text{....to } \infty\right]$$

$$= 120 + 240\left[\frac{\frac{4}{5}}{1-\frac{4}{5}}\right] = 120 + 240 \times 4 = 1080 \text{ m}$$

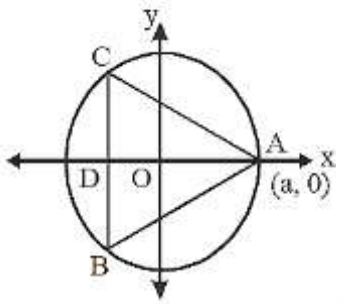
119. **(c)** Since the equilateral triangle is inscribed in the circle with centre at the origin, centroid lies on the origin.

So, $\frac{AO}{OD} = \frac{2}{1} \Rightarrow OD = \frac{1}{2}AO = \frac{a}{2}$

So, other vertices of triangle have coordinates,

$\left(-\frac{a}{2}, \frac{\sqrt{3}a}{2}\right)$ and $\left[-\frac{a}{2}, -\frac{\sqrt{3}}{2}a\right]$

$\left(-\frac{a}{2}, \frac{\sqrt{3}}{2}a\right)$



$\left(-\frac{a}{2}, \frac{\sqrt{3}}{2}a\right)$

∴ Equation of line BC is

$x = -\frac{a}{2} \Rightarrow 2x + a = 0$

120. **(a)** we have, $f(x) = x - |x - x^2| = x - |x(1 - x)|$

$= x - |x||1 - x|$,

∴ Continuity is to be checked at $x = 0$ and $x = 1$. At $x = 0$

$$\text{LHL} = \lim_{h\to 0} f(0 - h) = \lim_{h\to 0} -h - |-h||1 + h|$$
$$= \lim_{h\to 0} -h - h(1 + h) = 0$$

RHL $= \lim_{h\to 0} f(0+h) = \lim_{h\to 0} h - |h||1-h|$
$= \lim_{h\to 0} h - h(1-h) = 0$

and $f(0) = 0$
Since LHL = RHL = $f(0)$,
$\therefore f(x)$ is continuous at $x = 0$.
At $x = 1$

LHL $= \lim_{h\to 0} f(1-h) \quad = \lim_{h\to 0}(1-h) - |1-h||1-(1-h)|$
$= \lim_{h\to 0}(1-h) - h(1-h) = 1$

Similarly RHL $= \lim_{h\to 0} f(1+h) = 1$
and $f(1) = 1 - |1| \cdot |1-1| = 1$
$\therefore f(x)$ is continuous at $x = 1$
Hence $f(x)$ is continuous for all $x \in [-1,1]$

121. **(d)** Let $P(n): \frac{4^n}{n+1} < \frac{(2n)!}{(n!)^2}$

For $n = 2$,

$$P(2): \frac{4^2}{2+1} < \frac{4!}{(2)^2} \Rightarrow \frac{16}{3} < \frac{24}{4}$$

which is true.
Let for $n = m \geq 2$, $P(m)$ is true.
i.e. $\frac{4^m}{m+1} < \frac{(2m)!}{(m!)^2}$

Now, $\frac{4^{m+1}}{m+2} = \frac{4^m}{m+1} \cdot \frac{4(m+1)}{m+2}$

$$< \frac{(2m)!}{(m!)^2} \cdot \frac{4(m+1)}{(m+2)}$$
$$= \frac{(2m)!\,(2m+1)(2m+2)4(m+1)(m+1)^2}{(2m+1)(2m+2)(m!)^2(m+1)^2(m+2)}$$

$$= \frac{[2(m+1)]!}{[(m+1)!]^2} \cdot \frac{2(m+1)^2}{(2m+1)(m+2)}$$
$$< \frac{[2(m+1)]!}{[(m+1)!]^2}$$

Hence, for $n \geq 2$, $P(n)$ is true.

