

PART-I: PHYSICS

1. (b) Given,

Amplitude of magnetic field in an electromagnetic wave, $B = 6 \times 10^{-7} \text{ T}$

Relation between magnetic field (B) and electric field (E) of an electromagnetic wave is given by

$$\frac{E}{B} = c \Rightarrow E = Bc = 6 \times 10^{-7} \times 3 \times 10^8$$

$$(\because c = 3 \times 10^8 \text{ m/s})$$

$$\Rightarrow E = 180 \text{ Vm}^{-1}$$

2. (d) Power, $P = \frac{W}{t} = \frac{mgh}{t}$

$$\therefore \frac{P_1}{P_2} = \frac{\frac{m_1 gh}{t_1}}{\frac{m_2 gh}{t_2}} = \frac{m_1}{m_2} \frac{t_2}{t_1}$$

$$\therefore \frac{P_1}{P_2} = \frac{300 \times 2}{5 \times 50} = \frac{12}{5} = \frac{3\sqrt{x}}{\sqrt{x} + 1}$$

$$\text{So, } 12\sqrt{x} + 12 = 15\sqrt{x} \Rightarrow 3\sqrt{x} = 12$$

$$\therefore x = 16$$

3. (a) Given : Length of the dipole ($2l$) = 10 cm = 0.1 m or $l = 0.05 \text{ m}$

Charge on the dipole (q) = $500 \mu\text{C} = 500 \times 10^{-6} \text{ C}$ and distance of the point on the axis from the midpoint of the dipole (r) = $20 + 5 = 25 \text{ cm} = 0.25 \text{ m}$.

We know that the electric field intensity due to dipole on the given point (E)

$$= \frac{1}{4\pi\epsilon_0} \times \frac{2(q \cdot 2l)r}{(r^2 - l^2)^2}$$

$$= 9 \times 10^9 \times \frac{2(500 \times 10^{-6} \times 0.1) \times 0.25}{[(0.25)^2 - (0.05)^2]^2}$$

$$= 6.25 \times 10^7 \text{ N/C}$$

4. (a) Root mean square speed is given by

$$v_{rms} = \sqrt{\frac{3kT}{M}} \Rightarrow v_{rms} \propto \frac{1}{\sqrt{M}}$$

$$\therefore \frac{(v_{rms})_{O_2}}{(v_{rms})_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}}$$

$$\Rightarrow \frac{(v_{rms})_{O_2}}{(v_{rms})_{H_2}} = \sqrt{\frac{2}{32}} = \frac{1}{4}$$

$$\Rightarrow (v_{rms})_{H_2} = 4 \times (v_{rms})_{O_2}$$

$$(\because (v_{rms})_{O_2} = 160 \text{ m/s})$$

$$= 4 \times 160 = 640 \text{ m/s}$$

5. (c) $\frac{f_a}{f_l} = \frac{\left(\frac{\mu_g}{\mu_l} - 1\right)}{(\mu_g - 1)} = \frac{\left(\frac{1.5}{1.25} - 1\right)}{1.5 - 1} = \frac{\frac{1}{5}}{\frac{1}{2}} = \frac{2}{5}$

$$f_l = \frac{5}{2} f_a = \frac{5}{2} \times 2 = 5 \text{ cm}$$

6. (b) Dimensional formula of energy

$$E = \frac{1}{2} mv^2$$

$$[E] = [M^1 L^2 T^{-2}] \dots\dots(i)$$

Dimensional formula of gravitational constant

$$G = \frac{Fr^2}{m_1 m_2}$$

$$[G] = [M^{-1} L^3 T^{-2}] \dots\dots(ii)$$

From eq. (I) & (II)

$$\therefore \frac{E}{G} = \frac{[M^1 L^2 T^{-2}]}{[M^{-1} L^3 T^{-2}]} = [M^2 L^{-1} T^0]$$

7. (c) For photoelectric emission, incident light frequency should be greater than threshold frequency.

Frequency of incident light is 1.5 times the threshold frequency ν_0 .

$$\nu = \frac{3}{2} \nu_0$$

If frequency is halved,

$$\therefore \nu' = \frac{\nu}{2} = \frac{3}{4} \nu_0 \therefore \nu' < \nu_0$$

$\therefore$ No photoelectric emission will take place.

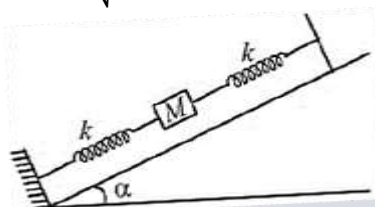
8. (b) Centre of mass of system depends upon position and masses of particle. Also, it depends upon relative distance between particles.

9. (c) The two springs are in parallel, so the effective spring constant is

$$k_{eq} = k_1 + k_2 = k + k = 2k$$

The period of oscillation,

$$T = 2\pi \sqrt{\frac{m}{k_{eq}}} = 2\pi \sqrt{\frac{m}{2k}}$$



Frequency of oscillation,

$$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{2k}{m}}$$

10. (c) The energy stored by a capacitor

$$U = \frac{1}{2} CV^2 \quad \dots(i)$$

V is the p.d. between two plates of the capacitor potential difference $V = E \cdot d$.

The capacitance of the parallel plate capacitor

$$C = \frac{A\epsilon_0}{d}$$

Substituting the value of C in equation(i)

$$U = \frac{1}{2} \frac{A\epsilon_0}{d} (Ed)^2 = \frac{1}{2} A\epsilon_0 E^2 d$$

11. (c) $v_x = \frac{dx}{dt} = \frac{d(12t)}{dt} = 12$

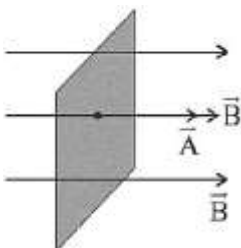
$$\text{and } v_y = \frac{dy}{dt} = \frac{d(5t - 8t^2)}{dt} = 5 - 16t.$$

$$\therefore v_{t=0} = \sqrt{v_x^2 + v_y^2} = \sqrt{12^2 + 5^2} = 13 \text{ m/s}$$

12. (a) Clearly angle between $\vec{B}$ and $\vec{A}$ is 0°

$$\text{so, } \phi(\text{flux}) = \vec{B} \cdot \vec{A} = BA \cos 0^\circ$$

$$= BA = 0.5 \times 1^2 = 0.5 \text{ weber}$$



13. (d) $\tan \theta = \frac{v^2}{rg}$

14. (c) For closed organ pipe, resonate frequency is odd multiple of fundamental frequency.

