1. (d)

$$V = 5x^2 + 10x - 4$$
$$E = \frac{-dV}{dx} = -(10x + 10)$$

At $x = 1$ m, $E = -20$ V/m.

2. (a)

given that

$v = 3 \times 10^5$ m/s, $B = 0.3$ T, and $e/m = 10^8$ C/kg

$$F = \frac{mv^2}{r} = qvB\sin\theta$$
$$r = \frac{mv}{qB\sin\theta}\{\because q/m = 10^8\}$$
$$r = \frac{3 \times 10^5}{10^8 \times 0.3\sin 30}$$

radius of curvature of its path

$r = 2$ cm

3. (a) Let $m = KF^aL^bT^c$

Substituting the dimensions of $[F] = [MLT^{-2}]$, $[L] = [L]$ and $[T] = [T]$ and comparing both side, we get $m = FL^{-1}T^2$

4. (b) Induced emf produced between the centre and a point on the disc is given by

$$e = \frac{1}{2}\omega BR^2$$

$\omega = 60$ rad/s, $B = 0.05$ Wb/m^2

and $R = 100$ cm $= 1$ m

We get $e = \frac{1}{2} \times 60 \times 0.05 \times (1)^2 = 1.5$ V

5. (b) For stone to be dropped from rising balloon of velocity 29 m/s, $u = -29$ m/s, $t = 10$ sec.

$$\therefore h = -29 \times 10 + \frac{1}{2} \times 9.8 \times 100$$
$$= -290 + 490 = 200 \text{ m}$$

6. (a) The phase angle between voltage V and current I is $\frac{\Pi}{2}$. Therefore, power factor $\cos\phi = \cos\left(\frac{\pi}{2}\right) = 0$. Hence the power consumed is zero.

7. (b) The bullets are fired at the same initial speed

$$\frac{H}{H'} = \frac{u^2\sin^2 60^\circ}{2g} \times \frac{2g}{u^2\sin^2 30^\circ} = \frac{\sin^2 60^\circ}{\sin^2 30^\circ}$$
$$= \frac{(\sqrt{3}/2)^2}{(1/2)^2} = \frac{3}{1}$$

8. (a)

$$_a\lambda_g = \frac{\lambda_a}{\mu} = \frac{5460}{1.5} = 3640\text{Å}$$

9. **(a)** Two springs on the L.H.S. of mass M are in series and two springs on the R.H.S. of mass M are in parallel. These combinations of springs will be considered in parallel to mass M. Thus effective spring constant,

$$K = \frac{2k \times 2k}{2k + 2k} + (k + 2k) = 4k$$

$$\therefore \text{ Frequency } v = \frac{1}{2\pi}\sqrt{\frac{K}{M}} = \frac{1}{2\pi}\sqrt{\frac{4k}{M}}.$$

10. **(a)** According to kepler's law of area

$$\frac{dA}{dt} = \frac{L}{2\,m}$$

For central forces, torque $= 0$
$\therefore$ L = constant

$$\therefore \frac{dA}{dt} = \text{ constant}$$

11. **(b)**

1 becquerel $= 1\text{dis/s}$
1 rutherford $= 10^6\text{dis/s}$
1 curie $= 3.7 \times 10^{10}\text{dis/s}$

12. **(c)**

Length of solenoid, $1 = 2$ m
Total number of turns, $N = 5 \times 1000 = 5000$
Number of turns per unit length,

$$n = \frac{N}{l} = \frac{5000}{2} = 2500\ \text{m}^{-1}$$

Current, $I = 5$ A
Now, $B = \mu_0 nI = 4\pi \times 10^{-7} \times 2500 \times 5$ T

$= 1.57 \times 10^{-2}$ T

13. **(c)** The tension in the above string can be divided into two components where the component opposite to weight will balance the weight while the component of this tension opposite to T_1 will be equal to T_1.
$T\sin\theta = 2\text{kgwt}$
$T\cos\theta = T_1$
The first equation gives
$T = 4\text{kgwt}$
So T_1 will be,

$T_1 = 2\sqrt{3}\text{kgwt}$

14. **(b)** Let we have a wire of length 1 , cross-sectional area A, and Resistivity ρ
Then Resistance of wire is

$$R=\rho\frac{1}{A}-(1)$$

The volume of cylindrical wire is the product of its length and cross-sectional area.

$$V=I.A$$

If length is increased by 10 percent, the new length I' is

$$I=|+10\% \text{ of } I=1.1\,|-(2)$$

After stretching the cross-sectional area will also change, as volume remains constant. Let the new cross-sectional area be A′

$$V=I\cdot A=IA\ldots\ (3)$$

Putting (2) in (3) we have

$$I\cdot A=1.1I\cdot A$$
$$A=\frac{A}{1.1}\ldots\ (4)$$

So, the Resistance of this stretched wire is

$$R=\rho\frac{1}{A}\cdots$$

Putting (2) and (4) in (5)

$$R=\rho\left(\frac{1.11}{A/1.1}\right)$$
$$\Rightarrow R=\rho\left(\frac{1.1\times 1.11}{A}\right)$$
$$\Rightarrow R=\rho\left(\frac{1.211}{A}\right)$$
$$\Rightarrow R=1.21\left(\rho\frac{1}{A}\right)-(6)$$

Comparing Eq(6) and Eq(1) We get $R=1.21R$
So, the resistance increases 1.21 times.

15. **(a)** The significant result deduced from the Rutherford's scatter ing is that whole of the positive charge is concentrated at the centre of atom i.e. nucleus.

16. **(d)** Let the velocity of the ball just when it leaves the hand is u then applying, $v^2-u^2=2$ as for upward journey

$$\Rightarrow -u^2=2(-10)\times 2$$
$$\Rightarrow u^2=40$$

Again applying $v^2-u^2=2$ as for the upward journey of the ball, when the ball is in the hands of the thrower,

$v^2 - u^2 = 2as$
$\Rightarrow 40 - 0 = 2(a)0.2$
$\Rightarrow a = 100\ m/s^2$
$\therefore F = ma = 0.2 \times 100 = 20\ N$
$\Rightarrow N - mg = 20$
$\Rightarrow N = 20 + 2 = 22\ N$

17. **(a)** Here, $E = 9\ V; V_z = 6; R_L = 1000\Omega$
and $R_s = 100\Omega$,

Potential drop across series resistor

$V = E - V_Z = 9 - 6 = 3\ V$

Current through series resistance R_S is

$$I = \frac{V}{R} = \frac{3}{100} = 0.03\ A$$

Current through load resistance R_L is

$$I_L = \frac{V_Z}{R_L} = \frac{6}{1000} = 0.006\ A$$

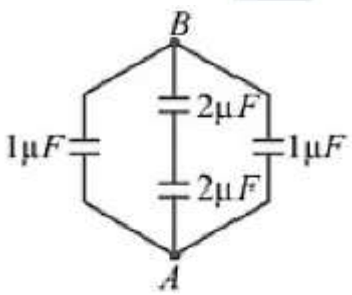
Current through Zener diode is

$I_Z = I - I_L = 0.03 - 0.006 = 0.024$amp.

Power dissipated in Zener diode is

$P_Z = V_Z I_Z = 6 \times 0.024 = 0.144$ Watt

18. **(a)** The equivalent circuit is shown in figure.



$$C_{AB} = 1 + \frac{2 \times 2}{2 + 2} + 1$$
$$C_{AB} = 3\mu F.$$

19. **(d)** If 1 is length of pendulum and θ be angular amplitude then from principle of conservation of energy (Initial total energy) $0 + mgh = \frac{1}{2}mv^2 + 0$ (Final total energy)

$$\Rightarrow v = \sqrt{2gh} = \sqrt{2gl(1 - \cos\theta)}$$

20. **(c)** Let G be resistance of galvanometer and i_g the current through it. Let V is maximum potential difference, then from Ohm's law

$$i_g = \frac{V}{G+R}$$
$$\Rightarrow R = \frac{V}{i_g} - G$$

Given, $G = 10\Omega, i_g = 0.01$ A

$$V = 10 \text{ volt}$$
$$\therefore R = \frac{10}{0.01} - 10 = 990\Omega$$

Thus, on connecting a resistance R of 990Ω in series with the galvanometer, the galvanometer will become a voltmeter of range zero to 10 V.
For the voltmeter, a high resistance is series with the galvanometer and so the resistance of a voltmeter is very high compared to that of galvanometer.

