1. **(a)** $eV_0 = h(v - v_0) = hv - hv_0$

$V_0 = \left(\frac{h}{e}\right)v - \left(\frac{h}{v}\right)v_0$

From Eq. (i), it follows that the slope of the graph is $\frac{h}{e}$.

Therefore,

$h = e \times \text{slope} = 1.6 \times 10^{-19} \times \frac{165-0}{(8-4)\times 10^{15}}$

$= 16 \times 10^{-19} \times \frac{16.5}{4} \times 10^{15} = 6.6 \times 10^{-34}\ \text{J} - \text{s}.$

2. **(b)** Given, first resonance column at deoth, $l_1 = 24$ cm

Second resonance column at depth, $l_2 = 78$ cm

Third resonance column at depth, $l_3 = ?$

$\text{As} l_1 + x = \frac{\lambda}{4} = 24.$

$l_2 + x = \frac{3\lambda}{4} = 78$

$l_3 + x = \frac{5\lambda}{4}$

From Eqs. (i) and (ii),

$x = \frac{l_2 - 3l_1}{2} = \frac{78-3(24)}{2} = 3.$

substituting l_1 and x value, we get

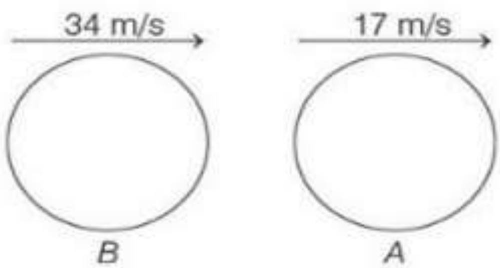
$l_3 = 5(24) + 4(3)$

$\Rightarrow l_3 = 132$ cm.

3. **(a)** Given, velocity of submarine A, $v_A 17$ m/s

Velocity of Submarine B, $v_B = 34$ m/s

Signal sent by submarine B is detected by submarine A can be shown as



Frequency of the signal, $f_0 = 600$Hz

speed of sound in water $v_s = 1500\ \text{ms}^{-1}$

Frequency received by submarine A is

$$f_1 = \left(\frac{v_S - v_A}{v_S - v_B}\right) f_0 = \left[\frac{1500 - 17}{1500 - 34}\right] \times 600\text{hz}$$

$$f_1 = \frac{1483}{1466} \times 600$$

Frequency received by submarine is

$$f_2 = \left(\frac{v_S + v_B}{v_{svA}}\right) f_1$$

Substituting given values and f_1 value from Eq. (i), we get

$$f_2 = \left(\frac{1500 + 34}{1500 + 17}\right) \times \left[\frac{1483}{1466} \times 600\right]$$

$$f_2 = 1.0112 \times 1.0115 \times 600$$

$$f_2 = 613.7\text{Hz}.$$

4. (c) The velocity if transverse waves is given by

$v = \sqrt{\frac{T}{m}}$ where, $T =$ Tension and $m =$ mass per unit length of the wire. If r is the radius if the wire and ρ its density then,

$$m = \pi r^2 \rho$$

$$\therefore v = \sqrt{\frac{T}{m}} = \sqrt{\frac{T}{\Pi r^2 \rho}}$$

$$\therefore v_A = \frac{\sqrt{T_A}}{r_A\sqrt{\Pi\rho}}$$

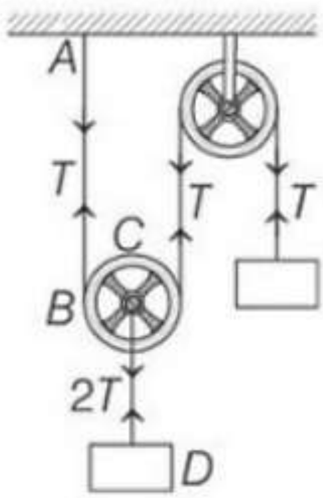
and $v_B = \frac{\sqrt{T_B}}{r_B\sqrt{\Pi\rho}}$

Now, $\frac{v_a}{v_B} = \sqrt{\frac{T_A}{T_B}} \cdot \frac{r_B}{r_A}$

$\because r_A = 2r_B \text{and}_A = \frac{1}{2}\ T_B$

Hence, $\frac{v_A}{v_B} = \frac{1}{2\sqrt{2}}$

5. (d)



Let the speed of wave in string be v and tension in string b e T. $v \propto \sqrt{T}$

$$\frac{v_{AB}}{v_{CD}} = \frac{\sqrt{T}}{2\,T} = \frac{1}{\sqrt{2}}$$

6. **(d)** Acceleration due to gravity at the surface of earth,

$$g = G\frac{M}{R^2} = \frac{4}{3}\Pi\rho GR$$

$$\left[\because M = \frac{4}{3}\pi R^3\rho\right]$$

Where, ρ = density of earth.

Acceleration due to gravity at depth d from the surface of earth,

$$g' = \frac{4}{3}\Pi\rho G(R-d)$$

From Eqs. (i) and (ii), we get

$$g' = g\left[1 - \frac{d}{R}\right]$$

7. **(c)** Given, $I = 6 + 10\sin\omega t$

$$I_{eff} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{\frac{1}{2}} = \left[\frac{1}{T}\int_0^T (6 + 10\sin\omega t)^2 dt\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{T}\int_0^T (36 + 120\sin\omega t + 100\sin^2\omega t)dt\right]^{\frac{1}{2}}$$

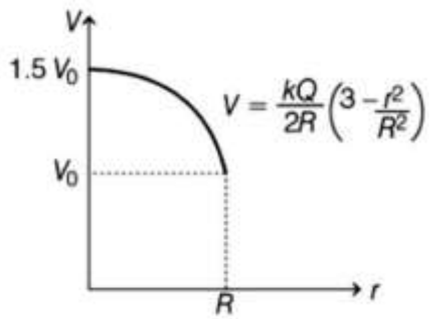
But as, $\frac{1}{T}\int_0^T \sin\omega t dt = 0$ and $\frac{1}{T}\int_0^T \sin^2 Wt = \frac{1}{2}$

$$\Rightarrow I_{eff} = \left[36 + \frac{1}{2}\times 100\right]^{\frac{1}{2}} = 0.97$$

Thus, $I_{eff} = 9.27$ A

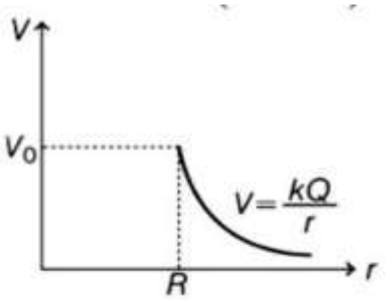
8. (d) Electric potential at centre $R=0$

$V_{centre} = 1.5V_{surface}$



where, $V_0=\frac{kQ}{R}$

$$V_{inside} = \frac{kQ}{2R}\left(3-\frac{r^2}{R^2}\right)$$



$$V_{outside} = k\frac{Q}{r} \propto \frac{1}{r}$$

9. (d) For an insulator, $E_g > 3eV$, that is why electron transition from valence band to conduction band is not possible. For semiconductor E_g is 0.2eV to 0.3eV, while for metals $E_g > 5eV$.

10. (b) Work done by the gun = total kinetic energy of the bullets

$$= n\frac{1}{2}mv^2 = 300 \times \frac{1}{2} \times 10 \times 10^{-3} \times (600)^2$$

$$= 150 \times 10 \times 10^{-3} \times 600 \times 600$$

Power of gun $= \frac{\text{work done}}{\text{Time taken}}$

$$= \frac{150 \times 10 \times 10^{-3} \times 600 \times 600}{60\ s}$$

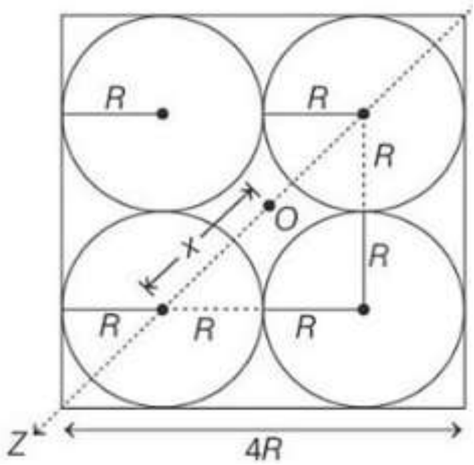
$= 9\ kW (\because 1 \text{ minute} = 60 \text{ seconds })$

11. (c) Area mass density, $\sigma = \frac{M}{16R^2} (\because \text{Area} = 4R \times 4R = 16R^2)$

Mass of each hole $m_1 = \sigma \Pi R^2 = \frac{M}{16R^2}\Pi R^2 = \frac{\Pi M}{16}$

Distance between centre of plate and centre of hole

$x = \frac{\sqrt{(2R)^2 + (2R)^2}}{2} = \frac{2\sqrt{2}R}{2}$



Moment of inertia of one hole at about Z-axis

$I_1 = \frac{1}{2}m_1R^2 + m_1x^2 = \frac{5\pi}{32}MR^2$

Moment of inertia of whole plate about Z-axis, $I = \frac{M(4R)^2}{6} = \frac{8}{3}MR^2$