122. **(b)** $\Delta = \begin{vmatrix} -a & 1 & 1 \\ 1 & -b & 1 \\ 1 & 1 & -c \end{vmatrix} = 0$ for non-zero solution

$\Rightarrow abc - a - b - c - 2 = 0$
$\Rightarrow abc = a + b + c + 2$

Now, $\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c}$

$$= \frac{3 + 2(a+b+c) + (ab+bc+ac)}{1 + (a+b+c) + (ab+bc+ac) + abc}$$

$$= \frac{3 + 2(a+b+c) + (ab+bc+ac)}{1 + 2(a+b+c) + 2 + ab + bc + ac} = 1$$

123. **(b)** A function $f(x)$ is said to be increasing function in $[a, b]$ if $f'(x) > 0$ in $[a, b]$.

Given $f(x) = x^x$

Differentiate equation (i)

$f'(x) = x^x(1 + \log x)$

Put $f'(x) = 0$

$0 = x^x(1 + \log x)$

$\Rightarrow x = 0, \log x = -1 \Rightarrow x = e^{-1}$

$\Rightarrow x = \frac{1}{e}, 0$

Now, in $\left[0, \frac{1}{e}\right], f'(x) > 0$

$\therefore$ $f(x)$ is increasing in interval $\left[0, \frac{1}{e}\right]$

124. **(d)**

Let $y = \dfrac{x}{x^2 - 5x + 9}$

$\Rightarrow$ $x^2y - (5y + 1)x + 9y = 0$

for real x, Discriminant $= b^2 - 4ac \geq 0$

$(5y + 1)^2 - 36y^2 \geq 0$

$\Rightarrow$ $(5y + 1 - 6y)(5y + 1 + 6y) \geq 0$

$\Rightarrow$ $(-y + 1)(11y + 1) \geq 0$

$\Rightarrow$ $(y - 1)(11y + 1) \leq 0 \Rightarrow y \in \left[\frac{-1}{11}, 1\right]$

125. **(d)** Putting $x = \frac{1}{y}$, we get

$$\mathrm{L} = \lim_{y \to 0}\left(\frac{a_1^y + a_2^y + \cdots..+a_n^y}{n}\right)^{n/y}$$

$(\because x \to \infty y \to 0)$

$$\therefore \log_e \mathrm{L} = \lim_{y \to 0} \frac{n}{y} \cdot \log_e \frac{1}{n}(a_1^y + a_2^y + \cdots..+a_n^y)\left(\frac{0}{0}\right)$$

$$= n\lim_{y \to 0} \left(\frac{\left(\dfrac{a_1^y \log a_1 + a_2^y \log a_2 + \cdots..+a_n^y \log a_n}{a_1^y + a_2^y + \cdots ... + a_n^y}\right)}{1}\right)$$

$= n \cdot \dfrac{\log(a_1 a_2 .... a_n)}{n}$ $[\log \mathrm{L} = \log(a_1 \cdot a_2 ..... a_n) \Rightarrow \mathrm{L} = a_1 \cdot a_2 \cdot a_3 ... . a_n$

126. **(d)** We have $\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18$

$\because xy = \frac{1}{7} \cdot \frac{1}{8} < 1$

$\therefore \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18}$

$\tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\times\frac{1}{8}}\right) + \tan^{-1}\frac{1}{18} = \tan^{-1}\frac{15}{55}\tan^{-1}\frac{1}{18}$

also $\frac{3}{11} \cdot \frac{1}{18} < 1$

$\tan^{-1}\frac{3}{11} + \tan^{-1}\frac{1}{18} = \tan^{-1}\left(\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{3}{11}\times\frac{1}{18}}\right)$