21. (b)

$$n' = \frac{nv}{v - v_s} \dots\dots\dots (1)$$
$$n'' = \frac{nv}{v + v_s} \dots\dots (2)$$

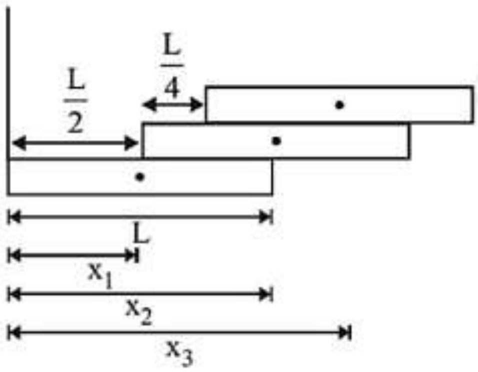
From (1) and (2), $\frac{n'}{n"} = \frac{v+v_s}{v-v_s}$..
According to question, $\frac{n'}{n'} = \frac{5}{4}$

$v_s = ? v = 330$ m/s.

From eq. (3) and (4)

$$\frac{5}{4} = \left[\frac{330 + v_s}{330 - v_s}\right] \Rightarrow 9v_s = 330$$
$$\therefore v_s = 36.6 \text{ m/s}$$

22. (d)



$$x_1 = \frac{L}{2}, x_2 = L, x_3 = \frac{5L}{4}$$
$$\therefore x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$
$$= \frac{M \times \frac{L}{2} + M \times L + M \times \frac{5L}{4}}{M + M + M} = \frac{11L}{12}$$

23. (b) The figure is showing I − V characteristics of non ohmic or non linear conductors.

24. (d)

$$\tan\theta = \frac{V}{H} = \frac{3}{4}\left[\because \tan 37^\circ = \frac{3}{4}\right]$$
$$\therefore V = \frac{3}{4}\mathrm{H}$$
$$\mathrm{V} = 6 \times 10^{-5}\ \mathrm{T}$$
$$\mathrm{H} = \frac{4}{3} \times 6 \times 10^{-5}\ \mathrm{T} = 8 \times 10^{-5}\ \mathrm{T}$$
$$\therefore \mathrm{B_{total}} = \sqrt{\mathrm{V}^2 + \mathrm{H}^2} = \sqrt{(36+64)} \times 10^{-5}$$
$$= 10 \times 10^{-5} = 10^{-4}\ \mathrm{T}$$

25. (d) Angular momentum will be conserved

$$\text{i.e., } \mathrm{I_1}\omega = \mathrm{I_1}\omega' + \mathrm{I_2}\omega'$$
$$\Rightarrow \omega' = \mathrm{I_1}\omega\mathrm{I_1} + \mathrm{I_2}$$

26. (b)

27. (c)

$$\frac{\mathrm{dQ}}{\mathrm{dt}} = \frac{\mathrm{kA(T_1 - T_2)}}{\mathrm{L}}$$

[$(\mathrm{T_1 - T_2})$ is the temperature difference]

28. (a)

29. (a)

$$\lambda = 667 \times 10^{-9}\ \mathrm{m}, \mathrm{P} = 9 \times 10^{-3}\ \mathrm{W}$$
$$\mathrm{P} = \frac{\mathrm{Nhc}}{\lambda}, \mathrm{N} = \text{ No. of photons emitted /sec.}$$
$$\mathrm{N} = \frac{9 \times 10^{-3} \times 667 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^{8}} = 3 \times 10^{16}/\mathrm{sec}$$

30. (b) From equation given,

$\omega = 100$

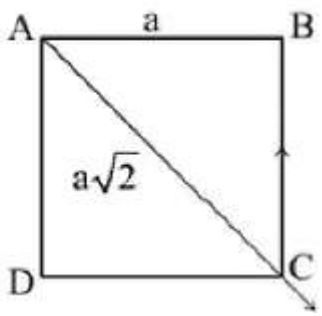
and $\mathrm{k} = 20$;

$$\mathrm{v} = \frac{\omega}{\mathrm{k}} = \frac{100}{20} = 5\ \mathrm{m/s}$$

31. (b) Volume $\mathrm{v} = \frac{\mathrm{m}}{\mathrm{d}}$ and using $\mathrm{PV}^\gamma = $ constant

$$\frac{\mathrm{P}'}{\mathrm{P}} = \frac{\mathrm{V}}{\mathrm{V}'} = \left(\frac{\mathrm{d}'}{\mathrm{d}}\right)^\gamma$$
$$\text{or } 128 = (32)^\gamma$$
$$\therefore \gamma = \frac{7}{5} = 1.4$$

32. (d)

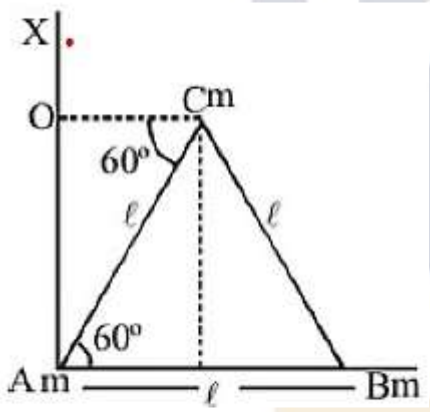


It is given that $\vec{F}_A + \vec{F}_B + \vec{F}_D = 0$
Where $\vec{F}_A, \vec{F}_B$ and $\vec{F}_D$ are the forces applied by charges placed at A, B and D on the charge placed at C.

$$\Rightarrow \vec{F}_B + \vec{F}_D = -\vec{F}_A$$
$$\left|\vec{F}_B + \vec{F}_D\right| = \sqrt{2}\frac{KqQ}{a^2}$$
$$\Rightarrow \left|\vec{F}_A\right| = \frac{KQ^2}{2a^2}$$
$$\therefore \sqrt{2}\frac{KqQ}{a^2} = -\frac{KQ}{2a^2}$$
$$\Rightarrow \frac{Q}{q} = -2\sqrt{2}$$

33. (d)

$$I_{AX} = m(AB)^2 + m(OC)^2 = ml^2 + m(l\cos 60^\circ)^2 = ml^2 + \frac{ml^2}{4} = \frac{5}{4}ml^2$$



34. (d)

Energy = charge × potential diff.
$E_{electron} = q_eV$ and $E_{proton} = q_p 4\,V$
de - Broglie wavelength $\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mE}}$

$$\lambda_e = \frac{h}{\sqrt{2m_eeV}} \text{ and } \lambda_p = \frac{h}{\sqrt{2m_pe4V}} \because (q_e = q_p)$$

$$\therefore \frac{\lambda_e}{\lambda_p} = \frac{\frac{h}{\sqrt{2m_eeV}}}{\frac{h}{\sqrt{2m_pe4V}}} = 2\sqrt{\frac{m_p}{m_e}}$$

35. (b)

36. (c)

Let ' M ' be the mass of the particle Now, $E_{initial} = E_{final}$

i.e. $\frac{GMm}{\sqrt{2}r} + 0 = \frac{GMm}{r} + \frac{1}{2}MV^2$

or, $\frac{1}{2}MV^2 = \frac{GMm}{r}\left[1 - \frac{1}{\sqrt{2}}\right]$

$\Rightarrow \frac{1}{2}V^2 = \frac{Gm}{r}\left[1 - \frac{1}{\sqrt{2}}\right]$ or,

$$V = \sqrt{\frac{2Gm}{r}\left(1 - \frac{1}{\sqrt{2}}\right)}$$

37. (c) Incident momentum, $p = \frac{E}{C}$

For perfectly reflecting surface with normal incidence

$$\Delta p = 2p = \frac{2E}{c}; F = \frac{\Delta p}{\Delta t} = \frac{2E}{ct}$$

38. (a)

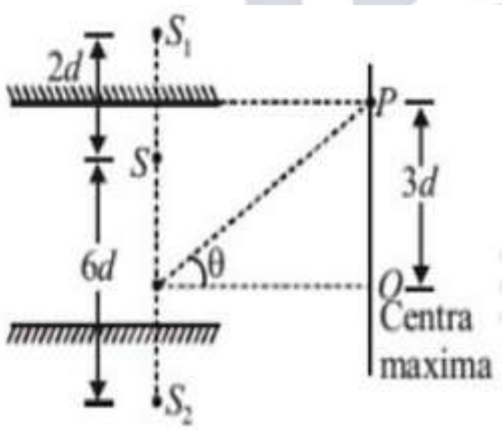
Compressibility $= \frac{1}{B} = \frac{\Delta V/V}{P}$.