Required moment of Inertia

$I_0 = I - 4I_1, = \left[\frac{8}{3} - 4\left(\frac{5\Pi}{32}\right)\right]MR^2 = \left[\frac{8}{3} - \frac{5\Pi}{8}\right]MR^2$

Given, $R = 5$ cm and $M = 1$ kg

So, $I_0 \left[\frac{8}{3} - \frac{5\Pi}{8}\right] 1 \times 25 \times 10^{-4} = 0.0017\ \text{kg} - \text{m}^2$

12. (b) According to the law of conservation of linear momentum at the highest point,

$mv\cos\theta = \frac{m}{2}(v\cos\theta) + \frac{m}{2}v'$

$\Rightarrow mv\cos\theta = \frac{m}{2}v\cos\theta = \frac{m}{2}v'$

$\Rightarrow v' = 2\left(v\cos\theta - \frac{v\cos\theta}{2}\right) = v\cos\theta$

13. (b) $I_{\text{initial}} = 0$ for $V_i \leq +1$ V

This diode conduct only beyond $V_i = +1$ V

$I_{\text{final}} = \frac{5}{250} = 0.02$ A

So, change in $I = 0.02$ A $= 20$ mA

14. (b) The de-Brogile wavelength, $\lambda = \frac{h}{mv}$

Here, $\lambda_e = \frac{h}{m_e \frac{c}{3}}$ and $\lambda_p = \frac{h}{m_p c}$

Given, $\lambda_e = \lambda_p$

So, $\frac{h}{m_e \frac{c}{3}} = \frac{h}{m_p c}$

$\Rightarrow \frac{m_e}{m_p} = 3$

Ratio of KE $= \frac{K_e}{K_p} = \frac{\frac{1}{2} m_e v_e{}^2}{\frac{1}{2} m_p v_p{}^2} = \frac{1}{3} \left(\because \frac{m_e}{m_p} = 3, v_e = \frac{c}{3} \text{ and } v_p = c \right)$

15. (b) Given,

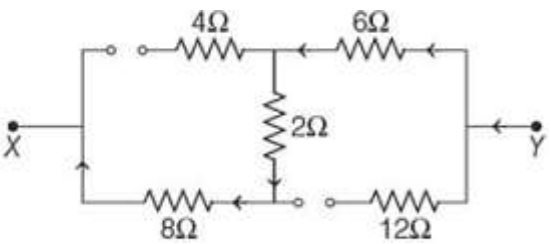
$R_1 = 8\Omega$
$R_2 = 2\Omega$
$L = 5\text{mH}$

$C = 50\mu F$
and $V_0 = 20\cos(2000t)$
Independence, $Z = \sqrt{R^2 + (X_L - X_C)^2}$
As, here, $X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10\Omega$
Similarly, $X_L = \frac{1}{\omega} C = \frac{1}{2000 \times 50 \times 10^{-6}} = 10\Omega$
$\because X_L = X_C$, Hence, $i_{max} = \frac{V_0}{Z} = \frac{20}{(8+2)} = 2 \text{ A}$
Hence, $i_{rms} = \frac{i_{max}}{\sqrt{2}} = \frac{2}{\sqrt{2}} = 1.41 \text{ A}$
and $V = R_2 i_{rms} = 1.41 \text{ A} \times 2$
$= 282 \text{ V}$

16. (a) As we know that, $\frac{E}{B} = c$

so, $B = \frac{E}{C} = \frac{8}{3} \times 10^8 = 266 \times 10^{-8}$

17. (c) According to the given figure X is at lower potential w.r.t γ. Hence, both diodes are in revere biasing, so, equivalent citcuit can be redrawn as follows



Equivalent resistance between X and Y

$R_{eq} = 8 + 2 + 6 = 16\Omega$

18. (d) Let the insulated container has n moles of the monoatomic gas. The loss of kinetic energy of the gas,

$$\Delta E_K = \frac{1}{2}(mv)v^2$$

If the change in temperature of the gas is ΔT, then heat gained by the gas is,

$$\Delta Q = \frac{3}{2}nR\Delta T$$

Now, according to question,

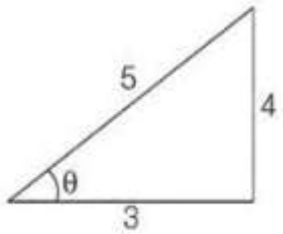
$$\Delta Q = \Delta E_K$$
$$\frac{3}{2}nR\Delta T = \frac{1}{2}mnv^2$$

$$\Rightarrow \Delta T = \frac{mv^2}{3R}$$

19. (b) Given, $v = 3\hat{i} + 4\hat{j}$

$$\therefore v = |v| = \sqrt{3^2 + 4^2} \Rightarrow v = 5\ m/s$$

From figure,



$$\sin\theta = \frac{4}{5} \text{ and } \cos\theta = \frac{3}{5}$$
$$\because R = \frac{v^2 \sin 2\theta}{g}$$
$$= \frac{v^2 \cdot 2\sin\theta \cdot \cos\theta}{g}$$
$$= \frac{5 \times 5 \times 2 \times \frac{4}{5} \times \frac{3}{5}}{10} = 24\ m$$

20. (b) Given, $V_{Cc} = 8\ V$

Voltage drop, $V_0 = \Delta I_C R_L = 0.6\ V$

Load Resistor, $R_L = 500\Omega$

Gain factor, $\alpha = 0.96$

$$\because \beta = \frac{\alpha}{1-\alpha} = \frac{0.96}{1-0.96} = 24$$

Now, $V_0 = \Delta IC_L$

$$0.6 = \Delta I_C \times 500$$

$\Delta I_C = \frac{0.6}{500} = 1.2 \times 10^{-3}$ A

Now, $\beta = \frac{\Delta I_C}{\Delta I_B}$

$\therefore$ Base current $\Delta I_B = \frac{\Delta I_C}{\beta} = \frac{1.2\times10^{-3}}{24} = 5 \times 10^{-5}$

$= 50 \times 10^{-6}$

$\Delta I_B = 50\mu A$

21. (d) Given, mass of solid sphere = 80 kg

radius of solid sphere, $R_s = 15$ m

radius of circular disc, $R_c = 20$ m

and time = 1 hour = 60 minutes = 60×60sec

$\therefore$ Moment if inertia of solid sphere, $I_s = \frac{2}{5}MR^2$

$= \frac{2}{5} \times 80 \times (15)^2$

$= 7200$ kg $-$ m^2

Similarly,

moment of inertia of the disc, $I_c = \frac{1}{2}MR_c{}^2$

$= \frac{1}{2} \times 80 \times (20)^2$

$= 16000$ kg $-$ m^2

Rate of change of moment of Inertia $= \frac{I_c - I_s}{t}$

$= \frac{16000 - 7200}{60 \times 60}$

$= \frac{22}{9}$ kg $-$ m^2 s^{-1}

22. (c) For permanent magnet we prefer a material with high retentivity and high coercivity. For electromagnet, we prefer high saturated magnetism, low coercivity and least possible area of hysteris loop.

Therefore, X is suitable for making parmanent magnet and γ for making electromagnet.

23. (a) From radioactive decay law,

$-\frac{dN}{dt} \propto N$ or $-\frac{dN}{dt} = \lambda N$

Thus, $A = -\frac{dN}{dt}$ or $A = \lambda N$

$\Rightarrow A = \lambda N_0 e^{-\lambda t}$

Where, $A_0 = \lambda N_0$ is the activity of the radioactive material at time, $t = 0$

At time, $t_1 \ldots \ldots A_1 = A_0 e^{-\lambda t_1} \ldots$ (ii)

At time, $t_2 \ldots \ldots A_2 = A_0 e^{-\lambda t_2} \ldots \ldots$ (iii)

On didviding Eq. (ii) by Eq. (iii), we have

$$\frac{A_1}{A_2} = \frac{e^{-\lambda t_1}}{e^{-\lambda t_2}} = e^{-\lambda(t_1 - t_2)}$$

$\Rightarrow A_1 = A_2 e^{-\lambda(t_1 - t_2)}$

24. (a)

25. (a) In the curve, $V \propto \frac{1}{T}$

$\Rightarrow V = m\frac{1}{T}$ where, $m =$ slope of curve.