$= \tan^{-1}\frac{65}{195} = \tan^{-1}\frac{1}{3} = \cot^{-1} 3$

127. **(a)**

128. **(b)**

$P(E) = P(2 \text{ or } 3 \text{ or } 5 \text{ or } 7)$
$= 0.23 + 0.12 + 0.20 + 0.07 = 0.62$
$P(F) = P(1 \text{ or } 2 \text{ or } 3)$
$= 0.15 + 0.23 + 0.12 = 0.50$
$P(E \cap F) = P(2 \text{ or } 3) = 0.23 + 0.12 = 0.35$
$\therefore P(EUF) = P(E) + P(F) - P(E \cap F)$
$= 0.62 + 0.50 - 0.35 = 0.77$

129. **(c)**

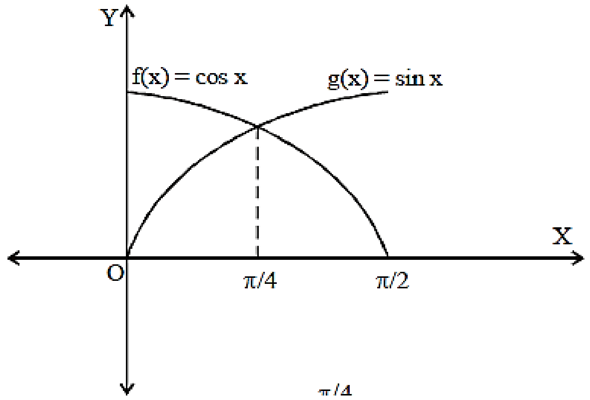
We have $\cos x + \cos 2x + \cos 3x = 0$
or $(\cos 3x + \cos x) + \cos 2x = 0$
or $2\cos 2x \cdot \cos x + \cos 2x = 0$
or $\cos 2x(2\cos x + 1) = 0$
We have, either $\cos 2x = 0$ or $2\cos x + 1 = 0$
If $\cos 2x = 0$, then $2x = (2m+1)\frac{\pi}{2}$
or $x = (2m+1)\frac{\pi}{4}, m \in I$
If $2\cos x + 1 = 0$, then $\cos x = -\frac{1}{2} = \cos\frac{2\pi}{3}$

$\Rightarrow x = 2n\pi \pm \frac{2\pi}{3}, n \in I$

Hence the required general solution are

$x = (2m+1)\frac{\pi}{4}$ and $x = 2n\pi \pm \frac{2\pi}{3}$,
$m \cdot n \in I$

130. **(b)** $y = |\cos x - \sin x|$



Required area $= 2\int_0^{\pi/4}(\cos x - \sin x)dx$

$= 2[\sin x + \cos x]_0^{\pi/4}$

$= 2\left[\frac{2}{\sqrt{2}} - 1\right] = (2\sqrt{2} - 2)$ sq. units

131. **(a)** We have,

$$\mathrm{Lf}'(0) = \lim_{h\to0}\frac{f(0-h)-f(0)}{-h} = \lim_{h\to0}\frac{-h\log\cosh}{-h\log(1+h^2)}$$

$$= \lim_{h\to0}\frac{\log\cosh}{\log(1+h^2)}\left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{h\to0}\frac{-\tan h}{2h/(1+h^2)} = -1/2$$

$$\mathrm{Rf}'(0) = \lim_{h\to0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to0}\frac{h\log\cosh}{h\log(1+h^2)}$$

$$= \lim_{h\to0}\frac{\log\cos h}{\log(1+h^2)}\left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{h\to0}\frac{-\tan h}{2h/(1+h^2)} = \frac{-1}{2}$$

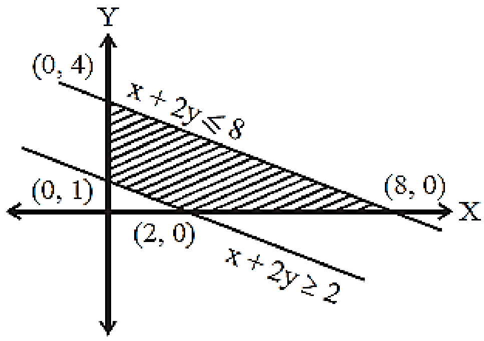
Since $\mathrm{Lf}'(0) = \mathrm{Rf}'(0)$, therefore f(x) is differentiable at $x = 0$

