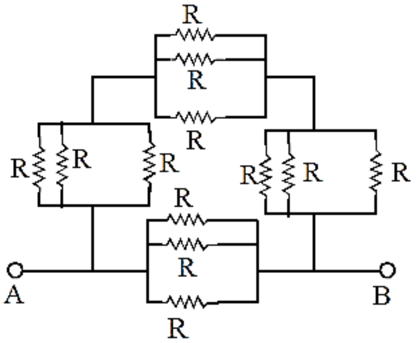
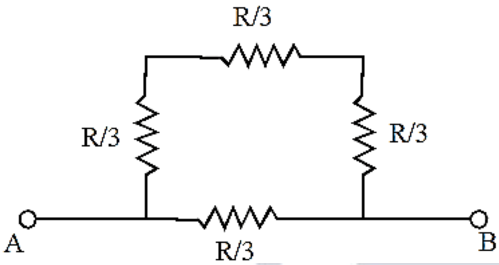
1. **(c)** The charged sphere is a conductor. Therefore, the field inside is zero and outside it is proportional to $1/r^2$.
2. **(c)** The direction of propagation of electromagnetic wave is perpendicular to the variation of electric field $\vec{E}$ as well as to the magnetic field $\vec{B}$.
3. **(b)** Young's modulus of wire does not vary with dimention of wire. It is a constant quantity.
4. **(d)** Redraw the given circuit,



$\Downarrow$



$$R_{net} \text{ between AB} = \frac{\frac{3R}{3} \times \frac{R}{3}}{\frac{3R}{3} + \frac{R}{3}} = \frac{R^2}{4R}$$

where, $R = 16\Omega$

$R_{net} = 4\Omega$

5. **(c)** According to Kepler's law of planetary motion, $T^2 \propto R^3$

$$\therefore T_2 = T_1 \left(\frac{R_2}{R_1}\right)^{3/2}$$

$$= 5 \times \left[\frac{4R}{R}\right]^{3/2} = 40 \text{ hours}$$

6. **(d)** $f' = f\left(\frac{v+v_0}{v-v_s}\right)$

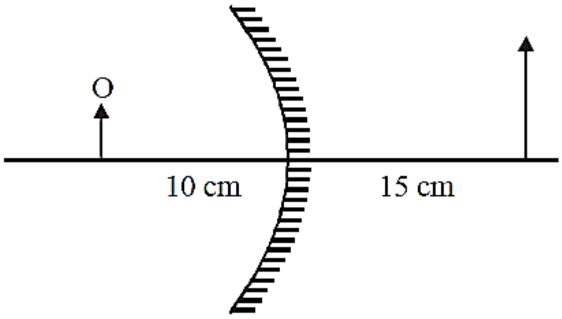
Here, $f = 600 \text{ Hz}, v_o = 15 \text{ m/s}$

$v_s = 20 \text{ m/s}, v = 340 \text{ m/s}$

$$f' = 600 \times \left[\frac{340 + 15}{340 - 20}\right]$$

$$\therefore f' = 600\left(\frac{355}{320}\right) \approx 666 \text{ Hz}$$

7. **(c)** Concave morror is used as a shaving mirror.



From question: $v = 15$ cm, $u = -10$ cm Radius of curvature, $R = 2f = ?$
Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

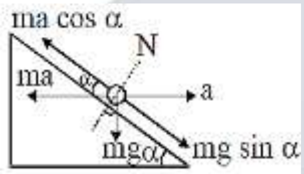
$$\frac{1}{15} + \frac{1}{(-10)} = \frac{1}{f} \Rightarrow f = -30 \text{ cm}$$

Therefore, radius of curvature,

$R = 2f = -60$ cm

8. **(c)** From free body diagram, For block to remain stationary,

$mg\sin\alpha = ma\cos\alpha$



$\Rightarrow a = g\tan\alpha$

9. **(a)** On increasing the temperature, angle of contact decreases.

10. **(b)** Forward bias opposes the potential barrier and if the applied voltage is more than knee voltage it cancels the potential barrier.

11. **(c)** In multiplication or division the final result should return as many significant figures as there are in the original number with the least significant figures. (Rounding off to three significant digits)

12. **(b)** Let ' n ' be the degree of freedom

$$C_v = \frac{n}{2}R$$
also, $C_p - C_v = R$
$$C_p = C_v + R$$
$$C_p = \frac{n}{2}R + R$$
$$C_p = \left(\frac{n}{2} + 1\right)R$$
so,
$$\gamma = \frac{C_p}{C_v} = \frac{\left(\frac{n}{2} + 1\right)R}{\left(\frac{n}{2}\right)R} = \left(1 + \frac{2}{n}\right)$$

13. **(c)** Since range on horizontal plane is

$$R = \frac{u^2 \sin 2\theta}{g}$$

so it is maximum when, $\sin 2\theta = 1$

$\theta = \frac{\pi}{4}$

14. (c)

$$\text{Mass} = 150\text{gm} = \frac{150}{1000}\text{ kg}$$
$$\text{Force} = \text{Mass} \times \text{acceleration}$$
$$= \frac{150}{1000} \times 20\text{ N} = 3\text{ N}$$
$$\text{Impulsive force} = F \cdot \Delta t = 3 \times 0.1 = 0.3\text{ N}$$

15. (b) Given, d = 10 m

$$\theta = 30^\circ$$
$$\mu = 0.5$$
$$\mathrm{F} = \sqrt{3}\mathrm{kN} = \sqrt{3} \times 10^3\text{ N}$$
$$\mathrm{W} = \mathrm{F_s} \mathrm{d} \cos\theta$$

Where,

$$\mathrm{F_s} = \mu\mathrm{F}$$
$$\mathrm{F_s} = 0.5 \times \sqrt{3}\mathrm{kN}$$
$$\mathrm{F_s} = 0.866\mathrm{kN}$$
$$\mathrm{F_s} = 866\text{ N}$$

So, $\mathrm{W} = 866 \times 10 \times \cos 30^\circ$

$$\mathrm{W} = \frac{866 \times 10 \times \sqrt{3}}{2}$$
$$\mathrm{W} = 7499.56\text{ J}$$
$$\mathrm{W} \simeq 7.5\text{ kJ}$$

16. (a)

$$\mathrm{W} = \vec{\mathrm{F}} \cdot \vec{\mathrm{s}} = (2\hat{\imath} + 3\hat{\jmath} + 4\hat{\mathrm{k}}) \cdot (3\hat{\imath} + 4\hat{\jmath} + 5\hat{\mathrm{k}})$$
$$= 2 \times 3 + 3 \times 4 + 4 \times 5 = 38\text{ J}$$
$$\mathrm{P} = \frac{\mathrm{W}}{\mathrm{t}} = \frac{38}{4} = 9.5\text{ W.}$$

17. (d)

$$v_e = \sqrt{\frac{2GM}{R}}$$

and, $v_e' = \sqrt{\frac{2GM}{(R+h)}} = \sqrt{\frac{2GM}{(R+R)}} = \frac{v_e}{\sqrt{2}}$

$$\therefore\ f = \frac{1}{\sqrt{2}}$$

18. (b) $\frac{dA}{dt} = \frac{L}{2\,m} = \text{Constant}$

19. (a) According to Bernoulli's theorem

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

According to the condition,

$$P_1 - P_2 = 3 \times 10^5, \frac{A_1}{A_2} = 5$$

From equation of continuity,

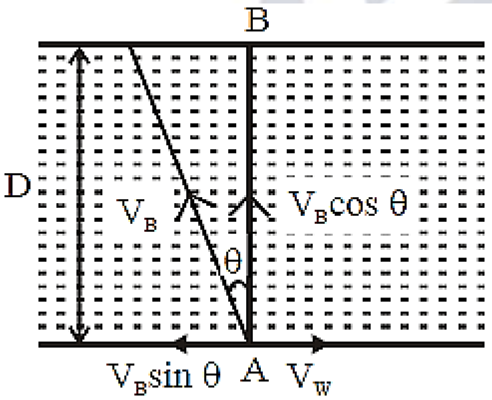
$$A_1 v_1 = A_2 v_2$$
$$\text{so, } \frac{A_1}{A_2} = \frac{v_2}{v_1} = 5 \Rightarrow v_2 = 5v_1$$

