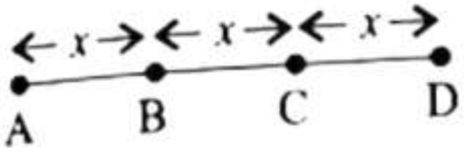
1. (b)

Consider,

AB = x

BC = x



$2x + CD = 3x \Rightarrow CD = 3x - 2x = x$

Average speed of the object $\langle v \rangle$

$$= \frac{\text{Total distance}}{\text{Total time}}$$

$$\langle v \rangle = \frac{3x}{\frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}} = \frac{3v_1v_2v_3}{v_2v_3 + v_1v_3 + v_1v_2}$$

2. (c) When temperature increases, more electrons excite to conduction band and hence conductivity increases, therefore resistance decreases.

3. (d) $N = N_0\left(\frac{1}{2}\right)^n N = \frac{N_0}{8}$

$$\frac{N_0}{8} = N_0\left(\frac{1}{2}\right)^n \Rightarrow \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^n$$

$n = 3$

3 half lives = 3 days

1 half life = 1 day

5 days = 5 half life

$$N = N_0\left(\frac{1}{2}\right)^n \Rightarrow 8 \times 10^{-3} = N_0\left(\frac{1}{2}\right)^5$$

$$\Rightarrow N_0 = 2^5 \times 8 \times 10^{-3} = 256\text{gm}$$

4. (c) If we assume electron in hydrogen atom takes energy 12.09eV from the incoming radiation, the maximum excited state

of electron will be $n = 3$. So, number of spectral lines is $\frac{3(3-1)}{2} = 3$.

Here we assume some part of energy $12.5\text{eV} - 12.09\text{eV} = 0.41\text{eV}$ get lost due to collision.

5. (b) We have

$V_f = V_i$

$\Rightarrow \frac{4}{3}\pi r_f^3 = 1000 \times \frac{4}{3}\pi r_i^3 \Rightarrow r_f^3 = 1000 r_i^3$

$\Rightarrow r_f = 10 r_i$

So, released energy

= Initial surface energy - final surface

energy

$= 1000 \times T \times 4\pi r_i^2 - T \times 4\pi r_f^2$

$= 4\pi T(1000 r_i^2 - r_f^2)$

$= 4\pi \times 0.07(1000 r_i^2 - 100 r_i^2)$

$= 4\pi \times 0.07 \times 900 r_i^2$

$= 4\pi \times 63 \times 10^{-6} = 7.92 \times 10^{-4}$ J

6. **(b)** Here, $q_1 = 1 \times 10^{-7}$C, q_2 and 2×10^{-7}C,

$r = 20 \text{ cm} = 20 \times 10^{-2}$ m

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{9 \times 10^9 \times 1 \times 10^{-7} \times 2 \times 10^{-7}}{(20 \times 10^{-2})^2}$$

$= 4.5 \times 10^{-3}$ N

7. **(c)** Work done $= \frac{1}{2}\frac{q^2}{C} = \frac{(8\times10^{-18})^2}{2\times100\times10^{-6}} = 32 \times 10^{-32}$ J

8. **(c)**

Let resistance of a wire R and length l.

$R = \frac{\rho \ell}{A} = 5\Omega$

∴ Volume of wire is constant in stretching

$V_i = V_f \Rightarrow A_i \ell_i = A_f \ell_f$

$A\ell = A'(5\ell) \Rightarrow A' = \frac{A}{5}$

$R_f = \frac{\rho \ell_f}{A_f} = \frac{\rho(5\ell)}{\left(\frac{A}{5}\right)} = 25\left(\frac{\rho \ell}{A}\right) = 25 \times 5 = 125\Omega$

9. **(b)** Given that uniform magnetic field, $\vec{B} = (2\hat{i} + 3\hat{j})_T$

Acceleration $\vec{a} = (\alpha\hat{i} - 4\hat{j})m/s^2$
We know that

$F = q(\vec{v} \times \vec{B}) \Rightarrow ma = q(\vec{v} \times \vec{B})$

Here, $\vec{a} \perp \vec{B}$, so, $\vec{a} \cdot \vec{B} = 0$

$(\alpha_i - 4\jmath)(2\imath + 3\jmath) = 0 \Rightarrow 2\alpha - 12 = 0 \Rightarrow \alpha = 6$

10. (d) De Broglie wavelength is $\lambda = \frac{h}{mv}$

$$\lambda_p = \lambda_e \Rightarrow \frac{h}{m_p v_p} = \frac{h}{m_e v_e}$$

$$m_e v_e = m_p v_p \Rightarrow p_e = p_p \ \therefore \frac{p_p}{p_e} = \frac{1}{1}$$

11. (b) Work done = Area under the curve

$$\Rightarrow W = \frac{1}{2} \times (4 - 2) \times (400 - 100) = \frac{1}{2}(2) \times 300$$

$W = 300\ J$

12. (d) To redirect the light that enters the telescope to the eyepiece or camer(a) The primary mirror of a reflecting telescopes gathers the light and reflects towards the secondary mirror which then reflect the light towards the eyepiece allowing the observer to see image. It has advantage of a large focal length in a short telescope.

13. (a)

As $\vec{M} = I\vec{A}$

$$\Rightarrow |\vec{M}| = \frac{e}{\frac{2\pi R}{v}}\pi R^2 \left[\because I = \frac{Q}{T} = \frac{e}{\frac{2\pi R}{v}}\right]$$

$$\Rightarrow |\vec{M}| = \frac{1}{2}evR \Rightarrow |\vec{M}| = \frac{mvR}{1} \cdot \frac{e}{2m}$$

$$\Rightarrow |\vec{M}| = \frac{eL}{2m} \Rightarrow |\vec{M}| = -\frac{e\vec{L}}{2m}$$

[$\because$ Here $\vec{M}$ and $\vec{L}$ will always be opposite]

14. (c) Intensity at a point in Young's double slit experiment is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi$$

Here $I_1 = I_2 = I_0$ (say)

At P

$$\therefore I_p = I_0 + I_0 + 2I_0\cos\frac{\pi}{3} = 2I_0 + 2I_0 \times \frac{1}{2} = 3I_0$$

At Q

$I_Q = I_0 + I_0 + 2I_0 \cos 90° = 2I_0$

$\frac{I_P}{I_Q} = \frac{3}{2}$

15. (c) From the ideal gas equation PV = nRT

Here, volume is constant $\therefore \frac{P_1}{T_1} = \frac{P_2}{T_2}$

Here, $T_1 = 27 + 273 = 300$ K

$P_1 = 270\text{kPa}$

$T_2 = 36 + 273 = 309$ K

$\Rightarrow P_2 = \frac{P_1}{T_1} \times T_2 = \frac{270 \times (309)}{300} = 278\text{kP(a)}$

16. (c) Let the distance from the mean position is X.