Here, $P = 100$ atm

Compressibility $= 4 \times 10^{-5}$ and $V = 100\ cm^3$

Hence, $\Delta V = 0.4\ cm^3$

39. (a)



At P, $\Delta x = \frac{(8\ d) \times 3\ d}{D}$

For 2 nd maxima, $\Delta x = 2\lambda$

$$\Rightarrow \frac{24\ d^2}{D} = 2\lambda$$

$$\Rightarrow \lambda = \frac{12\ d^2}{D}$$

40. (c) Volume of 8 small droplets = Volume of 1 big drop

$\Rightarrow \frac{4}{3}\Pi r^3 \times 8 = \frac{4}{3}\Pi R^3 \Rightarrow r = \frac{R}{(8)^{1/3}}$

Work done $=$ (Change in area) $\times$ surface tension

$= (4\Pi r^2 n - 4\Pi R^2)T$

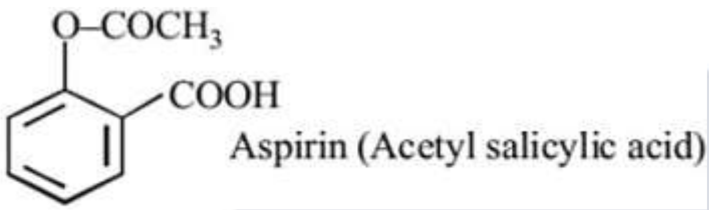
$\left\{4\Pi\left(\frac{R^2}{(8)^{2/3}}\right) \times 8 - 4\Pi R^2\right\}T = 4\Pi R^2 T$

41. **(d)** $Fe^{3+}(d^5)$ has 5 unpaired electrons therefore magneticmoment $= \sqrt{n(n+2)} = \sqrt{5(5+2)} = 5.91$ which is maximum among given options. As$Sc^{3+}, Ti^{3+}, Cr^{3+}$ and V^{3+} contain 0,1,3, and 2 number of unpaired electrons respectively.

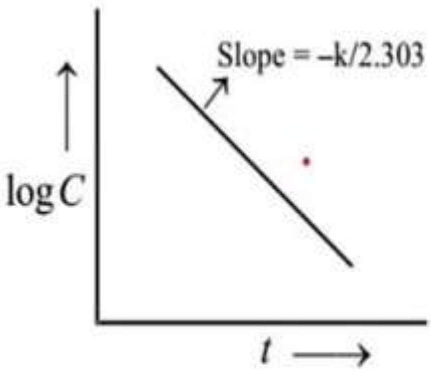
42. **(c)**

43. **(d)** Cisplatin $[Pt(NH_3)_2Cl_2]$ is used in cancer treatment.

44. **(a)**



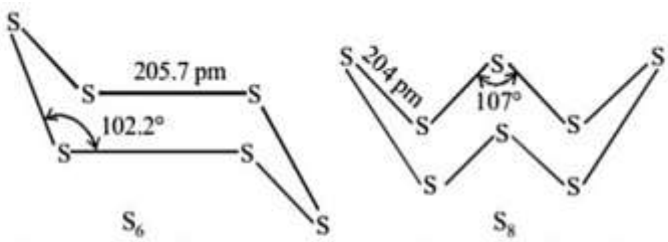
45. **(b)**



46. **(a)** Nitrogen due to small size is able to show pπ − pπ lateral overlap forming $N \equiv N$, rest elements due to bigger size are not able to show pπ - pπ lateral overlap.

47. **(a)** The S_6 molecule has a chair-form hexagon ring with the approx. same bond length as that in S_8, but with some what smaller bond angles i.e.

bond lengths are approx. same but bond angles are different.



48. **(c)** HF_2 is the only compound among the given options which does not contain any coordinate bond because it has hydrogen bonding.

$[H-F \cdot 0 \cdot H]^-$

49. (d) BF_3, BCl_3, BBr_3 are sp^2 hybridise(d) So, all have same structure and bond angle i.e. 120°.

50. (b)

$$
\begin{aligned}
& P_A^\circ = 7 \times 10^3 \\
& P_B^\circ = 12 \times 10^3 \\
& x_A' = 0.4; x_B' = 1 - 0.4 \\
& x_B' = 0.6 \\
& P_{\text{total}} = P^\circ{}_A x_A + P^\circ{}_B x_B' \\
& = 7 \times 10^3 \times 0.4 + 12 \times 10^3 \times 0.6 \\
& = (7 \times 0.4 + 12 \times 0.6) \times 10^3 = 10^4 \\
& x_A = \frac{P^\circ{}_A x_A'}{P_{\text{total}}} = \frac{7 \times 10^3 \times 0.4}{10^4} \\
& \therefore x_A = 0.28, x_B = 1 - 0.28 = 0.72
\end{aligned}
$$

51. (c) Completing the reaction, we get

$$NCl_3 + 3H_2O \rightarrow 3H\underset{x}{Cl}O + NH_3$$

52. (b) No. of lattice points = No. of octahedral voids $= \frac{1}{2} \times$ No. of tetrahedral voids in ccp structure

$\therefore$ No. of atoms of B $= 4$

No. of atoms of A $= \frac{1}{2} \times$ No. of octahedral voids

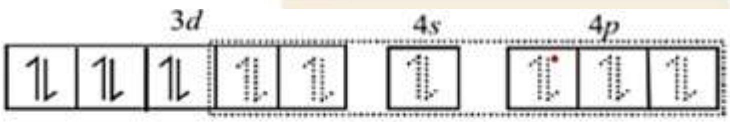
$= \frac{1}{2} \times 4 = 2$

No. of atoms of O $=$ All tetrahedral voids $= 2 \times$ No. of lattice points $= 2 \times 4 = 8$

Hence, $A : B : O = 1 : 2 : 4$

Therefore, the formula of the compound is AB_2O_4

53. (d) Complex X is $[Fe(en)_3]^{2+}$; as 'en' is a strong field ligand pairing of electrons will take place.
$[Fe(en)_3]^{2+}$:



d^2sp^3 hybridisation

Hence, hybridisation is d^2sp^3 and complex is diamagneti(c) As it has 3 bidentate symmetrical 'en' ligands so it will not show geometrical isomerism.

54. (c) $v = \frac{1}{h} \times IE \times \left[\frac{1}{{n_1}^2} - \frac{1}{n_2^2}\right]$

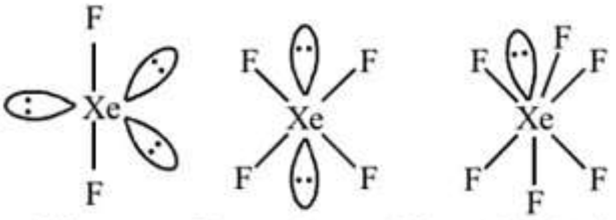
$$= \frac{2.18 \times 10^{-18}}{6.625 \times 10^{-34}} \times \left[\frac{1}{1} - \frac{1}{16}\right] = 3.08 \times 10^{15}\ s^{-1}$$

55. (b) Cetyltrimethyl ammonium bromide which is a germicide, is a popular cationic detergent.

56. (d) Sucralose is trichloro derivative of sucrose. It is stable at cooking temperture. It does not provide calories.

57.

(d) XeF_2 XeF_4 XeF_6



Linear Square planar Distorted octahedral

58. (d) HF form linear polymeric structure due to hydrogen bonding.

59. (d)

60. (c)

Metel ion	No. of unpaired ele(c)		Difference in the unpaired electrons
	High spin	Low spin	
$Ni^{2+}(3d^8)$	2	2	0
$Mn^{2+}(3d^5)$	5	1	4
$Fe(3d^6)$	4	0	4
$Co^{2+}(3d^7)$	3	1	2

61. (a) According to Freundlich adsorption isotherm

$$\frac{x}{m} \propto p^{\frac{1}{n}}; \frac{x}{m} = kp\frac{1}{n}$$

$$\text{Slope } = \frac{2}{3}$$

$$\log\frac{x}{m} = \log k + \frac{1}{n}\log p$$

$$\text{Slope } = \frac{1}{n} = \frac{2}{3}$$

$$\frac{x}{m} \propto p^{\frac{2}{3}}$$

62. (c) x has a conjugated diene system, w an isolated diene system, z a cumulated diene system, and y an antiaromatic system.