$\Rightarrow VT =$ constant

$\therefore TdV + Vd = 0.$

$$\Rightarrow dV = -\frac{V}{T}dT$$

As, $Q = \Delta U + W$

$CdT = C_V dT + pdV$

$$CdT = C_V dT - \frac{p}{T}VdT$$

$$\Rightarrow C = C_V - p\frac{V}{T}$$

$$= C_V - R\left(\because p\frac{V}{T} = R\right)$$

For diatomic gas, $C_V = \frac{5}{2}R$

$$\therefore C = C_V - R = \frac{5}{2}R - R = 3\frac{R}{2}$$

26. (d) Liquid volume coming out of the tube per second

$$V = \frac{p\pi r^4}{8\eta l}$$

$\Rightarrow V \propto r^4$

$$\Rightarrow V_2\ V_1 = \left[\frac{r_2}{r_1}\right]^4 \Rightarrow V_2 = V_1\left[\frac{125}{100}\right]^4 = V_1\left(\frac{5}{4}\right)^4$$

$\Rightarrow V_2 = V_1(244)$

so, now $\frac{\Delta V}{V} = \frac{V_2 - V_1}{V_1} = \frac{(244)V_1 - V_1}{V_1} = 144\%$

27. **(a)** Resistance of wire per unit length, $x = \frac{R}{L} = R\Omega m^{-1}$

$\because I = \frac{E_0}{R}$

Now, the potential drop across 50 cm length is 6 V, so

$\frac{E_0}{R} \times R \times \frac{50}{100} = 6$

$\Rightarrow E_0 = 12\text{ V}$

When S_2 closed, potential drop across 0.416 cm length,

$V_1 = \frac{E_0}{R} \times R \times 0.416 = 12 \times 12 \times 0.416 \approx 5\text{ V}$

Hence, $E - Ir = 5\text{ V}$

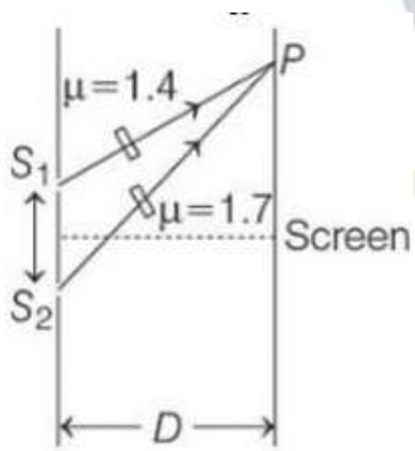
$\Rightarrow 6 - Ir = 5$

$\because I = \frac{5}{10}$

$\therefore 6 - 5 = \frac{5}{10} r$

$\Rightarrow r = 2\Omega$

28. **(b)** As, shift $\Delta x = \frac{\beta}{\lambda}(\mu - 1)t$



Shift due to one plate $\Delta x_1 (\mu_1 - 1)t$

Shift due to another path $\Delta x_2 = \frac{\beta}{\lambda}(\mu_2 - 1)t$

Net shift $\Delta x = \Delta x_2 - \Delta x_1 = \frac{\beta}{\lambda}(\mu_2 - \mu_1)t$

Also, it is given that $\Delta x = 6\beta$

Hence, $6\beta = \frac{\beta}{\lambda}(\mu_1 - \mu_2)t$

$$\Rightarrow t = 6\frac{\lambda}{(\mu_1 - \mu_2)} = 6 \times \frac{4000}{(1.7 - 1.4)} - 80000\text{Å}$$

$\Rightarrow t = 8 \times 10^{-6} = 8\mu m$

29. (d) Force on a moving charged particle in uniform magnetic field

$F = Bqv\sin\theta \; \ldots . \; (i)$

Since, charge particle moves along the axis odf circular current carrying loop, therefore, $\theta = 0^\circ$ or 180°

When $\theta = 0^\circ; f = Bqv\sin 0^\circ$

$F = 0$

When $\theta = 180^\circ, f = Bqv\sin 180^\circ$

$F = 0$

30. (c) The condition of achromatism is $W_1P_1 + W_2P_2 = 0$

$\Rightarrow W_1P_1 = -W_2P_2$

$$\Rightarrow \frac{W}{W_2} = \frac{P_2}{P_1}$$

Now, $P_1 + P_2 = 4D$

but, Power of converting lens,

$P_1 = 5D$

$\because$ Power of diverging lens

$P_2 = 4D - P_1$[From ii]

$= 4D - 5D = -D$

$\therefore$ From Eq. (i), we have

$$\frac{W_1}{W_2} = \frac{P_2}{P_1} = \frac{-(-D)}{5D} = \frac{1}{5} \Rightarrow \frac{W_1}{W_2} = \frac{1}{5}$$

31. (d)
32. (a)
33. (b)
34. (b)
35. (a)
36. (c)
37. (a)
38. (c)

39. (a)

40. (a)

41. (d) Protium (P), deuterium (D) and tritium (T) are the three isotopes of hydrogen. These isotopes follow the order in different contexts as shown below :

$T_2 > D_2 > P_2$ [Order of boiling point (BP)]

$T_2 > D_2 > P_2$ [Order of bond energy (BE)]

$T_2 = D_2 = P_2$ [Order of bond length (BL)]

$T_1 2$ Hence, all the given order are correct.

42.(b) The amino acid which gives yellow colour with ninhydrin in paper chromatography is proline.

Ninhydrin + Proline → Yellow-orange product

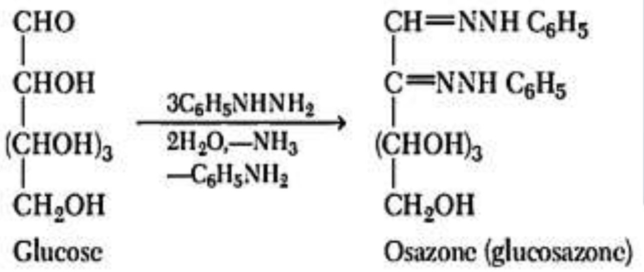
43.(b) In case of liquids, cohesive forces decrease and hence, their viscosity decreases while increasing the temperature. However, in case of gases, with the increase in temperature kinetic energy of gaseous molecules increases and the molecules become more mobile.

44.(a) The thermal stability of carbonates increases down the group. So, the order of thermal stability of carbonates of alkaline earth metals is

$BaCO_3 > CaCO_3 > MgCO_3 > BeCO_3$.

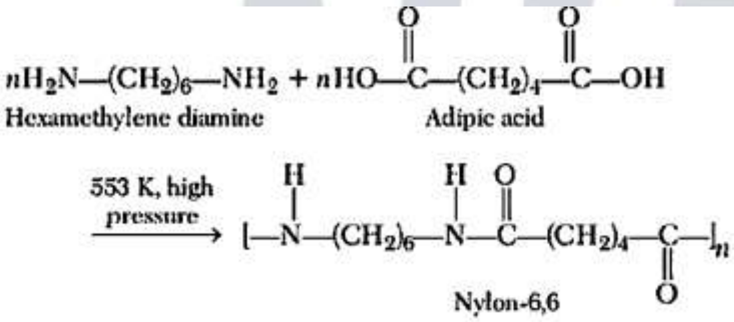
Therefore, $BaCO_3$ is thermodynamically most stable and $BeCO_3$ is least stable.

45.(a)



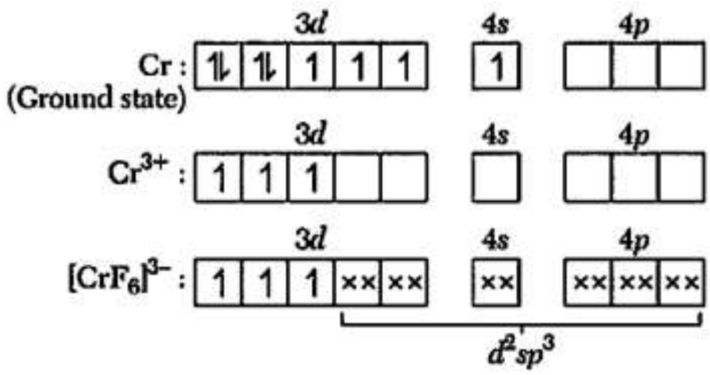
Hence, three molecules of phenyl hudrazine (C_6H_5NHNH2) is used to yield osazone.

46.(a) The monomer units of nylon −6,6 are hexamethylene diamine and adipic acid

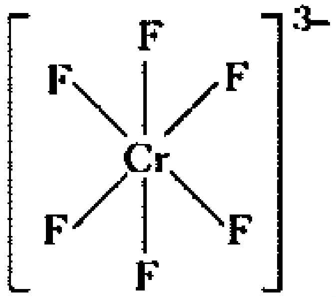


47.(c)

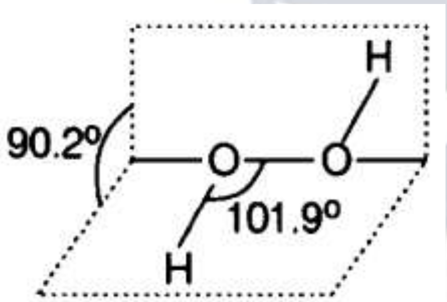
(c) The electronic configuration of Cr is $[Ar]3d^5 4s^1$.



$[CrF_6]^{3-}$ has d^2sp^3 - hybridization and octahedral geometry.



48. **(d)** Hydrogen peroxide (H_2O_2) is a non-planar molecule with twisted symmetrical structure. It has non-linear structure with an open book structure.



