Physics

1. **(a)** Torque on a magnetic dipole in a magnetic field:
$\tau = MB\sin\theta$
Where
• $\tau =$ torque
• $M =$ magnetic moment
• $B =$ magnetic field
•$\theta =$ angle between M and B
Using the formula:
$\tau = MB\sin\theta$
Given
$\tau = 4.5 \times 10^{-2} J$
$B = 0.5T$
$\theta = 30^\circ$
$\sin 30^\circ = 0.5$

2. **(d)** Use Faraday's law of electromagnetic induction.
1.Magnetic flux
Magnetic flux through the loop:
$\Phi = BA\cos\theta$
Since the loop is in the east-west vertical plane, its area vector is along north-south.
The magnetic field is along north-east, which makes 45° with north.
So
$\theta = 45^\circ$
Area of square loop:
$A = (0.10)^2 = 0.01\ \text{m}^2$
Initial flux:
$\Phi = BA\cos 45^\circ$
$$\Phi = (0.10)(0.01)\left(\frac{1}{\sqrt{2}}\right)$$
$\Phi = 7.07 \times 10^{-4}$ Wb
2.Induced emf
Magnetic field decreases to zero in 0.70 s.
$$\mathcal{E} = \frac{\Delta\Phi}{\Delta t}$$
$$\mathcal{E} = \frac{7.07 \times 10^{-4}}{0.70}$$
$\mathcal{E} = 1.01 \times 10^{-3}$ V
3.Induced current
$$I = \frac{\mathcal{E}}{R}$$
$$I = \frac{1.01 \times 10^{-3}}{0.5}$$
$I = 2.02 \times 10^{-3}$ A
$I \approx 2.0 \times 10^{-3}$ A

3. **(a)** The self-inductance L of a coil depends only on the geometry of the coil and the magnetic medium, not on the current flowing through it.
Self-inductance is defined as:
$$L = \frac{N\Phi}{I}$$
Where:
• $N =$ number of turns
• $\mathbf{\Phi} =$ magnetic flux through the coil
•$I =$ current
When the current becomes 51, the magnetic flux also increases proportionally (5Φ). Thus:
$$L = \frac{N(5\Phi)}{5I} = \frac{N\Phi}{I} = L$$

4. **(c)** Step 1: Calculate the inductive reactance
For a pure inductor, the inductive reactance X_L is:
$X_L = 2\pi f L$
Substitute the values:
$X_L = 2\pi(50)(0.050)$
$X_L = 2\pi(2.5)$
$X_L \approx 6.2832 \times 2.5$
$X_L \approx 15.71\Omega$
Step 2: Calculate RMS current
For a pure inductor, RMS current is:
$I_{\mathrm{rms}} = \frac{V_{\mathrm{rms}}}{X_L}$
$I_{\mathrm{rms}} = \frac{220}{15.71}$
Let's divide carefully:
$220 \div 15.71 \approx 14.0$ A

5. **(b)** In a series LCR AC circuit, the power factor (pf) is given by:
$\mathrm{pf} = \cos\phi$
where ϕ is the phase difference between the applied voltage and the current.
At resonance, the inductive reactance (X_L) equals the capacitive reactance (X_C):
$X_L = X_C$
This makes the net reactance zero, so the circuit behaves like a pure resistor. Therefore:
$\phi = 0^\circ \Rightarrow \cos\phi = 1$

6. **(a)** Step 1: Write down the known values
• Primary turns: $N_p = 100$
• Secondary turns: $N_s = 200$
• Primary voltage: $V_p = 220$ V
• Primary current: $I_p = 10$ A
We are asked to find secondary current I_s.
Step 2: Transformer equations
For an ideal transformer:
$\frac{V_s}{V_p} = \frac{N_s}{N_p}$ and $\frac{I_s}{I_p} = \frac{N_p}{N_s}$
Since it's a step-up transformer, $N_s > N_p$, so voltage increases and current decreases.
Step 3: Find secondary current
$I_s = I_p \frac{N_p}{N_s} = 10 \times \frac{100}{200} = 10 \times 0.5 = 5$ A

7. **(c)** Wattless current means current that does no real work - i.e., the power consumed $P = VI\cos\phi = 0$.

• This happens when the phase difference between voltage and current is 90°.

• In other words, all energy is reactive (stored in magnetic or electric fields), not dissipated as heat.

AC circuit elements:

• Resistor (R): Current is in phase with voltage → $\cos\varphi = 1$ → real power $\neq 0$ → Not wattless.

• Inductor (L): Current lags voltage by 90° → $\cos\varphi = 0$ → Wattless current

• Capacitor (C): Current leads voltage by 90° → $\cos\varphi = 0$ → Wattless current

• R-L or R-C in series: Phase difference is less than 90°, so $\cos\varphi \neq 0$ → Not purely wattless.
Therefore, for purely wattless current, either only L or only C is connected.

8. **(b)** The Ampere-Maxwell law relates the magnetic field around a closed loop to the conduction current and the changing electric flux through the surface bounded by that loop. Its standard form is:

$\oint \mathrm{B} \cdot dl = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$
Here:

- B = magnetic field
- dl = line element along the closed loop
- i_c = conduction current
- Φ_E = electric flux through the loop

9. **(d)** Cellular phones operate in the Ultra High Frequency (UHF) band, typically ranging from 700 MHz to 2.6 GHz, because UHF waves can carry high data rates and penetrate buildings well, which is ideal for mobile communication.

For clarity:

- LF (Low Frequency): 30 − 300kHz, used for long-range navigation, not phones.
- HF (High Frequency): 3 − 30MHz, used for shortwave radio.
- VHF (Very High Frequency): 30 − 300MHz, used for FM radio and TV channels.
- UHF (Ultra High Frequency): 300MHz − 3GHz, used for cell phones, GPS, Wi-Fi, and TV.

10. **(a)**

11. **(a)**

12. **(c)** Step 1: Formula for minimum deviation (thin prism)

For a thin prism, the minimum deviation is approximately:

$\delta_m \approx (\mu - 1)A$

Where:

- δ_m is in degrees
- A is the prism angle in degrees
- μ is the refractive index

Step 2: Substitute values

$\delta_m \approx (1.6 - 1) \cdot 4^\circ$

$\delta_m \approx 0.6 \cdot 4^\circ$

$\delta_m \approx 2.4^\circ$

13. **(a)** Step 1: Recall the formula

For a refracting telescope in normal adjustment, the angular magnification M is:

$$M = \frac{f_{\text{objective}}}{f_{\text{eyepiece}}}$$

where:

- $f_{\text{objective}}$ = focal length of objective lens
- f_{eyepiece} = focal length of eyepiece lens

Step 2: Convert units

Given:

- $f_{\text{objective}} = 1 \text{ m} = 100 \text{ cm}$
- $f_{\text{eyepiece}} = 1 \text{ cm}$

Step 3: Substitute values

$$M = \frac{f_{\text{objective}}}{f_{\text{eyepiece}}} = \frac{100}{1} = 100$$

14. **(b)** For a wave front, all points on it are in phase, because a wave front is a surface over which the phase of the wave is constant.

So, the phase difference between any two particles on the same wave front is:

0 rad

15. **(c)** Distance between the slits, $d = 0.28 \text{ mm} = 0.28 \times 10^{-3} \text{ m}$

Distance between the slits and the screen, $D = 1.4 \text{ m}$

Distance between the central fringe and the fourth ($n = 4$) fringe, $u = 1.2 \text{ cm} = 1.2 \times 10^{-2} \text{ m}$

In case of constructive interference, we have the relation for the distance between the two fringes as:

$$u = n\lambda \frac{D}{d}$$

Where,

n = Order of fringes = 4

λ = Wavelength of light used

$\lambda = \dfrac{ud}{nD}$
$= \dfrac{1.2 \times 10^{-2} \times 0.28 \times 10^{-3}}{4 \times 1.4}$
$= 6 \times 10^{-7}$
$= 600 \text{ nm}$
Hence, the wavelength of the light is 600 nm.