132. **(b)**

Given: $x + 2y \geq 2$

$x + 2y \leq 8$

and $x, y \geq 0$



For equation (1)

$\frac{x}{2}+\frac{y}{1}=1$

and for equation (2)

$\frac{x}{8}+\frac{y}{4}=1$

Given: $z=3x+2y$
At point $(2,0); z=3\times2+0=6$
At point $(0,1); z=3\times0+2\times1=2$
At point $(8,0); z=3\times8+2\times0=24$
At point $(0,4); z=3\times0+2\times4=8$
maximum value of z is 24 at point $(8,0)$.

133. (c)

$V=\pi r^2 h=$ constant. If k be the thickness of the sides then that of the top will be $(5/4)k$.
$\therefore\ S=(2\pi h)k+(\pi r^2)\cdot(5/4)k$
('S' is vol. of material used)
or $S=2\pi rk\cdot\frac{V}{\pi r^2}+\frac{5}{4}\pi r^2 k=k\left(\frac{2V}{r}+\frac{5}{4}\pi r^2\right)$
$\therefore \frac{dS}{dr}=k\left(-\frac{2V}{r^2}+\frac{5}{2}\pi r\right), \therefore r^3=4V/5\pi$
$\frac{d^2S}{dr^2}=k\left(\frac{4V}{r^3}+\frac{5}{2}\pi\right),=\frac{15}{2}k\pi=$ positive
When $r^3=4V/5\pi$ or $5\pi r^3=4\pi r^2 h. \therefore$
$\frac{r}{h}=\frac{4}{5}$.

134. (d) We have

$n(A\cup B\cup C)=n(A)+n(B)+n(C)$
$-n(A\cap B)-n(B\cap C)-n(C\cap A)$
$+n(A\cap B\cap C)$

$=10+15+20-8-9-n(C\cap A)$
$+n(A\cap B\cap C)$
$=28-\{n(C\cap A)-n(A\cap B\cap C)\}$ ... (i)
Since $n(C\cap A)\geq n(A\cap B\cap C)$
We have $n(C\cap A)-n(A\cap B\cap C)\geq 0$

From (i) and (ii)
$n(A\cup B\cup C)\leq 28$
Now, $n(A\cup B)=n(A)+n(B)-n(A\cap B)$

$=10+15-8=17$

and $n(B\cup C)=n(B)+n(C)-n(B\cap C)$

$=15+20-9=26$

Since, $n(A\cup B\cup C)\geq n(A\cup C)$ and $n(A\cup B\cup C)\geq n(B\cup C)$, we have
$n(A\cup B\cup C)\geq 17$ and $n(A\cup B\cup C)\geq 26$

Hence $n(A \cup B \cup C) \geq 26$
From (iii) and (iv) we obtain
$26 \leq n(A \cup B \cup C) \leq 28$
Also $n(A \cup B \cup C)$ is a positive integer
$\therefore n(A \cup B \cup C) = 26$ or 27 or 28

135. **(a)**

$$z = 1 + 2i \Rightarrow |z| = \sqrt{1+4} = \sqrt{5}$$
$$\therefore f(z) = \frac{7-z}{1-z^2} = \frac{7-1-2i}{1-(1+2i)^2}$$
$$= \frac{6-2i}{1-(1-4+4i)} = \frac{6-2i}{4-4i} = \frac{3-i}{2-2i}$$
$$\Rightarrow |f(z)| = \left|\frac{3-i}{2-2i}\right| = \frac{|3-i|}{|2-2i|}$$
$$= \frac{\sqrt{9+1}}{\sqrt{4+4}} = \frac{\sqrt{5}}{2} = \frac{|z|}{2}$$