49.(a) The correct match is

A → (4); B → (3); C → (2); D → (1)

(a) DDT → Pesticides

(b) $NaClO_3$ → Herbicides

(c) Cl_2 → Disinfectants

(d) PAN → Photochemical smog

50.(d)

	Bond order		
I.	O_2^{2-}	1	Weaker
II.	N_2	1	3
N_2^+	2.5	Stronger	

III.	NO^+	3	Stronger

Larger the bond order, stronger the bond forme(d)

51. (d)

$$[Co(NH_3)_5SO_4]Br \rightleftharpoons [Co(NH_3)_5SO_4]^+ + \underset{\substack{\text{Yellow ppt.} \\ \text{with } AgNO_3}}{\underset{\uparrow}{Br^-}}$$

$$[Co(NH_3)_5Br]SO_4 \rightleftharpoons [Co(NH_3)_5Br]^{2+} + \underset{\text{White ppt. with } BaCl_2}{\underset{\uparrow}{SO_4^{2-}}}$$

52. (d) $MS-2 = M + 32 \times 2 = M + 64$

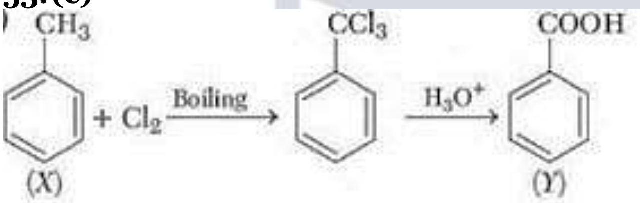
% of sulphur $= \left(\frac{64}{M+64}\right) \times 100 = 40.06$

$$M + 64 = \frac{6400}{40.06}$$

$M + 64 = 160$

$M = 160 - 64 = 96u$

53. (c)



54. (d) Inert pair effect is more pronounced in heavier members like Pb.
Hence, Pb (IV) compounds act as strong oxidising agents and are reduced to more stable Pb (II) compounds.

55. (b) Anti-fluorite structure refers to an anion array with tetrahedral cations and the compound having A_2 B formula have antifluorite structure. So, Na_2O has an anti-fluorite structure in which Na^+ions occupy all the tetrahedral voids and O^{2-} occupy half of the cubic holes.

56. (d) Sodium formate is present at the equivalence point. It is the salt of weak acid + strong base. So, final solution will be basic in nature.
As we know

$$pH = 7 + \frac{pK_a}{2} + \frac{\log C}{2}$$

where, C is concentration of salt.
Total volume of solution = 100 + 100 = 200 mL
Concentration of salt (C) $= 2 \times \frac{100}{200} = 1M$

$$pH = 7 + \frac{3.74}{2} + \frac{\log[1]}{2}$$
$$pH = 7 + 1.87 + 0$$
$$pH = 8.87$$

57. (a)

$$C_6H_5CH{=}CHCH_3 + H^+ \xrightarrow{\text{Slow}} C_6H_5{-}\overset{+}{C}H{-}CH_2{-}CH_3$$

Stable carbocation (Benzylic)

$$C_6H_5{-}\overset{+}{C}H{-}CH_2{-}CH_3 + Br^- \xrightarrow{\text{Fast}} C_6H_5{-}\underset{\displaystyle Br}{\underset{|}{C}H}{-}CH_2{-}CH_3$$

Addition product

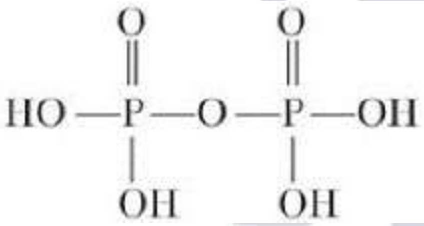
Electrophile addition reaction takes place via more stable carbocation.

58.(c) The structure of diborane is

In this structure, there are two $3C-2e^-$bonds. 3 atoms, $B-H-B$ share two electrons and form an angular geometry, leading bent bond, also known as banana bond.

59. (a)

60. **(a)** The structure of pyrophosphoric acid is



There are $P-OH$ bonds, $1P-O-P$ bond and two $P=O$ bonds.

61. (d)

(a) $2\,F^{3+} + 2e^- \rightarrow 2\,Fe^{2+}; E^\circ = 0.77$ V
$2I^- \rightarrow I_2 + 2e^-; E^\circ = -0.54$ V (sign of E° is reversed)
$2Fe^{3+} + 2I^- \rightarrow 2Fe^{2+} + I_2; E^\circ_{cel} = +0.23$ V
This reaction is feasible, since E° cell is positive.
(b) $Cu \rightarrow Cu^{2+} + 2e^-; E^\circ = -0.34$ V (sign of E° gas been reversed)
$2Ag^+ + 2e^- \rightarrow 2Ag; E^\circ = +0.80$ V

This reaction is feasible, since E° cell is positive.
(c) $2Fe^{3+} + 2e^- \rightarrow 2Fe^{2+}; E^\circ = +0.77$ V
$Cu \rightarrow Cu^{2+} + 2e^-; E^\circ = -34$ V (sign of E° is reversed)

This reaction is feasible, since E° cell is positive.
(d) $Ag \rightarrow Ag^+ + e^-; E^\circ = -0.80$ V (sign of E° is reversed)
$Fe^{3+} + e^- \rightarrow Fe^{2+}; E^\circ = +0.77$ V
$Ag + Fe^{3+} \rightarrow Ag^+ + Fe^{2+}; E^\circ = -0.03$ V
This reaction is not feasible, since $E^\circ{}_{cell}$ is negative.

62.(a)

$$NH_4OH \rightleftharpoons NH_4^{\ +} + OH^-$$
$$K_b = \frac{[NH_4^{\ +}][OH^-]}{[NH_4OH]}$$
$$1.8 \times 10^{-5} = \frac{[NH_4^{\ +}][0.01]}{[0.02]}$$
$$\therefore [NH_4^{\ +}] = 3.6 \times 10^{-5} M$$

63.(a) van't Hoff equation giving variation of equilibeium constant, K with temperature T is

$$\log_e K = -\frac{\Delta H^\circ}{RT} + \frac{\Delta S^\circ}{R}$$

Given, $\log_e K = -\frac{2000}{T} + 4.0$

$$\therefore \frac{\Delta S^\circ}{R} = 4$$
$$\Rightarrow \Delta S^\circ = 4R$$

64. (d)

65.(c) Free expansion, $W = 0$
Adiabatic process, $q = 0$
$\Delta U = q + W = 0$, this means that internal energy remains constant. Therefore, $\Delta T = 0$ in ideal gas there is no intermolecular attraction.
Hence, when such a gas expands under adiabatic conditions into a vacuum, no heat is absorbed or evolved, since no external work is done to separate the molecules.

66.(b) From the graph it is clear that, the value of ' Z ' decreases with increase of pressure. We can explain as follows on the basis of van der Waals' equation.
At high pressure, when ' p ' is large, V will be small and one cannot ignore ' b ' in comparison to V. However, the team $\frac{a}{V^2}$ may be considered negligible in comparison to ' p ' in van der Waals' equation.

$$\left(p + \frac{a}{V^2}\right)(V - b) = nRT$$
$$p(V - b) = nRT$$
$$\Rightarrow pV - pb = nRT$$
$$\text{or } p\frac{V}{n}RT = 1 + \frac{pb}{nRT}$$
$$\text{or } Z = 1 + \frac{pb}{nRT}$$

Thus, Z is greater than 1. As pressure is increased (at constant T), the factor $\frac{pb}{nRT}$ increases. This explains why after minima in the curves, Z increase continuously with pressure. Hence, the only incorrect statement is (b).

67. (b)

68. (c)

69.(a) The hydrogen atom has the configuration 1 s and 3 s, 3p and 3 d-orbitals will have same energy with respect to 1 s-orbital.

70.(b)

CH_4; bp = 4; sp^3 − hybridisation
NF_3; bp = 3; 1p = 1; sp^3 − hybridisation
NH_4^+; bp = 4; 1p = 0; sp^3 − hybridisation

H_2O; bp = 2; 1p = 2; sp^3 − hybridisation
Hence, all have the same slope.