$$\therefore (2n + 1)f_0 \leq 20,000$$

(f_0 is fundamental frequency = 1.5kHz)

$$\therefore n = 6$$

$\therefore$ Total number of overtone that can be heard = 7

15. (b) For long straight wire

$$B = \frac{\mu_0 i r}{2\pi R^2} \Rightarrow B \propto r, r < R$$

$$B = \frac{\mu_0 I}{2\pi r} \Rightarrow B \propto \frac{1}{r}$$

16. (a) Let

Initial length = ℓ_1

Final length = ℓ_2

Initial area = A_1

Final area = A_2

$\therefore$ Volume remains same

$$\therefore A_1 \ell_1 = A_2 \ell_2 \Rightarrow A_1 \ell_1 = A_2 \frac{\ell_1}{4}$$

$$\Rightarrow 4A_1 = A_2$$

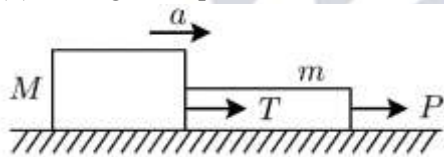
Initial resistance, $R_1 = \frac{\rho \ell_1}{A_1} = 160\Omega$ (given)

Final resistance, $R_2 = \frac{\rho \ell_2}{A_2}$

$$\therefore \frac{R_2}{R_1} = \frac{\ell_2 A_1}{\ell_1 A_2} = \frac{\ell_1}{4} \frac{A_1}{A_1 \ell_1}$$

$$\Rightarrow R_2 = \frac{1}{16} R_1 = \frac{1}{16} \times 160 = 10\Omega$$

17. (d) Taking the rope and the block as a system we get $P = (m + M)a$

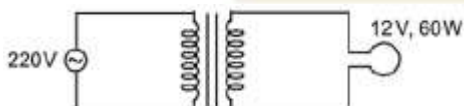


$$\therefore a = \frac{P}{m + M}$$

Taking the block as a system,
we get $T = Ma$

$$\therefore T = \frac{MP}{m + M}$$

18. (a)



A transformer to be ideal and for ideal transformer

$$P_{\text{input}} = P_{\text{output}}$$

$$\therefore (VI)_{\text{in}} = 60 \Rightarrow 220 \times I = 60$$

$$\therefore I = 0.27 \text{ A}$$

19. (b) $\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow \vec{r} \cdot \vec{\tau} = 0 \quad \vec{F} \cdot \vec{\tau} = 0$

Since, $\vec{\tau}$ is perpendicular to the plane of $\vec{r}$ and $\vec{F}$, hence the dot product of $\vec{\tau}$ with $\vec{r}$ and $\vec{F}$ is zero.

20. (d) It has been known that a nucleus of mass number A has radius

$$R = R_0 A^{1/3}$$

where $R_0 = 1.2 \times 10^{-15} \text{ m}$ and $A = \text{mass number}$

In case of ${}_{13}^{27}\text{Al}$, let nuclear radius be R_1

and for ${}_{32}^{125}\text{Te}$, nuclear radius be R_2

For ${}_{13}^{27}\text{Al}$, $R_1 = R_0(27)^{1/3} = 3R_0$

For ${}_{52}^{125}\text{Te}$, $R_2 = R_0(125)^{1/3} = 5R_0$

$$\frac{R_2}{R_1} = \frac{5R_0}{3R_0} \Rightarrow R_2 = \frac{5}{3}R_1 = \frac{5}{3} \times 3.6 = 6\text{fm}$$

21. (b) Given : Mass $m_1 = 4\text{ kg}$ and $m_2 = 6\text{ kg}$.

Acceleration of the system,

$$a = \frac{(m_2 - m_1)g}{(m_1 + m_2)} \text{ where } m_2 > m_1$$

$$\therefore a = \frac{(6 - 4)g}{6 + 4} = \frac{g}{5}$$

22. (c) No. of fringes (n) = $\frac{\text{length of segment (x)}}{\text{fringe width } (\beta)}$

$$\Rightarrow n = \frac{x}{\beta} \Rightarrow x = n\beta = \text{cons.}$$

$$\Rightarrow n_1\beta_1 = n_2\beta_2 \Rightarrow n_1 \frac{\lambda_1 D}{d} = n_2 \frac{\lambda_2 D}{d}$$

$$\Rightarrow n_1\lambda_1 = n_2\lambda_2 \Rightarrow 8 \times 600 = 400 \times n_2$$

$$\Rightarrow n_2 = 12$$

23. (b) Reverse resistance

$$= \frac{\Delta V}{\Delta I} = \frac{1}{0.5 \times 10^{-6}} = 2 \times 10^6 \Omega$$

24. (a) The terminal potential difference of a cell is given by $V + Ir = E$

$$V = V_A - V_B$$

$$\text{or } V = E - Ir$$

$$\Rightarrow \frac{dV}{dI} = -r, \text{ Also for, } i = 0 \text{ then } V = E$$

$$\therefore \text{slope} = -r, \text{ intercept} = E$$

25. (d) De Broglie wavelength is $\lambda = \frac{h}{mv}$

$$\lambda_p = \lambda_e \Rightarrow \frac{h}{m_p v_p} = \frac{h}{m_e v_e}$$

$$m_e v_e = m_p v_p \Rightarrow p_e = p_p \therefore \frac{p_p}{p_e} = \frac{1}{1}$$

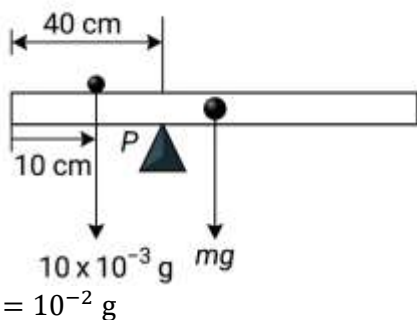
26. (b) Diamagnetic materials does not allow magnetic field lines to pass through them. Bar B represents diamagnetic materials.