From equation (i)

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$
$$\text{or } 3 \times 10^5 = \frac{1}{2} \times 1000(25v_1^2 - v_1^2)$$
$$\Rightarrow 600 = 24v_1 \Rightarrow v_1^2 = 25$$
$$\therefore v_1 = 5\text{ m/s}$$

20. (c) Work done = Area under curve $ACBDA$

21. (a)



From figure, $V_B \sin\theta = V_W$

$$\sin\theta = \frac{V_W}{V_B} = \frac{1}{2} \Rightarrow \theta = 30^\circ \; [\because V_B = 2\,V_W]$$

Time taken to cross the river.

$$t = \frac{D}{V_B\cos\theta} = \frac{D}{V_B\cos 30^\circ} = \frac{2D}{V_B\sqrt{3}}.$$

22. (a) The two springs are in parallel.

$\therefore$ Effective spring constant,

$k = k_1 + k_2$

Now, frequency of oscillation is given by

$$f = \frac{1}{2p}\sqrt{\frac{k}{m}}$$

or, $f = \frac{1}{2p}\sqrt{\frac{k_1+k_2}{m}}$

When both k_1 and k_2 are made four times their original values, the new frequency is given by

$$f' = \frac{1}{2p}\sqrt{\frac{4k_1 + 4k_2}{m}}$$

$$f' = \frac{1}{2\pi}\sqrt{\frac{4(k_1+k_2)}{m}} = 2\left(\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}\right) = 2f$$

23. (b) Drift velocity,

$$v_d = \frac{i}{neA} = \frac{J}{ne} = \frac{\sigma E}{ne} = \frac{E}{\rho ne}\frac{V}{\rho \ell \text{ ne}}$$

so $v_d \propto V$

24. (b) Amplitude of a damped oscillator

$A = A_0 e^{-bt/2m}$

Case 1 :-

When $t = 2$ s, $A = \frac{A_0}{3}$

$$\therefore \frac{A_0}{3} = A_0 e^{-2b/2m} \Rightarrow \frac{1}{3} = e^{-b/m} \ldots$$

Case 2 :-

When $t = 6$ s, $A = \frac{A_0}{n}$

$$\therefore \frac{A_0}{n} = A_0 e^{-6b/2m} \Rightarrow \frac{1}{n} = \left(e^{-b/m}\right)^3 \ldots$$

From (i) and (ii)

$$\frac{1}{n} = \left(\frac{1}{3}\right)^3 \Rightarrow \therefore n = 3^3$$

25. (d) Angular speed of electron in the nth orbit of Bohr H-atom is inversely proportional to n^3

$$\omega_n \propto \frac{1}{n^3}$$

26. (a) C = equivalent capacitance

$$\therefore \frac{1}{C} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} \Rightarrow C = 1\mu F$$

Charge on each capacitor in series circuit will be same.

$\therefore$ q $= CV = (1 \times 10^{-6}) \times 10 = 10\mu C$

$\therefore$ Charge across $3\mu F$ capacitor will be $10\mu C$.

27. (a)
28. (c)
29. (a) For open pipe, $n_1 = \frac{v}{2\ell}$, where n_1 is the fundamental frequency of open pipe. length of open pipe is,

$$\therefore \ell = \frac{v}{2n} = \frac{330}{2 \times 300} = \frac{11}{20}$$

Ist overtone of open pipe, $n_2 = 2n_1 = 2\left(\frac{v}{2l}\right)$
Ist overtone of closed pipe,

$$n_3 = 3n_1 = 3\left(\frac{v}{4l'}\right)$$

where, ℓ' = length of closed pipe As freq. of 1 st overtone of open pipe = freq. of 1st overtone of closed pipe

$$\therefore\ 2\frac{v}{2\ell} = 3\frac{v}{4\ell'} \Rightarrow \ell' = \frac{3\ell}{4} = \frac{3}{4} \times \frac{11}{20}$$
$$= 41.25 \text{ cm}$$

30. (d) $\beta = \frac{D\lambda}{d}$ and $\beta' = \frac{(2D)\lambda}{(d/2)} = 4\beta$
Thus the fringe width becomes four times.
31. (a) As we know, τ is change in angular momentum.
so, $\tau = \frac{20}{3}$ SI units
32. (a) Let q and q$'$ be the charges on spheres of radii x and $2x$ respectively.
Given, $q + q' = Q$
Surface charge densities are

$$\sigma = \frac{q}{4\pi x^2} \text{ and } \sigma = \frac{q'}{4\pi(2x)^2}$$

Given, $\sigma = \sigma'$

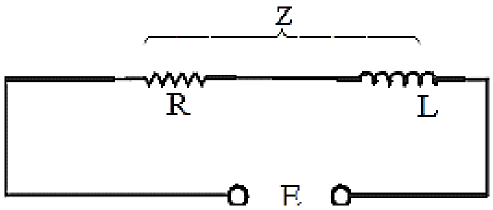
$$\therefore \frac{q}{4\pi x^2} = \frac{q'}{4\pi(2x)^2}$$
$$q' = 4q$$

From eq. (i), $q' = Q - q$ or, $4q = Q - q$

or, $Q = 5q$

$$\therefore q' = Q - q = Q - \frac{Q}{5} = \frac{4Q}{5}$$

33. (d) $L = 2H, E = 5$ volts, $R = 1\Omega$



$$I=\frac{E}{Z}$$
$$Z=\sqrt{R^2+(\omega L)^2}$$
$$I=\frac{5}{\sqrt{R^2+(\omega L)^2}}=\frac{5}{\sqrt{1+4\pi^2\times 50^2\times 4}}$$
$$=\frac{5}{\sqrt{1+(200\pi)^2}}\simeq\frac{5}{200\pi}$$
$$\text{Energy in inductor }=\frac{1}{2}LI^2$$
$$=\frac{1}{2}\times 2\times\left(\frac{5\times 5}{200\times 200\pi^2}\right)$$
$$=6.33\times 10^{-5}\text{ joules}$$

34. (b) $F=Bi\ell=2\times 1.2\times 0.5=1.2\text{ N}$

35. (b) Distance between two cars leaving from the station A is,

$$d=\frac{1}{6}\times 60=10\text{ km}$$

Man meets the first car after time,

$$t_1=\frac{60}{60+60}=\frac{1}{2}\text{ h}$$

He will meet the next car after time, $t_2=\frac{10}{60+60}=\frac{1}{12}$ h

In the remaining half an hour, the number of cars he will meet again is, $n=\frac{1/2}{1/12}=6$

∴ Total number of cars would be meet on route will be 7 .

36. (b) From $v=r\omega$, linear velocities (v) for particles at different distances (r) from the axis of rotation are different.

37. (a) For an SHM, the acceleration $a=-\omega^2 x$ where, ω is a constant $=\frac{2\pi}{T}$ $a=-\frac{4\pi^2}{T^2}\cdot x\Rightarrow\frac{aT}{x}\Rightarrow-\frac{4\pi^2}{T}$

The period of oscilation T is a constant.

∴ $\frac{aT}{x}$ is a constant.