71. (d) Given,
Mass of an organic compound = 0.20 g
Mass of AgBr = 0.12 g
Molecular mass of AgBr = 188gmol^{-1}
188 g of AgBr contains 80 g of bromine

$$\therefore 0.12 \text{ g of AgBr will contain } = \frac{80}{188} \times 0.12$$
$$= 0.05 \text{ g of bromine}$$
$$\therefore \text{ Percentage of bromine } = \frac{0.05}{0.20} \times 100$$
$$= 25\%$$

72. (d)
73. (b)
74. (d)
75. (c)
76. (b)
77. (a)
78. (b)
79. (b)
80. (c)
81. (a) Forthrightness and 'outspokenness' both have a same meaning, i.e. straight forward.
82.(b) 'Inexorable' means 'relentless', i.e. without any particular end.
83.(c) 'Then she bowed her face', is the correct passive form of the given sentence.
84. (d) The complex form of the sentence would be the child is spoiled when the rod is spared'.
85.(a) Retrograde means declining. Hence, 'progressive' is its correct antonym.
86. (b)
87.(d) Disinterested and selfless concern for the well-being of other is called Altruism.
88. (c) Code of diplomatic etiquette and precedence is called protocol.
89. (a) BADC is the correct sequence to from a logical paragraph
90. (c) CBAD is the correct sequence to form a meaningful and logical paragraph.
91. (c)
92.(d)
93.(d)
94. (b)
95. (c)
96.(a) Average marks of 50 students = 64
Sum of marks of 50 students
= Average × number of students

$= 64 \times 50 = 3200$

According to the question,
Correct sum of marks of 50 students

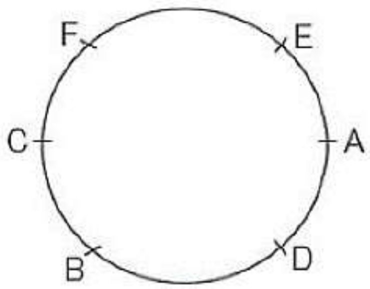
$$= 3200 - 38 - 42 + 83 + 24$$
$$= 3200 + 27$$
$$= 3227$$

∴ Correct average marks of 50 students

$= \dfrac{3227}{50} = 64.54$

97. (a)

98. **(a)** According to the question,



Hence, A sits immediate left of E.

99.(c) According to the question,

$E > B > C > A > D$

Hence, B's car was the 2nd costliest.

100. **(b)** If figure shown in option (b), is placed in the place of missing portion of the original figure, then it is completed as shown below

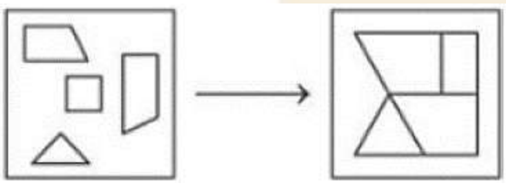
$E > B > C > A > D$

101. **(d)** Clearly, in the first row, the number of dots in the second figure is twice the number of dots in the first figure. Similarly, in the second row, the number of dots in the second figure must be twice the number of dots in the first figure.
So, number of dots in missing segment $= 2 \times 3 = 6$

102. **(d)** Either of the situation is possible. If Mr. Gopal was one of the member of 60^{wedge} employees, then he went on strike. If he was not in group of 60%, then he did not participate in the strike. Hence, either conclusion I or II follows.

103. **(d)** The statement I is talking about in a condition with the lawyer that they marry only fair girls. But it is not talking about any condition with Shobha. So, Shobha can marry either a lawyer or anyone else.

104. **(b)** Figure given in option (b) can be formed by joining the pieces given in question figure, as shown below.



105. **(a)** As, pressure measure in Borometer. Similarly, current measure in Ammeter.

106. **(c)**

$$4x^2 + 2x - 1 = 0$$
$$\therefore \alpha + \beta = -\frac{1}{2}, \alpha\beta = -\frac{1}{4} \ldots.$$

Also, $4\alpha^2 + 2\alpha - 1 = 0$ as α is a root and we have to prove that $\beta = 4\alpha^3 - 3\alpha$

$$= \alpha(1-2\alpha) - 2\alpha = -2\alpha^2 - 2\alpha$$
$$= -\frac{1}{2}[4\alpha^2 + 4\alpha] = -\frac{1}{2}[1 - 2\alpha + 4\alpha]$$
$$= -\frac{1}{2}(1+2\alpha) = -\frac{1}{2} - \alpha = \beta$$

Now, $\alpha + \beta = -\frac{1}{2}$ [from Eq. (i)]
Hence, $\beta = 4\alpha^3 - 3\alpha$.

107. (b)

108. (d) We have, $|w| = 2 \Rightarrow w = 2(\cos\theta + i\sin\theta)$

$$\therefore z = w - \frac{1}{w}$$
$$= 2(\cos\theta + i\sin\theta) - \frac{1}{2}(\cos\theta - i\sin\theta)$$
$$\Rightarrow z = \frac{3}{2}\cos\theta + \frac{5}{2}i\sin\theta$$
$$\Rightarrow |z| = \sqrt{\frac{9}{4} + \frac{25}{4}} = \frac{\sqrt{17}}{2} < 3$$

Hence, option (d) is correct.

109. (b)

$$1 - \cos(1-\cos x) = 2\sin^2\left(\frac{1-\cos x}{2}\right)$$
$$= 2\sin^2\left(\sin^2\frac{x}{2}\right)$$
$$\therefore \lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4} = \lim_{x\to 0}\frac{2\sin^2\left(\sin^2\frac{x}{2}\right)}{x^4}$$
$$= \lim_{x\to 0}\frac{2\sin^2\left(\sin^{2x}\frac{x}{2}\right)}{\left(\sin^2\frac{x}{2}\right)^2} \times \frac{\sin^4\frac{x}{2}}{\left(\frac{x}{2}\right)^4 \times 16}\left[\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$
$$= \frac{1}{8}$$

110. (d) Let d be the common difference of the AP. Then,

$$a_{10} = 3 \Rightarrow a_1 + 9d = 3$$
$$\Rightarrow 2 + 9d = 3 \Rightarrow d = \frac{1}{9}$$
$$\therefore a_4 = a_1 + 3d = 2 + \frac{1}{3} = \frac{7}{3}$$

Let D be the common difference of $\frac{1}{h_1}, \frac{1}{h_2}, \ldots\ldots - \frac{1}{h_{10}}$.

Then, $h_{10} = 3$

$$\Rightarrow \frac{1}{h_{10}} = \frac{1}{3} \Rightarrow \frac{1}{2} + 9D = \frac{1}{3}$$

$$\Rightarrow 9D = -\frac{1}{6} \Rightarrow D = -\frac{1}{54}$$

$$\therefore \frac{1}{h_7} = \frac{1}{h_1} + 6D = \frac{1}{2} - \frac{1}{9} = \frac{7}{18}$$

$$\Rightarrow h_7 = \frac{18}{7}$$

$$\therefore a_4 h_7 = \frac{7}{3} \times \frac{18}{7} = 6$$

111. **(a)** We have,

$$(1 + 5\sqrt{2}x)^9 + (1 - 5\sqrt{2}x)^9$$

$$= 2\{ {}^2C_0 + {}^9C_2(5\sqrt{2}x)^2 + \cdots . + {}^9C_8(5\sqrt{2}x)^8\}$$

Clearly, it has 5 terms

112. **(a)** Others than 2, remaining five places can be filled by 1 and 3 for each place. The number of ways for five places is $2 \times 2 \times 2 \times 2 \times 2 = 2^5$. For 2 , selecting 2 places out of 7 is ${}^7C_2$. Hence, the required number of ways is ${}^7C_2 \times 2^5$.

113. **(d)** Since, the system has no solution.

$$\begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & -1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 2(-2\lambda + 1) + 1(\lambda + 1) + 2(3) = 0$$

$$\Rightarrow -4\lambda + 2 + \lambda + 1 + 6 = 0$$

$$\Rightarrow 3\lambda = 9$$

$$\Rightarrow \lambda = 3$$

114. **(d)**

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 1(1 + 3) + 1(2 + 3) + 1(2 - 1) = 4 + 5 + 1 = 10$$

$$\Rightarrow \text{Adj}A = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}^T = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\Rightarrow B = A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\left[\because A^{-1} = \frac{1}{|A|}\text{Adj}A\right]$$

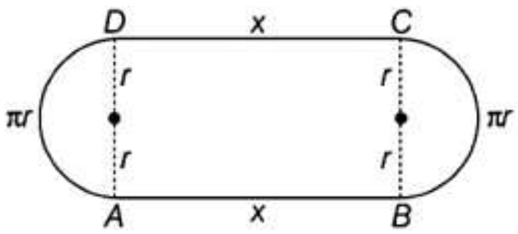
$$\Rightarrow 10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

Hence, $\alpha = 5$.