27. (c) $F_v = mg - F_b$

$$= \rho V g - \rho_l V g = V g (\rho - \rho_l)$$

$$\text{So, } \frac{F_{v1}}{F_{v2}} = \frac{V_1}{V_2} = \left(\frac{r}{2r}\right)^3 = \frac{1}{8}$$

28. (a)



$$= 10^{-2} \text{ g}$$

Let mass of scale be 'm'. Then,

$$\text{About 'P' } \tau_{\text{net}} = 0$$

$$\Rightarrow 2 \times 10^{-2} \text{ g} \times 30 - mg \times 10 = 0$$

$$\Rightarrow 6 \times 10^{-1} - 10m = 0 \Rightarrow 0.6 = 10m$$

$$\Rightarrow m = \frac{0.6}{10} \Rightarrow m = 0.06 \text{ kg}$$

$$29. (b) W_h = \frac{W_{\text{surface}}}{3} \Rightarrow Mg' = \frac{Mg}{3}$$

$$\Rightarrow \frac{g}{\left(1 + \frac{h}{R}\right)^2} = \frac{g}{3} \Rightarrow \left(1 + \frac{h}{R}\right) = \sqrt{3}$$

$$\Rightarrow \frac{h}{R} = \sqrt{3} - 1 \Rightarrow h = (\sqrt{3} - 1)R$$

$$\Rightarrow h = 0.732 \times 6400 \approx 4685 \text{ km}$$

30. (d) Let L is the original length of the rubber cord and K is the force constant of rubber cord. Final length = initial length + elongation

$$L' = L + \frac{F}{K}$$

For tension 4N

$$\ell_1 = L + \frac{4}{K} \quad \dots(i)$$

For tension 6 N

$$\ell_2 = L + \frac{6}{K} \quad \dots(ii)$$

Solving (i) and (ii) we get

$$L = 3\ell_1 - 2\ell_2 \text{ and } K = \frac{2}{\ell_2 - \ell_1}$$

when tension is 9 N , then

$$\text{length} = L + \frac{9}{K} = 2.5\ell_2 - 1.5\ell_1$$

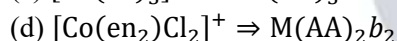
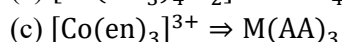
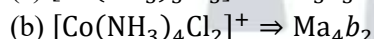
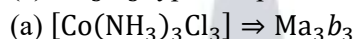
PART - II : CHEMISTRY

$$31. (d) \text{SO}_3(g) \rightleftharpoons \text{SO}_2(g) + \frac{1}{2}\text{O}_2(g)$$

On reversing the reaction and multiplying by 2 we get desired reaction. Thus,

$$K'_c = \left(\frac{1}{K_c}\right)^2 = \left(\frac{1}{4.9 \times 10^{-2}}\right)^2, K'_c = 416.49$$

32. (a) Ma_3b_3 type complexes show Facial Meridional isomerism

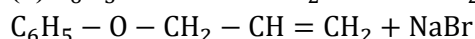
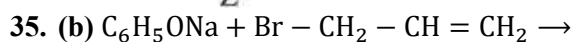
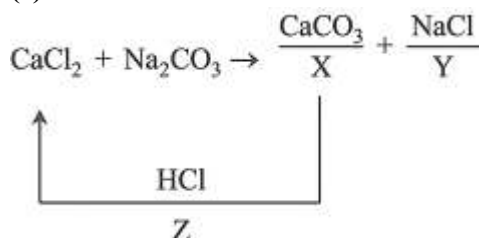


$a, b = \text{NH}_3, \text{Cl}^-$ (Monodentate ligands)

$\text{AA} = \text{en}$ (Bidentate ligand)

33. (c) Conductivity of electrolytic cell is affected by concentration of electrolyte, nature of electrolyte and nature of solvent.

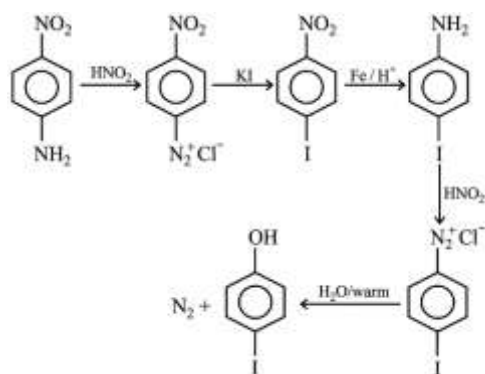
34. (c)



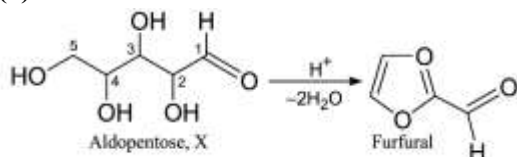
Allyl phenyl ether

In other three options -Br is attached to sp^2 - C atom.

36. (b)



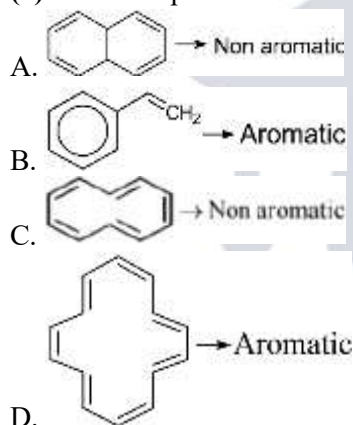
37. (a)



Since furfural is a five membered cyclic compound having an aldehydic group, so the parent sugar (X) should be an aldopentose. Reaction of aldose/ketose with resorcinol gives coloured compound (Seliwanoff test). Ketoses respond this test immediately while aldoses after sometime.

38. (c) Both differ in the arrangement of group in space, therefore grouped under stereo-isomerism.

39. (a) For a compound to be aromatic it should be planar and follow Huckel's rule of $(4n + 2)\pi$ electrons.



$$40. (c) t_{1/4} = \frac{2.303}{k} \log \frac{1}{3/4} = \frac{2.303}{k} \log \frac{4}{3}$$

$$= \frac{2.303}{k} (2 \times 0.301 - 0.4771) = \frac{0.29}{k}$$

41. (a) If the surface tension and vapour pressure are low and the substance solidifies on the surface then stability will be enhanced. Egg white possess all these properties.

42. (c) Density $(\rho) = \frac{PM}{RT}$ (1 bar = 0.987 atm)

$$\rho_{N_2} = \frac{4 \times 0.987 \text{ atm} \times 28 \text{ g/mol}}{R \times 300 \text{ K}}$$

Let the molar mass of gas be x

$$\rho_{\text{gas}} = \frac{2 \times 0.987 \text{ atm} \times x}{R \times 300 \text{ K}}$$

Given $\rho_{\text{gas}} = \rho_{N_2} \times 2$

$$\frac{2 \times 0.987 \text{ atm} \times x}{R \times 300 \text{ K}} = \frac{4 \times 0.987 \text{ atm} \times 28 \text{ g/mol}}{R \times 300} \times 2$$

$$\therefore x = 112 \text{ g/mol.}$$

43. (b) $S(g) + \frac{3}{2}O_2(g) \rightarrow SO_3(g) \Delta H_1 = -2x$

$SO_2(g) + \frac{1}{2}O_2(g) \rightarrow SO_3(g) \Delta H_2 = -y$

$$\Delta H_f(SO_2) = (\Delta H_1) - \Delta H_2 = (-2x) - (-y) = y -$$

2 x and is shorter than a single bond but longer than a double bond as bond order is 1.5