Given KE = PE

So, $\frac{1}{2}M\omega^2(A^2 - x^2) = \frac{1}{2}M\omega^2x^2A^2 - x^2 = x^2 \Rightarrow A^2 = 2 \times 2$

$\therefore x = \pm\frac{A}{\sqrt{2}}$

17. (b) Electric field in a certain region is given by,

$$\vec{E} = \frac{A}{x^2}\hat{\imath} + \frac{B}{y^3}\hat{\jmath}$$

$$\left[\frac{A}{x^2}\right] = NC^{-1} \Rightarrow [A] = Nm^2C^{-1}$$

$$\left[\frac{B}{y^3}\right] = NC^{-1} \Rightarrow [B] = Nm^3C^{-1}$$

18. (a) Mass of particle,

m = 500 g = 0.5 kg

velocity of a particle,

$\vec{v} = 2\hat{t} + 3t^2\hat{\jmath}$

$\vec{a} = \frac{d\vec{v}}{dt} = 2\hat{\imath} + 6\hat{t}$

at $t = 1, \vec{a} = 2\hat{\imath} + 6\hat{\jmath}$

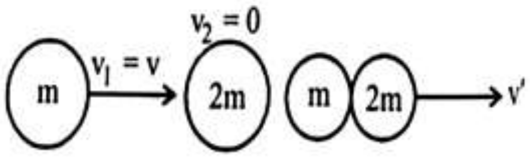
Force acting on the particle,

$\vec{F} = m\vec{a} = 0.5(2\hat{\imath} + 6\hat{\jmath}) = \hat{\imath} + 3\hat{\jmath}$

$\vec{F} = \hat{i} + x_j$

Hence x = 3

19. (c)



Applying conservation of linear momentum

$$\Rightarrow \vec{P}_i = \vec{P}_f (\because P = mv)$$
$$mv_1 + 2mv_2 = (m + 2m)v'$$
$$mv + 2\,m \times 0 = (3\,m)v'$$
$$\Rightarrow mv = 3mvv' \Rightarrow v' = \frac{v}{3}$$

20. (c) For time varying condition Maxwell's equation, $\oint \vec{E} \cdot \vec{dl} = -\frac{d\varphi_B}{dt}$

21. (a) The resonance frequency of LC oscillations circuit is

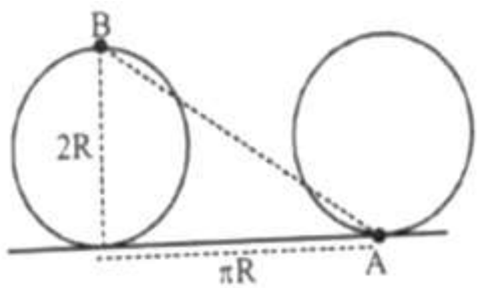
$$\omega = \frac{1}{\sqrt{L'C'}} \Rightarrow L' \rightarrow 2\,L$$
$$C' \rightarrow 8C$$
$$\omega = \frac{1}{\sqrt{2\,L \times 8C}} = \frac{1}{4\sqrt{LC}} 9\omega_0 = \frac{1}{\sqrt{LC}}$$
$$\omega = \frac{\omega_0}{4} \text{ So, } x = \frac{1}{4}$$

22. (b)

$$\text{As} \varepsilon_{|t=0.5\text{sec}} = -\frac{d\varphi}{dt}$$
$$= -A\frac{dB}{dt} [\because \theta = 0^\circ \Rightarrow \cos\theta = 1]$$
$$= -\pi \times \left(\frac{10}{\sqrt{\pi}}\right)^2 \times 10^{-4} \times \frac{0 - 0.5}{0.5} = 10^{-2}\text{ V} = 10\text{mV}$$

As $\frac{dB}{dt} =$ constant $\Rightarrow$ Induced emf will not change with time. So, $e\,|_{0.5\text{sec}} = e|_{0.25\text{sec}} = 10\text{mV}$

23. (a)



Displacement,

$BA = \sqrt{(2R)^2 + (\pi R)^2} = R\sqrt{4 + \pi^2}$

24. (c) Acceleration due to gravity,

$$g = \frac{GM}{R^2} = \frac{4}{3}\pi G\rho R$$
$$\therefore \frac{g_2}{g_1} = \frac{\rho_2}{\rho_1} \times \frac{R_2}{R_1} = \frac{1}{2} \times 1.5 = \frac{3}{4}$$

25. (d) $\Delta\ell = \frac{F\ell}{YA} = \frac{250\times100}{10^{10}\times6.25\times10^{-4}} = 40 \times 10^{-4}\text{ m}$
$= 4 \times 10^{-3}\text{ m}$

26. (a)

Given $M_{H_2} = 2; M_{O_2} = 32$

Speed of sound, $'v = \sqrt{\frac{\gamma RT}{M}}$

$\Rightarrow v \propto \frac{1}{\sqrt{M}}$

$$\therefore \frac{v_{H_2}}{v_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}} = 4:1$$

27. (a) Given,

Susceptibility of material, $\chi_m = 2 \times 10^{-2}$

Using $\mu_r = 1 + \chi_m = 1 + 0.02 = 1.02$

$B_{final} = \mu_r B_0$ (here, B_0 = initial magnetic field)

% increase in magnetic field

$$= \frac{B_{final} - B_0}{B_0} \times 100 = \frac{\mu_r B_0 - B_0 \times 100}{B_0}$$

$$= \frac{(\chi + 1) - 1 \times 100}{1} = 0.02 \times 100 = 2\%$$

28. (b)