136. **(b)** Let $f(x) = \cos^{-1}\left[\frac{1-(\log x)^2}{1+(\log x)^2}\right]$

Put $\log x = t$ in $f(x)$

$$\therefore f(x) = \cos^{-1}\left[\frac{1-t^2}{1+t^2}\right]$$

Now, put $t = \tan\theta$, we get

$$f(x) = \cos^{-1}\left[\frac{1-\tan^2\theta}{1+\tan^2\theta}\right]$$
$$= \cos^{-1}[\cos 2\theta] = 2\theta = 2\tan^{-1} t = 2$$

$\tan^{-1}(\log x)$

Diff. both side w.r.t ' x ', we get

$$f'(x) = 2 \cdot \frac{1}{1+(\log x)^2} \cdot \frac{1}{x}$$

Now,

$$f'(e) = 2 \cdot \frac{1}{1+(\log e)^2} \cdot \frac{1}{e} = \frac{1}{e}$$
$(\because \log e = 1)$

137. **(d)** Number form by using 1,2,3,4,5 = 5 ! = 120
Number formed by using 0,1,2,4,5

4	4	3	2	1

$= 4.4.3.2.1 = 96$

Total number formed, divisible by 3 (taking numbers without repetition) = 216
Statement 1 is false and statement 2 is true.

138. **(c)** Any tangent to parabola $y^2 = 8x$ is

$y = mx + \frac{2}{m}$

It touches the circle $x^2 + y^2 - 12x + 4 = 0$, if the length of perpendicular from the centre $(6,0)$ is equal to radius $\sqrt{32}$.

$$\therefore \frac{6\,m + \frac{2}{m}}{\sqrt{m^2 + 1}} = \pm\sqrt{32} \Rightarrow \left(3\,m + \frac{1}{m}\right)^2 = 8(\,m^2 + 1)$$
$$\Rightarrow (3\,m^2 + 1)^2 = 8(\,m^4 + m^2)$$
$$\Rightarrow m^4 - 2\,m^2 + 1 = 0 \Rightarrow m = \pm 1$$

Hence, the required tangents are $y = x + 2$ and $y = -x - 2$.

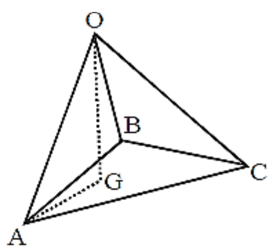
139. (a)

$$R(s)R(t) = \begin{bmatrix} \cos s & \sin s \\ -\sin s & \cos s \end{bmatrix} \times \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$$
$$= \begin{bmatrix} \cos s \cos t - \sin s \sin t & \cos s \sin t + \sin s \cos t \\ -\sin s \cos t - \cos s \sin t & -\sin s \sin t + \cos s \cos t \end{bmatrix}$$
$$= \begin{bmatrix} \cos(s+t) & \sin(s+t) \\ -\sin(s+t) & \cos(s+t) \end{bmatrix} = R(s+t)$$

140. (d)

$$\int x \log\left(1 + \tfrac{1}{x}\right) dx$$
$$= \log\left(1 + \tfrac{1}{x}\right) \cdot \tfrac{x^2}{2} - \int \tfrac{x}{x+1} \cdot \left(-\tfrac{1}{x^2}\right) \cdot \tfrac{x^2}{2} dx$$
$$= \tfrac{x^2}{2} \log\left(\tfrac{x+1}{x}\right) \cdot \tfrac{x^2}{2} + \tfrac{1}{2} \int \tfrac{x+1-1}{x+1} dx$$
$$= \tfrac{x^2}{2} \log\left(\tfrac{x+1}{x}\right) + \tfrac{1}{2} x - \tfrac{1}{2} \log(x+1) + c$$
$$= \left(\tfrac{x^2-1}{2}\right) \log(x+1) - \tfrac{x^2}{2} \log x + \tfrac{1}{2} x + c$$

141. (c) $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b} \& \overrightarrow{OC} = \vec{c}$ are unit vectors and equally inclined to each other at an acute angle θ.