63. (a)

64. (a)

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1 \times 10^3}$$

$= 3.97 \times 10^{-10}$ meter $= 0.397$ nanometer ≈ 0.40 nm

65. (b) Hypo solution is $Na_2\,S_2O_3$ solution which is used in photography for fixing films & prints. Photographic emulsions are made of AgBr. After developing, the film is put into hypo solution. This forms soluble complex with Ag.

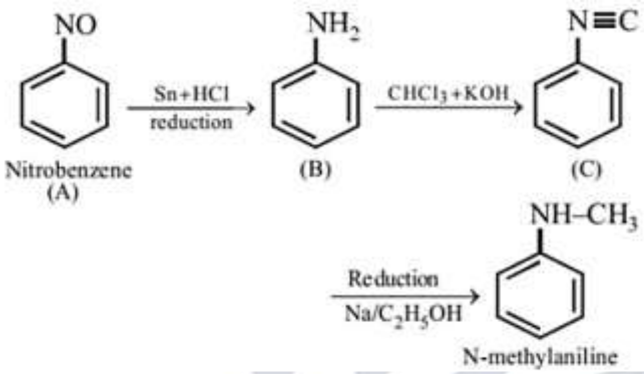
$$Na_2\,S_2O_3 + AgBr \rightarrow Ag_2\,S_2O_3 \xrightarrow{Na_2\,S_2O_3} Na_5[Ag(S_2O_3)_3]$$

66. (a)
67. (c)

C_2H_2 forms copper acetylide with ammonical Cu_2Cl_2

$$Cu_2Cl_2 + 2NH_4OH \rightarrow Cu_2O + H_2O + 2NH_4Cl$$
$$Cu_2O + C_2H_2 \rightarrow \underset{\text{(copper acetylide)}}{Cu_2C_2} + H_2O$$

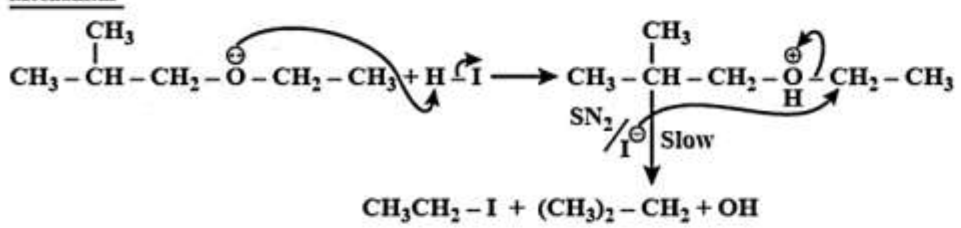
68. (d) Vitamin E is an antioxidant present in edible oils.
69. (c)



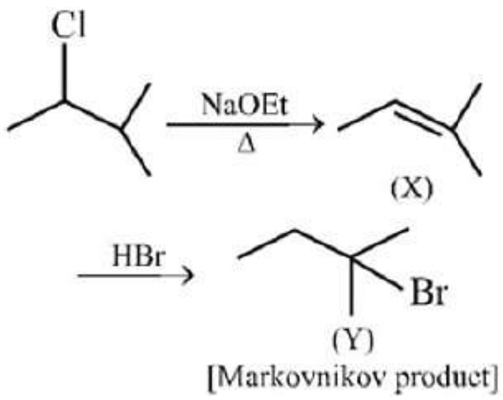
70. (c)

$$CH_3-\underset{}{\overset{CH_3}{\overset{|}{C}H}}-CH_2-O-CH_2-CH_3+HI \xrightarrow{\text{Heated}} CH_3-\overset{CH_3}{\overset{|}{C}H}-CH_2-OH+CH_3-CH_2-I$$

Mechanism



71. (c)



72. **(a)** Clean water has BOD value less than 5ppm. Polluted water has BOD value higher than 10ppm.

73. **(b)** Ionic compounds have high melting point. Greater the ionic character, more is melting point. HCl has least ionic character because of maximum electronegativity difference between the two constituent elements, (H and Cl) among CsF, HCl, HF and LiF

∴ HCl has minimum melting point.

74. **(b)** Under identical conditions, $\frac{r_1}{r_2}=\sqrt{\frac{M_2}{M_1}}$

As rate of diffusion is also inversely proportional to time, we will have, $\frac{t_2}{t_1}=\sqrt{\frac{M_2}{M_1}}$

(a) Thus, For He, $t_2=\sqrt{\frac{4}{2}}(5\text{ s})=5\sqrt{2}\text{ s}\neq 10\text{ s};$

(b) For $O_2, t_2=\sqrt{\frac{32}{2}}(5\text{ s})=20\text{ s}$

(c) For $CO, t_2=\sqrt{\frac{28}{2}}(5\text{ s})\neq 25\text{ s};$

(d) For $CO_2, t_2=\sqrt{\frac{44}{2}}(5\text{ s})\neq 55\text{ s}$

75. **(b)** $q=-w_{rev}=-\left(-2.303nRT\log\frac{P_1}{P_2}\right)$

$$=2.303\times\frac{40}{4}\times 2\times 300\log\frac{1}{10}=-13.82\text{kcal}$$

76. **(b)**

77. **(c)**

The balanced equation:

$2NH_3+OCl^-\rightarrow N_2H_4+Cl^-+H_2O$

78. **(d)** Li and Mg do not form solid bicarbonate, but react with N_2 to give nitrides.

$6Li+N_2\xrightarrow{\Delta}2Li_3N$

$3Mg+N_2\xrightarrow{\Delta}Mg_3N_2$

79. **(a)**

$B_2H_6+3O_2\rightarrow B_2O_3+3H_2O$

$B_2H_6+6H_2O\rightarrow 2H_3BO_3+6H_2$

80. **(b)**

81. **(d)** The correct sequence 'is 'If Socrates was innocent at the age of seventy it may be imagined now innocent St. John was at the age of seventeen'.

Therefore, correct combination is (d)

82. **(c)** The correct sequence is 'It was all very wonderful and glamorous here in the old places that seemed so ordinary'. Therefore correct combination is (c)

83. **(d)** The correct sequence is If you feed a dog or tame a bear by hand, they get their teeth into the meet and tear and pull at it until they bite a piece of or until they succeed in getting it all out of our hand'.

Therefore, correct combination is (d).

84. **(b)** To express general truths we use simple present tense, therefore, (b) is the correct choice.

85. **(b)** The simple past is used to indicate an action completed in the past.

Therefore option(b) is correct.

86. **(a)** Simple present tense is also used, instead to the simple future tense, therefore option (a) is correct.

87. **(d)** 'amenities' means 'pleasing acts' and 'courtesies' means 'polite behaviour'. Therefore correct synonym is (d).

88. **(c)** 'deflect' means 'turn aside' and 'pervert' means 'twist'.

Therefore correct synonym is (c).

89. **(c)** 'exorbitant' means 'beyond the limit' and 'excessive' means 'extreme'.

Therefore, correct synonym is (c).

90. **(d)** 'at' is used for things at rest.

91. **(c)** 'towards' is used to show the sense of direction.

92. **(a)** 'at' is used for things at rest.

93. **(b)** According to the passage, our world has a heritage of culture means, we have an inherited cultural tradition. Therefore option (b) is correct.

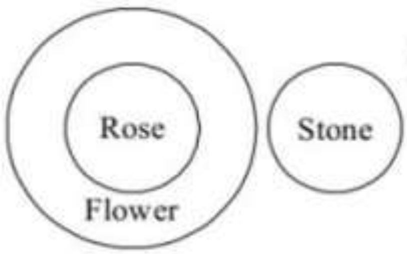
94. **(c)** According to the passage common people may be described as the less active and important people. Therefore option (c) is close to correct answer.

95. **(c)** According to the passage the persons who are not aware of history are the men ignorant of the past. Therefore option (c) is correct.