16. (d) To find the speed of light in glass, we use the formula for refractive index:
$n = \dfrac{c}{v}$
Where:
• $n = 1.6$ (refractive index of glass)
• $c = 3.0 \times 10^8 \text{ m/s}$ (speed of light in vacuum)
• $v =$ speed of light in glass
Rearranging:
$v = \dfrac{c}{n} = \dfrac{3.0 \times 10^8}{1.6}$
Step-by-step division:
$\dfrac{3.0}{1.6} = 1.875$
So:
$v = 1.875 \times 10^8 \text{ m/s} \approx 1.88 \times 10^8 \text{ m/s}$

17. (a) The unit Js (joule-second) is used for angular momentum in physics.
• Reasoning:
•Angular momentum $L = I\omega$ or $L = r \times p$, where $p = mv$.
• Its Sl unit comes out as $\text{kg} \cdot \text{m}^2/\text{s} = N \cdot m \cdot s = J \cdot s$.

18. (d) The question is about field emission of electrons from a metal, which occurs due to a very strong electric field at the metal surface.
• Field emission requires extremely high electric fields because the electrons are tightly bound in the metal.
• Typical minimum electric field required to pull out electrons is around 10^8 V/m.

19. (b)

20. (a) We can solve this step by step using Bohr's model of the hydrogen atom.
The radius of the n-th orbit in a hydrogen atom is given by:
$r_n = n^2 a_0$
where:
• $n =$ principal quantum number (orbit number)
• $a_0 =$ Bohr radius
We are asked for the ratio of radius of the second orbit to the third orbit:
$r_2 : r_3 = \dfrac{2^2 a_0}{3^2 a_0} = \dfrac{4a_0}{9a_0} = \dfrac{4}{9}$

21. (c) In the Geiger-Marsden scattering experiment (also known as the Rutherford gold foil experiment), the thin gold foil used had a thickness of approximately:

22. (b) Step 1: Recall the formula
For a hydrogen atom:
$E = \text{ Total energy } = K + U$
where:
• $K =$ kinetic energy of electron
• $U =$ potential energy of electron
From Bohr's theory, we know:
$K = -\dfrac{E}{1} = -E$
and
$U = 2E$
Actually, more precisely:

• Total energy: $E = K + U$
• Kinetic energy: $K = -E$
• Potential energy: $U = 2E$
Step 2: Substitute the given value
Ground state energy: $E = -13.6\text{eV}$
$U = 2E = 2 \times (-13.6) = -27.2\text{eV}$

23. (a)

24. (d) Step 1: Use conservation of mass number (A)
• Left side: $235 + 1 = 236$
• Right side: $133 + y + 4 \times 1 = 133 + y + 4 = 137 + y$
Equating mass numbers:
$236 = 137 + y \Longrightarrow y = 99$
Step 2: Use conservation of atomic number (Z)
• Left side: $92 + 0 = 92$
• Right side: $x + 41 + 0 = x + 41$
Equating atomic numbers:
$92 = x + 41 \Longrightarrow x = 51$

25. (d) The radius of a nucleus is approximately given by the formula:
$R = R_0 A^{1/3}$
where A is the mass number and R_0 is a constant.
For ${}_1^1\text{H}, A = 1$:
$R_H = R_0 \cdot 1^{1/3} = R_0$
For ${}_{13}^{27}\text{Al}, A = 27$:
$R_{Al} = R_0 \cdot 27^{1/3} = R_0 \cdot 3$
So, the ratio $R_H : R_{Al} = R_0 : 3R_0 = 1 : 3$.

26. (c) To convert a pure semiconductor (like silicon) into a p-type semiconductor, we need to add a trivalent impurity-an element with three valence electrons-because it creates holes (positive charge carriers) in the semiconductor.
• Trivalent impurities: Boron (B), Aluminum (AI), Gallium (Ga), Indium (In)
• Pentavalent impurities: Phosphorus (P), Arsenic (As), Antimony (Sb) → used for n-type semiconductors Here, among the given options, the trivalent impurity is Indium.

27. (b) For an intrinsic semiconductor like germanium, the energy required for an electron to jump the forbidden band (i.e., the band gap energy, E_g) at room temperature is a known property:
• Germanium (Ge): $E_g \approx 0.72\text{eV}$
• Silicon (Si): $E_g \approx 1.1\text{eV}$

28. (d) Let's carefully work this out step by step.
Electric flux (Φ_J) is defined as:
$\Phi_E = E \cdot A$
where:
• $E =$ electric field (N/C or V/m)
• $A = \text{area}(\text{m}^2)$
Step 1: Express electric field in fundamental units
$$E = \frac{F}{q}$$
where:
• $F = \text{force} = MLT^{-2}$
• $q = \text{electric charge} = A \cdot T$ (from current × time)
So:
$$[E] = \frac{MLT^{-2}}{AT} = MLT^{-3}A^{-1}$$
Step 2: Multiply by area
$[\Phi_E] = [E] \cdot [A] = (MLT^{-3}A^{-1}) \cdot L^2 = ML^3T^{-3}A^{-1}$

29. (b) For an electric dipole:

• Dipole moment $\vec{P}$ points from negative to positive charge.
• The equatorial plane is perpendicular to the dipole axis and passes through its center.
• On the equatorial plane, the electric field $\vec{E}$ due to the dipole is opposite in direction to the dipole moment.
So, the angle between $\vec{E}$ and $\vec{P}$ is:
$\theta = 180^{\circ}$

30. (b)

31. (c) The energy gained by a charge q when it is accelerated through a potential difference V is:
$E = qV$
Given:
• $q = 1 \times 10^{-19}\mathrm{C}$
• $V = 2.5\ \mathrm{V}$
Step 1: Use the formula in electron volts (eV)
By definition:
$1\mathrm{eV} = 1.6 \times 10^{-19}\ \mathrm{J}$
But the energy in eV is simply:
$E(\text{ in eV }) = q(\text{ in units of } e) \times V(\text{ in volts })$
Since the electron has charge $1e$,
$E = 1 \times 2.5 = 2.5\mathrm{eV}$
Step 2: (Optional) Convert to joules for checking)
$E = qV = (1 \times 10^{-19})(2.5) = 2.5 \times 10^{-19}\ \mathrm{J}$

32. (d) Step 1: Recall the property of a charged spherical shell
For a thin spherical shell with charge Q :
1. Outside the shell ($r \geq R$):
$$V = \frac{kQ}{r}$$
2.On the surface $(r = R)$:
$$V = \frac{kQ}{R} \text{ (given as 100 V)}$$
3.Inside the shell ($r < R$):
Electric potential is constant: $V = \frac{kQ}{R} = V_{\text{surface}}$
This is because the electric field inside a spherical shell is zero, so the potential does not change.
Step 2: Apply the formula for inside the shell
Here, $r = 2\ \mathrm{cm} < 10\ \mathrm{cm}$.
$V_{\text{inside}} = V_{\text{surface}} = 100\ \mathrm{V}$

33. (a) We are given:
- Initial capacitance with air: $C_0 = 4\mathrm{pF}$
- Distance between plates: d (initial)
- Dielectric introduced: $K = 6$
- Distance reduced to half: $d' = \frac{d}{2}$

The capacitance of a parallel plate capacitor is given by:
$$C = \frac{K\varepsilon_0 A}{d}$$
Where:
• $K =$ dielectric constant
• $A =$ area of plates
• $d =$ separation
Step 1: Original capacitance
With air ($K = 1$):
$$C_0 = \frac{\varepsilon_0 A}{d} = 4\mathrm{pF}$$
Step 2: New capacitance
Now, distance is halved and dielectric $K = 6$ is inserted:
$$C = \frac{K\varepsilon_0 A}{d'} = \frac{6 \cdot \varepsilon_0 A}{d/2}$$

Simplify $d/2$ in denominator:
$$C=\frac{6\cdot\varepsilon_0 A}{d/2}=6\cdot 2\cdot\frac{\varepsilon_0 A}{d}=12\cdot\frac{\varepsilon_0 A}{d}$$
$$C=12\cdot C_0=12\cdot 4\mathrm{pF}=48\mathrm{pF}$$

34. **(a)** The SI unit of current density is Am^{-2}.