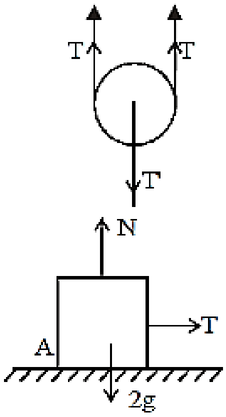
38. (a) As the inward magnetic field increases, its flux also increases into the page and so induced current in bigger loop will be anticlockwise. i.e., from D to C in bigger loop and then from B to A in smaller loop.

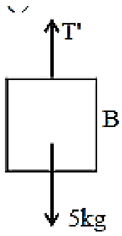
39. (a) Since A moves twice the distance moved by B.

If acceleration of B is ' a ', then acceleration of A is ' $2a$ '.

$T'-(T+T)=0$ (since pulley is massless)

$\Rightarrow T'=2T$





For 5 kg block

$5\text{ g} - \text{T}' = 5\text{a}$

for 2 kg block

$$\Rightarrow 5\text{ g} - 2\text{ T} = 5\text{a}$$
$$\text{T} = 2 \times (2\text{a}) = 4\text{a}$$

From equations (ii) and (iii),

$$5g - (2 \times 4a) = 5a$$
$$5g - 8a = 5a$$
$$5g = 13a$$
$$a = \frac{5g}{13}$$
$$a_A = 2a = \frac{10g}{13};\ a_B = a = \frac{5g}{13}$$

40. (a) Isotones means equal number of neutrons i.e., $(A - Z) = 74 - 34 = 71 - 31 = 40$

41. (a) Asper Arrhenius equation $\left(k = Ae^{-E_a/RT}\right)$, the rate constant increases exponentially with temperature.

42. (c) Mass of O_2 absorbed per gram of adsorbent

$$= \frac{3.6}{1.2} = 3$$

No. of moles of O_2 absorbed per gram of adsorbent $= \frac{3}{32}$
Volume of O_2 absorbed per gram of adsorbent

$$\text{PV} = \text{nRT}$$
$$\text{V} = \frac{\text{nRT}}{\text{P}}$$
$$= \frac{3}{32} \times \frac{0.0821 \times 273}{1} = 2.1$$

43. (b)

$$\underset{\text{(Impure)}}{Ni} + 4CO \xrightarrow{50^\circ \text{ to } 60\,^\circ C} \underset{\text{Volatile compound}}{[Ni(CO)_4]\uparrow}$$

$$\xrightarrow[\text{Thermal decomposition}]{200 \text{ to } 230\,^\circ C} \underset{\text{pure}}{Ni} + 4CO\uparrow$$

44. (a) $3Cl_2 + 2NaI \rightarrow 2NaCl + I_2$ I_2
gives violet colouration in $CHCl_3$.

$5Cl_2 + 6H_2O + I_2 \rightarrow$ HIO_3 $+ 10HCl$

Colourless

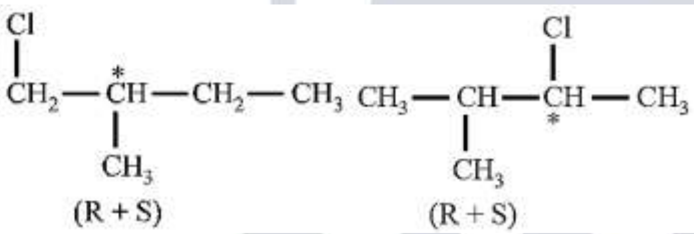
45. (d) Clathrate formation involves dipoleinduced dipole interaction.

46. (d)

$Eu(63) \quad = [Xe]4f^7 6s^2$
$Gd(64) \quad = [Xe]4f^7 5d^1 6s^2$
$Tb(65) \quad = [Xe]4f^9 6s^2$

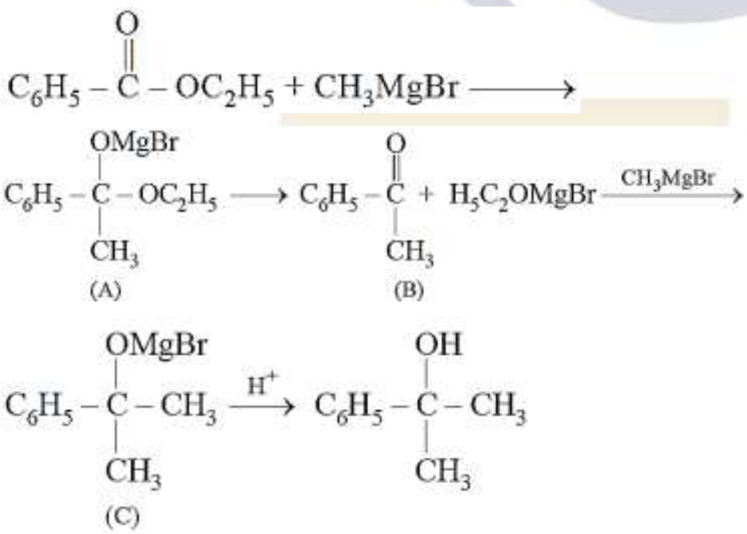
47. (a) As positive charge on the central metal atom increases, the less readily the metal can donate electron density into the π^* orbitals of CO ligand (donation of electron density into π^* orbitals of CO result in weakening of $C - O$ bond). Hence, the $C - O$ bond would be strongest in $[Mn(CO)_6]^+$.

48. (c)



Four monochloro derivatives are chiral.

49. (b)



50. (b)

51. (b) In compound (b), the lone pair of nitrogen is not involved in resonance therefore it is the strongest base.

52. (c) Sucrose, being a non-reducing sugar, does not reduce Benedict's solution. Remember that fructose has an α-hydroxy ketonic group, which is also reducing group (different from ordinary ketonic group)

53. (c) Zn^{+2} present in alternate tetrahedral void

$$= \frac{1}{2} \times 8 = 4$$

S^{2-} present in ccp $= 4$

$\therefore\ Zn_4\ S_4 = ZnS$ i.e., AB type compound.

54. (d) Glycine in alkaline solution exists as <u>anion</u> and migrates to <u>anode.</u>

$$\begin{array}{l} \quad\ \ NH_2 \longleftarrow \text{basic group} \\ \quad\ \ \ | \\ H - CH - COOH \longleftarrow \text{acidic group} \end{array}$$

Due to internal proton tranfer of $H^+$from the $-COOH$ group to the $-NH_2$, the amino acid exists as an ion with both a negative charge and a positive charge, called a Zwitter ion

$$\begin{array}{l} \quad\ \ NH_3^+ \\ \quad\ \ \ | \\ H-CH-COO^- \\ \quad \text{zwitter ion} \end{array}$$

Adding an alkali to glycine

$$\begin{array}{l} \quad NH_3^+ \\ \quad\ | \\ H-CH-COO^- + OH^- \end{array} \longrightarrow \begin{array}{l} \quad NH_2 \\ \quad\ | \\ H-CH-COO^- + H_2O \\ \quad\ \ \text{(anion)} \end{array}$$

Now, during electrophoresis, glycine moves towards anode.