44. (a) (E.N.) = $\begin{matrix} \text{Li} & \text{Be} & \text{B} & \text{C} & \text{N} & \text{O} & \text{F} \\ 1 & 1.5 & 2 & 2.5 & 3 & 3.5 & 4.0 \end{matrix}$

On pauling scale

(E.N.) = $\begin{matrix} \text{Na} & \text{Mg} & \text{Al} & \text{Si} & \text{P} & \text{S} & \text{Cl} \\ 0.9 & 1.2 & 1.5 & 1.8 & 2.1 & 2.5 & 3.0 \end{matrix}$

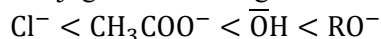
Correct order $\text{Mg} < \text{Al} < \text{B} < \text{N}$

45. (b) Strong acid have weak conjugate base.

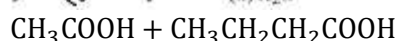
Acidic strength :



Conjugate base strength :



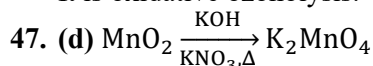
46. (a)



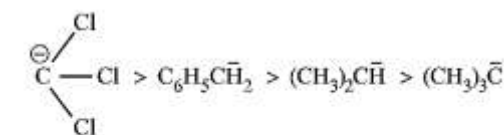
Acetic acid

Butanoic acid

It is oxidative ozonolysis.



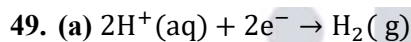
48. (b)



-ve charge
highly dispersed
due to -I effect

-M effect
delocalises
-ve charge

+I effect of CH_3 group
intensifies the
-ve charge



$E = E^\circ - \frac{0.059}{n} \log \frac{P_{\text{H}_2}}{[\text{H}^+]^2}$

For pure water, $[\text{H}^+] = 10^{-7} \text{M}$

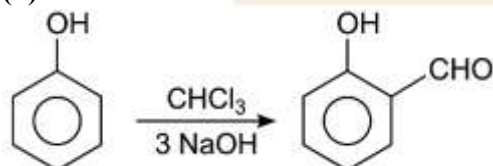
$0 = 0 - \frac{0.059}{2} \log \frac{P_{\text{H}_2}}{(10^{-7})^2}$

$\log \frac{P_{\text{H}_2}}{(10^{-7})^2} = 0 \Rightarrow \frac{P_{\text{H}_2}}{10^{-14}} = 1$

$P_{\text{H}_2} = 10^{-14} \text{ bar}$

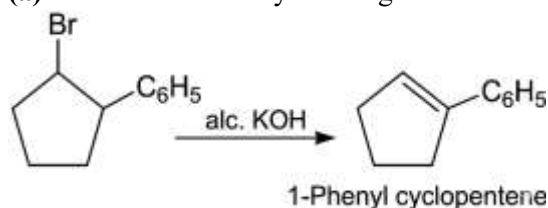
50. (d) Fluorine does not show variable oxidation state as it is the most electronegative element and shows only -1 state.

51. (d)



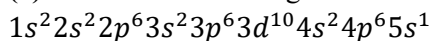
(Reimer-Tiemann reaction)

52. (a) The reaction is dehydrohalogenation



53. (c) In fcc, tetrahedral voids are located on the body diagonal at a distance of $\frac{\sqrt{3}a}{4}$ from the corner. Together they form a smaller cube of edge length $\frac{a}{2}$. Therefore, distance between centres of two nearest tetrahedral voids in the lattice is also $\frac{a}{2}$.

54. (a) The electronic configuration of Rubidium (Rb = 37) is



The last electron enters in 5s orbital.

$$\text{Hence, } n = 5, l = 0, m = 0, s = \pm \frac{1}{2}$$

55. (a)

ion / molecule	Number of e ⁻ in BMO	Number of e ⁻ in ABMO	Bond order
O ₂ ⁺	10	5	2.5
O ₂	10	6	2
O ₂ ⁻	10	7	1.5
O ₂ ²⁻	10	8	1

$$\text{Bond order: } O_2^{2-} < O_2^- < O_2 < O_2^+$$

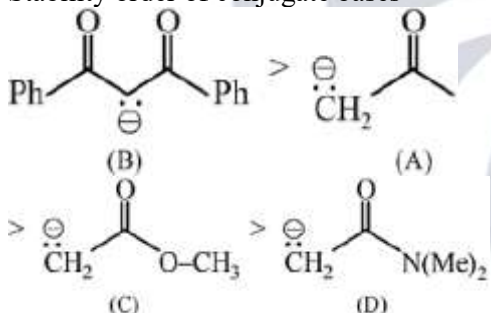
56. (c) Diphenhydramine is used as antihistamine.

57. (a) 2, 4-Dinitrophenylhydrazine reacts with aldehydes and ketones to form 2, 4 dinitrophenylhydrazone derivatives. It is a condensation reaction with elimination of water.

58. (a) Burning of fossil fuels (which contain sulphur and nitrogenous matter) such as coal and oil in power stations and furnaces produce sulphur dioxide and nitrogen oxides which causes acid rain.

59. (a) Acidity $\propto$ Stability of conjugate base

Stability order of conjugate bases



Thus increasing order of acidity is D

60. (a) Higher the number of milli moles of acid or base reacted higher will be temperature rise.

Option (a) n_{acid} or n_{base} reacted = 30mmol

Option (b) n_{acid} or n_{base} reacted = 30mmol

but less energy will be released by neutralisation reaction of weak acid hence option (b) can not be correct.

Option (c) $\Rightarrow$ 20 m mol

Option (d) $\Rightarrow$ 25 m mol

PART - III (A):

ENGLISH PROFICIENCY

61. (b) The idiom "face the music" means to accept the consequences of one's actions, especially criticism or punishment. In the sentence, the team must endure the coach's disapproval after a poor performance.

62. (a) The paragraph starts with **Q**, which introduces the idea that creativity grows in encouraging environments. **P** follows by highlighting the need to overcome fear. **S** shows how small actions build creative confidence, and **R** concludes by explaining the long-term benefits. Together, they build a logical, developmental flow from environment to habit to result.