We have $\frac{U_E}{U_T} = \frac{U_E}{U_E + U_B} = \frac{U_E}{2U_E} = \frac{1}{2}$

$\left[\because U_E = U_B = \frac{1}{2}E_0E_0^2 = \frac{B_0^2}{2\mu_0}\right]$

29. (a) Resultant intensity in Young's double slit experiment

$$I = 4I_0\cos^2\left(\frac{\Delta\varphi}{2}\right)$$

For path difference $\frac{\lambda}{4}$ phase difference,

$$\Delta\varphi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{4}$$
$$\therefore I_1 = 4I_0\cos^2\left(\frac{\pi}{4}\right)' = 2I_0$$

For path difference $\frac{\lambda}{3}$

$$I_2 = 4I_0\cos^2\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{3}\right) = I_0$$

$$\therefore \frac{I_1 + I_2}{I_0} = 3$$

30. **(d)** As $E(eV) = \frac{1240}{\lambda(nm)} = \frac{1240}{124.1} \simeq 10eV$ Only is transition (d), the energy gap is 10eV So, option (d) is correct $K_a = 0.001\left(\frac{\alpha^2}{1-\alpha}\right) = \frac{0.001\times\left(\frac{2}{19}\right)^2}{1-\left(\frac{2}{19}\right)}$

31. **(a)**
32. **(b)**
33. **(a)**
34. **(b)**
35. **(a)**
36. **(d)**
37. **(c)**
38. **(c)**
39. **(a)**
40. **(b)**
41. **(c)** Frenkel and Schottky defects are crystal defects. It arises due to dislodgement of cation or anion from their places in the crystal lattice.
42. **(d)** Given: Radius of hydrogen atom = 0.530Å, Number of excited state (n) = 2 and atomic number of hydrogen atom (Z) = 1. We know that the Bohr radius

$$(r) = \frac{n^2}{Z} \times \text{radius of atom} = \frac{(2)^2}{1} \times 0.530$$

$$= 4 \times 0.530 = 2.12\text{Å}$$

43. **(d)** The probability density of electrons in 2s orbital first increases then decreases and after that it increases again as distance increases from nucleus.
44. **(b)** Its valence shell has 5 electrons (ns^2, np^3). It belongs to 15 th group of the periodic table.
45. **(b)**
46. **(b)** The stability of the ionic bond depends upon the lattice energy which is expected to be more between Mg and F due to +2 charge on Mg atom.
47. **(a)** 7.5 mol

$$\Delta T_f = K_f m$$

$$\Delta T_f = K_f \frac{n_2 \times 1000}{w_1}$$

$$\Rightarrow 14 = 1.86 \times \frac{n_2 \times 1000}{1000}$$

$n_2 = 7.5$ mol

48. (d)

49.(c)

$$A_m = 1000 \times \frac{\kappa}{M}$$
$$= 1000 \times \frac{2 \times 10^{-5}}{0.001} = 20 \mathrm{Scm^2\ mol^{-1}}$$
$$\Rightarrow \alpha = \frac{\Lambda_m}{\Lambda_m^{\circ}} = \frac{20}{190} = \left(\frac{2}{19}\right)$$
$$\underset{0.001(1-\alpha)}{HA} \rightleftharpoons \underset{0.001\alpha}{H^+} + \underset{0.001\alpha}{A^-}$$
$$K_a = 0.001\left(\frac{\alpha^2}{1-\alpha}\right) = \frac{0.001 \times \left(\frac{2}{19}\right)^2}{1-\left(\frac{2}{19}\right)}$$
$$= 12.3 \times 10^{-6}$$

50.(a) As per Arrhenius equation $\left(k = Ae^{-E_a/RT}\right)$, the rate constant increases exponentially with temperature.

51. (a)

52. (d)

53.(a) At 900 – 1500 K (higher temperature range in the blast furnace)

Reaction which take place are:

$C + CO_2 \rightarrow 2CO$

$FeO + CO \rightarrow Fe + CO_2$

$CaO + SiO_2 \rightarrow CaSiO_3$ (Slag formation)

$FeO_3 + CO \rightarrow 2FeO + CO_2$ at 500 – 800 K

54. (a)

55.(d) Kinetic theory of gases proves all the given gas laws.

56.(b) Enthalpy of formation of C_2H_4, CO_2 and H_2O are 52, –394 and –286 kJ/mol respectively. (Given)

The reaction is

$C_2H_4 + 3O_2 \rightarrow 2CO_2 + 2H_2O$

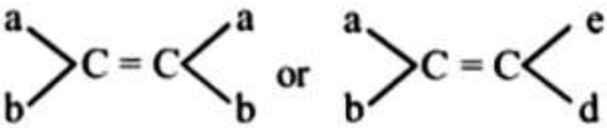
change in enthalpy,

$$(\Delta H) = \Delta H_{\text{products}} - \Delta H_{\text{reactants}}$$
$$= 2 \times (-394) + 2 \times (-286) - (52 + 0)$$
$$= -1412 \text{ kJ/mol}$$

57. (d)

58.(c) Photochemical smog contains nitrogen dioxide (NO2), Ozone (O3), PAN (peroxyacetylnitrate), and compounds containing –CHO group.

59.(c) The condition for geometrical isomerism is



$CH_2 = C(Cl)CH_3$ does not follow above mention condition

60. **(d)** Separating funnel is used when the two liquids are immiscible.

61. (b)

$2Al(s) + 2NaOH(aq) + 6H_2O(l) \rightarrow$

$2Na^+[Al(OH)_4]^-(aq) + 3H_2(g)$

62. (c)

63.(d) Down the group solubility of sulphate decreases. Thus, Ba^{2+} ions will precipitate out most easily.

64.(c) Ionic hydrides give the basic solution when it reacts with water, e.g.,

$LiH + H_2O \rightarrow LiOH + H_2$

65.(b) Tofranil is used for the treatment of antidepressant.