55. (b)

$$\underset{\text{Ethanamide}}{CH_3 - \overset{\overset{\displaystyle O}{||}}{C} - NH_2} \xrightarrow{P_2O_5} \underset{\text{Acetonitrile}}{CH_3 - C \equiv N}$$

56. (a)

HX	+	NaOH	$\longrightarrow$	NaX	+	H_2O
0.25		0.05		—		—
0.25 − 0.05 = 0.20		—		0.05		0.05

$$K_a = \frac{[H^+][X^-]}{[HX]}$$

$$10^{-5} = \frac{[H^+]0.05}{0.20}$$

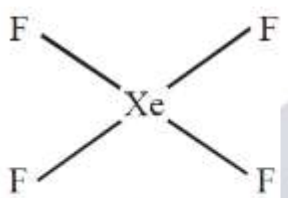
$$\frac{0.20}{0.05} \times 10^{-5} = [H^+]$$

$$4 \times 10^{-5} M = [H^+]$$

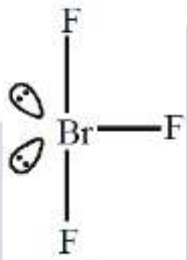
57. (c)

$$Pt|H_2(640\text{ mm})|HCl|H_2(510\text{ mm})|Pt$$
$$E^\circ_{cell} = 0$$
$$(p_1)H_2\ g) \longrightarrow 2H^+ + 2e^-$$
$$2H^+ + 2e^- \longrightarrow H_2(\ g)(p_2)$$
$$H_2(\ g) \longrightarrow H_2(\ g)$$
$$(p_1)\ (p_2)$$
$$E_{cell} = E^\circ_{cell} - \frac{0.0591}{n}\log\frac{p_2}{p_1}$$
$$= 0 - \frac{0.0591}{2}\log\frac{510}{640}$$
$$= 2.91 \times 10^{-3}\text{ V}$$

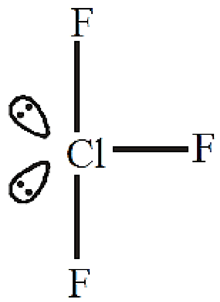
58. (a)



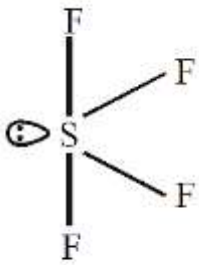
$\mu = 0$



$\mu \neq 0$



$\mu \neq 0$



$\mu \neq 0$

XeF_4 has zero net dipole moment

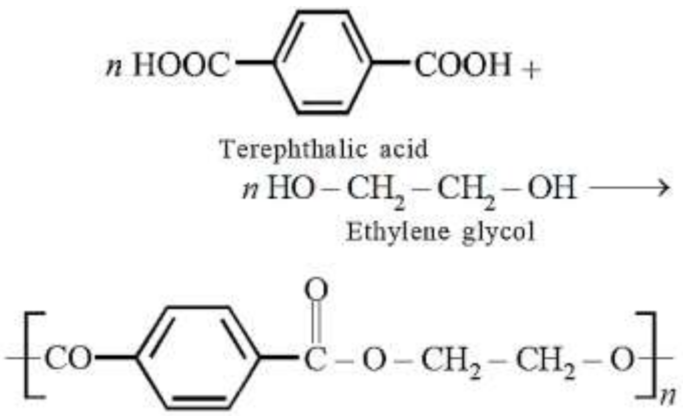
59. (a) Boron has the highest ionisation enthalpy amongst the following.
Ionisation enthalpy decreases down the group and increases across the period.

60. (a) Element with atomic number 7 has the smallest size and highest ionization enthalpy Nitrogen-Atomic Number 7

$N - 2s^2 2p^3$ $\boxed{\uparrow\downarrow}$ $\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$

N has a stable half-filled electronic configuration therefore it is difficult to remove electron and hence it has a high ionization enthalpy.

61. (a) Except dacron all are additive polymers. Terephthalic acid condenses with ethylene glycol to give dacron.



Dacron (polyester)

62. **(a)** Phenelzine is an antidepressant, while others are antacids.

63. **(b)** 1 molal solution means 1 mole of solute dissolved in 1000 g solvent.

$\therefore \quad n_{\text{solute}} = 1 \; w_{\text{solvent}} = 1000 \text{ g}$

$\therefore \quad n_{\text{solvent}} = \dfrac{1000}{18} = 55.56$

$\quad x_{\text{solute}} = \dfrac{1}{1+55.56} = 0.0177$

64. **(a)**

65. **(a)** Stability of carbocation $\propto$ no. of $\alpha - H$ present on carbocation.

66. **(a)** Only Lindlar's catalyst converts alkyne to alkene (cis addition) and alkenes with Baeyer's reagent give cis glycols.

67. **(b)** Lower oxidation state become more stable on moving down the group

$\mathrm{Al} < \mathrm{Ga} < \mathrm{ln} < \mathrm{Tl}$

68. **(a)** $\mathrm{Be(OH)_2}$ is amphoteric while $\mathrm{Ca(OH)_2}$, $\mathrm{Sr(OH)_2}$ and $\mathrm{Ba(OH)_2}$ are all basic.

69. **(a)** $\mathrm{H_2O_2} + 2\mathrm{KI} \longrightarrow \mathrm{I_2}$, O.S. of $\mathrm{I^-}(-1)$ changes to $\mathrm{I_2}$ (Zero) There is increase in oxidation number, hence oxidation.

70. **(d)** The balanced equation is

$$\mathrm{K_2Cr_2O_7 + 4KCl + 6H_2SO_4 \longrightarrow 2CrO_2Cl_2 + 6KHSO_4 + 3H_2O}$$

71. **(a)** $\frac{[\mathrm{B}]}{[\mathrm{A}]} = 2, \frac{[\mathrm{C}]}{[\mathrm{B}]} = 4$ and $\left[\frac{\mathrm{D}}{\mathrm{C}}\right] = 6$

Multiply the three equations,

$$2 \times 4 \times 6 = \frac{[\mathrm{D}]}{[\mathrm{A}]} = \mathrm{K_c}$$

72. **(d)** $\Delta\mathrm{G} = \Delta\mathrm{H} - \mathrm{T}\Delta\mathrm{S} = +\mathrm{ve}$

for spontaneous change, $\Delta\mathrm{H} < 0, \Delta\mathrm{S} > 0$

for non-spontaneous change, $\Delta\mathrm{H} > 0, \Delta\mathrm{S} < 0$

73. **(b)** $\frac{\mathrm{r_1}}{\mathrm{r_2}} = \sqrt{\frac{\mathrm{d_2}}{\mathrm{d_1}}} = \sqrt{\frac{16}{1}} = 4:1$

74. **(a)**

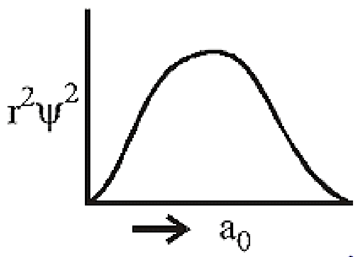
$$2\mathrm{PCl_5} \rightleftharpoons \mathrm{PCl_4^+} + \mathrm{PCl_6^-}$$

$sp^3d \; sp^3 \; sp^3d^2$

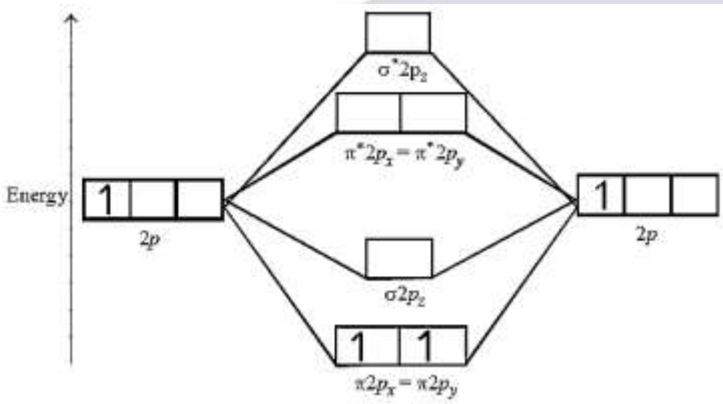
75. **(a)**
76. **(c)** This is Avogadro's hypothesis. According to this, equal volume of all gases contain equal no. of molecules under similar condition of temperature and pressure.
77. **(a)** Eq. of A = Eq. of B

$$\frac{m_A}{E_A} = \frac{m_B}{E_B}; E_A = \frac{m_A}{m_B} \times E_B$$

78. **(c)** For $4p$ electron $n = 4, l = 1, m = -1, 0, +1$ and $s = +1/2$ or $-1/2$
79. **(c)** $l = 2$ represent d orbital for which



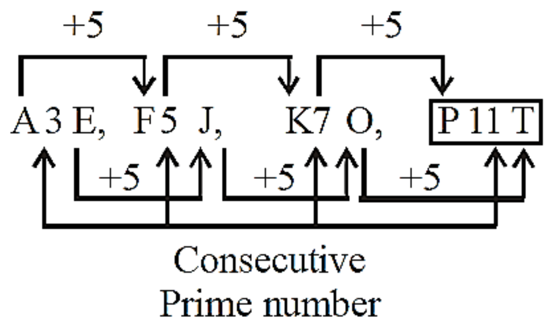
80. **(a)** B_2 is paramagnetic due to the presence of unpaired electron in $\pi 2p_x = \pi 2p_y$ orbital.