63. (b) *S* introduces the idea that even vivid memories shift over time. *P* explains why - memories are reconstructed. *Q* then supports this with scientific findings. *R* concludes with the real-world impact of this phenomenon. The sequence builds logically from subtle memory change to explanation, then evidence, then consequence.
64. (c) "Wrangle" means to argue or quarrel noisily. The opposite of that is "agree," which means to share the same opinion or come to a mutual understanding. The other options are synonyms of "wrangle."
65. (d) Gaudy means excessively bright, showy, or tastelessly flashy. The word "flashy" best captures that sense. The other options either imply subtlety (modest, elegant) or lack of color/brightness (dull).
66. (c) The contrast is between real feelings and "performative" acts, especially common online. "Performative outrage" (acting angry just to make a point or show off) is a well-known phrase, fitting best here. The other choices don't match the collocation or context as closely.
67. (b) The sentence suggests the resignation had a strategic purpose ("protecting the company's reputation"). "Tactical" fits this nuance, while the others either imply negative traits or unrelated emotional context.
68. (b) This sentence uses an inversion ("Hardly had..."), so the rest must follow correct tense. The verb "starts" (present tense) is incorrect; it should be "started" (past tense) to maintain consistency.
69. (b) Tense consistency is the issue. It should be "she had left" to reflect the past perfect action that occurred before the remembering. So, the correct phrase is: "she remembered she had left."
70. (b) The verb "know" is a stative verb, and such verbs are not used in continuous tenses. Therefore, "has known" (present perfect) is the correct form.

PART - III (B) :

LOGICAL REASONING

71. (d) $\sqrt{AFI} = \sqrt{169} = 13 = M$

$\sqrt{ADD} = \sqrt{144} = 12 = L$

Similarly,

$\sqrt{ABA} = \sqrt{121} = 11 = K$

72. (b)

As,

A K P : 1121256

↓ ↓ ↓
1² 11² 16²

Similarly,

L N O : 144196225

↓ ↓ ↓
12² 14² 15²

73. (a)

74. (d)

S U P E R F L U O U S
1st 4th 7th 11th

So, possible word with S, E, L, S is LE SS

75. (b) $225 = (15)^2 D(8 + 7)^2$

$49 = (7)^2 D(3 + 4)^2$

$121 = (11)^2 D(6 + 5)^2$

So missing number is 5 .

76. (a)

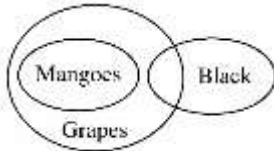
P A L E and E A R T H
 ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
 5 2 9 3 3 2 6 8 1

So,
 P E A R L
 ↓ ↓ ↓ ↓ ↓
 5 3 2 6 9

77. (a) The sequence is:

fggf/gff/fgf/gffg/f

78. (e)

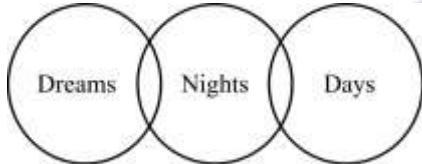


Conclusion:-

(I) Some mangoes being black is a possibility

(II) There is a possibility that some mangoes are not black

79. (b)



Conclusions:-

(I) All days are either nights or dreams x

(II) Some days are nights

80. (a) The pattern is as follows :

$$(7 \times 3 \times 2) = 42$$

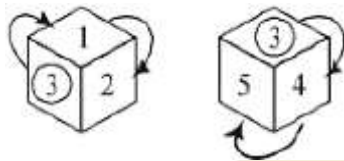
$$(3 \times 4 \times 7) = 84$$

$$(10 \times 7 \times x) = 140$$

$$\Rightarrow \therefore x = \frac{140}{70} = 2$$

$\therefore$ The missing number is = 2.

81. (c)



From figure (ii) and (iii)

1 will be opposite of 4

2 will be opposite of 5

3 will be opposite of 6 missing number

82.(c)

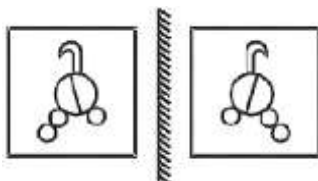


83. (c)

84. (d)

85. (a) Options (b) and (c), (d) have line of symmetry.

86. (c)



87. (b) Meaning order

= unlock vehicle, start vehicle, drive vehicle, Reach destination, Parking of vehicle

88. (c) In the every next fig. each symbol is shifted to its next top position along the same diagonal and symbol at top position comes to bottom position in every next fig.

89. (c) Shraddha's mother's father's son P Shraddha's maternal uncle. Shraddha's maternal uncle is husband of Deepika's sister. Deepika is 'aunt' of Shraddha.

90. (a) The next term is quintuple because quintuple means five times.

PART - IV : MATHEMATICS

91. (c) $A \cup B \cup C = \{0,1,2,3,4,5,6,7,8,9,10\}$

92. (c) Since in (c) each element is associated with unique element. While in (a) element b is associated with two elements, in (b) element a is associated with three elements and in (d) element b is associated with two elements so relation given in option (c) is function.

93. (a) We have, $\sin A = \frac{3}{5}$

$$\Rightarrow \cos A = \sqrt{1 - \sin^2 A}$$

$$= \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\text{and } \tan A = \frac{\sin A}{\cos A} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

$$\text{Now, } \sin 2A = 2 \sin A \cdot \cos A$$

$$= 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$$

$$\cos 2A = 1 - 2 \sin^2 A = 1 - 2 \times \frac{9}{25}$$

$$= 1 - \frac{18}{25} = \frac{7}{25} \text{ and } \tan 2A = \frac{24}{7}$$

94. (a) $2 \tan^2 \theta = \sec^2 \theta = 1 + \tan^2 \theta$

$$\Rightarrow \tan^2 \theta = 1 = (1)^2$$

$$= \tan^2 \frac{\pi}{4}$$

$$\theta = n\pi \pm \frac{\pi}{4}, n \in \mathbb{I}$$

95. (b) $\frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{(1-i)^2}{1-i^2} = \frac{1+i^2-2i}{2} = -i$

$$\therefore (-i)^{100} = (i)^{100} = (i^4)^{25} = 1$$

$$\Rightarrow 1 = a + ib \Rightarrow a = 1, b = 0$$

96. (b) For $x^2 + ax + b = 0$

$$\text{Sum of roots} = a = \frac{-1}{\alpha^2} - \frac{1}{\beta^2} - 2$$

$$\text{Product of roots} = b = \frac{1}{\alpha^2} + \frac{1}{\beta^2} + 1 + \frac{1}{\alpha^2 \beta^2}$$

$$a + b = \frac{1}{(\alpha\beta)^2} - 1 = \frac{1}{6} - 1 = -\frac{5}{6} [\because \alpha\beta = \sqrt{6}]$$

$$x^2 - \left(-\frac{5}{6} - 2\right)x + \left(2 - \frac{5}{6}\right) = 0$$

$$\Rightarrow 6x^2 + 17x + 7 = 0$$

$$x = -\frac{7}{3}, x = -\frac{1}{2} \text{ are the roots}$$

Both roots are real and negative.