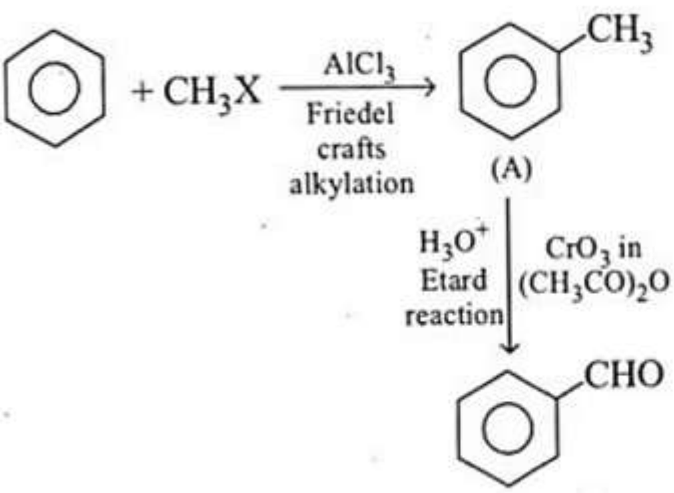
66.(a) Melamine plastic crockery is a copolymer of HCHO and Melamine.

67. (d)

68. **(b)** The α-helix structure is formed when the chain of α-amino acids coils as a right handed screw (called α-helix) because of the formation of hydrogen bonds between amide groups of the same peptide chain, i.e., NH group in one unit is linked to carbonyl oxygen of the third unit by hydrogen bonding. This hydrogen bonding between different units is responsible for holding helix in a position.

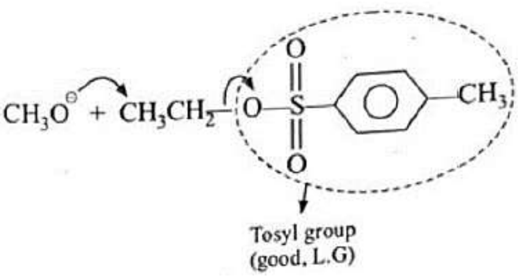
69.(b) Inductive effect, steric hinderance and solvation effect the basic strength of amines.

70.(b)



71. (c)

72. (c) At 443 K compound in (d) will produce propene. In (a) alkene will be produced as tertiary halide and strong base favours elimination. In (b) reaction is not possible at room temperature as due to resonance $C - Cl$ bond has partial double bond character which is very difficult to break.



$\rightarrow OTs^{\ominus} + CH_3OCH_2CH_3$

73. (c) S_N2 and S_N1 same, if $C^{\oplus}$ not rearrange.
74. (a)
75. (a) Ligands can be arranged in a series in the orders of increasing field strength as given below:
Weak field ligands:

$$I^- < Br^- < S^{2-} < SCN < Cl^- < N_3^- \cdot F^-$$
$$< \text{Urea}, OH^- < \text{oxalate}$$

Strong field ligands

$$O^{--} < H_2O < NCS^- < EDTA < Py, NH_3 <$$
$$\text{en} = SO_3^- < \text{bipy. Phen} < NO_2^- < CH_3^-$$
$$< C_6H_5^- < CN^- < CO$$

Such a series is termed as spectrochemical series. It is an experimentally determined series based on the absorption of light by complexes with different ligands.

76. (d) Due to some backbonding by sidewise overlapping between d-orbitals of metal and p orbital of carbon, the $Fe - C$ bond in $[Fe(CO)_5]$ hasboth σ and Π character.
77. (c) In laboratory, manganese (II) ion salt is oxidised to permanganate ion in aqueous solution by peroxodisulphate.

$2Mn^{2+} + 5\ S_2O_8^{2-} + 8H_2O \rightarrow$

peroxodisulphate ion

$2MnO_4^- + 10SO_4^{2-} + 16H^+$

78. (c) Boron in B_2H_6 is electron deficient
79. (a)

	Fe	Al	Br
$E_{Red}\ ^\circ$	0.77	-1.66	1.08
E°_{Oxi}	-0.77	1.66	-1.08

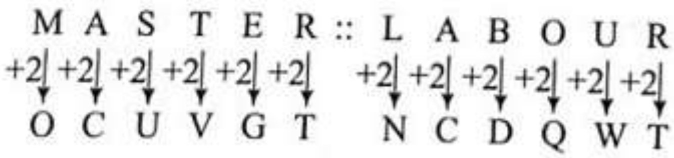
Hence, reducing power $Al > Fe^{2+} > Br^-$

80. (b)
81. (c)
82.(b) Retribution is punishment, contempt is feeling of disgust and grudge is an ill feeling.
83.(c) Sumptuous means sufficient or more in quantity, meagre means very little.
84. (b) Pertinent means relevant.
85.(a) Cold and couldn't go out are suggestive that it was very cold because of which we couldn't go out. So the word substitute for very is 'so' and because is 'that'.
86. (b) Faced with the challenge of the intrusion of colonial culture and ideology at attempt to reinvigorate traditional institutions and realize the potential of traditional culture in the pre-independence India developed during the nineteenth century.
87.(a) A diversified use of natural gas is emerging adding a new dimension to the traditional use of gas as a heating or power generation fuel by converting gas into amongst other products, high quality diesel transportation fuel virtually free of sulphur.
88. (b) Music is often linked to moo(d) A certain song can make us feel happy, sad, energetic, or relaxe (d)

89. **(a)** Music helps one feel different emotions. Based on the mood, a certain song can make us feel happy, sad, energetic, or relaxe(d)

90. **(d)** All forms of music may have therapeutic effects, although music from one's own culture may be most effective.

91. (b)



92. (a)

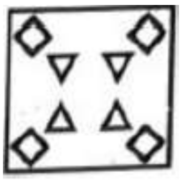
93. (c)

94. (b)

95. (d)

96. (b)

97. (d)



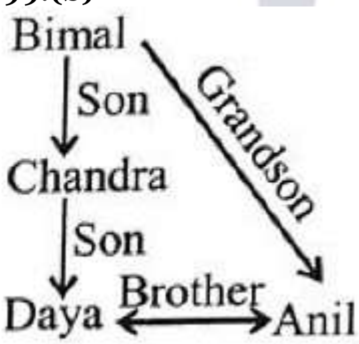
98. **(b)** In this code the alphabets are coded as follows

SISTER UNCLE BOY

535301 84670 129

If we apply this method, the code comes out to be 185336

99.(b)



100. **(b)** Except option (b) all given pairs are synonyms to each other.