115. **(a)**

$$
\begin{aligned}
y &= \cos^{-1}\left(\frac{7}{2}(1+\cos 2x)+\sqrt{(\sin^2 x-48\cos^2 x)\sin x}\right.\\
&= \cos^{-1}\left((7\cos x)(\cos x)+\sqrt{1-49\cos^2 x}\sqrt{1-\cos^2 x}\right)\\
&= \cos^{-1}(\cos x)-\cos^{-1}(7\cos x)[\because \cos x<7\cos x]\\
&= x-\cos^{-1}(7\cos x)
\end{aligned}
$$

116. (c)



$$
\begin{aligned}
&\text{Perimeter } = 440\text{ft}\\
&\Rightarrow 2x+\pi r+\pi r=440\\
&2x+2\pi r=440\\
&A = \text{ Area of the rectangular portion } = x2r\\
&A = x\frac{(440-2x)}{\pi}=\frac{1}{\pi}(440x-2x^2)\\
&\text{Let } \frac{dA}{dx}=\frac{1}{\pi}(440-4x)=0\\
&\Rightarrow x=110 \text{ for which } \frac{d^2A}{dx^2}<0\\
&\Rightarrow A \text{ is maximum when } x=110\\
&\Rightarrow 2r=\frac{440-2x}{\pi}=\frac{440-220}{\frac{22}{7}}=70\\
&\therefore r=35\text{ft and } x=110\text{ft}
\end{aligned}
$$

117. (a) Given that

$$
\begin{aligned}
&\left(\frac{dy}{dx}\right)\tan x = y\sec^2 x+\sin x\\
&\tan x\left(\frac{dy}{dx}\right)-y\sec^2 x=\sin x\\
&\frac{dy}{dx}+\left(\frac{-\sec^2 x}{\tan x}\right)y=\frac{\sin x}{\tan x}
\end{aligned}
$$

Here, $P=\frac{\sec^2 x}{\tan x}$ and $Q=\cos x$

$$
\begin{aligned}
&IF = e^{\int Pdx}=e^{\int \frac{-\sec^2 x}{\tan x}dx}\\
&IF = e^{-\log(\tan x)}=e^{\log\cot x}=\cot x
\end{aligned}
$$

Now, $y\times IF=\int (Q\times IF)dx+C$

$$
\begin{aligned}
&y\cdot\cot x=\int \cos x\cdot\cot x+C\\
&y\cdot\cot x=\int \frac{1-\sin^2 x}{\sin x}+C\\
&y\cdot\cot x=\log|\operatorname{cosec} x-\cot x|+\cos x+C
\end{aligned}
$$

118. **(d)** Solving the given equations,

$$(mx+c)^2 = 4ax - a^3$$
$$\Rightarrow m^2x^2 + 2mc \cdot x + c^2 = m4ax - 4a^3$$
$$\Rightarrow m^2x^2 + (2mc - 4a)x + c^2 + 4a^3 = 0$$

Since the straight line touches the parabola at a point,

$$\text{so, the discriminant } = 0$$
$$\Rightarrow (2mc-4a)^2 - 4m^2(c^2+4a^3) = 0$$
$$\Rightarrow 4m^2c^2 - 16amc + 16a^2 - 4m^2c^2 - 16a^3m^2 = 0$$
$$\Rightarrow -mc + a - a^2m^2 = 0$$
$$\Rightarrow mc = a - a^2m^2 \Rightarrow c = \frac{a}{m} - a^2m$$

119. **(c)** For the parabola $y^2 = 4ax$, the equation of normal at $P(am^2, -2am)$ is $y = mx - 2am - a^3$.
Here, $m = \tan 60^\circ = \sqrt{3}$

$\therefore P \equiv \left(a(\sqrt{3})^2, -2a(\sqrt{3})\right) \equiv (6, -4\sqrt{3}) (\because a = 2)$

Thus, P satisfies $2x - y - 12 - 4\sqrt{3} = 0$

120. **(c)**

$$I = \int \frac{1}{\left(\frac{x-1}{x+2}\right)^{\frac{3}{4}}(x+2)^2} dx$$

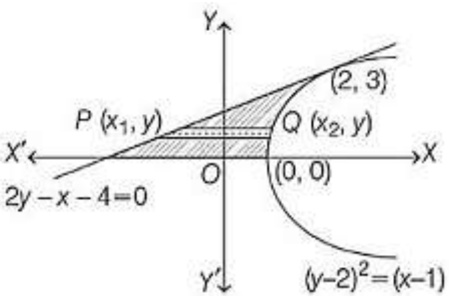
$$\text{Let } \frac{x-1}{x+2} = t \Rightarrow \frac{3dx}{(x+2)^2} = dt$$

$$\Rightarrow I = \frac{1}{3}\int \frac{1}{t^{\frac{3}{4}}} dt = \frac{1}{3}\left(\frac{t^{\frac{1}{4}}}{\frac{1}{4}}\right) + C$$

$$= \frac{4}{3} + C = \frac{1}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}} + C$$

121. **(c)** The equation of the parabola is $y^2 - 4y - x + 5 = 0$
The equation of tangent at (2,3) is

$$3y - 2(y+3) - \frac{x+2}{2} + 5 = 0$$
$$2y - x - 4 = 0$$



$\therefore$ Required area A is given by

$$A=\int_0^3 (x_2-x_1)dy$$

$$\Rightarrow A=\int_0^3 [\{(y-2)^2+1\}-\{2y-4\}]dy$$

$$\Rightarrow A=\int_0^3 (y^2-6y+9)dy=\int_0^3 (3-y)^2dy$$

$$=-\left[\frac{(3-y)^3}{3}\right]_0^3=9$$

122. **(d)** Given that, $\left|\frac{\hat{u}+\hat{v}}{2}+\hat{u}\times\hat{v}\right|=1$

$$\Rightarrow \left|\frac{\hat{u}+\hat{v}}{2}+\hat{u}\times\hat{v}\right|^2=1$$

$$\Rightarrow \frac{2+2\cos\theta}{4}+\sin^2\theta=1[\because \hat{u}\cdot(\hat{u}\times\hat{v})=\hat{v}(\hat{u}\times\hat{v})=0]$$

$$\Rightarrow \cos^2\frac{\theta}{2}=\cos 2\theta$$

$$\Rightarrow \theta=n\pi\pm\frac{\theta}{2}, n\in z$$

$$\Rightarrow \theta=\frac{2\pi}{3}$$

$$\Rightarrow |\hat{u}\times\hat{v}|=\sin\frac{2\pi}{3}=\sin\frac{\pi}{3}=\left|\frac{\hat{u}-\hat{v}}{2}\right|$$

123. **(b)** Let P be the probability of getting an even number. Then by hypothesis the probability of getting an odd number is 3P. Since the events of getting an even number and an odd number are mutually exclusive and exhaustive.

$\therefore P+3P=1\Rightarrow P-1/4$

Thus, the probability of getting an odd number in a single throw is $3/4$ and that of an even number is $1/4$.
If the die is thrown twice, then the sum of the numbers in two throws is even if both the numbers are even or both are odd.
$\therefore$ Required probability $=\frac{3}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{1}{4}=\frac{10}{16}=\frac{5}{8}$.

124. **(b)**

$$2\cos\theta[\cos 120^\circ+\cos 2\theta]=1$$

$$\Rightarrow 2\cos\theta\left(-\frac{1}{2}+2\cos^2\theta-1\right)=1$$

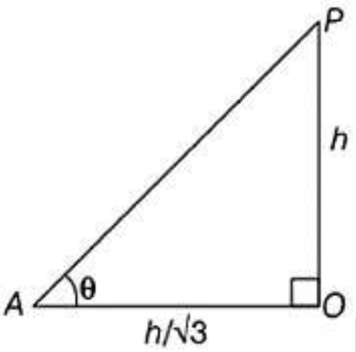
$$\Rightarrow 4\cos^3\theta-3\cos\theta-1=0$$

$$\Rightarrow 3\theta=2n\pi \text{ or } \theta=\frac{2n\pi}{3}, n\in z$$

Given the values so that 2n does not exceed 18
Hence, the sum $=\frac{2\pi}{3}\Sigma_1^9 n=\frac{2\pi}{3}\times\frac{9(9+1)}{2}=30\pi$

125. **(c)** We have,

$$16^{\sin^2\theta} + 16^{\cos^2\theta} = 10$$
$$\Rightarrow 16^{\sin^2\theta} + 16^{1-\sin^2\theta} = 10$$
$$x + \frac{16}{x} = 10, \text{ where } x = 16^{\sin^2\theta}$$
$$\Rightarrow x^2 - 10x + 16 = 0 \Rightarrow x = 2,8$$
$$\therefore 16^{\sin^2\theta} = 2,8 \Rightarrow 2^{4\sin^2\theta} = 2, 2^3$$
$$\Rightarrow 4\sin^2\theta = 1,3 \Rightarrow \sin^2\theta = \frac{1}{4}, \left(\frac{\sqrt{3}}{2}\right)^2$$
$$\Rightarrow \sin\theta = \frac{1}{2}, \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{3}$$



Let the altitude of the sum be θ. Then,

$$\tan\theta = \frac{h}{\frac{h}{\sqrt{3}}} \Rightarrow \tan\theta = \sqrt{3}$$
$$\Rightarrow \theta = \frac{\pi}{3} \Rightarrow \theta = 2\alpha.$$

126. **(b)** Suppose the parabola $y = x^2 + px + q$ cuts X-axis at $A(\alpha, 0)$ and $B(\beta, 0)$. Then, α, β are roots of the equation $x^2 + px + q = 0$

$\therefore \alpha + \beta = -p$ and $\alpha.\beta = q$

The parabola $y = x^2 + px + q$ cuts Y-axis at $(0, q)$.
Let the equation of the circle passing through A, B and C be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \ldots.$$
$$\therefore \alpha^2 + 2g\alpha + c = 0 \ldots\ldots(ii)$$
$$\beta^2 + 2g\beta + c = 0 \ldots\ (iii)$$
$$\text{and } q^2 + 2fq + c = 0 \ldots(iv)$$

Subtracting Eq. (iii) from Eq. (ii), we get

$$\alpha + \beta + 2g = 0$$
$$\Rightarrow g = \frac{p}{2}$$

Adding Eqs. (ii) and (iii), we get

$$\alpha^2 + \beta^2 + 2g(\alpha + \beta) + 2c = 0$$
$$(\alpha + \beta)^2 - 2\alpha\beta + 2g(\alpha + \beta) + 2c = 0$$

$p^2 - 2q - p^2 + 2c = 0\left[\because \alpha + \beta = \text{ pand } g = \frac{p}{2}\right]$

Putting $c = q$ in Eq. (iv), we get $f = -\left(\frac{q+1}{2}\right)$

Substituting the values of g, f and c in Eq. (i), we obtain the equation of family of circles passing through A, B and C as

$$x^2 + y^2 + px - (q+1)y + q = 0$$
$$x^2 + y^2 + px - (q+1)y + q = 0$$

Clearly, it passes through (0,1).