M.O diagram for B_2 molecule

81. **(a)** The word Garrulous (Adjective) means talkative; talking a lot.
82. **(b)** The word Tinsel (Noun/Adjective) means: strips of shiny material like metal used as decorations.
83. **(a)** The word Labyrinth (Noun) means: a place that has many confusing paths or passage. The correct synonym will be 'meandering' which means, 'to have a lot of curves on a path'.
84. **(b)** Knack means a clever way of doing something.
85. **(d)** Pernicious means highly injurious or destructive.
86. **(d)** Opulence means wealthy.
87. **(a)** In good friendships, we receive as much as we give.
88. **(b)** Empathy means the ability to show and understand the feelings of others.
89. **(c)** A strong friendship helps us gain acceptance and tolerance.
90. **(d)** The very first line of the passage states that friendships and relationships grow when they are nurtured just like nurturing a plant.
91. **(a)** Dependent on = needing somebody / something in order to survive or be successful; affected or decided by something.
92. **(b)** Take your leave = to say good bye.
93. **(b)** Here, indefinite article i.e., 'about a plane crash' should be used. No particular incident is evident here.
94. **(b)** 'With a View to' should be followed by gerund i.e., surveying.
95. **(a)**

96. (b)
97. (c)
98. (d)
99. (c) Medicine is given to patient. Similarly, Education is given to student.
100. (d)



101. (a) Except 5 , all numbers are perfect square numbers.
102. (a) As,

TIRED $= 20 + 9 + 18 + 5 + 4 = 56$
BRAIN $= 2 + 18 + 1 + 9 + 14 = 44$
Similarly,
LAZY $= 12 + 1 + 26 + 25 = 64$

103. (b)

$8 \times 8 \times 88 = 5632$
$7 \times 7 \times 77 = 3773$

Similarly, $6 \times 6 \times ? = 3132$

$$\therefore ? = \frac{3132}{6 \times 6} = 87$$

104. (b)
105. (d) Mohan is son of Ram Lal and uncle of Ram and Rekha. Mithun is uncle of Sharat who is son of Rekha. Rekha is niece of Mohan. Therefore, Mithun is brother of Rekha's husband.
106. (c) Given line is $x\cos\alpha + y\sin\alpha = P$
Any tangent to the ellipse is

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$$

Comparing (1) and (2)

$$\frac{\cos\theta}{a\cos\alpha} = \frac{\sin\theta}{b\sin\alpha} = \frac{1}{P}$$

$$\Rightarrow \cos\theta = \frac{a\cos\alpha}{P} \text{ and } \sin\theta = \frac{b\sin\alpha}{P}$$

Eliminate $\theta, \cos^2\theta + \sin^2\theta$

$$= \frac{a^2\cos^2\alpha}{P^2} + \frac{b^2\sin^2\alpha}{P^2},$$

or $a^2\cos^2\alpha + b^2\sin^2\alpha = P^2$

107. (c) As $a_1, a_2, a_3, \ldots \ldots, a_n$, are in A.P. we get,

$a_2 - a_1 = a_3 - a_2 =$

$. = a_n - a_{n-1} = d$ (say)

Now, $\frac{1}{\sqrt{a_1}+\sqrt{a_2}} = \frac{\sqrt{a_1}-\sqrt{a_2}}{a_1-a_2} = \frac{\sqrt{a_1}-\sqrt{a_2}}{-d}$

Similarly,

$$\frac{1}{\sqrt{a_2}+\sqrt{a_3}} = \frac{\sqrt{a_2}-\sqrt{a_3}}{-d}, \ldots, \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} = \frac{\sqrt{a_{n-1}}-\sqrt{a_n}}{-d}$$

$$\therefore \frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \cdots . + \frac{1}{\sqrt{a_n}+\sqrt{a_{n-1}}}$$
$$= \frac{\sqrt{a_1}-\sqrt{a_2}+\sqrt{a_2}-\sqrt{a_3}+\cdots \ldots \ldots + \sqrt{a_{n-1}}-\sqrt{a_n}}{-d}$$
$$= \frac{\sqrt{a_1}-\sqrt{a_n}}{-d} = -\frac{1}{d}\left[\frac{a_1-a_n}{\sqrt{a_1}+\sqrt{a_n}}\right]$$
$$= -\frac{1}{d}\left[\frac{a_1-\{a_1+(n-1)d\}}{\sqrt{a_1}+\sqrt{a_n}}\right]$$

[formula for n^{th} term]

$$= -\frac{1}{d}\left[\frac{-(n-1)d}{\sqrt{a_1}+\sqrt{a_n}}\right] = \frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

108. **(b)** Given differential equation is :

$x\cos x dy/dx + y(x\sin x + \cos x) = 1$

Dividing both the sides by $x\cos x$,

$$\Rightarrow \frac{dy}{dx} + \frac{xy\sin x}{x\cos x} + \frac{y\cos x}{x\cos x} = \frac{1}{x\cos x}$$
$$\Rightarrow \frac{dy}{dx} + y\tan x + \frac{y}{x} = \frac{1}{x\cos x}$$
$$\Rightarrow \frac{dy}{dx} + \left(\tan x + \frac{1}{x}\right)y = \frac{\sec x}{x}$$

which is of the form $\frac{dy}{dx} + Py = Q$

Here, $P = \tan x + \frac{1}{x}$ and $Q = \frac{\sec x}{x}$

Integrating factor $= e^{\int P dx} = e^{\int \tan x + \frac{1}{x} dx}$

$= e^{(\log\sec x + \log x)} = e^{\log(\sec x \cdot x)} = x\sec x$

109. **(c)** Equation of the first line L_1 is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and that of the second line L_2 is $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1}$ Clearly, these lines are not parallel (the ratios of D.R. are not equal). Anypoint P on the first line is $(1+2\lambda, 2+3\lambda, 3+4\lambda)$ and any point Q on the second line is $(4+5\mu, 1+2\mu, \mu)$. If these two points P and Q are identical then.

$$1 + 2\lambda = 4 + 5\mu$$
$$2 + 3\lambda = 1 + 2\mu$$
$$3 + 4\lambda = \mu$$

From (2) and (3), we get $\lambda = \mu = -1$, which also satisfies (1). Thus the two lines L_1 and L_2; entersect and the coordinates of the point of intersection are $(-1, -1, -1)$.