$\therefore$ ABC is an equilateral triangle
and

$$AB = \sqrt{OA^2 + OB^2 - 2OA \cdot OB \cdot \cos\theta}$$
$$= \sqrt{2 - 2\cos\theta} = \sqrt{2}\sqrt{1 - \cos\theta}$$

$\therefore$ Area of $\triangle$ ABC

$$= \frac{\sqrt{3}}{4} AB^2 = \frac{\sqrt{3}}{4} \cdot 2(1 - \cos\theta) = \frac{\sqrt{3}}{2}(1 - \cos\theta)$$

If G is the centroid of the $\triangle$ ABC, then

$$OG = \frac{1}{3}|\vec{a}+\vec{b}+\vec{c}|$$
$$= \frac{1}{3}\sqrt{a^2+b^2+c^2+2\vec{a}\cdot\vec{b}+2\vec{b}\cdot\vec{c}+2\vec{c}\cdot\vec{a}}$$
$$= \frac{1}{\sqrt{3}}\sqrt{1+2\cos\theta}$$

$\therefore [\vec{a} \quad b \quad \vec{c}] =$ Volume of parallelopiped

$$= OG \times 2ar(\Delta ABC)$$
$$= 2\cdot\frac{1}{\sqrt{3}}\sqrt{1+2\cos\theta}\times\frac{\sqrt{3}}{2}(1-\cos\theta)$$
$$= (1-\cos\theta)\sqrt{1+2\cos\theta}$$

142. **(b)** The given product

$$= 2^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+\cdots} = 2^{s} \text{ (say)}$$

Now $S = \frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+\cdots$.

$$\Rightarrow \frac{1}{2}S = \frac{1}{8}+\frac{2}{16}+\frac{3}{32}+\cdots$$

Apply; $(1)-(2)$

$$\Rightarrow \frac{1}{2}S = \frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots..$$
$$= \frac{1/4}{1-1/2} = \frac{1}{2} \quad \therefore S = 1$$
$$\Rightarrow \text{ Product } = 2^1 = 2$$

143. **(a)**

$$\frac{{}^{n}C_r}{{}^{r+3}C_r} = 3!\frac{1}{(r+3)(r+2)}\cdot\frac{{}^{n}C_r}{(r+1)}$$
$$= 3!\frac{1}{(r+3)(r+2)}\cdot\frac{{}^{n+1}C_{r+1}}{(n+1)} \text{ [See Formulae]}$$
$$= 3!\frac{1}{(r+3)(n+1)}\frac{{}^{n+1}C_{r+1}}{r+2}$$
$$= 3!\frac{1}{(r+3)(n+1)}\frac{n+2}{n+2}$$
$$= \frac{3!}{(n+1)(n+2)}\cdot\frac{n+2C_{r+2}}{r+3}$$
$$= \frac{3!}{(n+1)(n+2)(n+3)}\,{}^{n+3}C_{r+3}$$
$$\therefore \sum_{r=0}^{n}(-1)^r\frac{{}^{n}C_r}{{}^{r+3}C_r}$$

$$= \frac{6}{(n+1)(n+2)(n+3)} \sum_{r=0}^{n} (-1)^r \, {}^{n+3}C_{r+3}$$

$$= \frac{6}{(n+1)(n+2)(n+3)} [{}^{n+3}C_3 - {}^{n+3}C_4 + \cdots . + (-1)^{n} {}^{n+3}C_{n+3}]$$

