Physics

1. **(d)** In an LCR AC circuit, the maximum (peak) voltage $V_{\max}$ and RMS voltage V_{rms} are related by:

$$V_{\text{rms}} = \frac{V_{\max}}{\sqrt{2}} \Longrightarrow V_{\max} = \sqrt{2}V_{\text{rms}} \approx 1.414V_{\text{rms}}$$

Step 1: Convert to percentage

$$\frac{V_{\max}}{V_{\text{rms}}} \times 100\% = 1.414 \times 100\% \approx 141.4\%$$

2. **(a)** The power factor of an AC circuit is:

$$\text{Power factor } = \cos\phi = \frac{R}{Z} \quad \text{(for series RLC)}$$

• At resonance in a series LCR circuit, the inductive reactance X_L cancels the capacitive reactance X_C :

$X_L - X_C = 0 \Longrightarrow Z = R$

• So Power factor $= R/Z = R/R = 1$

• For other circuits (CR, LR, only L), resonance does not occur in the same way.

3. **(d)** We are asked: peak current when a 12 W bulb is connected to 24 V AC .

Step 1: Find RMS current

Power consumed by the bulb:

$$P = V_{\text{rms}} \cdot I_{\text{rms}} \Longrightarrow I_{\text{rms}} = \frac{P}{V_{\text{rms}}}$$

$$I_{\text{rms}} = \frac{12}{24} = 0.5 \text{ A}$$

Step 2: Find peak current

For AC:

$$I_{\text{peak}} = \sqrt{2}I_{\text{rms}} = \sqrt{2} \cdot 0.5 = \frac{\sqrt{2}}{2} \text{ A} \approx 0.707 \text{ A}$$

4. **(b)** The type of radiation used in medicine to destroy cancer cells is gamma rays.

• Gamma rays are highly penetrating and can kill cancerous cells effectively.

5. **(c)** We are asked to find the refractive index n of a medium.

Step 1: Recall formula

$$n = \frac{c}{v}$$

• $c = 3 \times 10^8 \text{ m/s}$

• $v = 200 \times 10^8 \text{ cm/s} = 2 \times 10^8 \text{ m/s}$ (convert cm → m)

Step 2: Calculate

$$n = \frac{c}{v} = \frac{3 \times 10^8}{2 \times 10^8} = 1.5$$

6. **(a)** We are asked about the power of a combination of a convex lens and concave lens of equal focal length.

Step 1: Recall formula for power

Power of a lens:

$$P = \frac{100}{f(\text{ cm})} \quad \text{(in diopters)}$$

• Convex lens: $f_1 = +25 \text{ cm} \Longrightarrow P_1 = +\frac{100}{25} = +4\text{D}$

• Concave lens: $f_2 = -25 \text{ cm} \Longrightarrow P_2 = -\frac{100}{25} = -4\text{D}$

Step 2: Power of combination

$P_{\text{total}} = P_1 + P_2 = +4 + (-4) = 0\text{D}$

7. **(d)** We are asked: angle of incidence on a prism of minimum deviation.

Step 1: Recall the formula for minimum deviation

For a prism of apex angle A and minimum deviation $\delta_{\min}$, the angle of incidence i is related to the angle of refraction inside the prism r by:

$$\delta_{\min} = 2i - A$$

$$i = \frac{\delta_{\min} + A}{2}$$

Step 2: Given values

• Equilateral prism → $A = 60^\circ$
• Minimum deviation → $\delta_{\text{min}} = 46^\circ$
$i = \frac{46^\circ + 60^\circ}{2} = \frac{106^\circ}{2} = 53^\circ$

8. **(b)** For a compound microscope, the total magnification is:
$M = M_{\text{objective}} \times M_{\text{eyepiece}}$ and $M_{\text{objective}} \approx \frac{L}{f_{\text{objective}}}$
• Here, $L =$ tube length, $f_{\text{objective}} =$ focal length of objective.
Step 1: Effect of increasing L
• $M_{\text{objective}} = L/f_{\text{objective}}$
• So if tube length L increases, the magnification increases proportionally.
• Eyepiece magnification is nearly constant for normal adjustment.

9. **(c)** We are asked about intensity due to superposition of two waves of same intensity I_0 and same phase difference ϕ.
Step 1: Formula for resultant intensity
If two waves of same amplitude A interfere with phase difference ϕ :
$I \propto A_{\text{res}}^2 = (A_1 + A_2)^2$
• For same amplitude $A_1 = A_2 = A$:
$A_{\text{res}}^2 = (2A\cos(\phi/2))^2 = 4A^2\cos^2(\phi/2)$
$\Rightarrow I \propto \cos^2(\phi/2)$

10. **(d)** For light diverging from a point source:
• The wave spreads out in all directions, forming a spherical wavefront.
• Intensity decreases with distance, but the question asks about the wavefront shape.