110. **(c)** We have $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$

$$\Rightarrow ydx - xdy = ay^2dx + ady$$
$$\Rightarrow y(1-ay)dx = (x+a)dy$$
$$\Rightarrow \frac{dx}{x+a} - \frac{dy}{y(1-ay)} = 0$$

Integrating, we get

$$\log(x+a) - \log y + \log(1-ay) = \log C$$

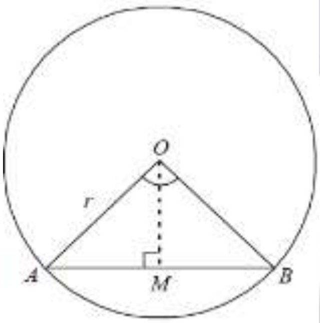
or $\log\frac{(a+x)(1-ay)}{y} = \log C$ i.e. $(x+a)(1-ay) = Cy$

Since the curve passes through $\left(a, -\frac{1}{a}\right)$

$\therefore 2a \times (1+1) = -\frac{C}{a}$ i.e $C = -4a^2$

So, $(x+a)(1-ay) = -4a^2y$

111. **(c)**



Equation of given circle is $x^2 + y^2 = 4$
Its centre, $O = (0,0)$ and radius, $r = 2$
Draw $OM \perp AB$
Clearly M is the mid-point of AB which subtends a right angle at O.
In $\triangle$ AOB, OA = OB radius

$$\therefore \angle A = \angle B = \frac{\pi}{4}$$

and in $\triangle OMA, \sin A = \frac{OM}{OA}$

$$\sin\frac{\pi}{4} = \frac{OM}{2}$$
$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{OM}{2}$$
$$\Rightarrow OM = \sqrt{2}$$

Let $M = (x, y)$ then $OM = \sqrt{x^2 + y^2}$
From (1) and (2), $x^2 + y^2 = 2$
This is the required equation of locus.

112. **(c)**

$$I=\int_1^2[x^2]dx-\int_1^2[x]^2dx$$
$$=\int_1^{\sqrt2}dx+\int_{\sqrt2}^{\sqrt3}2dx+\int_{\sqrt3}^{2}3dx-\int_1^2 1dx$$
$$=4-\sqrt2-\sqrt3$$

113. **(c)**

$$\frac{1+\sin A-\cos A}{1+\sin A+\cos A}$$
$$=\frac{2\sin^2\frac{A}{2}+2\sin\frac{A}{2}\cos\frac{A}{2}}{2\cos^2\frac{A}{2}+2\sin\frac{A}{2}\cos\frac{A}{2}}$$
$$=\frac{2\sin\frac{A}{2}\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)}{2\cos\frac{A}{2}\left(\cos\frac{A}{2}+\sin\frac{A}{2}\right)}=\tan\frac{A}{2}$$

114. **(c)** Given $x\sqrt{1+y}+y\sqrt{1+x}=0$

$\Rightarrow x\sqrt{1+y}=-y\sqrt{1+x}$

Squaring both sides, we get

$$x^2(1+y)=y^2(1+x)$$
$$\Rightarrow x^2-y^2+x^2y-xy^2=0\Rightarrow(x-y)(x+y+xy)=0$$
$$\Rightarrow y=x\text{ or }y(1+x)=-x\Rightarrow y=x\text{ or }y=-\frac{x}{1+x}$$
$$\Rightarrow\frac{dy}{dx}=\frac{-(1+x)\cdot1+x\cdot1}{(1+x)^2}=\frac{-1}{(1+x)^2}$$

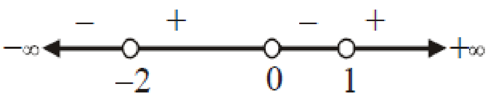
115. **(b)** Given: $f(x)=3x^4+4x^3-12x^2+12$

Differentiating with respect to x, we get

$f'(x)=12x^3+12x^2-24x$

For f(x) to be increasing

$$f'(x)>0\Rightarrow12x^3+12x^2-24x>0$$
$$\Rightarrow12x(x^2+x-2)>0$$
$$\Rightarrow12x(x-1)(x+2)>0$$
$$\Rightarrow x(x-1)(x+2)>0$$
$$\Rightarrow-2<x<0\text{ or }x>1$$



It means $x\in(-2,0)\cup(1,\infty)$.
Hence f(x) is increasing in $(-2,0)$ and $(1,\infty)$

116. **(c)** Consider $\frac{x}{2}+\frac{y}{4}\geq1,\frac{x}{3}+\frac{y}{2}\leq1,$

$x,y\geq0$ convert them into equation and solve them and draw the graph of these equations we get

$y=1$ and $x=3/2$

117. **(b)** Here, $a = 2\hat{i} + 5\hat{j} - 3\hat{k},\ N = 6\hat{i} - 3\hat{j} + 2\hat{k}$ and $d = 4$.

Therefore, the distance of the point $(2,5,-3)$ from the given plane is

$$\frac{|(2\hat{\imath} + 5\hat{\jmath} - 3\hat{k}) \cdot (6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) - 4|}{|6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}|}$$
$$= \frac{|12 - 15 - 6 - 4|}{\sqrt{36 + 9 + 4}} = \frac{13}{7}\left(\because \text{ distance } = \left|\frac{a \cdot N - d}{N}\right|\right)$$

118. **(a)**

119. **(a)**

120. **(c)** Since, $(e^x + 1)ydy = (y + 1)e^x dx$

$$\Rightarrow \frac{dx}{dy} = \frac{y}{1+y} + \frac{y}{(1+y)e^x}$$
$$\Rightarrow \frac{dx}{dy} = \left(\frac{y}{1+y}\right)\left(\frac{e^x + 1}{e^x}\right)$$
$$\Rightarrow \left(\frac{y}{1+y}\right) dy = \left(\frac{e^x + 1}{e^x}\right) dx$$

After integrating on both sides, we have

$$\int \frac{y}{1+y} dy = \int \frac{e^x}{1+e^x} dx$$
$$\Rightarrow \int 1dy - \int \frac{1}{1+y} dy = \int \frac{e^x}{1+e^x} dx$$
$$\Rightarrow y - \log|(1+y)| = \log|(1+e^x) + \log k$$
Hence $y = \log[k(1+y)(1+e^x)]$

121. **(c)** Equation of parabola is

$$y^2 = 4ax$$
$$\Rightarrow 2y\frac{dy}{dx} = 4a \text{ (On differentiating w.r.t}' x') $$

$$\therefore \frac{dy}{dx} = \frac{2a}{y}, \quad \text{[slope of tangent]}$$

So, slope of normal $= -\left(\frac{dx}{dy}\right)_{(at^2, 2at)}$

$$= -\left(\frac{y}{2a}\right) = -\frac{2at}{2a} = -t$$

122. **(a)** Let $I = \int_0^{\frac{\pi}{2}} x\sin^2 x\cos^2 x dx$

From the definite integral property

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

we have

$$I=\int_0^{\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)\sin^2 x\cos^2 x dx$$

$$\because \cos^2 x=\sin^2\left(\frac{\pi}{2}-x\right)\&\sin^2 x=\cos^2\left(\frac{\pi}{2}-x\right)\Big)$$

By adding (i) and (ii)

$$2I=\frac{\pi}{2}\int_0^{\frac{\pi}{2}}\sin^2 x\cos^2 x dx$$

$$\text{or } 2I=\frac{\pi}{8}\int_0^{\frac{\pi}{2}}\sin^2 2x dx$$

$$[\because \sin 2x=2\sin x\cos x]$$

$$=\frac{\pi}{8}\int_0^{\frac{\pi}{2}}(1-\cos 4x)dx(\because \cos 2\theta=1-2\sin^2\theta)$$

$$\Rightarrow 2I=\frac{\pi}{8}\left[x-\frac{\sin 4x}{4}\right]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I=\frac{\pi}{8}\left[\frac{\pi}{2}-0\right]\Rightarrow I=\frac{\pi^2}{32}$$

123. (d)

$$\begin{vmatrix}6i & -3i & 1\\ 4 & 3i & -1\\ 20 & 3 & i\end{vmatrix}$$

$$=6i[3i^2+3]+3i[4i+20]+1[12-60i]$$

$$=6i[-3+3]+12i^2+60i+12-60i$$

$$=-12+12=0=x+iy$$

$$\therefore x=0$$

124. (a)

$$\operatorname{Lim}_{x\to 0}\left\{\frac{\log_e(1+x)}{x^2}+\frac{x-1}{x}\right\}$$

$$=_{x\to 0}\frac{\log_e(1+x)+x^2-x}{x^2}$$

$$=\operatorname{Lim}_{x\to 0}\frac{\left(x-\frac{1}{2}x^2+\frac{1}{3}x^3-\cdots\ldots\right)+x^2-x}{x^2}=\frac{1}{2}$$