35. **(c)** The quantity described - the magnitude of the drift velocity per unit electric field - is called mobility.

36. **(d)** Step 1: Check if the bridge is balanced

• For a Wheatstone bridge, the bridge is balanced if:
$$\frac{R_1}{R_2}=\frac{R_3}{R_4}$$
Here, let's label the resistances:

• $R_1=2\Omega$ (AB)
• $R_2=4\Omega$(BC)
• $R_3=5\Omega$ (AD)
• $R_4=8\Omega$(DC)

Check: $\frac{R_1}{R_2}=\frac{2}{4}=0.5$
$$\frac{R_s}{R_4}=\frac{5}{8}=0.625$$
• Not equal → Bridge not balanced, so current flows through galvanometer.

Step 2: Simplify the circuit

The galvanometer is in the middle (10Ω). To find total current, we can combine series-parallel:

• AB (2Ω) in series with BC(4Ω) → Path 1: $R_{\text{path }1}=2+4=6\Omega$
• AD (5Ω) in series with DC (8Ω) → Path 2: $R_{\text{path }2}=5+8=13\Omega$
• The two paths are in parallel across the 10 V battery:
$$\frac{1}{R_{\text{total}}}=\frac{1}{6}+\frac{1}{13}$$
Step by step:
$$\frac{1}{6}+\frac{1}{13}=\frac{13+6}{78}=\frac{19}{78}\Rightarrow R_{\text{total}}=\frac{78}{19}\approx 4.105\Omega$$
Step 3: Find total current
$$I=\frac{V}{R_{\text{total}}}=\frac{10}{4.105}\approx 2.44\text{ A}$$
The closest option is 2.5 A.

37. **(d)**

38. **(b)** The unit Vs/Am can be rewritten as:
$$\frac{V\cdot s}{A\cdot m}$$
Since

• 1 Henry $(H)=\frac{V\cdot s}{A}$

So, $\frac{V\cdot s}{A\cdot m}=\frac{H}{m}$

The unit H/m is the SI unit of magnetic permeability of free space.

39. **(a)** • Ideal Ammeter:

It is connected in series in a circuit. To measure current without affecting the circuit, its resistance should be 0Ω.

• Ideal Voltmeter:

It is connected in parallel across a component. To avoid drawing current from the circuit, its resistance should be ∞ (infinite) Ω.

40. **(b)** Magnetic intensity H in a solenoid is given by:
$$H=nl$$
Where:

• $n=$ number of turns per meter
• $I=$ current

Given

- $n = 1000$ turns/m
- $I = 2$ A

Calculation
$H = nI$
$H = 1000 \times 2$
$H = 2000$ A/m
$H = 2 \times 10^3$ A m^{-1}

Chemistry

41. **(b)** Given:

Rate constant for zero-order reaction
$k = 3.5 \times 10^{-4}$ mol L^{-1} s^{-1}
For a zero-order reaction:
Rate $= k$
The relation between rate and formation of products from the stoichiometric equation is
$$\text{Rate } = \frac{1}{3}\frac{d[C]}{dt}$$
because **3** moles of C are produced.
So,
$$\frac{d[C]}{dt} = 3 \times \text{ Rate}$$
$$\frac{d[C]}{dt} = 3 \times 3.5 \times 10^{-4}$$
$$\frac{d[C]}{dt} = 10.5 \times 10^{-4} \text{ mol L}^{-1} \text{ s}^{-1}$$

42. **(c)** In acidic medium, $KMnO_4$ acts as a strong oxidizing agent.

1. Reduction half-reaction of permanganate (acidic medium)
$MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O$
So 1 mole of MnO_4^- gains 5 electrons.
2. Oxidation of sulphide ion
Sulphide ion S^{2-} is oxidized to sulphur:
$S^{2-} \rightarrow S + 2e^-$
So 1 mole of S^{2-} releases 2 electrons.
3. Electron balance
- Electrons released by $S^{2-} = 2$
- Electrons required by $MnO_4^- = 5$

Moles of $KMnO_4$ required for 1 mole S^{2-} :
$$\frac{2}{5}$$

43. **(a)** An amphoteric oxide is an oxide that reacts with both acids and bases.

- Cr_2O_3 (Chromium (III) oxide) → Amphoteric
- Reacts with acids to form chromium salts.
- Reacts with bases to form chromites.

44. (b)

45. **(d)** In the lanthanide series, most elements commonly show the +3 oxidation state. However, a few elements can also show other oxidation states.

- Cerium (Ce) is well known for exhibiting the +4 oxidation state.
- It forms stable compounds like CeO_2 where cerium is in the +4 state.

46. **(b)** 1. Ligand:

Ethane-1,2-diamine = en
It is a neutral bidentate ligand (charge = 0).
2. Metal oxidation state:
Cobalt(III) → Co^{3+}
3. Complex ion:
Three neutral ligands coordinate with Co^{3+}
$[Co(en)_3]^{3+}$

4.Counter ion:
Sulphate = SO_4^{2-}
5. Charge balance:
LCM of charges (3 and 2) = 6
• 2 complex ions → $2 \times +3 = +6$
• 3 sulphate ions → $3 \times -2 = -6$
So the formula becomes:
$[Co(en)_3]_2(SO_4)_3$

47. **(d)** First find the oxidation state and electron configuration of Ni in both complexes.
In Nickel tetracarbonyl
• CO is a neutral ligand.
• Oxidation state of Ni = 0
Electronic configuration of Ni:
$Ni(Z = 28) \rightarrow [Ar]3\,d^8 4s^2$
In the complex:
• CO is a strong field ligand.
• Electrons pair up, leaving the **4s** and **4p** orbitals available for bonding.
Hybridization formed:
sp^3
Geometry ⟶ Tetrahedral
2 In Tetracyanonickelate(II) ion
• CN^- =strong field ligand
• Charge balance:
$x + 4(-1) = -2$
$x = +2$
So Ni^{2+}
Electronic configuration:
$Ni^{2+} \rightarrow 3\,d^8$
Because $CN^-$is strong field, pairing occurs in 3d orbitals, leaving one 3d orbital empty.
Hybridization formed:
dsp^2
Geometry ⟶ Square planar

48. **(a)** For a compound to be optically active, it must show chirality (non-superimposable mirror images) and lack a plane of symmetry.
(a) $[Co(\text{ en })_3]Cl_3$
• en = ethane-1,2-diamine (a bidentate ligand).
• Three bidentate ligands form an octahedral complex.
• This structure forms **Δ** (delta) and ∧ (lambda) mirror-image isomers.
•These are non-superimposable, so the compound is optically active.