101. **(a)**

102. **(a)**

103. **(a)**

104. **(b)**

105. **(c)**

106. **(b)**

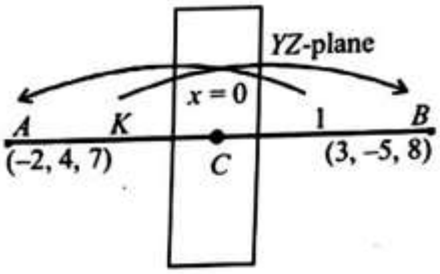
We have $\sec^2\theta = \frac{4}{3} \Rightarrow \cos^2\theta = \frac{3}{4}$

$\Rightarrow \cos^2\theta = \cos^2\left(\frac{\pi}{6}\right) \Rightarrow$

$\theta = n\pi \pm \frac{\pi}{6}$

107. **(b)**

108. **(b)** Let the points be $A(-2,4,7)$ and $B(3,-5,8)$ on YZ-plane, x-coordinate $= 0$.



Let the ratio be $K:1$.

The coordinates of C

are $\left(\frac{3K-2}{K+1}, \frac{-5K+4}{K+1}, \frac{8K+7}{K+1}\right)$

Clearly $\frac{3K-2}{K+1} = 0 \Rightarrow 3K = 2 \Rightarrow K = \frac{2}{3}$

Hence required ratio is $2:3$.

109. **(c)** We have

$\min n(A \cup B) = \max\{n(A), n(B)\} = \max\{3,6\} = 6$

$\operatorname{Max} n(A \cup B) = n(A) + n(B) = 9;$

$\therefore 6 \leq n(A \cup B) \leq 9$

110. **(d)** Let the statement $P(n)$ be defined as

$P(n): \frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is a natural number for all $n \in N$.

Step I: For $n = 1, P(1): \frac{1}{5} + \frac{1}{3} + \frac{7}{15} = 1 \in N$

Hence, it is true for $n = 1$

Step II : Let it is true for $n = k$,

i.e. $\frac{k^5}{5} + \frac{k^3}{3} + \frac{7k}{15} = \lambda \in N$

Step III : For $n = k + 1$

$$\frac{(k+1)^5}{5} + \frac{(k+1)^3}{3} + \frac{7(k+1)}{15}$$

$$= \frac{1}{5}(k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1)$$

$$+ \frac{1}{3}(k^3 + 3k^2 + 3k + 1) + \frac{7}{15}k + \frac{7}{15}$$

$$= \left(\frac{k^5}{5} + \frac{k^3}{3} + \frac{7}{15}k\right) + (k^4 + 2k^3 + 3k^2 + 2k)$$

$$+ \frac{1}{5} + \frac{1}{3} + \frac{7}{15}$$

$$= \lambda + k^4 + 2k^3 + 3k^2 + 2k + 1$$

[using equation (i)]

which is a natural number, since $\lambda k \in N$. Therefore, $P(k+1)$ is true, when $P(k)$ is true, Hence, from the principle of mathematical induction, the statement is true for all -natural numbers n.

111. **(c)** By using formula for finding the roots viz: $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$, we get

$$x = \frac{(r-q) \pm \sqrt{(q-r)^2 - 4(r-p)(p-q)}}{2(p-q)}$$

$$\Rightarrow x = \frac{(r-q) \pm (q+r-2p)}{2(p-q)} = \frac{r-p}{p-q}, 1$$

112. **(a)** The given lines are:

$y = (2-\sqrt{3})x + 5$ and $y = (2+\sqrt{3})x - 7$

Therefore, slope of first line $= m_1 = 2 - \sqrt{3}$ and slope of second line $= m_2 = 2 + \sqrt{3}$

$$\therefore \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{2+\sqrt{3}-2+\sqrt{3}}{1+(4-3)}\right|$$

$$= \left|\frac{2\sqrt{3}}{2}\right| = \sqrt{3} = \tan\frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3} = 60^\circ$$

113. **(c)** $f(x)$ is defined if $3x^2 - 4x + 5 \geq 0$

$$\Rightarrow 3\left[x^2 - \frac{4}{3}x + \frac{5}{3}\right] \geq 0 \Rightarrow 3\left[\left(x - \frac{2}{3}\right)^2 + \frac{11}{9}\right] \geq 0$$

Which is true for all real x: Domain of $f(x) = (-\infty, \infty)$

Let $y = \sqrt{3x^2 - 4x + 5}$

$\Rightarrow y^2 = 3x^2 - 4x + 5$ i.e. $3x^2 - 4x + (5 - y^2) = 0$

For x to be real, $16 - 12(5 - y^2) \geq 0 \Rightarrow y \geq \sqrt{\frac{11}{3}}$

$\therefore$ Range of $y = \left[\sqrt{\frac{11}{3}}, \infty\right)$

114. **(b)**

$$f(x) = \frac{x}{\sqrt{1+x^2}}$$

$$fof = \frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\frac{x^2}{1+x^2}}} = \frac{x}{\sqrt{2x^2+1}}$$

$$fofof = \frac{\frac{x}{\sqrt{2x^2+1}}}{\sqrt{1+\frac{x^2}{2x^2+1}}} = \frac{x}{\sqrt{1+3x^2}}$$

115. **(c)** Let $y = e^{x^3}, z = \log x$

On differentiating w.r.t.x, we get

$\frac{dy}{dx} = e^{x^3}(3x^2) = 3x^2e^{x^3}$ and $\frac{dz}{dx} = \frac{1}{x}$

$$\therefore \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{3x^2e^{x^3}}{\left(\frac{1}{x}\right)} = 3x^3e^{x^3}$$

116. **(a)** Equation of line AB is $y - 3 = \frac{6-3}{7-3}(x-3)$

$$\Rightarrow 3x - 4y + 3 = 0 \Rightarrow \frac{x}{-1} + \frac{y}{3/4} = 1$$

Hence required length is $\sqrt{(-1)^2 + \left(\frac{3}{4}\right)^2} = \frac{5}{4}$.