96. **(d)**

97. **(a)**

98. **(a)**

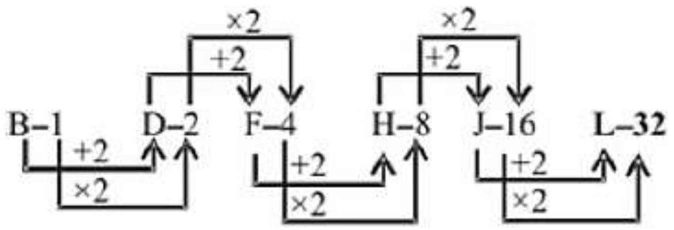


All roses are flower. Stone is neither Flower nor Rose.

99. **(c)**

100. **(d)**

101. **(b)**



102. **(c)** FLOURISH

IOUFHLRS (Vowels arranged alphabetically)

HNT FHLRS (Vowels replaced by previous letter) HNTGIMST (Consonants replaced by Next Letter)

So M is the answer

103. **(d)** The female members in the family are mother, wives of 3 married son's unmarried daughterand 2 daughters of each of the two sons. $(1+3+1+2\times 2)=9$

104. **(b)**

105. **(d)**

$\left(\frac{3}{2}-1\right)=\frac{1}{2}\Rightarrow 1^{\text{st}}$ row

$\left(\frac{8}{3}-2\right)=\frac{2}{3}\Rightarrow 2^{\text{nd}}$ row

$\left(\frac{19}{5}-3\right)=\frac{4}{5}\Rightarrow 3^{\text{rd}}$ row

106. **(c)** $a=25, d=22-25=-3$

Let n be the no. of terms

Sum = 116;

$\text{Sum}=\frac{n}{2}[2a+(n-1)d]$

$116=\frac{n}{2}[50+(n-1)(-3)]$

or $232=n[50-3n+3]=n[53-3n]=-3n^2+53n$

$\Rightarrow 3n^2-53+232=0$

$\Rightarrow (n-8)(3n-29)=0$

$\Rightarrow n=8$ or $n=\frac{29}{3}$,

$n \neq \frac{29}{3}$

$\therefore n = 8$

$\therefore$ Now, $T_8 = a + (8-1)d = 25 + 7 \times (-3) = 25 - 21 = 4$

$\therefore$ Last term $= 4$

107. **(c)** Given parallel lines are

$3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$

Required distance $= \frac{|7-5|}{\sqrt{(3)^2+(-4)^2}} = \frac{2}{5}$

$\Rightarrow a = 2, b = 5$

108. **(a)** We have, $\lim_{x\to 0} \frac{a^{\sin x}-1}{\sin x} = \lim_{y\to 0} \frac{a^y-1}{y}$

$= \log a$, where $y = \sin x$

$\because x \to 0 \Rightarrow y = \sin x \to 0$

109. **(b)**

$$\int \frac{\sin x}{\sin(x-\alpha)} dx = \int \frac{\sin(x-\alpha+\alpha}{\sin(x-\alpha)} dx$$
$$= \int \frac{\sin(x-\alpha)\cos\alpha + \cos(x-\alpha)\sin\alpha}{\sin(x-\alpha)} dx$$
$$= \int \{\cos\alpha + \sin\alpha\cot(x-\alpha)\}dx$$
$$= (\cos\alpha)x + (\sin\alpha)\log\sin(x-\alpha) + C$$
$$\therefore A = \cos\alpha, B = \sin\alpha$$

110. **(d)** $f(x)$ is defined for $x^{12} - x^9 + x^4 - x + 1 > 0$

$\Rightarrow x^4(x^8+1) - x(x^8+1) + 1 > 0$

$\Rightarrow (x^8+1)x(x^3-1) + 1 > 0$

If $x \geq 1$ or $x \leq -1$, then the above expres - sion is positive.

If $-1 < x \leq 0$, the above inequality still holds.

If $0 < x < 1$, then

$x^{12} - x(x^8 +) + (x^4 + 1) > 0$

$\because [x^4 + 1 > x^8 + 1$ and so $x^4 + 1 > x(x^8 + 1)]$

The domain of $f = (-\infty, \infty)$

111. **(c)** Let a be the major axis and b, the minor axis of the ellipse, then 3 minor axis = major axis.

$\Rightarrow 3\text{ b} = \text{a}$

Eccentricity is given by:

$b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 9b^2(1 - e^2)$

$\Rightarrow \frac{1}{9} = (1 - e^2)$

$\Rightarrow e^2 = \frac{8}{9}$

$\Rightarrow e = \frac{2\sqrt{2}}{3}$

112. **(a)** Sum of odd numbers between 1 and 1000 , which is divisible by

$3 = 3 + 9 + 15 + 21 + 27 + \cdots . +999 = S$ (let)

$\therefore$ Let n be the number of terms in series and a is first term.

$\therefore l = a + (n - 1)d$

where l is last term and d is is common difference.

$999 = 3 + (n - 1) \times 6$

$n - 1 = \frac{999 - 3}{6} = \frac{996}{6}$

$\Rightarrow n - 1 = 166$

$\Rightarrow n = 167$

$\therefore S = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{167}{2}[2 \times 3 + (167 - 1) \times 6]$

$= \frac{167}{2}[1002] = 167 \times 501 = 83667$

113. **(c)** The numbers between 999 and 10000 are all 4-digit numbers. The number of 4 - digit numbers formed by digits 0,2,3,6,7,8 is ${}^6P_4 = 360$.

But here those numbers are also involved which begin from 0 . So, we take those numbers as three-digit numbers.

Taking initial digit 0 , the number of ways to fill remaining 3 places from five digits 2,3,6,7,8 are ${}^5P_3 = 60$

So, the required numbers $= 360 - 60 = 300$.

114. **(a)** Let $P(0,7,10), Q(-1,6,6)$ and $R(-4,9,6)$ be the vertices of a triangle

Here, $PQ = \sqrt{1 + 1 + 16} = 3\sqrt{2}$

$QR = \sqrt{9+9+0} = 3\sqrt{2}$

$PR = \sqrt{16+4+16} = 6$

Now, $PQ^2 + QR^2 = (3\sqrt{2})^2 + (3\sqrt{2})^2 = 36 = (PR)^2$

Therefore, $\triangle$ PQR is a right -angled triangle at Q. Also, $OQ = QR$.

Hence, $\triangle$ PQR is a right -angled isosceles triangle.

115. **(c)** Let α, β be the roots of the equation.

$\therefore \alpha + \beta = a - 2$ and $\alpha\beta = -(a+1)$

Now, $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (a-2)^2 + 2(a+1) = (a-1)^2 + 5$

$\therefore \alpha^2 + \beta^2$ will be minimum if $(a-1)^2 = 0$

i.e. $a = 1$.

116. **(b)**

117. **(c)** Let feet of perpendicular is

$(2\alpha, 3\alpha + 2, 4\alpha + 3)$

$\Rightarrow$ Direction ratio of the $\perp$ line is

$2\alpha - 3, 3\alpha + 3, 4\alpha - 8$. and

$\Rightarrow$ Direction ratio of the line are 2,3,4

$\Rightarrow 2(2\alpha - 3) + 3(3\alpha + 3) + 4(4\alpha - 8) = 0$

$\Rightarrow \alpha = 1$

$\Rightarrow$ Feet of $\perp$ is (2,5,7)

$\Rightarrow$ Length $\perp$ is $\sqrt{1^2 + 6^2 + 4^2} = \sqrt{53}$

118. **(a)** According to the condition,

$\frac{dy}{dx} = \frac{y}{x} - \cos^2\frac{y}{x} \ldots$ (i)

This is a homogeneous differential equation Substituting $y = vx$, we get

$$v + x\frac{dv}{dx} = v - \cos^2 v$$

$$\Rightarrow x\frac{dv}{dx} = -\cos^2 v$$

$$\Rightarrow \int \sec^2 v dv = -\int \frac{dx}{x}$$

$\Rightarrow \tan v = -\log x + C$

$\Rightarrow \tan\frac{y}{x} + \log x = C$

Substituting $x = 1, y = \frac{\Pi}{4}$, we get $C = 1$

Thus, we get

$\tan\left(\frac{y}{x}\right) + \log x = 1$

which is the required solution,

119. **(b)** $\underline{1 - \cos\alpha + \sin\alpha}$

$1 + \sin\alpha$

$$= \frac{1 - \cos\alpha + \sin\alpha}{1 + \sin\alpha} \cdot \frac{1 + \cos\alpha + \sin\alpha}{1 + \cos\alpha + \sin\alpha}$$

$$= \frac{(1 + \sin\alpha)^2 - \cos^2\alpha}{(1 + \sin\alpha)(1 + \cos\alpha + \sin\alpha)}$$

$$= \frac{(1 + \sin^2\alpha + 2\sin\alpha) - (1 - \sin^2\alpha)}{(1 + \sin\alpha)(1 + \cos\alpha + \sin\alpha)}$$

$$= \frac{2\sin\alpha}{1 + \cos\alpha + \sin\alpha} = y$$

120. **(a)** The $(r + 1)$ th term in the expansion of $\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$ is given by

$$T_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \left(\frac{3}{2x^2}\right)^r$$

$$= {}^{10}C_r \frac{x^{5-\left(\frac{r}{2}\right)}}{3^{5-\left(\frac{r}{2}\right)}} \cdot \frac{3^r}{2^r x^{2r}}$$

$$= {}^{10}C_r \frac{3\left(\frac{3r}{2}\right) - 5}{2^r} x - \left(\frac{5r}{2}\right)$$

For T_{r+1} to be independent of x, we must have $5 - \left(\frac{5r}{2}\right) = 0$

or $r = 2$.