11. **(b)** We are asked to find the wavelength in Young's double slit experiment.
Step 1: Recall fringe formula
Distance between m -th bright fringe and central maximum:
$y_m = \frac{m\lambda L}{d}$
Where:
• $m = 6$ (6th bright fringe)
• $L = 1.8$ m (screen distance)
• $d = 0.54$ mm $= 0.54 \times 10^{-3}$ m (slit separation)
• $y_6 = 1.2$ cm $= 0.012$ m (distance from central bright fringe)
Step 2: Solve for λ
$y_6 = \frac{6\lambda L}{d} \Longrightarrow \lambda = \frac{y_6 d}{6L}$
$\lambda = \frac{0.012 \cdot 0.54 \times 10^{-3}}{6 \cdot 1.8}$
$\lambda = \frac{6.48 \times 10^{-6}}{10.8} \approx 6 \times 10^{-7}$ m $= 600$ nm

12. **(a)** The question is asking about the minimum electric field required to pull electrons out of a metal, which is basically the field needed for electron emission due to field emission (cold emission).
For metals, this occurs when the field is extremely strong, on the order of 10^9 V/cm. This comes from the relation between work function (ϕ) and field (E) for tunneling of electrons:
$E \sim \frac{\phi}{ea} \sim 10^9$ V/cm
where $a \sim 10^{-8}$ cm is the atomic spacing.

13. **(a)** Energy of one photon is given by Planck's relation:
$E = h\nu$
Where
$h = 6.63 \times 10^{-34}$Js
$\nu = 6 \times 10^{14}$ Hz
So, $E = (6.63 \times 10^{-34})(6 \times 10^{14})$
$E = 3.978 \times 10^{-19}$ J

Power of the laser:
$P = 4 \times 10^{-3}$ W
Number of photons emitted per second:
$$N = \frac{P}{E}$$
$$N = \frac{4 \times 10^{-3}}{3.978 \times 10^{-19}}$$
$N \approx 1.0 \times 10^{16}$

14. (b) The de-Broglie wavelength is given by
$$\lambda = \frac{h}{mv}$$
Given
$h = 6.6 \times 10^{-34}$Js
$m = 0.033$ kg
$v = 1$ km/s $= 1000$ m/s
Step 1: Substitute values
$$\lambda = \frac{6.6 \times 10^{-34}}{0.033 \times 1000}$$
Step 2: Simplify denominator
$0.033 \times 1000 = 33$
$$\lambda = \frac{6.6 \times 10^{-34}}{33}$$
Step 3: Calculate
$\lambda = 0.2 \times 10^{-34}$
$\lambda = 2 \times 10^{-35}$ m

15. (a) According to Bohr's quantization rule, the orbital angular momentum of an electron is:
$$L = \frac{nh}{2\pi}$$
Step 1: Identify quantum number
Third excited state means:
Ground state $n = 1$
First excited $n = 2$
Second excited $n = 3$
Third excited $n = 4$
Step 2: Substitute values
$$L = \frac{4 \times 6.63 \times 10^{-34}}{2\pi}$$
$$L = \frac{26.52 \times 10^{-34}}{6.283}$$
Step 3: Calculate
$L \approx 4.2 \times 10^{-34}$ kg m^2 s^{-1}

16. (d) For the hydrogen atom in ground state:
Total energy needed to remove the electron = 13.6 eV.
In Bohr's model:
Potential Energy $U = -2K$
Total Energy $E = K + U = -K$
Thus, Kinetic Energy $K = 13.6$eV.
Convert eV to joules:
$K = 13.6 \times 1.6 \times 10^{-19} = 2.176 \times 10^{-18} J$
Using kinetic energy formula:
$$K = \frac{1}{2}mv^2$$
$$2.176 \times 10^{-18} = \frac{1}{2}(9.1 \times 10^{-31})v^2$$

$v^2 = \frac{2 \times 2.176 \times 10^{-18}}{9.1 \times 10^{-31}}$
$v^2 \approx 4.78 \times 10^{12}$
$v \approx 2.18 \times 10^6$ m/s
$v \approx 2.2 \times 10^6$ m/s