$$= \frac{6}{(n+1)(n+2)(n+3)} [{}^{n+3}C_0 - {}^{n+3}C_1 + {}^{n+3}C_2]$$

$$= \frac{[\because {}^{n+3}C_0 - {}^{n+3}C_1 + \cdots . + (-1)^{n+3} \times {}^{n+3}C_{n+3} = 0]}{(n+1)(n+2)(n+3)} \left(1 - n - 3 + \frac{(n+3)(n+2)}{2}\right)$$

$$= \frac{3}{(n+1)(n+2)(n+3)} (n^2 + 3n + 2) = \frac{3}{n+3}$$

$$\text{Given, } \frac{3}{n+3} = \frac{3}{a+3}$$

$$\Rightarrow n = a \Rightarrow a - n = 0$$

144. **(c)** $\begin{vmatrix} p & q-y & r-z \\ p-x & q & r-z \\ p-x & q-y & r \end{vmatrix} = 0$

Apply $R_1 \to R_1 - R_3$ and $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix} x & 0 & -z \\ 0 & y & -z \\ p-x & q-y & r \end{vmatrix} = 0$$

$\Rightarrow x[yr + z(q-y)] - z[0 - y(p-x)] = 0$

[Expansion along first row]

$\Rightarrow xyr + xzq - xzy + yzp - zyx = 0$

$\Rightarrow xyr + zxq + yzp = 2xyz \Rightarrow \frac{p}{x} + \frac{q}{y} + \frac{r}{z} = 2$

145. **(d)** Let $A_i (i = 2,3,4,5)$ be the event that urn contains 2,3,4,5 white balls and let B be the event that two white balls have been drawn then we have to find $P(A_5/B)$.

Since the four events A_2, A_3, A_4 and A_5 are equally likely we have $P(A_2) = P(A_3) = P$

$(A_4) = P(A_5) = \frac{1}{4}$.

$P(B/A_2)$ is probability of event that the urn contains 2 white balls and both have been drawn.

$\therefore P(B/A_2) = \frac{{}^2C_2}{{}^5C_2} = \frac{1}{10}$

Similarly, $P(B/A_3) = \frac{{}^3C_2}{{}^5C_2} = \frac{3}{10}$,

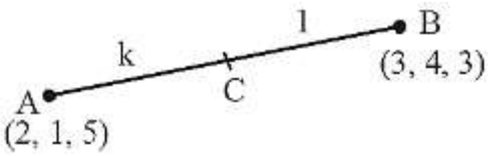
$$P(B/A_4) = \frac{{}^4C_2}{{}^5C_2} = \frac{3}{5}$$

$$P(B/A_5) = \frac{{}^5C_2}{{}^5C_2} = 1.$$

By Baye's theorem,

$$P(A_5/B) = \frac{P(A_5)P(B/A_5)}{P(A_2)P(B/A_2) + P(A_3)P(B/A_3)}$$
$$+P(A_4)(B/A_4) + P(A_5)P(B/A_5)$$
$$= \frac{\frac{1}{4}\cdot 1}{\frac{1}{4}\left[\frac{1}{10} + \frac{3}{10} + \frac{3}{5} + 1\right]} = \frac{10}{20} = \frac{1}{2}.$$

146. **(b)** As given plane $x + y - z = \frac{1}{2}$ divides the line joining the points A(2,1,5) and B(3,4,3) at a point C in the ratiok: 1.



Then coordinates of C

$$\left(\frac{3k+2}{k+1}, \frac{4k+1}{k+1}, \frac{3k+5}{k+1}\right)$$

Point C lies on the plane,
$\Rightarrow$ Coordinates of C must satisfy the equation of plane.
So, $\left(\frac{3k+2}{k+1}\right) + \left(\frac{4k+1}{k+1}\right) - \left(\frac{3k+5}{k+1}\right) = \frac{1}{2}$

$$\Rightarrow 3k + 2 + 4k + 1 - 3k - 5 = \frac{1}{2}(k+1)$$
$$\Rightarrow 4k - 2 = \frac{1}{2}(k+1)$$
$$\Rightarrow 8k - 4 = k + 1 \Rightarrow 7k = 5$$
$$\Rightarrow k = \frac{5}{7}$$

Ratio is 5: 7.