97. (a) $D = 4 - 16k \geq 0 \Rightarrow k \leq \frac{1}{4}$

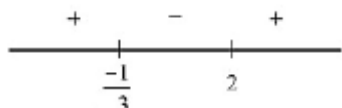
$$f(-1) = 6 + k > 0 \Rightarrow k > -6$$

$$f(1) = 2 + k > 0 \Rightarrow k > -2$$

$$\therefore k \in \left(-2, \frac{1}{4}\right]$$

98. (c) $\frac{4x-1}{3x+1} \geq 1 \Rightarrow \frac{4x-1}{3x+1} - 1 \geq 0 \Rightarrow \frac{x-2}{3x+1} \geq 0$

By using wavy-curve method



$$\therefore x \leq \frac{-1}{3}, x \geq 2 \text{ but } x \neq \frac{-1}{3}$$

$$\therefore x < \frac{-1}{3}, x \geq 2,$$

$$\therefore \left(-\infty, -\frac{1}{3}\right) \cup [2, \infty)$$

99. (b) Total number

= (First two digit identical) +

(Last two digit identical)

$$= 9 \times 1 \times 9 + 9 \times 9 \times 1 = 81 + 81 = 162$$

100. (c) Total number of 3-digit numbers having at least one of their digits as 5 = Total number of 3-digit numbers

- (Total number of 3-digit numbers in which 5 does not appear at all)

$$= 9 \times 10 \times 10 - 8 \times 9 \times 9$$

$$= 900 - 648 = 252$$

101. (a) $\left(x^2 + \frac{1}{x^2}\right)^n$

$$\text{General term } T_{r+1} = {}^nC_r (x^2)^{n-r} \left(\frac{1}{x^2}\right)^r$$

$$= {}^nC_r \cdot x^{2n-5r}$$

To find coefficient of x , $2n - 5r = 1$

$$\text{Given } {}^nC_r = {}^nC_{23} \Rightarrow r = 23 \text{ or } n - r = 23$$

$$\therefore n = 58 \text{ or } n = 38$$

Minimum value is $n = 38$

102. (a) $\therefore a, b, c$ are in GP.

$$\therefore \frac{b}{a} = \frac{c}{b} = r \Rightarrow \frac{b^2}{a^2} = \frac{c^2}{b^2} = r^2$$

$\Rightarrow a^2, b^2, c^2$ are in G.P.

103. (c) $A = \frac{1}{1-r^a} \Rightarrow 1 - r^a = \frac{1}{A}$

$$\Rightarrow r^a = 1 - \frac{1}{A} = \frac{A-1}{A}$$

$$B = \frac{1}{1-r^b} \Rightarrow 1 - r^b = \frac{1}{B} \Rightarrow r^b = 1 - \frac{1}{B} = \frac{B-1}{B}$$

$$\therefore a \log r = \log \left(\frac{A-1}{A}\right) \text{ and}$$

$$b \log r = \log \left(\frac{B-1}{B}\right)$$

$$\therefore \frac{a}{b} = \frac{\log \left(\frac{A-1}{A}\right)}{\log \left(\frac{B-1}{B}\right)} = \log_{\frac{B-1}{B}} \left(\frac{A-1}{A}\right)$$

104. (b) We have, $PA = PB \Rightarrow (PA)^2 = (PB)^2$

$$\Rightarrow [x - (a+b)]^2 + [y - (b-a)]^2$$

$$= [x - (a-b)]^2 + [y - (a+b)]^2$$

$$\begin{aligned} &\Rightarrow [(x-a)-b]^2 + [(y-b)+a]^2 \\ &\Rightarrow [(x-a)+b]^2 + [(y-b)-a]^2 \\ &\Rightarrow [(x-a)+b]^2 - [(x-a)-b]^2 \\ &= [(y-b)+a]^2 - [(y-b)-a]^2 \\ &\Rightarrow 4b(x-a) = 4a(y-b) \Rightarrow bx = ay \dots\dots(i) \end{aligned}$$

Also, $P(a, b)$ satisfies the condition (i), so it can be (a, b) .

105. (c) Let intercept on x -axis and y -axis be a and b respectively so that the equation of line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

But $a = b$ so; $x + y = a$

Also it passes through $(2, 4)$

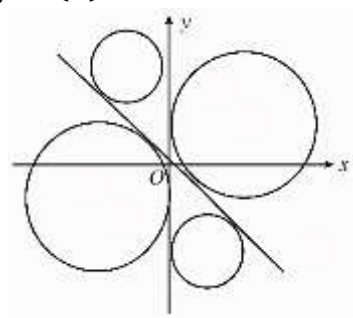
(given)

$$\text{Thus } 2 + 4 = a \Rightarrow a = 6$$

Now, the reqd. equation of the straight line $x + y = 6$ or, $x + y - 6 = 0$.

106. (c)

107. (d)



Let (h, k) is centre of circle

$$\left| \frac{h-k}{\sqrt{2}} \right| = |h|, k^2 - h^2 + 2hk = 0$$

Then, equation of locus is $y^2 - x^2 + 2xy = 0$

108. (b) The tangent at the vertex.

109. (d) Foot of perpendicular from $(6, 5, 8)$ on Y -axis is $(0, 5, 9)$.

$$\text{Req. distance} = \sqrt{(6-0)^2 + (5-5)^2 + (8-0)^2}$$

$$= 10 \text{ unit} \Rightarrow 5\lambda = 10 \Rightarrow \lambda = \frac{10}{5} = 2$$

110. (d) We have

$$\begin{aligned} &\lim_{x \rightarrow \frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{2^{\frac{5}{2}} - [(\cos x + \sin x)^2]^{\frac{5}{2}}}{2 - (1 + \sin 2x)} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{(1 + \sin 2x)^{\frac{5}{2}} - 2^{\frac{5}{2}}}{(1 + \sin 2x) - 2} \\ &= \lim_{t \rightarrow 2} \frac{t^{\frac{5}{2}} - 2^{\frac{5}{2}}}{t - 2}, \text{ where } t = 1 + \sin 2x \\ &= \frac{5}{2} \times (2)^{\frac{5}{2}-1} = 5\sqrt{2} \end{aligned}$$

111. (c) $n(S) = \frac{7!}{3!}, n(E) = 5!$

$$\text{So, } P(E) = \frac{5!}{7!/3!} = \frac{1}{7}$$

112. (a) Mean and $SD\sigma$ of the combined group are

$$m = \frac{63 \times 27.6 + 26 \times 19.2}{63 + 26} = 25.1$$

Thus, AM is decreased by $27.6 - 25.1 = 2.5$.

$$\sigma^2 = \frac{63 \times (7.1)^2 + 26 \times (6.2)^2}{89} + \frac{63(25.1 - 27.6)^2 + 26(25.1 - 19.2)^2}{89} \Rightarrow \sigma = 7.8$$

(approx.)