117. **(d)** Case I: $x \geq 0$, then Eq. (i) becomes

$$2^x + 2^x \geq 2\sqrt{2}$$
$$\Rightarrow 2^x \geq \sqrt{2} \Rightarrow x \geq \frac{1}{2}$$

Case II: $x < 0$, then Eq. (i) becomes

$$2^x + 2^{-x} \geq 2\sqrt{2}$$
$$\Rightarrow t + \frac{1}{t} \geq 2\sqrt{2}, \text{ where } 2^x = t$$
$$\Rightarrow t^2 - 2\sqrt{2}t + 1 \geq 0$$
$$\Rightarrow x \leq \log_2(\sqrt{2}-1)$$

Also, $0 < \sqrt{2} - 1 < 1, \log_2(\sqrt{2}-1) < 0$.

$\therefore$ The solution is

$\left(-\infty, \log_2(\sqrt{2}-1))\right] \cup \left[\frac{1}{2}, \infty\right)$.

118. **(a)**

$$y=\sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}}$$

$$\Rightarrow y=\sqrt{\frac{2\cos^2\theta}{2\sin^2\theta}}=\sqrt{\cot^2\theta}$$

$$\Rightarrow y=\cot\theta$$

Differentiate w.r.t. ' θ ', we get: $\frac{dy}{d\theta}=-\operatorname{cosec}^2\theta$

Now, $\left(\frac{dy}{d\theta}\right)_{\theta=\frac{3\pi}{4}}=-\operatorname{cosec}^2\left(\frac{3\pi}{4}\right)$

$$=-\operatorname{cosec}^2\left(\pi-\frac{\pi}{4}\right)=-\operatorname{cosec}^2\frac{\pi}{4}=-2$$

119. **(b)** Since, $\frac{dy}{dx}=\frac{y+1}{x-1}$

$$\Rightarrow -\frac{dy}{y+1}=\frac{dx}{x-1}$$

After integrating on both sides, we have

$\log(y+1)=\log(x-1)-\log C$

$C(y+1)=(x-1)$

$$C=\frac{x-1}{y+1}$$

If $x=1$, then $y=2$, so $C=0$.

Therefore, $x-1=0$

Hence, there is only one solution.

120. **(b)** Here $n(S)=6^2=36$

Let E be the event "getting sum more than 7 " i.e. sum of pair of dice = 8,9,10,11,12

$$\text{i.e. } E=\left\{\begin{matrix}(2,6) & (3,5) & (4,4) & (5,3) & (6,2)\\ (3,6) & (4,5) & (5,4) & (6,3) & \\ (4,6) & (5,5) & (6,4) & & \\ (5,6) & (6,5) & (6,6) & & \end{matrix}\right\}$$

$\therefore n(E)=15. \therefore$ Req. prob $=\frac{n(E)}{n(S)}=\frac{15}{36}=\frac{5}{12}$

121. **(c)** Total no. of cards = 52

13 cards are diamonds and 4 cards are king. There is only one card which is a king of diamond

$\therefore P$ (card is diamond) $=\frac{13}{52}$, P (card is king)

$$=\frac{4}{52}$$

P (card is king of diamond) $=\frac{1}{52}$

$\therefore P$ (card is diamond or king)

$$=\frac{13}{52}+\frac{4}{52}-\frac{1}{52}=\frac{16}{52}=\frac{4}{13}$$

122. (b)

$$kA=\begin{bmatrix}0 & 2k\\ 3k & -4k\end{bmatrix}=\begin{bmatrix}0 & 3a\\ 2b & 24\end{bmatrix}$$

$\Rightarrow k=-6, a=-4$ and $b=-9$

123. (c)

124. (b) Sum of infinite terms of series $a+ar+ar^2+$

$\ldots.=15\ldots$ (i)

$$\therefore \frac{a}{1-r}=15$$

Sequence formed by square of terms:

$$\text{Sum }=\frac{a^2}{1-r^2}=150$$
$$a^2+a^2r^2+\cdots-=150$$
$$\Rightarrow \frac{a}{1-r}\cdot\frac{a}{1+r}=15\Rightarrow 15\cdot\frac{a}{1+r}=150$$

On dividing equation (i) by (ii)

$$\frac{1+r}{1-r}=\frac{15}{10}$$

or $r=\frac{1}{5}\Rightarrow a=12$

Now series: $ar^2+ar^4+ar^6+\cdots$

$$\text{Sum}=\frac{ar^2}{1-r^2}=\frac{12\cdot\left(\frac{1}{25}\right)}{1-\frac{1}{25}}=\frac{1}{2}$$

125. (a) Given $f(x)=\frac{4x^2+1}{x}$ Thus $f'(x)=4-\frac{1}{x^2}$ $f(x)$ will be decreasing if $f'(x)<0$

Thus $4-\frac{1}{x^2}<0\Rightarrow\frac{1}{x^2}>4\Rightarrow\frac{-1}{2}<x<\frac{1}{2}$ Thus interval in which $f(x)$ is decreasing, is $\left(-\frac{1}{2},\frac{1}{2}\right)$

126. (c)

$$\int e^x \frac{(1+\sin x)}{(1+\cos x)} dx$$
$$= \int e^x \left[\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right] dx$$
$$= \frac{1}{2}\int e^x \sec^2\frac{x}{2} dx + \int e^x \tan\frac{x}{2} dx$$
$$= e^x \tan\frac{x}{2} + C$$

But $I = e^x f(x) + C$ (given)

$\therefore f(x) = \tan\frac{x}{2}$

127. **(b)** $\because x + y = e^{xy}$

Differentiating w.r.t. x, we get

$$1 + \frac{dy}{dx} = e^{xy}\left[y + x\frac{dy}{dx}\right]$$
$$\Rightarrow \frac{dy}{dx}(1 - xe^{xy}) = ye^{xy} - 1$$
$$\Rightarrow \frac{dy}{dx} = \frac{ye^{xy} - 1}{1 - xe^{xy}} \because \frac{dy}{dx} = \infty$$