127. **(c)** We can arrange the letters H, A, A, A in $4!/3! = 4$ WAYS

If one possible arrangement is XXXX

Then, we can arrange, V, N at any of the two places marked with 0 in the following arrangement

$0 \times 0 \times 0 \times 0 \times 0$

Thus, we can arrange V and N in ${}^5P_2 = 20$ ways. Thus, the number of ways in which letters can be arranged is $4 \times 20 = 80$

128. **(d)** It is given that a_1, a_2, a_3 ... are in HP.

Therefore, $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots \dots \dots$ are in AP

Let d be the common difference of the AP.

$$\therefore \frac{1}{a_n} = \frac{1}{a_1} + (n-1)d \text{ and } \frac{1}{a_{20}} = \frac{1}{a_1} + 19d$$
$$\Rightarrow \frac{1}{a_n} = \frac{1}{5} + (n-1)d \text{ and } \frac{1}{25} = \frac{1}{5} + 19d$$
$$\Rightarrow \frac{1}{a_n} = \frac{1}{5} + (n-1)d \text{ and } d = \frac{-4}{19 \times 25}$$
$$\Rightarrow \frac{1}{a_n} = \frac{1}{5} - \frac{4(n-1)}{19 \times 25}$$
$$\Rightarrow \frac{1}{a_n} = \frac{95 - 4n + 4}{19 \times 25}$$
$$\Rightarrow a_n = \frac{99 - 4n}{19 \times 25}$$

NoW, $a_n < 0$

$$\Rightarrow \frac{99 - 4n}{19 \times 25} < 0$$
$$\Rightarrow 99 - 4n < 0$$
$$\Rightarrow 4n > 99$$
$$\Rightarrow n > 24\frac{3}{4} \Rightarrow n \geq 25$$

129. **(b)** Given plane $3x + y + 2z + 6 = 0$ and line $\frac{x - \frac{1}{3}}{\frac{2b}{3}} = \frac{y-3}{-1} = \frac{z-1}{a}$

Since, plane is parallel to line, then

$$3\left(\frac{2b}{3}\right)+(1)(-1)+2(a)=0$$
$$2b-1+2a=0$$
$$\Rightarrow a+b=\frac{1}{2}$$

Now, $3a+3b=\frac{3}{2}$

130. (d)

Since, $a+b=-p, ab=1 \ldots$ (i)
and $c+d=-q, cd=1$
Now $(a-c)(b-c)$ and $(a+d)(b+d)$ are
the roots of $x^2+ax+\beta=0$

$$(a-c)(b)(a+d)(b+d)=\beta$$
$$\Rightarrow (ab-ac-bc+c^2)(ab+ad+bd+d^2)=\beta$$
$$\Rightarrow \{1-c(a+b)+c^2\}\{1+d(a+b)+d^2\}=\beta$$
$$\Rightarrow (1+pc+c^2)(1-pd+d^2)=\beta$$
$$\Rightarrow 1-pd+d^2+pc-p^2cd+pcd^2+c^2-pc^2d+c^2d^2=\beta$$
$$\Rightarrow 1-pd+d^2+pc-p^2+pd+c^2-pc+1=\beta[\because cd=1]$$
$$\Rightarrow 2+d^2+c^2-p^2=\beta$$
$$\Rightarrow 2cd+c^2+d^2-p^2=\beta$$
$$(c+d)^2-p^2=\beta$$
$$q^2-p^2=\beta[\because (c+d)=-q]$$

131. (c)
132. (a)

$$\text{Let } y=(1+x)^{\frac{1}{x}}$$
$$\Rightarrow \log y=\frac{1}{x}\log(1+x)=\frac{1}{x}\left[x=\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots.\right]$$
$$=1-\frac{x}{2}+\frac{x^2}{3}-\frac{x^3}{4}+\cdots$$
$$y=e^{\left(1-\frac{x}{2}+\frac{x^2}{3}+..\right)}=ee^{\left(-\frac{x}{2}+\frac{x^2}{3}..\right)}$$
$$=e\left[1+\left(-\frac{x}{2}+\frac{x^2}{3}-\cdots\right)+\frac{1}{2!}\left(-\frac{x}{2}+\frac{x^2}{3}-\cdots\right)^2+\cdots\right]$$
$$y-e+\frac{1}{2}ex=ex^2\left[\frac{1}{3}+0(x)+\frac{1}{2}\left(-\frac{1}{2}+0(x)\right)^2+\cdots\right]$$
$[\because 0(x)$ in terms containing $x]$
$$\lim\frac{y-e+\frac{1}{2}ex}{x^2}=e\left[\frac{1}{3}+\frac{1}{8}\right]=\frac{11}{24}e$$

133. (a) Let $P\left(t, 4t-\frac{20}{5}\right)$ be a point on the line
$4x-5y=20$. Then the chord of contact of tangents drawn from P to the circle $x^2+y^2=9$ is

$$tx\left(4t-\frac{20}{5}\right)y=9..$$

Let θ(h, k) be the mid-point of this chord of contact, then its equation is also $hx + ky = h^2 + k^2$ ... (ii) Clearly Eqs. (i) and (ii) represent the same line

$$\therefore \frac{t}{h} = \frac{4t-20}{5k} = \frac{9}{h^2+k^2}$$
$$\Rightarrow \frac{t}{h} = \frac{9}{h^2+k^2} \text{ and } \frac{t}{h} = \frac{4t-20}{5k}$$
$$\Rightarrow t = 9\frac{h}{h^2+k^2} \text{ and } t = \frac{20h}{4h-5k}$$
$$\Rightarrow 9\frac{h}{h^2+k^2} = \frac{20h}{4h-5k}$$
$$\Rightarrow h\{20(h^2+k^2) - 36h + 45k\} = 0$$
$$x = 0, \text{or } [20(x^2+y^2) - 36x + 45y] = 0$$

134. (b)

$$f(x) = \int \frac{x^2 dx}{(1+x^2)(1+\sqrt{1+x^2})}$$
Let $x = \tan\theta$
$$\Rightarrow dx = \sec^2\theta d\theta = (1+x^2)d\theta$$
$$f(x) = \int \frac{x^2 dx}{(1+x^2)(1+\sqrt{1+x^2})}$$
$$= \int \frac{\tan^2\theta \sec^2\theta d\theta}{\sec^2\theta(1+\sec\theta)} = \int \frac{\tan^2\theta d\theta}{1+\sec\theta}$$
$$= \int \frac{\sin^2\theta d\theta}{\cos\theta(1+\cos\theta)}$$
$$= \int \frac{1-\cos^2\theta d\theta}{\cos\theta(1+\cos\theta)}$$
$$= \int \frac{1-\cos^\theta d\theta}{\cos\theta(1+\cos\theta)}$$
$$= \int \frac{(1-\cos\theta)d\theta}{\cos\theta}$$
$$= \int \sec\theta d\theta - \int d\theta$$
$$= \log\left(x+\sqrt{1+x^2}\right) - \tan^{-1}x + C$$
$$\therefore f(0) = \log(0+\sqrt{1+0}) - \tan^{-1}(0) + C$$
$$0 = \log 1 - 0 + C$$
$$\Rightarrow C = 0$$
$$\therefore f(1) = \log\left(1+\sqrt{1+1^2}\right) - \tan^{-1}(1)$$
$$= \log(1+\sqrt{2}) - \frac{\pi}{4}$$

135. (c) Let the two unknown items be x and y. Then,

$$\text{Mean} = 4 \Rightarrow \frac{1+2+6+x+y}{5} = 4$$
$$\Rightarrow x+y = 11 \ldots \text{(i)}$$
$$\text{and variance} = 5.2$$
$$\Rightarrow \frac{1^2+2^2+6^2+x^2+y^2}{5} - (\text{mean})^2 = 5.2$$
$$\Rightarrow 41+x^2+y^2 = 5(5.2+16)$$
$$\Rightarrow 41+x^2+y^2 = 106$$
$$\Rightarrow x^2+y^2 = 65 \ldots \text{(ii)}$$