17. (c) For a hydrogen atom in Bohr's model:
Relations between energies are:
Potential Energy $U = -2K$
Total Energy $E = K + U = -K$
Given:
$E = -13.6\text{eV}$
Step 1: Find kinetic energy
$K = -E = -(-13.6) = +13.6\text{eV}$
Step 2: Find potential energy
$U = -2K = -2(13.6) = -27.2\text{eV}$

18. (d) For a head-on collision of two deuterons, the potential barrier equals the electrostatic potential energy when their surfaces just touch.
Formula:
$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$
Here
$q_1 = q_2 = e = 1.6 \times 10^{-19} C$
Radius of each deuteron $= 2\text{fm} = 2 \times 10^{-15}$ m
Distance between centres when touching
$r = 2R = 4\text{fm} = 4 \times 10^{-15}$ m
Also
$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$
Calculation
$U = 9 \times 10^9 \times \frac{(1.6 \times 10^{-19})^2}{4 \times 10^{-15}}$
$(1.6 \times 10^{-19})^2 = 2.56 \times 10^{-38}$
$U = \frac{9 \times 2.56 \times 10^{-29}}{4 \times 10^{-15}}$
$U = 5.76 \times 10^{-14}$ J

19. (b)

20. (a) Isotones are nuclei having same number of neutrons but different atomic numbers (Z).
Number of neutrons $N = A - Z$
${}^{198}_{80}\text{Hg} \rightarrow \text{N} = 198 - 80 = 118$
${}^{197}_{79}Au \rightarrow N = 197 - 79 = 118$
Neutrons same → Isotones

21. (d) Determine the Diode Bias
An ideal diode acts as a short circuit (zero resistance) when forward-biased and an open circuit (infinite resistance) when reverse-biased.
The anode (p-side) is connected to a potential of $+\mathbf{6V}$.
The cathode (n-side) is connected toward a potential of $+\mathbf{5V}$.
Since the anode potential (6 V) is higher than the cathode side potential (5 V), the diode is forward-biased.
Calculate the Current (I)
Because the diode is ideal and forward-biased, it has zero voltage drop. Therefore, the entire potential difference falls across the 100Ω resistor.
Using Ohm's Law:
$I = \frac{V_{\text{anode}} - V_{\text{cathode}}}{R}$
Substituting the given values:

$I = \frac{6\text{ V} - 5\text{ V}}{100\Omega}$

$I = \frac{1\text{ V}}{100\Omega}$

$I = 0.01\text{ A}$

Convert to Milliamperes

To match the options, convert the current from Amperes to milliamperes (mA):

$I = 0.01 \times 1000\text{ mA} = 10\text{ mA}$

22. (d) When a reverse bias is applied to a p-n junction:

The depletion region widens.

Majority carriers are pulled away from the junction, so their current decreases.

The potential barrier increases.

Therefore, reverse bias raises the potential barrier.

23. (b) Time constant of an RC filter circuit:

$\tau = RC$

Given:

$R = 200\Omega$

$C = 15\mu F = 15 \times 10^{-6} F$

$\tau = 200 \times 15 \times 10^{-6}$

$\tau = 3000 \times 10^{-6}$

$\tau = 3 \times 10^{-3}\text{ s}$

$\tau = 3\text{ ms}$

24. (b) Electric field due to a point charge:

$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

Step 1: Given field

For charge $2q$ at distance r :

$E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$

Step 2: Field due to spherical shell

A uniformly charged spherical shell behaves like a point charge at its centre for points outside it ($r \gg R$).

Here

Charge = $\boldsymbol{q}$

Distance = $\frac{r}{2}$

$E' = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r/2)^2}$

$E' = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2/4}$

$E' = \frac{4q}{4\pi\varepsilon_0 r^2}$

Step 3: Compare with E

$E = \frac{2q}{4\pi\varepsilon_0 r^2}$

$E' = 2E$

25. (a) Using Gauss's law:

$\Phi = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$

Electric flux through a closed surface depends only on the total charge enclosed.