147. **(c)**

(c) Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$
Then,

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2 - x)}}{\sqrt{\sin(\pi/2 - x)} + \sqrt{\cos(\pi/2 - x)}} dx$$
$$\Rightarrow I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

Adding (i) and (ii), we get 2I

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$$

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx = \int_0^{\pi/2} 1.dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0$$

$$\Rightarrow I = \frac{\pi}{4} \Rightarrow \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx = \frac{\pi}{4}$$

148. **(a)** Let the required vector be

$\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$, then

$\vec{v} \cdot (\hat{i} + \hat{j} - 3\hat{k}) = 0 \Rightarrow x + y - 3z = 0$

$\vec{v} \cdot (\hat{i} + 3\hat{j} - 2\hat{k}) = 5 \Rightarrow x + 3y - 2z = 5$

and $\vec{v} \cdot (2\hat{i} + \hat{j} + 4\hat{k}) = 8 \Rightarrow 2x + y + 4z = 8$

Subtracting (ii) from (i), we have

$-2y - z = -5 \Rightarrow 2y + z = 5$

Multiply (ii) by 2 and subtracting (iii) from it, we obtain

$5y - 8z = 2$

Multiply (iv) by 8 and adding (v) to it, we have

$21y = 42 \Rightarrow y = 2$

Substituting $y = 2 \text{in(iv)}$, we get

$2 \times 2 + z = 5 \Rightarrow z = 5 - 4 = 1$

Substituting these values in (i), we get

$x + 2 - 3 = 0 \Rightarrow x = 3 - 2 = 1$

Hence, the required vector is

$\vec{v} = x\hat{i} + y\hat{j} + z\hat{k} = \hat{i} + 2\hat{j} + \hat{k}$

149. **(c)** Equation of lines are $\frac{x}{a} - \frac{y}{b} = 1$ and $\frac{x}{b} - \frac{y}{a} = 1$

$\Rightarrow m_1 = \frac{b}{a}$ and $m_2 = \frac{a}{b}$

Therefore

$$\theta = \tan^{-1} \frac{\frac{b}{a} - \frac{a}{b}}{1 + \frac{b}{a} \cdot \frac{a}{b}} = \tan^{-1} \frac{b^2 - a^2}{2ab}$$

150. **(d)** Given points are $A(k, 1, -1), B(2k, 0, 2)$
and $C(2 + 2k, k, 1)$
Let r_1 = length of line
$AB = \sqrt{(2k - k)^2 + (0 - 1)^2 + (2 + 1)^2}$
$= \sqrt{k^2 + 10}$
and r_2 = length of line BC

$= \sqrt{(2)^2 + k^2 + (-1)^2}$
$= \sqrt{k^2 + 5}$
Now, let ℓ_1, m_1, n_1 be direction-cosines of line AB and ℓ_2, m_2, n_2 be the direction cosines of BC.
Since AB is perpendicular to BC

$\therefore \ell_1\ell_2 + m_1m_2 + n_1n_2 = 0$

Now, $\ell_1 = \frac{k}{\sqrt{k^2+10}}, m_1 = \frac{-1}{\sqrt{k^2+10}},$

$n_1 = \dfrac{3}{\sqrt{k^2 + 10}}$

So, $\ell_1\ell_2 + m_1m_2 + n_1n_2 = 0$

$$\Rightarrow \frac{2k}{\sqrt{k^2+10}}\sqrt{k^2+5} - \frac{k}{\sqrt{k^2+10}\sqrt{k^2+5}} - \frac{3}{\sqrt{k^2+10}\sqrt{k^2+5}} = 0$$
$$\Rightarrow 2k - k - 3 = 0$$
$$\Rightarrow k = 3$$

For $k = 3$, AB is perpendicular to BC.