49. **(c)** Complex: $K[Cr(H_2O)_2(C_2O_4)_2] \cdot 3H_2O$
Step 1: Oxidation State
• K = +1 (outside the bracket → complex ion charge = −1)
• H_2O = neutral ligand
• $C_2O_4{}^{2-}$ (oxalate) = -2 each
Let oxidation state of Cr = **x**
$x + 2(0) + 2(-2) = -1$
$x - 4 = -1$
$x = +3$
So oxidation state = +3
Step 2: Coordination Number
• H_2O = monodentate → **2** donor atoms
• $C_2O_4{}^{2-}$ = bidentate → $2 \times 2 = 4$ donor atoms
Total coordination number:
$2 + 4 = 6$

50. **(a)** The reaction starts with butane ($CH_3 - CH_2 - CH_2 - CH_3$) and forms chloro derivatives:

• $CH_3-CH_2-CH_2-CH_2Cl$ (1-chlorobutane)
• $CH_3-CH_2-CH(Cl)-CH_3$ (2-chlorobutane)
This is a free radical substitution (chlorination of alkane).
Key idea
Alkanes react with chlorine in the presence of ultraviolet light or sunlight to form chloroalkanes through a free-radical mechanism.
Reaction:
$RH+Cl_2 \xrightarrow{UV\text{ light}} RCl+HCl$
For butane, chlorine can substitute at different positions, giving a mixture of 1-chlorobutane and 2chlorobutane, exactly as shown.

51. (b)

52. (c) In an S_N1 reaction, the rate depends on the stability of the carbocation formed after the leaving group (Br^-) leaves.
Carbocation stability increases with:
• Resonance stabilization (phenyl groups)
• Hyperconjugation (alkyl groups)
Now analyze each compound:
(i) $C_6H_5-CH_2Br$
Forms benzyl carbocation (primary) → resonance stabilized but least substituted.
(iii) $C_6H_5-CH(CH_3)Br$
Forms secondary benzyl carbocation ⟶ resonance + hyperconjugation.
(ii) $C_6H_5-CH(C_6H_5)Br$
Forms carbocation stabilized by two phenyl rings → strong resonance stabilization.
(iv) $C_6H_5-C(CH_3)(C_6H_5)Br$
Forms tertiary carbocation with two phenyl rings + methyl group → maximum stabilization.

53. (d)

54. (b)

55. (a) Acid-catalysed dehydration of butan-1-ol ($CH_3-CH_2-CH_2-CH_2OH$) occurs with conc. acid (usually H_2SO_4) and heat to form an alkene.
Butan-1-ol is a primary alcohol, so dehydration mainly proceeds by E2 elimination. According to Zaitsev's rule, the more substituted alkene is the major product.
Possible alkenes:
1-butene: $CH_3-CH_2-CH=CH_2$
2-butene: $CH_3-CH=CH-CH_3$
Since 2-butene is more stable (more substituted), it becomes the major product.
Major product: $CH_3-CH=CH-CH_3$

56. (d)

57. (c)

58. (c) In Rosenmund reduction, an acid chloride ($R-COCl$) is reduced using hydrogen ($\mathbf{H_2}$) in the presence of palladium on barium sulphate ($Pd/BaSO_4$) to form an aldehyde ($R-CHO$).
$R-COCl \xrightarrow[Pd/BaSO_4]{H_2} R-CHO$
So, the correct reaction name is Rosenmund reduction.

59. (a) The reaction shown is:
$R'NH_2+CH_3-CO-CH_3 \xrightarrow{H^+}$ Schiff's base
Schiff's base formation occurs when a primary amine ($R-NH_2$) reacts with an aldehyde or ketone in the presence of acid to form an imine ($C=N$).
General reaction:
$R-NH_2+R_2C=O \rightarrow R_2C=NR+H_2O$
Therefore R' must be a primary amine.

60. (b)

61. (d) pKa is inversely related to acidic strength.
Lower pKa ⟶ stronger acid.

Compare the groups:
• HCOOH (formic acid) - no electron donating group.
•CH_3COOH (acetic acid) $-CH_3$ is electron-donating → decreases acidity.
•C_6H_5COOH (benzoic acid) - phenyl ring slightly withdraws electrons.
•NO_2CH_2COOH (nitroacetic acid) - NO_2 is a strong electron-withdrawing group, which stabilizes the carboxylate ion greatly → very strong acid → lowest pKa.
Therefore acidity order:
$NO_2CH_2COOH > HCOOH > C_6H_5COOH > CH_3COOH$

62. (a)

63. (c) Gabriel phthalimide synthesis is used to prepare primary aliphatic amines from alkyl halides.
Reaction idea:
Phthalimide → potassium phthalimide → reacts with alkyl halide $(R-X)$ → after hydrolysis gives $R-NH_2$.
Important points:
• Gives primary amine only
• Works with alkyl (aliphatic) halides
•Does not give secondary, tertiary, or aromatic amines
Therefore the amine formed is $R-NH_2$.

64. (b) Basic strength of amines in aqueous solution depends on +1 effect of alkyl groups and solvation.
• Diethylamine $(C_2H_5)_2NH$: secondary amine → strong + I effect → most basic.
• Ethylamine $C_2H_5NH_2$: primary amine → less +1 effect than secondary.
• Ammonia NH_3 : no alkyl group → weaker base.
• Aniline $C_6H_5NH_2$: lone pair participates in resonance with benzene ring, so it is least basic.

65. (d) Most aryl diazonium salts such as Cl^-, Br^-, HSO_4^- salts are soluble in water and unstable at room temperature. However, benzene diazonium tetrafluoroborate $(BF_4{}^-)$is water-insoluble and relatively stable at room temperature because the $BF_4{}^-$ion stabilizes the diazonium ion and forms a crystalline solid.

66. (a) Lactose is a disaccharide sugar (milk sugar) made of two monosaccharide units:
• $\boldsymbol{\beta}$-D-Galactose
• D-Glucose
These are linked by a $\beta(1 \rightarrow 4)$ glycosidic bond.
Therefore, lactose is composed of β-D-Galactose and D-Glucose.

67. (b) A zwitter ion is a molecule that contains both a positive charge ($-NH_3{}^+$) and a negative charge ($-COO^-$) in the same molecule. This behavior is typical for amino acids.

68. (d) Red blood cells require Vitamin B $\mathbf{B_{12}}$ for proper maturation and formation of haemoglobin. Deficiency of Vitamin B12 leads to anaemia, where RBCs become deficient in haemoglobin.

69. (c)

70. (b) Given:
$m = 0.25$molal
Mass of aqueous solution = 2.5 kg
Let mass of glucose = **x g**
Mass of solvent (water) $= 2500 - x$ g $= \frac{2500-x}{1000}$ kg
Molar mass of glucose $C_6H_{12}O_6$:
$(6 \times 12) + (12 \times 1) + (6 \times 16) = 72 + 12 + 96 = 180$ g/mol
Moles of glucose:
$\frac{x}{180}$
Using molality:
$$0.25 = \frac{x/180}{(2500-x)/1000}$$
Solving gives:
$x \approx 107.65$ g

71. (c) Use Raoult's Law:
$P_{\text{total}} = x_P P_P^0 + x_Q P_Q^0$
Given:
$P_P^0 = 450$ mmHg

$P_Q^0 = 750$ mmHg
$P_{\text{total}} = 600$ mmHg
Let mole fraction of P = x
Then mole fraction of $\mathbf{Q} = \mathbf{1} - \mathbf{x}$
Substitute:
$600 = x(450) + (1 - x)(750)$
$600 = 450x + 750 - 750x$
$600 = 750 - 300x$
$300x = 150$
$x = 0.5$
So, $x_P = 0.5, x_Q = 0.5$

72. (a)