Thus, the 3rd term is independent of x and is equal to

$${}^{10}C_2 \frac{3^{3-5}}{2^2} = \frac{10 \times 9}{2} \times \frac{3^{-2}}{4} = \frac{5}{4}$$

121. **(b)** $2^x + 2^y = 2^{x+y}$

Differentiating both sides

$$\ln 2.2^x + \ln 2.2^y \frac{dy}{dx} = \ln 2.2^{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$2^x + 2y\frac{dy}{dx} = 2^{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$2^x + 2^y\frac{dy}{dx} = 2^{x+y} + 2^{x+y}\frac{dy}{dx}$$

$$(2^y - 2^{x+y})\frac{dy}{dx} = (2^{x+y} - 2^x)$$

$$\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y - 2^{x+y}}$$

$$\frac{dy}{dx} = \frac{2^x(2^y - 1)}{2^y(1 - 2^x)}$$

$$\frac{dy}{dx} = 2^{x-y}\left(\frac{2^y - 1}{1 - 2^x}\right)$$

122. **(c)** Let $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$

Put $t = \tan\theta$, we get

$x = \frac{1-\tan^2\theta}{1+\tan^2\theta}$ and $y = \frac{2\tan\theta}{1+\tan^2\theta}$

$\Rightarrow x = \cos 2\theta$ and $y = \sin 2\theta$

$\therefore \frac{dx}{d\theta} = -2\sin 2\theta$ and $\frac{dy}{d\theta} = 2\cos 2\theta$

Now, $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = -\frac{\cos 2\theta}{\sin 2\theta} = -\frac{x}{y}$

123. **(d)** The equation of the given curve is

$y = 4x^3 - 2x^5$

$$\frac{dy}{dx} = 12x^2 - 10x^4$$

Therefore, the slope of the tangent at point (x, y) is $12x^2 - 10x^4$.

The equation of the tangent at (x, y) is given by $Y - y = (12x^2 - 10x^4)(X - x)$ ... (i)

When, the tangent passes through the origin $(0,0)$, then $X = Y = 0$

Therefore, eq. (i) reduce to

$-y = (12x^2 - 10x^4)(-x)$

$\Rightarrow y = 12x^3 - 10x^5$

Also, we have $y = 4x^3 - 2x^5$
$\therefore 12x^3 - 10x^5$

$= 4x^3 - 2x^5$

$\Rightarrow 8x^5 - 8x^3 = 0$

$\Rightarrow x^5 - x^3 = 0$

$\Rightarrow x^3(x^2 - 1) = 0$

$\Rightarrow x = 0, \pm 1$

When, $x = 0$,

$y = 4(0)^3 - 2(0)^5 = 0$

When, $x = 1$,

$y = 4(1)^3 - 2(1)^5 = 2$

When, $x = -1$,

$y = 4(-1)^3 - 2(-1)^5 = -2$

Hence, the require points are $(0,0), (1,2)$ and $(-1,-2)$.

124. **(c)** Let r be the radius of the sphere and Δr be the error in measuring the radius. Then, $r = 9$ cm and $\Delta r = 0.03$ cm Let V be the volume of the sphere. Then,

$$V = \frac{4}{3}\Pi r^3$$

$$\Rightarrow \frac{dV}{dr} = 4\Pi r^2$$

$$\Rightarrow \left(\frac{dV}{dr}\right)_{r=9} = 4\Pi \times 9^2 = 324\Pi$$

Let ΔV be the error in V due to error Δr in r.

Then,

$$\Delta V = \frac{dV}{dr}\Delta r$$

$$\Rightarrow \Delta V = 324\Pi \times 0.03 = 9.72\pi cm^3$$

125. **(c)** Let $z = x + iy$, then its conjugate $\overline{z} = x - iy$

Given that $z^2 = (\overline{z})^2$

$\Rightarrow x^2 - y^2 + 2ixy = x^2 - y^2 - 2ixy$

$\Rightarrow 4ixy = 0$

126. **(b)** If f(x) is an even function then

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

Here $f(x) = x^8 - x^4 + x^2 + 1$ is an even function, therefore $a = 4$.

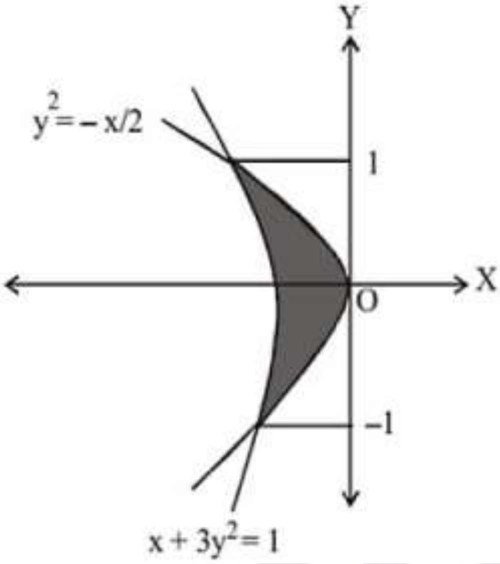
127. **(d)** We have, $x + 2y^2 = 0$

$\Rightarrow y^2 = -\frac{x}{2} \ldots$ (i)

a parabola with vertex $(0,0)$ and $x + 3y^2 = 1$

$\Rightarrow y^2 = \frac{1-x}{3} = -\left(\frac{x-1}{3}\right) \ldots$ (ii), a parabola with vertex $(1,0)$

Solving (i) and (ii), we get $y = \pm 1$



$A = \int_{-1}^{1}\left((1 - 3y^2) - (-2y^2)\right)dy$

$= 2\int_{0}^{1}(1 - y^2)dy = 2\left[y - \frac{y^3}{3}\right]_0^1$

$= \frac{4}{3}$ sq. units

128. **(a)**

We have $\frac{d}{dx}\left(\tan^{-1}\sqrt{\frac{1+\cos\frac{x}{2}}{1-\cos\frac{x}{2}}}\right)$

$$=\frac{d}{dx}\left[\tan^{-1}\sqrt{\frac{1+\left(2\cos^2\frac{x}{4}-1\right)}{1-\left(1-2\sin^2\frac{x}{4}\right)}}\right]$$

$$=\frac{d}{dx}\left(\tan^{-1}\sqrt{\frac{2\cos^2\frac{x}{4}}{2\sin^2\frac{x}{4}}}\right)$$

$$=\frac{d}{dx}\left(\tan^{-1}\sqrt{\cot^2\frac{x}{4}}\right)=\frac{d}{dx}\left(\tan^{-1}\cot\frac{x}{4}\right)$$

$$=\frac{d}{dx}\left(\tan^{-1}\tan\left(\frac{\pi}{2}-\frac{x}{4}\right)\right)=\frac{d}{dx}\left(\frac{\pi}{2}-\frac{x}{4}\right)=0-\frac{1}{4}=-\frac{1}{4}$$

129. **(a)** $\because f(x)=\left(\frac{e^{2x}-1}{e^{2x}+1}\right)$

$\therefore f(-x)=\frac{e^{-2x}-1}{e^{-2x}+1}=\frac{1-e^{2x}}{1+e^{2x}}$

$\Rightarrow f(-x)=\frac{-(e^{2x}-1)}{e^{2x}+1}=-f(x)$

$\therefore$ f(x) is an odd function.