113. (d) We have, $f(x) = 2x - 3, g(x) = x^3 + 5$
 $(f \circ g)x = f(x^3 + 5) = 2(x^3 + 5) - 3 = 2x^3 + 7$
 Let $y = (f \circ g)x = 2x^3 + 7$
 $\Rightarrow x = \left(\frac{y-7}{2}\right)^{1/3}$

$$\Rightarrow (f \circ g)^{-1}x = \left(\frac{x-7}{2}\right)^{1/3}$$

114. (a) Given $\tan^{-1}(x+1) + \cot^{-1}(x-1)$

$$= \sin^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{3}{5}\right)$$

$$\Rightarrow \tan^{-1}(x+1) + \tan^{-1}\left(\frac{1}{x-1}\right)$$

$$= \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{4}{3} = 2\tan^{-1}\frac{4}{3}$$

$$\Rightarrow \tan^{-1}\left[\frac{x+1 + \frac{1}{x-1}}{1 - (x+1)\left(\frac{1}{x-1}\right)}\right] = \tan^{-1}\left[\frac{2 \times \frac{4}{3}}{1 - \frac{16}{9}}\right]$$

$$\Rightarrow \tan^{-1}\left(-\frac{x^2}{2}\right) = \tan^{-1}\left(-\frac{24}{7}\right)$$

$$\Rightarrow \frac{x^2}{2} = \frac{24}{7} \Rightarrow x^2 = \frac{48}{7} \Rightarrow x = 4\sqrt{\frac{3}{7}}$$

115. (b) $|A| = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 0 & 0 & 2 \end{vmatrix} = 2$

$$\therefore |\text{adj}(\text{adj}A)| = |A|^{(n-1)^2} = |A|^{2^2}$$

[$\therefore$ Here $n = 3$]

$$= 2^4 = 16$$

116. (b) The system of equation has no solution.

$$D = \begin{vmatrix} 3 \sin 3\theta & -1 & 1 \\ 3 \cos 2\theta & 4 & 3 \\ 6 & 7 & 7 \end{vmatrix} = 0$$

$$21 \sin 3\theta + 42 \cos 2\theta - 42 = 0$$

$$\sin 3\theta + 2 \cos 2\theta - 2 = 0$$

$$4 \sin^3 \theta + 4 \sin^2 \theta - 3 \sin \theta = 0$$

$$\sin \theta [4 \sin^2 \theta + 4 \sin \theta - 3] = 0$$

Number of solutions are 7 in $(0, 4\pi)$

117. (b) $x = b \cos^{-1} \sqrt{\frac{y}{b}} + \sqrt{by - y^2}$

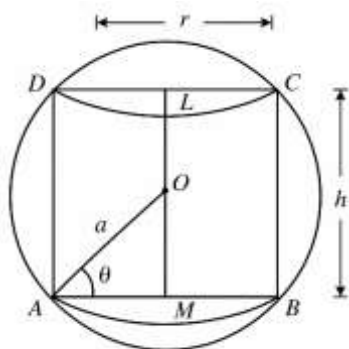
$$\Rightarrow \frac{dx}{dy} = -b \frac{1}{\sqrt{1 - \frac{y}{b}}} \cdot \frac{1}{2\sqrt{\frac{y}{b}}} \cdot \frac{1}{b} + \frac{b - 2y}{2\sqrt{by - y^2}}$$

$$= \frac{-b}{2\sqrt{by - y^2}} + \frac{b}{2\sqrt{by - y^2}} - \frac{2y}{2\sqrt{by - y^2}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{by - y^2}}{y} = -\sqrt{\frac{b}{y}} - 1$$

118. (d) $\lim_{x \rightarrow 4^-} f(x) = \lim_{h \rightarrow 0} f(4 - h)$
 $= \lim_{h \rightarrow 0} \frac{4 - h - 4}{|4 - h - 4|} + a = \lim_{h \rightarrow 0} -\frac{h}{h} + a = a - 1.$
 $= \lim_{x \rightarrow 4^+} f(x) = \lim_{h \rightarrow 0} f(4 + h)$
 $= \lim_{h \rightarrow 0} \frac{4 + h - 4}{|4 + h - 4|} + b = b + 1$
 and $f(4) = a + b$
 Since $f(x)$ is continuous at $x = 4$
 Therefore $\lim_{x \rightarrow 4^-} f(x) = f(4) = \lim_{x \rightarrow 4^+} f(x)$
 $\Rightarrow a - 1 = a + b = b + 1 \Rightarrow b = -1$ and $a = 1.$

119. (b)



If a is the radius of sphere and h the height of cylinder, then from adjacent figure,
 $r^2 + (h^2/4) = a^2$ or $h^2 = 4(a^2 - r^2)$

Now,

$$V = \pi r^2 h = \pi \left(a^2 - \frac{1}{4} h^2 \right) h = \pi \left(a^2 h - \frac{1}{4} h^3 \right)$$

$$\Rightarrow \frac{dV}{dh} = \pi \left(a^2 - \frac{3}{4} h^2 \right) = 0 \text{ (for maximum or minimum)}$$

This gives $h = (2/\sqrt{3})a$ for which
 $d^2V/dh^2 = -6h/4 < 0.$

Hence, V is maximum when $h = 2a/\sqrt{3}.$

120. (a) $\int \left(x + 1 + \frac{1}{2(x-1)} - \frac{2x}{4(x^2+1)} \right) dx$
 $= \frac{x^2}{2} + x + \frac{1}{2} \log(x+1) - \frac{1}{4} \log(x^2+1) + c$