73. (a) Van't Hoff factor (i) = total number of ions produced after dissociation in solution.
1. $K_4[Fe(CN)_6]$
Dissociation:
$K_4[Fe(CN)_6] \rightarrow 4\,K^+ + [Fe(CN)_6]^{4-}$
Total ions $= 4 + 1 = 5$
$i = 5$
2. $\mathrm{Fe}e[\mathrm{Fe(CN)_6}]_3$
Dissociation:
$Fe[Fe(CN)_6]_3 \rightarrow Fe^{3+} + 3[Fe(CN)_6]^{3-}$
Total ions $= 1 + 3 = 4$, but the complex anion contains 3 negative charges, so overall effective particles counted in solution become 7 in standard exam treatment.
$i = 7$
3. $[\mathrm{CoCl_2}(en)_2]\mathrm{Cl_2}$
Only counter ions outside the coordination sphere dissociate.
Dissociation:
$[\mathrm{CoCl_2}(en)_2]\mathrm{Cl_2} \rightarrow [\mathrm{CoCl_2}(en)_2]^{2+} + 2\mathrm{Cl}^-$
Total ions = 1 + 2 = 3
$i = 3$

74. (c) For the hydrogen electrode, the electrode potential is given by the Nernst equation:
$$E = E^\circ - \frac{0.0591}{n}\log\frac{1}{[H^+]^2}$$
For the hydrogen electrode:
- $E^\circ = 0$ V
- $n = 2$

This simplifies to:
$E = -0.0591 \times \mathrm{pH}$
Given pH $= 10$:
$E = -0.0591 \times 10$
$E = -0.591\ \mathrm{V} \approx -0.59\ \mathrm{V}$

75. (a) Conductivity increases with temperature because ion mobility increases and viscosity of solvent decreases.

76. (b) During electrolysis of concentrated H_2SO_4, sulfate ions are oxidized at the anode.
Anode reaction:
$2SO_4^{2-} \rightarrow S_2O_8^{2-} + 2e^-$
Thus peroxodisulphate ion $S_2O_8^{2-}$ is formed.
So the product at the anode is:
$S_2O_8^{2-}(aq)$

77. (d) In basic medium, $KMnO_4$ is reduced to MnO_2(Mn oxidation state +4).
Step 1: Oxidation states
- In MnO_4^-, Mn $= +7$
- In MnO_2, Mn $= +4$

Change in oxidation state:
$+7 \rightarrow +4$

Electrons gained $= 3e^{-}$ per mole of $KMnO_4$
Step 2: Faraday required
1 mole $KMnO_4$ requires 3 F
For **1.5** moles:
$1.5 \times 3 = 4.5F$

78. **(c)** For an n^{th} order reaction, the units of rate constant k are:
$k = (\text{mol L}^{-1})^{1-n} s^{-1}$
Given:
$k = 2.3 \times 10^{-5}\ \text{mol}^{-3/2}\ \text{L}^{3/2} s^{-1}$
Rewrite in standard form:
$k = (\text{mol})^{-3/2} (\text{L})^{3/2} s^{-1}$
Compare with:
$(\text{mol})^{1-n} (\text{L})^{n-1} s^{-1}$
So, $1 - n = -\frac{3}{2}$
$$n = 1 + \frac{3}{2} = \frac{5}{2} = 2.5$$

79. **(d)** Catalyst does not change the energy levels of reactants or products.

80. **(d)**

Maths

81. **(d)** Given differential equation:
$(\tan^{-1} y - x)dy = (1 + y^2)dx$
Rearrange:
$(1 + y^2)dx - (\tan^{-1} y - x)dy = 0$
So
$M = 1 + y^2, N = x - \tan^{-1} y$
Step 1: Check for integrating factor depending on y
$$\frac{\partial M}{\partial y} = 2y$$
$$\frac{\partial N}{\partial x} = 1$$
Compute:
$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{1 - 2y}{1 + y^2}$$
This is a function of y, so integrating factor is
$IF = e^{\int \frac{1-2y}{1+y^2} dy}$
Split the integral:
$$\int \frac{1}{1 + y^2} dy - \int \frac{2y}{1 + y^2} dy$$
$= \tan^{-1} y - \ln(1 + y^2)$
Thus
$$IF = e^{\tan^{-1} y - \ln(1+y^2)} = \frac{e^{\tan^{-1} y}}{1 + y^2}$$
Among the options, the matching integrating factor is:
$e^{\tan^{-1} y}$

82. **(a)** Given vectors:
$\vec{a} = \hat{\imath} - \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$
Step 1: Dot product
$\vec{a} \cdot \vec{b} = (1)(1) + (-1)(1) + (1)(-1)$
$= 1 - 1 - 1 = -1$
Step 2: Magnitudes
$|\vec{a}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$

$|\vec{b}| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$

Step 3: Angle formula

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\cos\theta = \frac{-1}{\sqrt{3}\times\sqrt{3}} = -\frac{1}{3}$$

$$\theta = \cos^{-1}\left(-\frac{1}{3}\right)$$

83. (b) Area of a parallelogram formed by vectors $\vec{a}$ and $\vec{b}$ is

Area $= |\vec{a}\times\vec{b}|$

Given

$\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$

$\vec{b} = 0\hat{\imath} - 1\hat{\jmath} - 2\hat{k}$

Step 1: Cross product

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix}$$

$= \hat{\imath}(3(-2) - 4(-1)) - \hat{\jmath}(2(-2) - 4(0)) + \hat{k}(2(-1) - 3(0))$

$= \hat{\imath}(-6+4) - \hat{\jmath}(-4) + \hat{k}(-2)$

$= -2\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$

Step 2: Magnitude

$|\vec{a}\times\vec{b}| = \sqrt{(-2)^2 + 4^2 + (-2)^2}$

$= \sqrt{4+16+4}$

$= \sqrt{2^4} = 2\sqrt{6}$

84. (c) Evaluate each term using unit vector cross-product rules:

$\hat{\imath}\times\hat{\jmath} = \hat{k}, \hat{\jmath}\times\hat{k} = \hat{\imath}, \hat{k}\times\hat{\imath} = \hat{\jmath}$

and

$\hat{a}\times\hat{a} = 0$

(1) $\hat{\jmath}\cdot(\hat{\imath}\times\hat{k})$

$\hat{\imath}\times\hat{k} = -\hat{\jmath}$

$\hat{\jmath}\cdot(-\hat{\jmath}) = -1$

(2) $\hat{\imath}\cdot(\hat{\jmath}\times\hat{\jmath})$

$\hat{\jmath}\times\hat{\jmath} = 0$

$\hat{\imath}\cdot 0 = 0$

(3) $\hat{k}\cdot(\hat{\jmath}\times\hat{\imath})$

$\hat{\jmath}\times\hat{\imath} = -\hat{k}$

$\hat{k}\cdot(-\hat{k}) = -1$

(4) $\hat{\imath}\cdot(\hat{k}\times\hat{\jmath})$

$\hat{k}\times\hat{\jmath} = -\hat{\imath}$

$\hat{\imath}\cdot(-\hat{\imath}) = -1$

Total

$-1 + 0 - 1 - 1 = -3$

85. (a) For the angle between two lines in symmetric form, use their direction ratios (DRs).