Arrangement

15 charges q placed on the x-axis

Distance between charges = $0.5R$

One charge is at the centre of the sphere

Sphere radius = $1.5R$

Positions of charges from the centre:

$0, \pm 0.5R, \pm 1R, \pm 1.5R, \pm 2R, \pm 2.5R, \pm 3R, \pm 3.5R$
Inside the sphere ($r < 1.5R$) :
$0, \pm 0.5R, \pm 1R$
Total charges inside = 5
$Q_{\text{enclosed}} = 5q$
Electric flux
$\Phi = \frac{5q}{\varepsilon_0}$

26. (c) Force on a charge in an electric field:
$F = qE$
Using Newton's second law:
$F = ma$
Given mass $= 2m$
$qE = (2m)a$
$a = \frac{qE}{2m}$
Since the charge starts from rest, velocity after time t :
$v = at$
For $t = n$ seconds:
$v = \frac{qE}{2m} \times n$
$v = \frac{qEn}{2m}$

27. (d) Electric potential at a point is the algebraic sum of potentials due to all charges.
$V = k\frac{q}{r}$
Geometry
Square side $= 2l$
Point A is the midpoint of the side joining the two +q charges.
Distances from A:
From the two **+q** charges
$r_1 = r_2 = l$
Potential due to them:
$V_+ = k\frac{q}{l} + k\frac{q}{l} = \frac{2kq}{l}$
From the two -q charges
Distance from A to each bottom charge:
$r = \sqrt{l^2 + (2l)^2} = \sqrt{5}l$
Potential due to them:
$V_- = -k\frac{q}{\sqrt{5}l} - k\frac{q}{\sqrt{5}l} = -\frac{2kq}{\sqrt{5}l}$
Total potential
$V = \frac{2kq}{l} - \frac{2kq}{\sqrt{5}l}$
$V = \frac{2kq}{l}\left(1 - \frac{1}{\sqrt{5}}\right)$

28. (b) For a spherical conductor:
Potential inside (and on surface):
$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = \frac{kQ}{R}$
Electric field on the surface:
$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} = \frac{kQ}{R^2}$
Ratio

$$\frac{V}{E}=\frac{\frac{kQ}{R}}{\frac{kQ}{R^2}}$$

$$\frac{V}{E}=R$$

29. **(a)** In a parallel plate capacitor, the electric field between the plates is uniform (same everywhere between the plates).

Electric force on a charge:

$F=qE$

Since:

Both particles are electrons (same charge q)

Electric field E is same at P and Q

Therefore the force on both electrons is the same.

30. **(d)** Current in a conductor:

$I=nqAv_d$

where

n = number density of electrons

$\boldsymbol{q}$ = charge of electron

A = area of cross-section

v_d = drift velocity

Initially:

$I=nqAv_d$

Now

Area becomes $2A$ and current becomes $2I$.

$2I=nq(2A)v_d'$

Substitute $I=nqAv_d$:

$2(nqAv_d)=2nqAv_d'$

$v_d'=v_d$

31. **(c)**

32. **(b)**

33. **(a)** Magnetic field at the centre of a circular ring:

$$B_c=\frac{\mu_0 I}{2R}$$

Magnetic field at a point on the axis at distance x :

$$B=\frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}$$

Given

$x=2\sqrt{2}R$

Step 1: Substitute x

$R^2+x^2=R^2+(2\sqrt{2}R)^2$

$=R^2+8R^2=9R^2$

$(R^2+x^2)^{3/2}=(9R^2)^{3/2}$

$=27R^3$

Step 2: Magnetic field on axis

$$B_a=\frac{\mu_0 I R^2}{2\times 27R^3}$$

$$B_a=\frac{\mu_0 I}{54R}$$

Step 3: Ratio

$$\frac{B_c}{B_a}=\frac{\frac{\mu_0 I}{2R}}{\frac{\mu_0 I}{54R}}$$

$=27$

$B_c:B_a=27:1$

34. (c) Current sensitivity of a moving coil galvanometer

Current sensitivity $= \frac{\theta}{I}$

where

$\theta =$ angular deflection (dimensionless)

I = current

So the dimensional formula:

[Current sensitivity] $= \frac{1}{[I]}$

Since dimension of current $I = [A]$,

[Current sensitivity] $= [A^{-1}]$

35. (a) Force per unit length on a current carrying conductor in a magnetic field:

$\frac{F}{L} = IB\sin\theta$

Where

$I = 1A$

$B = 3 \times 10^{-5}T$

$\theta =$ angle between current and magnetic field.

The horizontal component of Earth's magnetic field is from South → North. The current in the conductor is also South → North.

So the angle between them:

$\theta = 0^\circ$

$\sin 0^\circ = 0$

Therefore,

$\frac{F}{L} = IB\sin 0 = 0$

36. (b)

37. (b) A paramagnetic substance is weakly attracted by a magnetic field.

In a non-uniform magnetic field, it experiences a force toward the region where the magnetic field is stronger.

Therefore, it moves from weak magnetic field to strong magnetic field.

38. (a) Use Lenz's Law: induced current flows in a direction that opposes the change in magnetic flux.

Figure (a):

Magnetic field is increasing.