Again, $f'(x)=\frac{4e^{2x}}{(1+e^{2x})^2}>0, \forall x\in R$

$\Rightarrow$ f(x) is an increasing function.

130. **(a)** We have, $\int_0^{\pi}\frac{1}{5+4\cos x}dx$

$$=\int_0^{\pi}\frac{1}{5+4\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$=\int_0^{\pi}\frac{1+\tan^2\frac{x}{2}}{5\left(1+\tan^2\frac{x}{2}\right)+4\left(1-\tan^2\frac{x}{2}\right)}dx$$

$$=\int_0^{\pi}\frac{1+\tan^2\frac{x}{2}}{9+\tan^2\frac{x}{2}}dx=\int_0^{\pi}\frac{\sec^2\frac{x}{2}}{9+\tan^2\frac{x}{2}}dx$$

Let $\tan\frac{x}{2}=t\Rightarrow\frac{1}{2}\sec^2\frac{x}{2}dx=dt$

Also, $x=0\Rightarrow t=0$ and $x=\Pi\Rightarrow t=\infty$

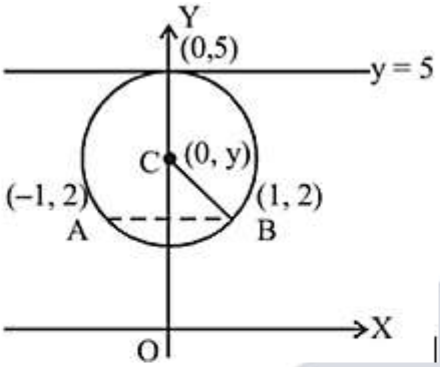
$$\therefore I=\int_0^{\infty}\frac{dt}{9+t^2}$$

$$\therefore I=2\int_0^{\infty}\frac{dt}{9+t^2}$$

$$\therefore I=\frac{2}{3}\left[\tan^{-1}\frac{t}{3}\right]_0^{\infty}=\frac{2}{3}[\tan^{-1}\infty-\tan^{-1}0]$$

$$=\frac{2}{3}\left(\frac{\pi}{2}-0\right)=\frac{\pi}{3}$$

131. **(a)**



The centre of the circle is on the perpendicular bisector of the line joining $(-1,2)$ and $(1,2)$, which is the y - axis. The ordinate of the centre is given by

$$(5-y)^2=1+(y-2)^2$$

$$\Rightarrow y=\frac{10}{3}$$

Hence, eq. of the circle is :

$$x^2+\left(y-\frac{10}{3}\right)^2=\left(\frac{5}{3}\right)^2$$

$$\Rightarrow 9x^2+9y^2-60y+75=0$$

132. **(a)** Expression $=(1-x)^5\cdot(1+x)^4(1+x^2)^4$

$=(1-x)(1-x^2)^4(1+x^2)^4$

$=(1-x)(1-x^4)^4$

$\therefore$ Coefficient of $x^{13}=-{}^4C_3(-1)^3=4$

133. **(b)** The given differential equation can be written as

$$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$$

Now, eq. (i) is a linear differential equation of the form

$$\frac{dx}{dy}+P_1x=Q_1$$

where $P_1=\frac{1}{1+y^2}$ and $Q_1=\frac{\tan^{-1}y}{1+y^2}$

Therefore, I.F $= e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$

Thus, the solution of the given differential equation is given by

$$xe^{\tan^{-1} y} = \int \left(\frac{\tan^{-1} y}{1+y^2}\right) e^{\tan^{-1} y} dy + C$$

Let $I = \int \left(\frac{\tan^{-1} y}{1+y^2}\right) e^{\tan^{-1} y} dy$

On substituting $\tan^{-1} y = t$, so that

$\left(\frac{1}{1+y^2}\right) dy = dt$, we get

$I = \int te^t dt = e^t - \int 1 \cdot e^t dt = te^t - e^t = e^t(t-1)$

or $I = e^{\tan^{-1} y}(\tan^{-1} y - 1)$

On substituting the value of 1 in equation (i1), we get

$x \cdot e^{\tan^{-1} y} =. e^{\tan^{-1} y}(\tan^{-1} y - 1) + C$

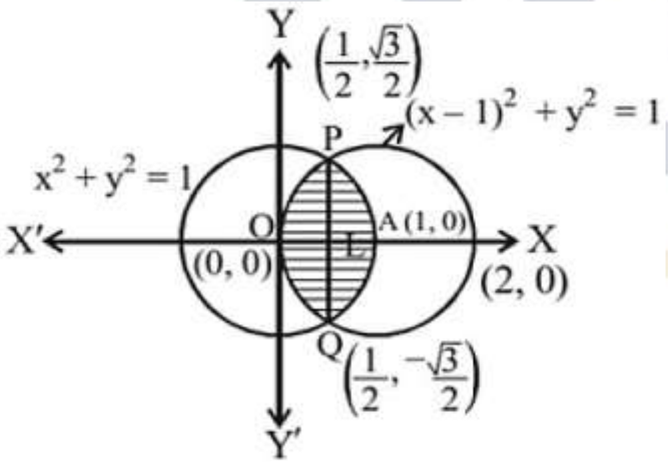
or $x = (\tan^{-1} y - 1) + Ce^{\tan^{-1} y}$

which is the general solution of the given differntial equation.

134. **(a)** Given circles are $x^2 + y^2 = 1$ ... (i)

and $(x-1)^2 + y^2 = 1$ ... (ii)

Centre of (i) is $O(0,0)$ and radius $= 1$



Both these circle are symmetrical about x-axis solving (i) and (ii), we get, $-2x + 1 = 0 \Rightarrow x = \frac{1}{2}$

then $y^2 = 1 - \left(\frac{1}{2}\right)^2 = 34$

$$\Rightarrow y = \frac{\sqrt{3}}{2}$$

∴ The points of intersection are

$P\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ and $Q\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$

It is clear from the figure that the shaded portion in region whose area is require(d)

$\therefore$ Required area = area OQAPO

= 2 × area of the region OLAP

= 2 × (area of the region OLPO + area of LAPL)

$$= 2\left[\int_0^{\frac{1}{2}}\sqrt{1-(x-1)^2}dx + \int_{\frac{1}{2}}^{1}\sqrt{1-x^2}dx\right]$$

$$= 2\left[\frac{(x-1)\sqrt{1-(x-1)^2}}{2} + \frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}} + 2\left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}$$

$$= -\frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \sin^{-1}\left(\frac{-1}{2}\right) - \sin^{-1}(-1) + 0 + \sin^{-1}(1) - \left(\frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \sin^{-1}\left(\frac{1}{2}\right)\right)$$

$= \left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$ sq. units.

135. **(d)** Since, sum $= 4$ and second term $= \frac{3}{4}$

$\Rightarrow \frac{a}{1-r} = 4$, and $ar = \frac{3}{4}$

$$\Rightarrow \frac{a}{1-\frac{3}{4a}} = 4$$

$\Rightarrow (a-1)(a-3) = 0$

$\Rightarrow a = 1$ or $a = 3$

136. **(b)** $\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$

Let $xe^x = t$

$$\Rightarrow (xe^x + e^x) = \frac{dt}{dx}$$

$$\Rightarrow dx = \frac{dt}{e^x(x+1)}$$

$$\therefore \int \frac{e^x(1+x)}{\cos^2(e^x x)}dx = \int \frac{e^x(1+x)}{\cos^2 t} \times \frac{dt}{e^x(1+x)} = \int \frac{1}{\cos^2 t}dt = \int \sec^2 t dt = \tan t + C = \tan(x^x) + C$$

137. **(d)** $B + C = \frac{\pi}{2} - A \Rightarrow \tan(B+C) = \cot A$

$$\Rightarrow \frac{\tan B + \tan C}{1 - \tan B \tan C} = \cot A$$

$\Rightarrow \tan A \tan B + \tan B \tan C + \tan C \tan A = 1$

$\Rightarrow \tan A \tan B \tan C(\cot C + \cot A + \cot B) = 1$

$\Rightarrow \cot A + \cot B + \cot C = \cot A \cot B \cot C$

$$\begin{aligned}
&\text{Again } \cos 2A + \cos 2B + \cos 2C \\
&= 2\cos(A+B)\cos(A-B) + \cos 2C \\
&= 2\cos\left(\frac{\pi}{2} - C\right)\cos(A-B) + 1 - 2\sin^2 C \\
&= 2\sin C[\cos(A-B) - \sin C] + 1 \\
&= 2\sin C\left[\cos(A-B) - \sin\frac{\Pi}{2} - (A+B)\right] + 1 \\
&= 2\sin C[\cos(A-B) - \cos(A+B)] + 1 \\
&= 4\sin A \sin B \sin C + 1
\end{aligned}$$