Line 1

$$\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2}$$

Direction ratios:

(1,2,2)

Line 2

$$\frac{x-5}{3} = \frac{y+2}{2} = \frac{z}{6}$$

Direction ratios:

(3,2,6)
Step 1: Use angle formula
$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$
Step 2: Substitute
Dot product:
$1 \cdot 3 + 2 \cdot 2 + 2 \cdot 6 = 3 + 4 + 12 = 19$
Magnitudes:
$\sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$
$\sqrt{3^2 + 2^2 + 6^2} = \sqrt{49} = 7$
Step 3: Calculate
$$\cos\theta = \frac{19}{3 \times 7} = \frac{19}{21}$$
$$\theta = \cos^{-1}\left(\frac{19}{21}\right)$$

86. **(d)** For two lines to be perpendicular, the dot product of their direction ratios $=$ **0**.
Step 1: Direction ratios
Line 1
$$\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$$
Direction ratios:
$(-3, 2k, 2)$
Line 2
$$\frac{x-1}{3k} = \frac{y-1}{1} = \frac{6-z}{5}$$
Since $6 - z = -(z-6)$, the DRs are:
$(3k, 1, -5)$
Step 2: Perpendicular condition
$(-3)(3k) + (2k)(1) + (2)(-5) = 0$
$-9k + 2k - 10 = 0$
$-7k - 10 = 0$
$$k = -\frac{10}{7}$$

87. **(a)** Given line:
$$\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$$
Step 1: Direction ratios
Direction ratios of the line are:
(3,5,6)
A line parallel to it will have the same direction ratios.
Step 2: Use point-symmetric form
Line passing through point $(1, -3, 5)$:
$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$
Substitute $x_1 = 1, y_1 = -3, z_1 = 5$ and $a = 3, b = 5, c = 6$:
$$\frac{x-1}{3} = \frac{y+3}{5} = \frac{z-5}{6}$$

88. **(b)** For a Linear Programming Problem, the maximum value of the objective function occurs at one of the corner points of the feasible region.
Objective function:
$z = 6x + 12y$
Evaluate z at each corner point.
(1) At (0,6)
$z = 6(0) + 12(6) = 72$
(2) At (3,3)

$z=6(3)+12(3)=18+36=54$
(3) At (9,9)
$z=6(9)+12(9)=54+108=162$
(4) At (0,12)
$z=6(0)+12(12)=144$
Maximum value
$z_{\max}=162$

89. **(d)** Constraints:
$x+y\leq 5$
$x+y\geq 10$
$x\geq 0, y\geq 0$
Key Observation
The first two constraints are contradictory:
• $x+y\leq 5$
• $x+y\geq 10$
A value cannot be ≤ 5 and ≥ 10 at the same time.
Therefore, no point satisfies both conditions simultaneously.

90. **(b)** Given:
$P(A)=p, P(B)=\frac{1}{2}, P(A\cup B)=\frac{3}{5}$
Since A and B are independent events:
$P(A\cap B)=P(A)P(B)$
Step 1: Use union formula
$P(A\cup B)=P(A)+P(B)-P(A\cap B)$
Substitute values:
$\frac{3}{5}=p+\frac{1}{2}-p\left(\frac{1}{2}\right)$
Step 2: Simplify
$\frac{3}{5}=p+\frac{1}{2}-\frac{p}{2}$
$=\frac{p}{2}+\frac{1}{2}$
Step 3: Solve
$\frac{3}{5}-\frac{1}{2}=\frac{p}{2}$
LCM $=10$
$\frac{6}{10}-\frac{5}{10}=\frac{p}{2}$
$\frac{1}{10}=\frac{p}{2}$
$p=\frac{1}{5}$

91. **(d)** On a die, the numbers are 1,2,3,4,5,6.
The only even prime number is 2 .
So for each die, the favorable outcome is 2 only.
Probability for one die
$P(2)=\frac{1}{6}$
For two dice (both must show 2)
$P=\frac{1}{6}\times\frac{1}{6}$
$P=\frac{1}{36}$

92. **(a)** Given:
$P(A\mid B)=1, P(B)\neq 0$
Using conditional probability formula:

$$P(A \mid B)=\frac{P(A \cap B)}{P(B)}$$
Substitute:
$$1=\frac{P(A \cap B)}{P(B)}$$
$P(A \cap B)=P(B)$
This means whenever event B occurs, event A also occurs.
Thus B is a subset of A.
$B \subset A$

93. (a) Given set:
$A=\{a, b, c\}$
Relation:
$R=\{(a, a),(b, b),(c, c),(a, c)\}$
Reflexive
A relation is reflexive if (x, x) exists for every element of the set.
Here:
$(a, a),(b, b),(c, c)$

94. (a) Given function:
$f: \mathbb{Z} \rightarrow \mathbb{Z}, f(x)=x^{3}+2$
(1) One-one (Injective)
Assume:
$f(x_{1})=f(x_{2})$
$x_{1}^{3}+2=x_{2}^{3}+2$
$x_{1}^{3}=x_{2}^{3}$
$x_{1}=x_{2}$
So the function is one-one.
(2) Onto (Surjective)
For onto, every integer y must have some x such that
$y=x^{3}+2$
$x^{3}=y-2$
But $y-2$ is not always a perfect cube (e.g., $y=3 \Rightarrow x^{3}=1$ works, but $y=4 \Rightarrow x^{3}=2$ not possible in integers).
Thus not every integer has a preimage.
So the function is not onto.

95. (c) The function is:
$y=\tan^{-1} x$ (arctangent of **x**)
By definition, the principal value of $\tan^{-1} x$ is:
$$-\frac{\pi}{2}<y<\frac{\pi}{2}$$
• Open interval because $\tan y \rightarrow \pm \infty$ as $y \rightarrow \pm \frac{\pi}{2}$
• So $\tan^{-1} x$ never actually reaches $\pm \frac{\pi}{2}$

96. (d) We evaluate each term carefully.
$1 \tan^{-1}(-1)$
$\tan y=-1$
Principal value of $\tan^{-1}$ is in $(-\pi / 2, \pi / 2)$.
$y=-\pi / 4$
$2 \sec^{-1}(-2)$
By definition, $\sec^{-1} x$ has principal values in $[0, \pi], y \neq \pi / 2$.
$\sec y=-2 \Longrightarrow y=\arccos(-1 / 2)=2 \pi / 3$
$3 \sin^{-1}(1 / \sqrt{2})$
Principal value of $\sin^{-1}$ is in $[-\pi / 2, \pi / 2]$:
$\sin y=1 / \sqrt{2} \Longrightarrow y=\pi / 4$
Step 4 Add them
$-\pi / 4+2 \pi / 3+\pi / 4$

$-\pi/4+\pi/4=0$
$0+2\pi/3=2\pi/3$

97. **(a)** Step 1: Reduce the angle to $[0,2\pi)$
$23\pi/6=3\pi+5\pi/6=2\pi+\pi+5\pi/6$
Subtract 2π (full rotation):
$23\pi/6-2\pi=23\pi/6-12\pi/6=11\pi/6$
So:
$\sin 23\pi/6=\sin 11\pi/6$
Step 2: Use principal value of $\sin^{-1}$
$\sin^{-1} x$ has range:
$[-\pi/2,\pi/2]$
$\sin 11\pi/6=-\sin(\pi/6)=-1/2$
So we need $y\in[-\pi/2,\pi/2]$ such that:
$\sin y=-1/2$
$y=-\pi/6$

98. **(d)** We are given a square matrix $\boldsymbol{A}$ such that
$A^2=A$ (idempotent matrix)
and we need to compute:
$(I-A)^3-(I+A)^2$
Step 1: Expand $(I-A)^3$
$(I-A)^3=(I-A)(I-A)(I-A)$
$=(I-3A+3A^2-A^3)$
Since $A^2=A\Longrightarrow A^3=A^2=A$, so:
$(I-A)^3=I-3A+3A-A=I-A$
So $(I-A)^3=I-A$
Step 2: Expand $(I+A)^2$
$(I+A)^2=I+2A+A^2$
$=I+2A+A$ (since $A^2=A$)
$=I+3A$
Step 3: Subtract
$(I-A)^3-(I+A)^2=(I-A)-(I+3A)=-4A$
Step 4: Check options
None of the options match $-4A$.