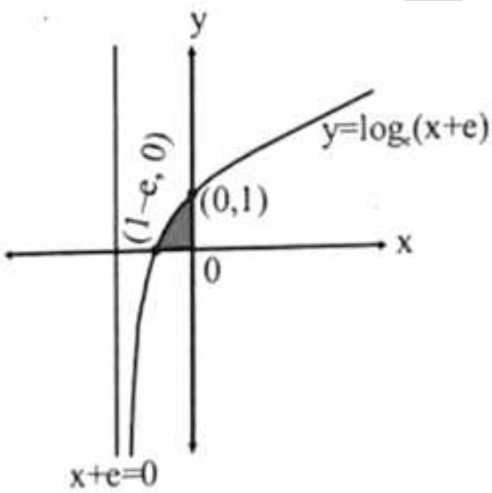
as tangent is parallel to Y-axis

$\Rightarrow 1 - xe^{xy} = 0$

$\therefore xe^{xy} = 1$

This holds, when $x = 1$ and $y = 0$

128. **(a)**



Required area $A = \int_{1-e}^{0} y dx = \int_{1-e}^{0} \log_e(x+e) dx$ put $x + e = t \Rightarrow dx = dt$ also when $x = 1 - e, t = 1$ and when $x = 0, t = e$

$\therefore A = \int_1^e \log_e t dt = [t\log_e t - t]_1^e$

$e - e - 0 + 1 = 1$

129. **(d)** Given $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{a} \cdot \vec{b} = 1$

130. **(c)**

$$\text{Let } l = \lim_{x \to 0} \frac{[\sin x]}{x}; \text{RHL} = \lim_{x \to 0^+} \frac{\sin x}{x} = 1$$

$$\text{LHL} = \lim_{x \to 0^-} \left(\frac{-\sin x}{x}\right) = -\left(\lim_{x \to 0^-} \frac{\sin x}{x}\right) = -1$$

As, LHL ≠ RHL Hence, limit l does not exist.

131. **(d)** Two planes are coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

Applying $C_2 \to C_2 + C_1, C_3 \to C_3 + C_1$

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 1-k \\ k & k+2 & 1+k \end{vmatrix} = 0$$

$$\Rightarrow 1[2 + 2k - (k+2)(1-k)] = 0$$

$$\Rightarrow 2 + 2k - (-k^2 - k + 2) = 0$$

$$k^2 + 3k = 0 \Rightarrow k(k+3) = 0$$

132. **(d)** Given expression is $p \Leftrightarrow (q \Rightarrow p)$

$$\sim (p \leftrightarrow (q \to p))$$

$$\sim (p \leftrightarrow q) = (p \wedge \sim q) \vee (q \wedge \sim p)$$

$$\sim (p \leftrightarrow (q \to p))$$

$$= (p \wedge \sim (q \to p)) \vee ((q \to p) \wedge \sim p)$$

$$(p \wedge \sim (q \to p)) = p \wedge (q \wedge \sim p)$$

$$= (p \wedge \sim p) \wedge q = c$$

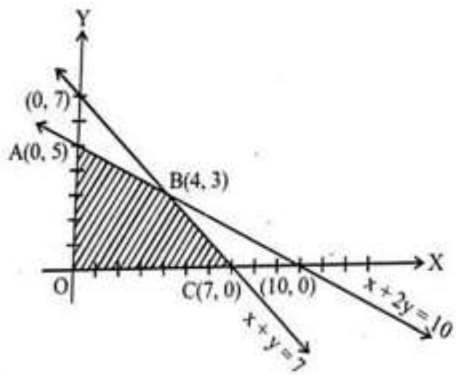
$$(q \to p) \wedge \sim p = (\sim q \vee p) \wedge \sim p$$

$$= \sim p \wedge (\sim q \vee p)$$

$$= (\sim p \wedge \sim q) \vee (\sim p \wedge p) = \sim p \wedge \sim q$$

$$\sim (p \leftrightarrow (q \to p)) = c \vee (\sim p \wedge \sim q) = \sim p \wedge \sim q$$

133. **(c)** Change the inequalities into equations and draw the graph of lines, thus we get the required feasible region as shown below.



The region bounded by the vertices $A(0,5), B(4,3), C(7,0)$.
The objective function is maximum at $C(7,0)$ and $\operatorname{Max} z = 5 \times 7 + 2 \times 0 = 35$

134. (a) Arithmetic mean $\bar{x}$ of 4,7,8,9,10,12,13,17 is

$$\bar{x} = \frac{4+7+8+9+10+12+13+17}{8} = \frac{80}{8} = 10$$

$\Sigma|x_i - \bar{x}| = 6+3+2+1+0+2+3+7 = 24$

∴ Mean deviation about mean

$= \text{M.(d)}\ (\bar{x}) = \frac{\Sigma|x_i-\bar{x}|}{n} = \frac{24}{8} = 3$

135. (d)

136. (b) Let E_1, E_2 and A be the events defined as follows:

E_1 = red ball is transferred from bag P to bag Q

E_2 = blue ball is transferred from bag P to bag Q

A = the ball drawn from bag Q is blue

As the bag P contains 6 red and 4 blue balls,

$P(E_1) = \frac{6}{10} = \frac{3}{5}$ and $P(E_2) = \frac{4}{10} = \frac{2}{5}$

Note that E_1 and E_2 are mutually exclusive and exhaustive events.

When E_1 has occurred i.e., a red ball has already been transferred from bag P to Q, then bag Q will contain 6 red and 6 blue balls, So, $P(A \mid E_1) = 6/12 = 1/2$

When E_2 has occurred i.e., a blue ball has already been transferred from bag P to Q, then bag Q will contain 5 red and 7 blue balls, So, $P(A \mid E_2) = 7/12$

By using law of total probability, we get

$$\begin{aligned} P(a) &= P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2) \\ &= \frac{3}{5} \times \frac{1}{2} + \frac{2}{5} \times \frac{7}{12} = \frac{8}{15} \end{aligned}$$

137. (d)

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \tan^{-1}\left\{\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\times\frac{2}{9}}\right\}$$

$$= \tan^{-1}\left\{\frac{9+8}{36-2}\right\} = \tan^{-1}\left(\frac{1}{2}\right)$$

138. (a) General term $= T_{r+1}$

$$= {}^{10}C_r \left(\frac{10}{x}\right)^{10-r} \cdot \left(\frac{x}{10}\right)^r$$

Here $n = 10$, which is an even number.