On solving Eqs. (i) and (ii), we get

$x = 4, y = 7$ or $x = 7, y = 4$

136. **(a)** Since, the first 11 terms are Ap, $d = 2$

$\therefore a_{11} = a + 10d = a + 20$

The middle term of AP is

$T_6 = a + 5d = a + 10$

For the next 11 terms in GP

$r = 2$

$\therefore$ The middle term of GP is $b(2)^5$ where, b is the first term of a GP which is the last term of AP

$b(2)^5 = (a + 20)32$

According to the given condition,

$$\Rightarrow a + 10 = (a + 20)32$$
$$\Rightarrow 32a = 10 - 640$$
$$a = -\frac{630}{31}$$

$\therefore$ Middle term of entire sequence is 11 th term

$$\therefore T_{11} = \frac{-630}{31} + 10 \times d$$
$$= \frac{-630}{31} + 10 \times 2 = \frac{-10}{31}$$

137. **(b)** As the given system of equations has a non-trivial solution.

$$\Delta = \begin{vmatrix} p & a & a \\ b & q & b \\ c & c & r \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$\Delta = \begin{vmatrix} p & a-p & a-p \\ b & q-b & 0 \\ c & 0 & r-c \end{vmatrix} = 0$$

Expanding along C_3, we get

$$(a-p)\begin{vmatrix} b & q-b \\ c & 0 \end{vmatrix} + (r-c)\begin{vmatrix} p & a-p \\ b & q-b \end{vmatrix} = 0$$
$$\Rightarrow (a-p)(-c)(q-b) + (r-c)$$
$$\{p(q-b) - b(a-p)\} = 0$$
$$\Rightarrow (p-a)(q-b)c + p(r-c)(q-b) + b(r-c)(p-a) = 0$$

Dividing by $(p-a)(q-b)(r-c)$, we get

$$\frac{c}{r-c} + \frac{p}{p-a} + \frac{b}{q-b} = 0$$
$$\Rightarrow \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = \frac{q-b}{q-b} + \frac{r-c}{r-c} = 2$$

138. **(a)** We have, $\frac{1}{2}gof(x) = 2x^2 - 5x + 2$

$$\Rightarrow g(f(x)) = 4x^2 - 10x + 4$$
$$\Rightarrow (f(x))^2 + f(x) - 2 = 4x^2 - 10x + 4$$
$$\Rightarrow (f(x))^2 + f(x) - (4x^2 - 10x + 6) = 0$$
$$\Rightarrow f(x) = \frac{-1 \pm \sqrt{1 + 4(4x^2 - 10x + 6)}}{2}$$
$$\Rightarrow f(x) = \frac{-1 \pm \sqrt{16x^2 - 40x + 25}}{2}$$
$$\Rightarrow f(x) = \frac{-1 \pm (4x-5)}{2} = 2x - 3, -2x + 2$$

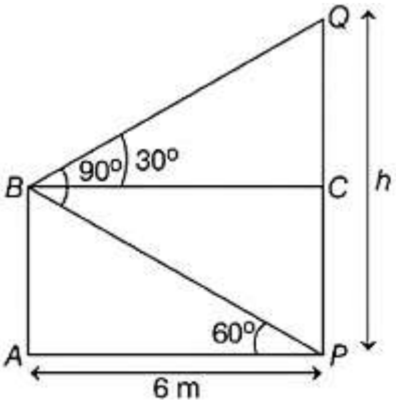
Hence, $f(x) = 2x - 3$

139. **(a)**

$$\left[\frac{1 + \sin\frac{\pi}{8} + i\cos\frac{\pi}{8}}{1 + \sin\frac{\pi}{8} - i\cos\frac{\pi}{8}}\right]^n = \left[\frac{1 + \cos\alpha + i\sin\alpha}{1 + \cos\alpha - i\sin\alpha}\right]^n$$
$$\left[\because \alpha = \frac{\pi}{2} - \frac{\pi}{8}\right]$$
$$= \left[\frac{2\cos^2\frac{\alpha}{2} + 2i\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2} - 2i\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}\right]^n$$
$$= \left[\frac{\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2} - i\sin\frac{\alpha}{2}}\right]^n = \left(e^{2i\frac{\alpha}{2}}\right)^n = e^{in\alpha}$$
$$= e^{in\left(\frac{3\pi}{8}\right)} = \cos\frac{3n\pi}{8} + i\sin\frac{3n\pi}{8}$$

For $n = 4$ we get imaginary part.

140. **(a)** Let PQ be the house subtending a right angle at the window B of opposite house AB



In $\triangle$ PAB, we have

$$\tan 60^\circ = \frac{AB}{6} = AB = 6\sqrt{3}\text{ m}$$

In $\triangle$ CBQ, we have

$$\tan 30^\circ = \frac{h - CP}{BC}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h - 6\sqrt{3}}{6} [\because AB = CP, BC = AP]$$
$$\Rightarrow h = 6\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$$
$$\Rightarrow h = 8\sqrt{3}\text{ m}$$

141. (c)

$$V = \frac{4}{2}\pi r^3 \Rightarrow 4500\pi = \frac{4\pi r^3}{3}$$
$$\Rightarrow r = 15\text{ m}$$
After 49 min $= (4500 - 49.72)\pi$
$$= 972\pi m^3$$
$$\Rightarrow 952\pi = \frac{4}{3}\pi r^3$$
$$\Rightarrow r^3 = 3 \times 243 = 3 \times 3^5$$
$$r = 9$$
$$\frac{dv}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$$
$$72\pi = 4\pi \times 9 \times 9\left(\frac{dr}{dt}\right)$$
$$\frac{dr}{dt} = \left(\frac{2}{9}\right)$$

142. (d)

$$\frac{\sin 3B}{\sin B}=\frac{3\sin B-4\sin^3 B}{\sin B}$$
$$=3-4\sin^2 B$$
$$=3-4(1-\cos^2 B)$$
$$=-1+\frac{4(a^2+c^2-b^2)^2}{4(ac)^2}$$
$$=-1+\frac{\left(\frac{a^2+c^2}{2}\right)^2}{(ac)^2}$$
$$=-1+\frac{(a^2+c^2)^2}{4(ac)^2}$$
$$=\left(\frac{c^2-a^2}{2ac}\right)^2$$

143. **(b)** Given, sum of the coefficient $=1024$
i.e. $2^n=1024=2^{10}$

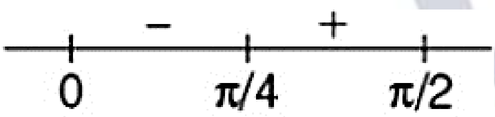
$\Rightarrow n=10$

Since, n is even, so greatest coefficient

$={}^{n}C_{\frac{n}{2}}={}^{10}C_5=252$

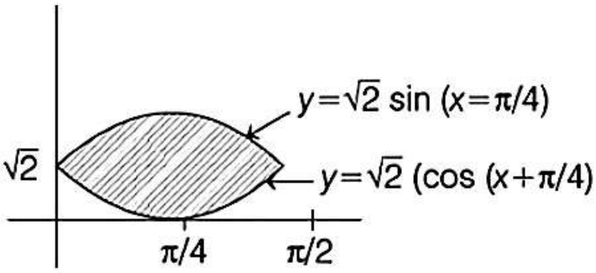
144. **(b)** Given, $y=\sin x+\cos x, x\in\left[0,\frac{\pi}{2}\right]$

$$\frac{dy}{dx}=\cos x-\sin x$$
$$y=|\cos x-\sin x|$$
$$=\begin{bmatrix}\cos x-\sin x & x\in\left[0,\frac{\pi}{4}\right]\\ \sin x-\cos x & x\in\left[\frac{\pi}{4},\frac{\pi}{2}\right]\end{bmatrix}$$



∴ Required area

$$=\int_0^{\frac{\pi}{4}}|(\sin x+\cos x)-(\cos x-\sin x)|dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}|2\cos x|dx$$
$$=\int_0^{\frac{\pi}{4}}|2\sin x|dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}|2\cos x|dx$$
$$=2\left[-\frac{1}{\sqrt{2}}+1+1-\frac{1}{\sqrt{2}}\right]$$



$$= 2\left(2 - \frac{2}{\sqrt{2}}\right)$$
$$= 2(2 - \sqrt{2})$$
$$= 4 - 2\sqrt{2}$$
$$= 2\sqrt{2}(\sqrt{2} - 1)$$

145. (c)

$$\frac{\sin\alpha - \sin\gamma}{\cos\gamma - \cos\alpha} = \frac{2\cos\frac{\alpha+\gamma}{2}\sin\alpha - \frac{\gamma}{2}}{2\sin\frac{\alpha+\gamma}{2}\cdot\sin\frac{\alpha-\gamma}{2}}$$

$$= \cot\alpha + \frac{\gamma}{2}$$

$$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right.$$

$$\left.\text{and } \cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)\right]$$

But α, β, γ are in AP

$$\Rightarrow \frac{\alpha+\gamma}{2} = \beta$$

So, required value is $\cot\beta$.

146. (a)
147. (c)
148. (b)
149. (d)
150. (b)