138. **(b)**

$$\begin{aligned}
&\text{Consider } \lim_{x\to 0} \frac{2\sin^2 3x}{x^2} \\
&= 2 \cdot \lim_{x\to 0}\left[\frac{\sin 3x}{x}\right]^2 = 2 \cdot \lim_{x\to 0}\left[3\frac{\sin 3x}{3x}\right]^2 \\
&= 2.9\lim_{x\to 0}\left(\frac{\sin 3x}{3x}\right)^2 = 18 \times 1 = 18
\end{aligned}$$

139. **(c)** Putting $H = \frac{2ab}{a+b}$, we have

$$\frac{1}{H-a} + \frac{1}{H-b}$$

$$= \frac{1}{\left(\frac{2ab}{a+b} - a\right)} + \frac{1}{\left(\frac{2ab}{a+b} - b\right)} = \frac{a+b}{ab-a^2} + \frac{a+b}{ab-b^2}$$

$$= \left(\frac{a+b}{b-a}\right)\left(\frac{1}{a} - \frac{1}{b}\right) = \left(\frac{a+b}{b-a}\right)\left(\frac{b-a}{ab}\right) = \frac{a+b}{ab} = \frac{1}{a} + \frac{1}{b}$$

140. **(b)** The desired probability

$$= \frac{\frac{10}{100} \times \frac{70}{100}}{\frac{10}{100} \times \frac{70}{100} + \frac{5}{100} \times \frac{30}{100}} = \frac{14}{17}$$

141. **(c)**

$$\begin{aligned}
&y^2 = x(2-x)^2 \Rightarrow y^2 = x^3 - 4x^2 + 4x \ldots \text{(i)} \\
&\Rightarrow 2y\frac{dy}{dx} = 3x^2 - 8x + 4 \\
&\Rightarrow \frac{dy}{dx} = \frac{3x^2 - 8x + 4}{2y} \\
&\Rightarrow \left[\frac{dy}{dx}\right]_P = \frac{3 - 8 + 4}{2} = -\frac{1}{2}
\end{aligned}$$

$\therefore$ Equation of tangent at P is: $y-1=-\frac{1}{2}(x-1)$

$\Rightarrow x+2y-3=0$

Using $y=\frac{3-x}{2}$ in (i), we get: $\left(\frac{3-x}{2}\right)^2$

$$=x^3-4x^2+4x$$
$$\Rightarrow 4x^3-17x^2+22x-9=0..(ii)$$

which has two roots 1,1 (Because of (ii) being tangent at $(1,1)$).
Sum of 3 roots $=\frac{17}{4}$
$\therefore$ 3 rd root $=\frac{17}{4}-2=\frac{9}{4}$
Then, $y=\frac{3-\frac{9}{4}}{2}=\frac{3}{8}$
$\therefore$ Q is $\left(\frac{9}{4},\frac{3}{8}\right)$

142. **(d)** Given $\alpha=45^\circ,\beta=60^\circ,\gamma=?$

$\because \cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\therefore \cos^2\gamma=1-\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$

$\Rightarrow \gamma=60^\circ$ or 120°

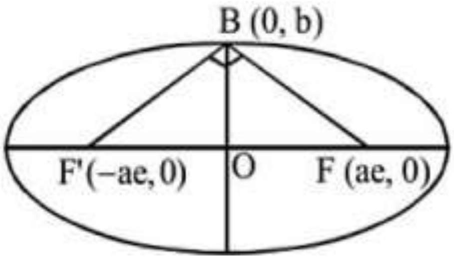
143. **(a)** $\because \angle FBF'=90^\circ$

$\Rightarrow FB^2+F'B^2=FF'^2$

$\therefore \left(\sqrt{a^2e^2+b^2}\right)^2+\left(\sqrt{a^2e^2+b^2}\right)^2=(2ae)^2$

$\Rightarrow 2(a^2e^2+b^2)=4a^2e^2$

$\Rightarrow e^2=\frac{b^2}{a^2}$



Also $e^2=1-\frac{b^2}{a^2}=1-e^2$

$\Rightarrow 2e^2=1$

$\Rightarrow e=\frac{1}{\sqrt{2}}$.

144. **(a)**

$$\cos[2\cos^{-1}x+\sin^{-1}x]$$
$$=\cos[\cos^{-1}x+\cos^{-1}x+\sin^{-1}x]$$
$$=\cos\left[\cos^{-1}x+\frac{\pi}{2}\right]=-\sin[\cos^{-1}x]$$
$$=-\sin\left[\sin^{-1}\sqrt{1-x^2}\right]=-\sqrt{1-x^2}$$
$$=-\sqrt{1-\left(\frac{1}{5}\right)^2}=-\sqrt{\frac{24}{25}}=-\frac{2\sqrt{6}}{5}$$

145. **(a)** Since $\vec{a}=m\vec{b}$ for some scalar m i.e.,

$$\vec{a}=m\left(6\hat{\imath}-8\hat{\jmath}-\frac{15}{2}\hat{k}\right)$$

$$\Rightarrow |a|=|m|\sqrt{36+64+\frac{225}{4}}$$

$$\Rightarrow 50=\frac{25}{2}|m|$$

$$\Rightarrow |m|=4$$

$$\Rightarrow m=\pm 4$$

Since, a makes an acute angle with the positive direction of Z - axis, so its z component must be positive and hence, m must be -4 .

$$\therefore a=-4\left(6\hat{\imath}+8\hat{\jmath}-\frac{15}{2}\hat{k}\right)=-24\hat{\imath}+32\hat{\jmath}+30\hat{k}$$

146. **(b)** $np=4$

$npq=2$

$$\Rightarrow q=\frac{1}{2},p=\frac{1}{2},n=8$$

$$P(X=1)={}^{8}C_{1}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^7=8\cdot\frac{1}{2^8}=\frac{1}{2^5}=\frac{1}{32}$$

147. **(a)** We prove the statement p is true by contrapositive method and by direct metho(d)

Direct method: For any real number x and y,

$x=y$

$\Rightarrow 2x=2y$

$\Rightarrow 2x+a=2y+a$ a for some $a\in Z$

Contrapositive method: The contrapositive statement of p is "For any real numbers x, y if $2x+a\neq 2y+a$, where $a\in Z$, then $x\neq y$."

Given, $2x+a\neq 2y+a$

$\Rightarrow 2x \neq 2y$

$\Rightarrow x \neq y$

Hence, the given statement is true.

148. (a)

$$z_1 = \sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\Pi}{4}\right] = \sqrt{2}\left[\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right] = 1 + i$$

$$|z_1| = \sqrt{2}$$

and $z_2 = \sqrt{3}\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right] = \sqrt{3}\left[\frac{1}{2} + i\frac{\sqrt{3}}{2}\right]$

$$|z_2| = \sqrt{\frac{3}{4} + \frac{9}{4}} = \sqrt{3}$$

$$|z_1 z_2| = |z_1||z_2| = \sqrt{2} \cdot \sqrt{3} = 6$$

149. (b) $\lim_{x\to 0}\sqrt{\frac{x-\sin x}{x+\sin^2 x}} = \lim_{x\to 0}\sqrt{\frac{1-\frac{\sin x}{x}}{1+\frac{\sin^2 x}{x}}}$

$$= \lim_{x\to 0}\sqrt{\frac{1-\frac{\sin x}{x}}{1+\left(\frac{\sin x}{x}\right)\sin x}} = \sqrt{\frac{1-1}{1+1\times 0}} = 0$$

150. (d) We have,

$$\frac{3x+1}{(x-3)(x-5)} = \frac{-5}{x-3} + \frac{B}{x-5}$$

$3x + 1 = -5(x-5) + B(x-3)$

Put $x = 5$

$3(5) + 1 = B(5-3)$

$16 = 2B$ or $B = 8$