Now, $\left[\frac{10}{2}+1\right]^{th}$ term i.e. 6^{th} term is the middle term.

Hence, middle term $= T_6$

$$T_{5+1} = {}^{10}C_5 \left(\frac{10}{x}\right)^{10-5} \left(\frac{x}{10}\right)^5$$
$$= {}^{10}C_5 \left(\frac{10}{x}\right)^5 \left(\frac{x}{10}\right)^5 = {}^{10}C_5$$

139. **(d)**

140. **(b)** Equation of tangent of parabola $y = x^2$ be

$tx = y + at^2$

$y = tx - \frac{t^2}{4}$

Solve with $y = -(x-2)^2$

$$tx - \frac{t^2}{4} = -(x-2)^2$$
$$x^2 + x(t-4) - \frac{t^2}{4} + 4 = 0$$

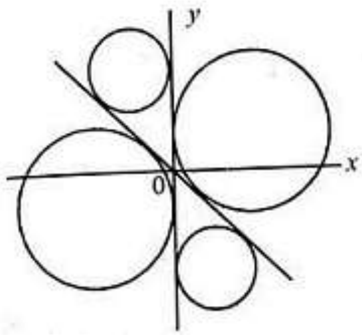
Here, Discriminant $= 0$.

$(t-4)^2 - 4 \cdot \left(4 - \frac{t^2}{4}\right) = 0 \Rightarrow t^2 - 4t = 0 \Rightarrow t = 0$

or $t = 4$

Put value of t in eq. (i), then $y = 4(x-1)$.

141. **(d)**



Let (h, k) is centre of circle

$\left|\frac{h-k}{\sqrt{2}}\right| = |h|, k^2 - h^2 + 2hk = 0$

142. **(b)**

Since, $f(x) = \tan^{-1}(\sin x + \cos x)$

$$\therefore f'(x) = \frac{1}{1 + (\sin x + \cos x)^2}(\cos x - \sin x)$$

$$= \frac{\sqrt{2}\cos\left(x + \frac{\pi}{4}\right)}{1 + (\sin x + \cos x)^2}$$

$f(x)$ is increasing if $f'(x) > 0 \Rightarrow \cos\left(x + \frac{\pi}{4}\right) > 0$

$$\Rightarrow -\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2} \Rightarrow -\frac{3\pi}{2} < x + \frac{\pi}{4}$$

143. **(b)**

144. **(a)**

$$i^{57} + \frac{1}{i^{25}} = (i^4)^{14} \cdot i + \frac{1}{(i^4)^6 \cdot i} = i + \frac{1}{i} = i - i = 0$$

145. **(d)** 4 is a root of $x^2 + px + 12 = 0$

$\Rightarrow 16 + 4p + 12 = 0 \Rightarrow p = -7$

Now, the equation $x^2 + px + q = 0$ has equal roots.

$$\therefore p^2 - 4q = 0 \Rightarrow q = \frac{p^2}{4} = \frac{49}{4}$$

So $I = \int_0^1 e^t dt = [e^t]_0^1 = e - 1$

146. **(a)** Suppose,

$$I = \int \frac{x+3}{(x+4)^2} e^x dx = \int \frac{(x+4)1}{(x+4)^2} e^x dx$$

$$= \int \frac{e^x}{(x+4)} - \int \frac{e^x}{(x+4)^2} dx$$

$$= \int e^x \left(\frac{1}{(x+4)} - \frac{1}{(x+4)^2}\right) dx$$

$$= e^x \frac{1}{(x+4)} + C$$

$\left[\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + C\right]$

147. **(c)**

148. **(a)** $\frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$

Lines passes through the points

$\vec{a}_1 = (3,2,1)$ and $\vec{a}_2 = (-3,6,5)$,

$\vec{b}_1 = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{b}_1 = 2\hat{i} + \hat{j} - 3k, \vec{a}_2 - \vec{a}_1 = 6\hat{i} - 4j - 4\hat{k}$

Shortest distance $= \frac{|(\vec{a}_2-\vec{a}_1)(\vec{b}_1\times\vec{b}_2)|}{|(\vec{b}_1\times\vec{b}_2)|}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 2 & 1 & 3 \end{vmatrix} = 10\hat{i} - 8\hat{j} - 4\hat{k}$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 60 + 32 + 16 = 108$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{100 + 64 + 16} = \sqrt{180}$$

$$S \cdot D = \frac{108}{\sqrt{180}} = \frac{108}{6\sqrt{5}} = \frac{18}{\sqrt{5}}$$

149. (d)

$$P(B) = \frac{3}{5}, P(A \mid B) = \frac{1}{2} \text{ and}$$
$$P(A \cup B) = \frac{4}{5}$$
$$P(A \cap B) = P(A \mid B)P(B) = \frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A) = \frac{4}{5} - \frac{3}{10} = \frac{1}{2} \cdot P(A) = 1 - P(A) = \frac{1}{2}$$

We know, $P(A \cap B) + P(A' \cap B) = P(B)$
[as $A \cap B$ and $A \cap B$ are mutually exclusive events]

$$\Rightarrow \frac{3}{10} + P(A' \cap B) = \frac{3}{5}$$
$$\Rightarrow P(A' \cap B) = \frac{3}{5} - \frac{3}{10} = \frac{3}{10}$$

Now, $P(A' \cup B) = P(A) + P(B) - P(A' \cap B)$

$$= \frac{1}{2} + \frac{3}{5} - \frac{3}{10} = \frac{5 + 6 - 3}{10} = \frac{4}{5}$$

$$P((A \cup B)) = 1 - P(A \cup B) = 1 - \frac{4}{5} = \frac{1}{5}$$
$$\therefore P((A \cup B)) + P(A' \cup B) = \frac{1}{5} + \frac{4}{5} = 1$$

150. (c)