125. (a)

$$2\cos^2 x+3\sin x-3=0$$

$$2-2\sin^2 x+3\sin x-3=0$$

$$\Rightarrow (2\sin x-1)(\sin x-1)=0$$

$$\Rightarrow \sin x=\frac{1}{2}\text{ or }\sin x=1$$

$$\Rightarrow x=\frac{p}{6},\frac{5p}{6},\frac{p}{2},\text{i.e. }30^\circ,150^\circ,90^\circ.$$

126. **(b)**

127. **(d)**

$$y = \tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{3/2}}\right) = \tan^{-1}\left(\frac{\sqrt{x}-x}{1+\sqrt{x}\cdot x}\right)$$
$$= \tan^{-1}(\sqrt{x}) - \tan^{-1}(x)$$

On differentiating w.r.t. x, we get

$$y' = \frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}} - \frac{1}{1+x^2}$$
$$\Rightarrow y'(1) = \frac{1}{2}\cdot\frac{1}{2} - \frac{1}{2} = -\frac{1}{4}$$

128. **(b)** The diagonal $= R$

Thus the area of rectangle

$$= \frac{1}{2}\times R\times R = \frac{R^2}{2}$$

129. **(b)** From the given problem: $P(A\cup B) = \frac{3}{4}$,

$$P(A\cap B) = \frac{1}{4}$$
$$P(A^c) = \frac{2}{3} = 1 - P(A) \Rightarrow P(A) = 1 - \frac{2}{3} = \frac{1}{3}$$
$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$
$$\Rightarrow P(B) = P(A\cup B) + P(A\cap B) - P(A)$$
$$= \frac{3}{4} + \frac{1}{4} - \frac{1}{3} = 1 - \frac{1}{3} = \frac{2}{3}$$

130. **(b)**

$$\lim_{x\to 0^+}\frac{e^{e/x}-e^{-e/x}}{e^{1/x}+e^{-1/x}}$$
$$= \lim_{x\to 0^+}\frac{e^{\frac{x}{x}}\left(1-e^{-2e/x}\right)}{\left(1+e^{-2/x}\right)} = +\infty$$
$$\lim_{x\to 0^-}\frac{e^{e/x}-e^{-e/x}}{e^{1/x}+e^{-1/x}} = \lim_{x\to 0^-}\frac{e^{-e/x}\left(e^{2e/x}-1\right)}{e^{-1/x}\left(e^{2/x}+1\right)}$$
$$= \lim_{x\to 0^+} e^{-\left(\frac{e-1}{x}\right)}\left(\frac{e^{2e/x}-1}{e^{2/x}+1}\right) = -\infty$$

Limit doesn't exist, so f(x) is not continuous at 0 .

131. **(b)**

Put $x = \cos 2\theta$

$\therefore\ I = \int \cos\{2\tan^{-1}\tan\theta\}(-2\sin 2\theta)d\theta$

$= -\int \sin 4\theta d\theta = \frac{1}{4}\cos 4\theta + c$

$= \frac{1}{4}(2x^2 - 1) + c = \frac{1}{2}x^2 + k$

132. **(b)**

$T = S_1 \Rightarrow x(4) + y(3) - 4(x+4) = 16 + 9 - 32$
$\Rightarrow 3y - 9 = 0 \Rightarrow y = 3$

133. **(d)** The roots of equation are 2 and 3

$\therefore\ g = \sqrt{xy} = 2 \Rightarrow xy = 4$

$G = \sqrt{(x+1)(y+1)} = 3 \Rightarrow (x+1)(y+1) = 9$

$\therefore\ x = y = 2$

134. **(c)**

$\frac{e^{7x} + e^{x}}{e^{3x}} = e^{4x} + e^{-2x}$

$= \left[1 + 4x + \frac{(4x)^2}{2!} + \cdots \ldots\right] + \left[1 + (-2x) + \frac{(-2x)^2}{2!} + \cdots \ldots\right]$

$\therefore$ coeff. of $x^n = \frac{4^n}{n!} + \frac{(-2)^n}{n!}$

135. **(d)** Equation of pair of tangents is given by $SS_1 = T^2$,

or $S = x^2 + y^2 + 20(x+y) + 20, S_1 = 20$

$T = 10(x+y) + 20 = 0$

$\therefore\ SS_1 = T^2$

$\Rightarrow 20(x^2 + y^2 + 20(x+y) + 20)$

$= 10^2(x+y+2)^2$

$\Rightarrow 4x^2 + 4y^2 + 10xy = 0$

$\Rightarrow 2x^2 + 2y^2 + 5xy = 0$

136. **(c)** $a = 25, d = 22 - 25 = -3.$

Let n be the no. of terms

Sum $= 116$; Sum $= \frac{n}{2}[2a + (n-1)d]$

$116 = \frac{n}{2}[50 + (n-1)(-3)]$

or $232 = n[50 - 3n + 3] = n[53 - 3n]$

$= -3n^2 + 53n$

$\Rightarrow 3n^2 - 53 + 232 = 0 \Rightarrow (n-8)(3n-29) = 0$

$\Rightarrow n = 8$ or $n = \frac{29}{3}, n^1\frac{29}{3}\ \ \therefore n = 8$

$\therefore$ Now, $T_8 = a + (8-1)d = 25 + 7 \times (-3)$

$= 25 - 21$

$\therefore$ Last term $= 4$

137. (d)

$2a = 1 + P$ and $g^2 = P$
$\Rightarrow g^2 = 2a - 1 \Rightarrow 1 - 2a + g^2 = 0$

138. (b)

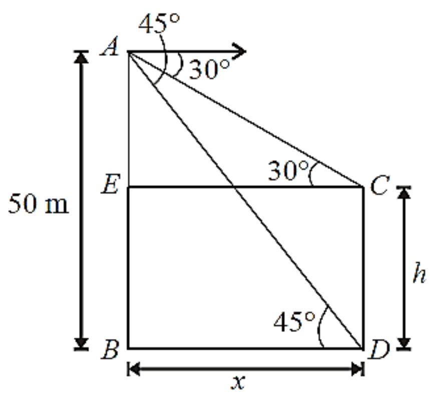
$$y = \sin x = e^x$$
$$\Rightarrow \frac{dy}{dx} = \cos x + e^x$$
$$\Rightarrow \frac{dx}{dy} = \frac{1}{\cos x + e^x}$$
$$\therefore \frac{d^2x}{dy^2} = -\frac{1}{(\cos x + e^x)^2}[-\sin x + e^x]\frac{dx}{dy}$$
$$= -\frac{(e^x - \sin x)}{(\cos x + e^x)} \times \frac{1}{\cos x + e^x}$$
$$= \frac{-(e^x - \sin x)}{(\cos x + e^x)^3} = \frac{\sin x - e^x}{(\cos x + e^x)^3}$$

139. (b)
$$4x^2 - 9y^2 = 1$$
$$\frac{x^2}{\left(\frac{1}{2}\right)^2} - \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1$$

eccentricity, e $= \sqrt{1 + \frac{\left(\frac{1}{3}\right)^2}{\left(\frac{1}{2}\right)^2}} = \frac{\sqrt{13}}{3}$

foci $= \left(\pm\frac{1}{2} \times \frac{\sqrt{13}}{3}, 0\right) = \left(\pm\frac{\sqrt{13}}{6}, 0\right)$

140. (d) Let height of the tower be h m and distance between tower and cliff be x m.