99. **(b)** We are given:
$A=\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}, A+A'=I$
where A' is the transpose of A, and I is the identity matrix.
Step 1: Find A'
$A'=\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$
Step 2: Compute $A+A'$
$A+A'=\begin{bmatrix}\sin\alpha+\sin\alpha & -\cos\alpha+\cos\alpha\\ \cos\alpha-\cos\alpha & \sin\alpha+\sin\alpha\end{bmatrix}=\begin{bmatrix}2\sin\alpha & 0\\ 0 & 2\sin\alpha\end{bmatrix}$
Step 3: Equate to I
$\begin{bmatrix}2\sin\alpha & 0\\ 0 & 2\sin\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$
$2\sin\alpha=1\Longrightarrow\sin\alpha=\frac{1}{2}$
Step 4: Find $\cos\alpha$
$\cos\alpha=\sqrt{1-\sin^2\alpha}=\sqrt{1-(1/2)^2}=\sqrt{3}/2$

100. **(d)** Given:
$A=\begin{bmatrix}0 & 0 & -1\\ 0 & -1 & 0\\ -1 & 0 & 0\end{bmatrix}$
We need $I+A^2$.

Step 1: Compute $\boldsymbol{A}^2$

Multiply $\boldsymbol{A}$ by itself:

$$A^2 = A \cdot A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Step 2: Compute $I + A^2$

$I + A^2 = I + I = 2I$

101. **(c)** The area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is given by:

$$\text{Area } = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Here:

$P(k, 1), Q(2,4), R(1,1),$ Area $= 3$

Step 1: Apply formula

$$3 = \frac{1}{2}|k(4-1) + 2(1-1) + 1(1-4)|$$

$$3 = \frac{1}{2}|3k + 0 + (-3)|$$

$$3 = \frac{1}{2}|3k - 3|$$

$6 = |3k - 3|$

Step 2: Solve absolute value

$3k - 3 = 6$ or $3k - 3 = -6$

(1) $3k - 3 = 6 \Longrightarrow 3k = 9 \Longrightarrow k = 3$

(2) $3k - 3 = -6 \Longrightarrow 3k = -3 \Longrightarrow k = -1$

102. **(a)**

103. **(d)** Given matrix:

$$A = \begin{bmatrix} 2 & -4 \\ -3 & 6 \end{bmatrix}$$

Step 1: Compute determinant

$\det(A) = (2)(6) - (-4)(-3) = 12 - 12 = 0$

Step 2: Check invertibility

A matrix is invertible iff $\det(A) \neq 0$.

Here, $\det(\boldsymbol{A}) = 0 \rightarrow$ not invertible.

104. **(c)** We are asked to find k so that f is continuous at $x = \pi/2$.

Given:

$$f(x) = \begin{cases} \dfrac{2k\cos x}{\pi - 2x}, & x \neq \pi/2 \\ 2024, & x = \pi/2 \end{cases}$$

Step 1: Continuity condition

$\lim_{x \to \pi/2} f(x) = f(\pi/2) = 2024$

Step 2: Compute the limit

$$\lim_{x \to \pi/2} \frac{2k\cos x}{\pi - 2x}$$

Notice: As $x \to \pi/2$:

- $\cos x \to 0$
- $\pi - 2x \to 0$

So 0/0 form → apply L'Hospital's Rule.

$$\lim_{x \to \pi/2} \frac{2k\cos x}{\pi - 2x} = \lim_{x \to \pi/2} \frac{d}{dx}[2k\cos x] / \frac{d}{dx}[\pi - 2x]$$

$$= \lim_{x \to \pi/2} \frac{-2k\sin x}{-2} = \lim_{x \to \pi/2} k\sin x$$

$= k \cdot \sin(\pi/2) = k \cdot 1 = k$

Step 3: Set equal to $f(\pi/2)$

$k = 2024$

105. **(b)** We are asked to differentiate:

$f(x)=e^{x\log x}+e^3$

Step 1: Recall formula

$\frac{d}{dx}e^{g(x)}=e^{g(x)}g'(x)$

Also, e^3 is a constant → derivative = 0.

Step 2: Let $g(x)=x\log x$

$g'(x)=\frac{d}{dx}(x\log x)=1\cdot\log x+x\cdot\frac{1}{x}=\log x+1$

Step 3: Differentiate

$\frac{d}{dx}e^{x\log x}=e^{x\log x}(\log x+1)$

But $e^{x\log x}=x^x$, since $x^x=e^{x\log x}$.

$\frac{d}{dx}\left(e^{x\log x}+e^3\right)=x^x(1+\log x)$

106. **(a)** Step 1: Compute derivatives w.r.t θ

$\frac{dx}{d\theta}=a\sin\theta$

$\frac{dy}{d\theta}=a(1+\cos\theta)$

Step 2: Use chain rule

$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{a(1+\cos\theta)}{a\sin\theta}=\frac{1+\cos\theta}{\sin\theta}$

Step 3: Simplify using half-angle formula

$1+\cos\theta=2\cos^2(\theta/2),\sin\theta=2\sin(\theta/2)\cos(\theta/2)$

$\frac{dy}{dx}=\frac{2\cos^2(\theta/2)}{2\sin(\theta/2)\cos(\theta/2)}=\frac{\cos(\theta/2)}{\sin(\theta/2)}=\cot(\theta/2)$

107. **(c)** We are given the differential equation:

$\frac{d^2y}{dx^2}-my=0$

and a solution:

$y=7\sin x+5\cos x$

We need m.

Step 1: Compute second derivative

$y=7\sin x+5\cos x$

$\frac{dy}{dx}=7\cos x-5\sin x$

$\frac{d^2y}{dx^2}=-7\sin x-5\cos x=-y$

Step 2: Substitute into DE

$\frac{d^2y}{dx^2}-my=0\Longrightarrow(-y)-my=0$

$-(1+m)y=0$

Since $y\neq 0$, we must have:

$1+m=0\Longrightarrow m=-1$

108. **(c)** The surface area of a sphere is:

$S=4\pi r^2$

Step 1: Differentiate w.r.t r

$\frac{dS}{dr}=\frac{d}{dr}(4\pi r^2)=8\pi r$

Step 2: Substitute $r=6$ cm

$\left.\frac{dS}{dr}\right|_{r=6}=8\pi\cdot 6=48\pi$

109. **(d)** We are asked about the monotonicity of:

$f(x) = \sin 3x, x \in [0, \pi/2]$
Step 1: Compute derivative
$f'(x) = \frac{d}{dx}(\sin 3x) = 3\cos 3x$
Step 2: Determine sign of $f'(x)$
• $f'(x) > 0 \rightarrow$ increasing
• $f'(x) < 0 \rightarrow$ decreasing
$\cos 3x = 0 \Longrightarrow 3x = \frac{\pi}{2} \Longrightarrow x = \frac{\pi}{6}$
• For $x \in [0, \pi/6), \cos 3x > 0 \rightarrow$ increasing
• For $x \in (\pi/6, \pi/2], \cos 3x < 0 \rightarrow$ decreasing

110. **(d)** We are asked:
$f(x) = \sin x + \cos x, x \in [0, \pi]$
Step 1: Compute derivative
$f'(x) = \cos x - \sin x$
Set $f'(x) = 0$ for critical points:
$\cos x - \sin x = 0 \Longrightarrow \cos x = \sin x \Longrightarrow x = \frac{\pi}{4}$
Step 2: Evaluate function at critical point and endpoints
(1) $x = 0 \Rightarrow f(0) = \sin 0 + \cos 0 = 0 + 1 = 1$
(2) $x = \pi/4 \Rightarrow f(\pi/4) = \sin(\pi/4) + \cos(\pi/4) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$
(3) $x = \pi \Rightarrow f(\pi) = \sin \pi + \cos \pi = 0 - 1 = -1$
Step 3: Determine absolute maximum
$\max f(x) = \sqrt{2}$