121. (b) Now we are given that the integral

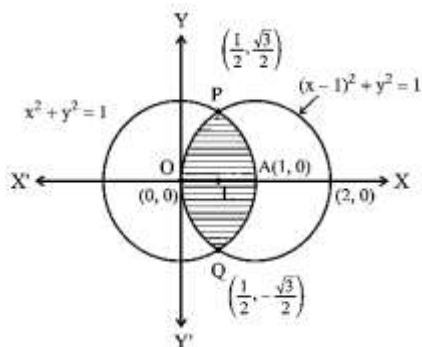
$$I = \int_0^{\frac{\pi}{2}} \frac{dx}{3 + 2\sin x + \cos x} = \int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2} \cdot dx}{2\tan^2 \frac{x}{2} + 4\tan \frac{x}{2} + 4}$$

$$\text{Put } \tan \frac{x}{2} = t \Rightarrow \frac{1}{2} \sec^2 \left(\frac{x}{2} \right) dx = dt$$

$$\text{Now } I = \int_0^1 \frac{dt}{(t+1)^2 + 1} = \tan^{-1}(t+1) \Big|_0^1$$

$$= \tan^{-1} 2 - \frac{\pi}{4}$$

122. (a) Given circles are $x^2 + y^2 = 1$ (i)
 and $(x-1)^2 + y^2 = 1$ (ii)
 Centre of (i) is $O(0,0)$ and radius = 1



Both these circle are symmetrical about x -axis solving (i) and (ii), we get, $-2x + 1 = 0 \Rightarrow x = \frac{1}{2}$

$$\text{then } y^2 = 1 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} \Rightarrow y = \frac{\sqrt{3}}{2}$$

$\therefore$ The points of intersection are

$$P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \text{ and } Q\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

It is clear from the figure that the shaded portion in region whose area is required.

$\therefore$ Required area = area OQAPO

= $2 \times$ area of the region OLAP

= $2 \times$ (area of the region OLPO + area of LAPL)

$$= 2 \left[\int_0^{1/2} \sqrt{1 - (x-1)^2} dx + \int_{1/2}^1 \sqrt{1 - x^2} dx \right]$$

$$= 2 \left[\frac{(x-1)\sqrt{1 - (x-1)^2}}{2} + \frac{1}{2} \sin^{-1}(x-1) \right]_{1/2}^{1/2}$$

$$+ 2 \left[\frac{x\sqrt{1 - x^2}}{2} + \frac{1}{2} \sin^{-1} x \right]_{1/2}^1$$

$$= -\frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \sin^{-1}\left(\frac{-1}{2}\right) - \sin^{-1}(-1) + 0$$

$$+ \sin^{-1}(1) - \left(\frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \sin^{-1}\left(\frac{1}{2}\right) \right)$$

$$= \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right) \text{ sq. units.}$$

123. (a) $I \cdot F = e^{\int \frac{1}{x \log} - \log x}$

$$y \cdot \log x = \int \frac{\log x}{x} dx$$

$$\Rightarrow y = \frac{1}{2} \log x + c(\log x)^{-1}$$

124. (a) $\int \frac{x}{x^2-1} dx = \int \frac{y}{1+y^2} dy$

$$\Rightarrow x^2 - 1 = C(1 + y^2)$$

125. (d) Three vectors are $\vec{u}, \vec{v}, \vec{w}$ Since, the dot product is commutative and the position of dot and cross can be interchanged in scalar triple product

$$\therefore \vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{v} \times \vec{w}) \cdot \vec{u} \text{ and}$$

$$(\vec{u} \times \vec{v}) \cdot \vec{w} = \vec{u} \cdot (\vec{v} \times \vec{w})$$

$$\Rightarrow \vec{v} \cdot (\vec{u} \times \vec{w}) = (\vec{u} \times \vec{w}) \cdot \vec{v}$$

$$\text{and option (d) gives : } \vec{u} \cdot (\vec{w} \times \vec{v}) = -\vec{u} \cdot (\vec{v} \times \vec{w})$$

126. (a) Equating the components in

$$\alpha(\hat{i} + 2\hat{j} + 3\hat{k}) + \beta(2\hat{i} + 3\hat{j} + \hat{k}) + \gamma(3\hat{i} + \hat{j} + 2\hat{k})$$

$$= -3(\hat{i} - \hat{k}), \text{ we have}$$

$$\alpha + 2\beta + 3\gamma = -3 \quad \dots(i)$$

$$2\alpha + 3\beta + \gamma = 0 \quad \dots(ii)$$

$$3\alpha + \beta + 2\gamma = 3 \quad \dots(iii)$$

Solving the equations (i), (ii), & (iii), we get

$$\alpha = 2, \beta = -1, \gamma = -1.$$

127. (c) The direction cosines of the line are $\cos \theta, \cos \beta, \cos \theta$

$$\therefore \cos^2 \theta + \cos^2 \beta + \cos^2 \theta = 1$$

$$\Rightarrow 2\cos^2 \theta = \sin^2 \beta = 3\sin^2 \theta$$

$$\Rightarrow 2\cos^2 \theta = 3 - 3\cos^2 \theta \therefore \cos^2 \theta = \frac{3}{5}$$

128. (d) The required plane is

$$(2 + 3\lambda)x + (-8 - 5\lambda)y + (4 + 4\lambda)z + P - 10\lambda = 0$$

Compare the coefficients with the plan

$$\text{We get, } 4 + 4\lambda = 0 \Rightarrow \lambda = -1$$

$$x + 3y + 0z + 13 = 0$$

Then we get $p = 3$.

129. (a) Given that $P(A | B) = \frac{1}{7} \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{1}{7}$

$$\Rightarrow P(B) = \frac{7}{9}$$

$$P(B | A) = \frac{2}{5} \Rightarrow \frac{P(A \cap B)}{P(A)} = \frac{2}{5} \Rightarrow P(A) = \frac{5}{18}$$

$$\text{We know that } P(A' \cup B) = 1 - P(A \cup B) + P(B)$$

$$= 1 - P(A) + P(A \cap B) = \frac{5}{6}$$

$$P(A' \cap B') = 1 - P(A \cup B)$$

$$= 1 - P(A) - P(B) + P(A \cap B) = \frac{1}{18}$$

$\Rightarrow \text{Both } (S1) \text{ and } (S2) \text{ are true.}$

130. (a) Since, A and B are independent events.

$$\therefore P(A \cap B) = P(A)P(B)$$

Further since, $A \cap C, B \cap C, A \cap B \cap C$ are subsets of C, we have, $P(A \cap C) \leq P(C) = 0$

$$P(B \cap C) \leq P(C) = 0$$

$$\text{and } P(A \cap B \cap C) \leq P(C) = 0$$

$$\Rightarrow P(A \cap C) = 0 = P(A)P(C)$$

$$P(B \cap C) = 0 = P(B)P(C)$$

$$P(A \cap B \cap C) = 0 = P(A)P(B)P(C).$$

Clearly A, B, C are pairwise independent as well as mutually independent. Thus, A, B, C are independent events.