$\therefore CD = h, BD = x$

In $\triangle$ ABD, $\tan 45^\circ = \frac{AB}{BD}$ or $1 = \frac{50}{x}$ $x = 50$
In $\triangle$ AEC

$$\tan 30^\circ = \frac{AE}{EC} = \frac{AB - EB}{EC} = \frac{AB - DC}{BD}$$
$$(\because EB = DC, EC = BD)$$
$$\frac{1}{\sqrt{3}} = \frac{50 - h}{x} \text{ or } x = 50\sqrt{3} - h\sqrt{3}$$

or $50 = 50\sqrt{3} - h$ [From(i), x = 50]
or $h\sqrt{3} = 50\sqrt{3} - 50$
or $h = \frac{50(\sqrt{3}-1)}{\sqrt{3}} = 50\left(1 - \frac{1}{\sqrt{3}}\right)$

$$\therefore h = 50\left(1 - \frac{\sqrt{3}}{3}\right)$$

141. **(a)** General term of the given binomial series is given by:

$$T_{r+1} = {}^{10}C_r \left\{\frac{x^{1/2}}{3}\right\}^{10-r} \cdot \{x^{-1/4}\}^r$$

Put $r = 4$, we get

$$T_5 = {}^{10}C_4 \cdot \frac{1}{3^6} x^3 \cdot x^{-1}$$
$$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} \cdot \frac{1}{3^6} x^2 = \frac{70}{243} x^2$$

Thus coefficient of $x^2 = \frac{70}{243}$.

142. **(c)** Two circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ cuts orthogonally if $2g_1g_2 + 2f_1f_2 = c_1 + c_2$
Given equations of two circles are

$$x^2 + y^2 + 2\lambda x + 6y + 1 = 0$$
$$x^2 + y^2 + 4x + 2y = 0$$

On comparing (i) and (ii) with original equation, we get

$g_1 = \lambda, f_1 = 3, c_1 = 1$ and $g_2 = 2, f_2 = 1, c_2 = 0$

So, from orthogonality condition, we have

$$4\lambda + 6 = 1 \Rightarrow 4\lambda = -5$$
$$\therefore \lambda = \frac{-5}{4}$$

143. **(a)** We know, scalar triple product

$[\vec{a}\vec{b}\vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c}) \equiv (\vec{a}\times\vec{b})\cdot\vec{c}$

Consider $[\vec{a}+\vec{b}\vec{b}+\vec{c}\vec{c}+\vec{a}]$

$= (\vec{a}+\vec{b})\cdot\{(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\}$

$= (\vec{a}+\vec{b})\cdot\{(\vec{b}\times\vec{c})+(\vec{b}\times\vec{a})+(\vec{c}\times\vec{c})+(\vec{c}\times\vec{a})\}$

$= (\vec{a}+\vec{b})\cdot\{(\vec{b}\times\vec{c})+(\vec{b}\times\vec{a})+(\vec{c}\times\vec{a})\}$

$(\because \vec{c}\times\vec{c}=0)$

$= \vec{a}\cdot(\vec{b}\times\vec{c})+\vec{a}\cdot(\vec{b}\times\vec{a})+\vec{a}\cdot(\vec{c}\times\vec{a})$

$+\vec{b}\cdot(\vec{b}\times\vec{c})+\vec{b}\cdot(\vec{b}\times\vec{a})+\vec{b}\cdot(\vec{c}\times\vec{a})$

$= [\vec{a}\vec{b}\vec{c}]+[\vec{a}\vec{b}\vec{a}]+[\vec{a}\vec{c}\vec{a}]+[\vec{b}\vec{b}\vec{c}]$

$+[\vec{b}\vec{b}\vec{a}]+[\vec{b}\vec{c}\vec{a}]$

(By definition of scalar triple product)

$[\vec{a}\vec{a}\vec{b}]=0, [\vec{a}\vec{b}\vec{a}]=0$ and $[\vec{b}\vec{a}\vec{a}]=0$

$= [\vec{a}\quad\vec{b}\quad\vec{c}]+[\vec{b}\quad\vec{c}\quad\vec{a}] = 2[\vec{a}\quad\vec{b}\quad\vec{c}]$

144. **(b)**

Given that $\mathrm{f(x)} = (\mathrm{a}-\mathrm{x}^{\mathrm{n}})^{1/\mathrm{n}}$

$\therefore$ fof $(x) = \left[a-\{(a-x^n)^{1/n}\}^n\right]^{1/n}$

$= [a-(a-x^n)]^{1/n}$

$= [x^n]^{1/n} = x$

145. **(a)**

Sum $= \frac{8}{9}[9+99+999+\cdots \text{n terms}]$

$= \frac{8}{9}[(10-1)+(100-1)+(1000-1)+\cdots.\text{n terms}]$

$= \frac{8}{9}[(10+10^2+10^3+\cdots.+10^n)-n]$

$= \frac{8}{9}\left[\frac{10(10^n-1)}{10-1}-n\right]$

$= \frac{8}{81}[10^{n+1}-9n-10]$

146. **(a)**

Let $z = x+iy$

$\therefore |z+3-i| = |(x+3)+i(y-1)| = 1$

$\Rightarrow \sqrt{(x+3)^2+(y-1)^2} = 1$

$\because \arg z = \pi \Rightarrow \tan^{-1}\frac{y}{x} = \pi$

$\Rightarrow \frac{y}{x} = \tan\pi = 0 \Rightarrow y = 0$

From equations (i) and (ii), we get

$x=-3, y=0 \therefore z=-3$

$\Rightarrow |z| = |-3| = 3$

147. **(b)** Let E_1, E_2 and A be the events defined as follows:

E_1 = red ball is transferred from bag P to bag Q

E_2 = blue ball is transferred from bag P to bag Q

A = the ball drawn from bag Q is blue

As the bag P contains 6 red and 4 blue balls, $P(E_1) = \frac{6}{10} = \frac{3}{5}$ and $P(E_2) = \frac{4}{10} = \frac{2}{5}$

Note that E_1 and E_2 are mutually exclusive and exhaustive events.

When E_1 has occurred i.e., a red ball has already been transferred from bag P to Q, then bag Q will contain 6 red and 6 blue balls, So, $P(A \mid E_1) = \frac{6}{12} = \frac{1}{2}$

When E_2 has occurred i.e., a blue ball has already been transferred from bag P to Q, then bag Q will contain 5 red and 7 blue balls, So, $P(A \mid E_2) = \frac{7}{12}$

By using law of total probability, we get

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)$$
$$= \frac{3}{5} \times \frac{1}{2} + \frac{2}{5} \times \frac{7}{12} = \frac{8}{15}$$

148. **(a)** Total number of 4-digit numbers

$= 5 \times 5 \times 5 \times 5 = 625$

(as each place can be filled by anyone of the numbers 1,2,3,4 and 5)

Numbers in which no two digits are identical $= 5 \times 4 \times 3 \times 2 = 120$ (i.e. repetition not allowed) (as 1^{st} place can be filled in 5 different ways, 2^{nd} place can be filled in 4 different ways and so on) Number of 4-digits numbers in which at least 2 digits are identical $= 625 - 120 = 505$

149. **(c)** $D = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix} = 0; D_1 = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \neq 0$

$\Rightarrow$ Given system, does not have any solution.

$\Rightarrow$ No solution

150. **(d)** Let $A(3, y), B(2,7), C(-1,4)$ and $D(0,6)$ be the given points.

$$m_1 = \text{ slope of AB} = \frac{7-y}{2-3} = (y-7)$$
$$m_2 = \text{ slope of CD} = \frac{6-4}{0-(-1)} = 2$$

Since AB and CD are parallel. $\therefore m_1 = m_2 \Rightarrow y = 9.$