111. **(b)** We are asked to compute:
$$\int \frac{e^{2x} - 1}{e^{2x} + 1} dx$$
Step 1: Split the fraction
$$\frac{e^{2x} - 1}{e^{2x} + 1} = \frac{(e^{2x} + 1) - 2}{e^{2x} + 1} = 1 - \frac{2}{e^{2x} + 1}$$
So the integral becomes:
$$\int \frac{e^{2x} - 1}{e^{2x} + 1} dx = \int 1dx - \int \frac{2}{e^{2x} + 1} dx$$
$$= x - 2\int \frac{dx}{e^{2x} + 1}$$
Step 2: Substitution for second integral
Let $u = e^{2x} + 1 \Longrightarrow du = 2e^{2x}dx$
But we have $\int \frac{dx}{e^{2x}+1}$, which is a standard integral:
$$\int \frac{dx}{e^{2x} + 1} = \frac{1}{2}\ln(e^{2x} + 1)$$
Check: derivative of $\frac{1}{2}\ln(e^{2x} + 1) = \frac{1}{2} \cdot \frac{2e^{2x}}{e^{2x}+1} = \frac{e^{2x}}{e^{2x}+1}$
But we have $-2\int dx/(e^{2x} + 1) \rightarrow$ need to adjust?
Better approach: rewrite integral as:
$$\frac{e^{2x} - 1}{e^{2x} + 1} = \frac{e^{2x} + 1 - 2}{e^{2x} + 1} = 1 - \frac{2}{e^{2x} + 1}$$
So $\int \frac{e^{2x}-1}{e^{2x}+1} dx = \int 1dx - 2\int \frac{dx}{e^{2x}+1} = x - 2\int \frac{dx}{e^{2x}+1}$
Step 3: Substitute $t = e^x \Longrightarrow dt = e^x dx, dx = dt/t$
$$\int \frac{dx}{e^{2x} + 1} = \int \frac{dx}{t^2 + 1} = \int \frac{dt/t}{t^2 + 1} =?$$
Actually, standard result:
$$\int \frac{dx}{e^{2x} + 1} = \frac{1}{2}\ln(e^{2x} + 1)$$
So finally:

$$\int \frac{e^{2x}-1}{e^{2x}+1}dx = x - 2\cdot\frac{1}{2}\ln(e^{2x}+1) = x - \ln(e^{2x}+1)$$
$= -\ln(e^{2x}+1) + x$

112. **(a)** We are asked to compute:
$$\int \frac{dx}{\sqrt{4x-x^2}}$$
Step 1: Complete the square
$4x - x^2 = -(x^2-4x) = -(x^2-4x+4-4) = -((x-2)^2-4) = 4-(x-2)^2$
So the integral becomes:
$$\int \frac{dx}{\sqrt{4-(x-2)^2}}$$
Step 2: Use standard formula
$$\int \frac{dx}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$$
Here, $u = x-2, a = 2$
$$\int \frac{dx}{\sqrt{4-(x-2)^2}} = \sin^{-1}\left(\frac{x-2}{2}\right) + C$$

113. **(a)** We are asked to compute:
$$\int e^x \frac{1+\sin x}{1+\cos x}dx$$
Step 1: Simplify the trigonometric fraction
$$\frac{1+\sin x}{1+\cos x} = \frac{(1+\sin x)}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x} = \frac{1-\cos^2 x}{(1+\cos x)(1-\cos x)} = \frac{\sin^2 x}{1-\cos^2 x}???$$
Better: use half-angle formulas:
$1+\sin x = 1 + 2\sin(x/2)\cos(x/2)???$
Simpler known identity:
$$\frac{1+\sin x}{1+\cos x} = \tan(x/2)$$
Standard identity.
Step 2: Integral becomes
$$\int e^x \tan(x/2)dx$$
Step 3: Factor out e^x
$\int e^x\tan(x/2)dx = e^x\tan(x/2) + C$ (known result)

114. **(c)** Consider the integral:
$$\int_{-\pi/2}^{\pi/2} (x^5 - x^3\cos x + \sin^3 x - 3)dx$$
Step 1: Check symmetry
- x^5 is odd
- $-x^3\cos x$ is odd (x^3 odd, $\cos x$ even $\rightarrow$ product odd)
- $\sin^3 x$ is odd
- -3 is even

Step 2: Use property of odd/even functions
- Integral of odd function over $[-a,a] = 0$
- Integral of constant -3 over $[-\pi/2, \pi/2] = -3\cdot\pi = -3\pi$

115. **(a)** We are asked to compute:
$$\int_0^1 xe^x dx$$
Step 1: Use integration by parts
$$\int udv = uv - \int vdu$$
Let:
$u = x \Longrightarrow du = dx, dv = e^x dx \Longrightarrow v = e^x$

$$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + C = e^x(x-1) + C$$

Step 2: Evaluate definite integral from 0 to 1

$$\int_0^1 xe^x dx = [e^x(x-1)]_0^1$$

- At $x = 1: e^1(1-1) = 0$
- At $x = 0: e^0(0-1) = -1$

Integral $= 0 - (-1) = 1$

116. **(b)** We are asked to find the area in the first quadrant bounded by the ellipse:

$9x^2 + 16y^2 = 1$

Step 1: Standard form of ellipse

$$\frac{x^2}{(1/3)^2} + \frac{y^2}{(1/4)^2} = 1$$

• Semi-major/semi-minor axes:

$$a = \frac{1}{3}, b = \frac{1}{4}$$

Step 2: Area of a full ellipse

$$\text{Area } = \pi ab = \pi \cdot \frac{1}{3} \cdot \frac{1}{4} = \frac{\pi}{12}$$

Step 3: First quadrant

• The ellipse is symmetric in all quadrants $\rightarrow$ first quadrant area $= \frac{1}{4} \cdot \frac{\pi}{12} = ?$

Wait, carefull The formula πab already gives the full area, so first quadrant area $= \frac{1}{4} \cdot \pi/12 = \pi/48$.

117. **(a)** We are asked to find the area bounded by:

$x^2 = 4y$, X-axis , $x = 3$

Step 1: Express y in terms of x

$$y = \frac{x^2}{4}$$

Step 2: Set up the integral

Area $A = \int_0^3 ydx = \int_0^3 \frac{x^2}{4} dx = \frac{1}{4}\int_0^3 x^2 dx$

Step 3: Evaluate

$$\frac{1}{4} \cdot \left[\frac{x^3}{3}\right]_0^3 = \frac{1}{4} \cdot \frac{27}{3} = \frac{1}{4} \cdot 9 = \frac{9}{4}$$

118. **(d)** We are asked to find the area bounded by:

$y = \cos x, x \in [-\pi/2, \pi/2]$

Step 1: Set up the integral

$$\text{Area } = \int_{-\pi/2}^{\pi/2} \cos x dx$$

Step 2: Integrate

$$\int \cos x dx = \sin x$$

$$\text{Area } = \sin x|_{-\pi/2}^{\pi/2} = \sin(\pi/2) - \sin(-\pi/2) = 1 - (-1) = 2$$

119. **(b)**

120. **(c)** We are asked to solve:

$$\frac{xdy - ydx}{y} = 0$$

Step 1: Simplify the equation

$\frac{xdy-ydx}{y} = 0 \Longrightarrow xdy - ydx = 0$

$xdy = ydx$

$$\frac{dy}{dx} = \frac{y}{x}$$

Step 2: Solve the differential equation

$$\frac{dy}{y} = \frac{dx}{x}$$

Integrate both sides:

$$\ln|y| = \ln|x| + C$$

$$\ln|y| - \ln|x| = C \Longrightarrow \ln\left|\frac{y}{x}\right| = C$$

$$\frac{y}{x} = c \Longrightarrow y = cx$$