The induced current will create a magnetic field opposite to the applied field to oppose the increase.

Using the right-hand rule, this gives clockwise current (seen from top).

Figure (b):

Magnetic field is decreasing.

The induced current will try to maintain the original magnetic field direction.

So the current must produce a field in the same direction as the original field, giving anticlockwise current (seen from top).

39. (c) Mutual inductance relation:

$$M = \frac{\Delta(N\Phi)}{\Delta I}$$

where $\Delta(N\Phi)$ is the change in flux linkage.

So,

$\Delta(N\Phi) = M\Delta I$

Given:

$M = 2H$

Current changes from 0 to 30 A

$\Delta I = 30$ A

$\Delta(N\Phi) = 2 \times 30$

$\Delta(N\Phi) = 60$ Wb

40. (d) In an AC generator, induced emf varies as:

$\varepsilon = \varepsilon_0 \sin(\omega t)$

Given: $\varepsilon = 0$ at $t = 0$, which matches the sine function.
The maximum value occurs when:
$\sin(\omega t) = 1$
$\omega t = \frac{\pi}{2}$
$t = \frac{\pi}{2\omega}$

41. (a) For a zero-order reaction, the half-life is given by:
$t_{1/2} = \frac{[A]_0}{2k}$
Time for complete reaction (i.e., [A] → 0):
$t_{\text{complete}} = \frac{[A]_0}{k} = 2t_{1/2}$
Given: $t_{1/2} = 10$ min
$t_{\text{complete}} = 2 \times 10 = 20$ min

42. (b) For radioactive decay, the relation between remaining fraction and time is:
$N = N_0 e^{-\lambda t}$
or in terms of half-life:
$t = \frac{t_{1/2}}{\ln 2} \ln \frac{N_0}{N}$
Here:
$t_{1/2} = 5730\text{yr}$
$N/N_0 = 80/100$
So the formula:
$t = \frac{5730}{\ln 2} \ln \frac{100}{80}$
Since $\ln 2 \approx 0.693 \approx 0.3$ (approximation in options), we write:
$t = \frac{5730}{0.3} \log \frac{100}{80}$

43. (c) Arrhenius equation:
$k = Ae^{-E_a/(RT)}$
Where:
k = rate constant
E_a = activation energy
T = temperature
Key points:
Increase in temperature T → exponent $-E_a/RT$ becomes less negative → rate increases.
Decrease in activation energy E_a → exponent becomes less negative → rate increases.
Therefore: Increase in temperature OR decrease in activation energy will increase rate of reaction.

44. (a) We are given the rates of change of concentrations:
$$\text{rate} = -6\frac{d[A]}{dt} = -4\frac{d[B]}{dt} = 3\frac{d[C]}{dt} = 4\frac{d[D]}{dt}$$
For a general reaction:
$aA + bB \rightarrow cC + dD$
Rate law in terms of stoichiometry:
$$\text{rate} = -\frac{1}{a}\frac{d[A]}{dt} = -\frac{1}{b}\frac{d[B]}{dt} = \frac{1}{c}\frac{d[C]}{dt} = \frac{1}{d}\frac{d[D]}{dt}$$
Step 1: Determine ratios
Given:
$-\frac{d[A]}{dt} : -\frac{d[B]}{dt} : \frac{d[C]}{dt} : \frac{d[D]}{dt} = 6:4:3:4$
Take ratios of stoichiometric coefficients:
$a:b:c:d = ?$
Use formula:
$a:b:c:d = \frac{1}{\text{rate of change}}$

$a \propto 1/6$
$b \propto 1/4$
$c \propto 1/3$
$d \propto 1/4$
Multiply all by **12** to get smallest integers:
$a = 12/6 = 2$
$b = 12/4 = 3$
$c = 12/3 = 4$
$d = 12/4 = 3$
Step 2: Write reaction
$2A + 3B \rightarrow 4C + 3D$

45. **(d)** The highest oxidation state of an element is stabilized by ligands that can form multiple bonds or delocalize charge.
Oxygen can form multiple bonds (double bonds) in oxides like $\mathrm{Mn_2O_7}$, which stabilizes the +7 oxidation state. Fluorine can only form single bonds due to its small size and high electronegativity, so the highest stable fluoride of manganese is $\mathrm{MnF_4}$ (+4 oxidation state).

46. **(b)** We are asked to find a metal whose divalent ion has "spin-only" magnetic moment $\mu_s = \sqrt{35}B$ (Bohr magneton).
Step 1: Spin-only formula
$\mu_s = \sqrt{n(n+2)}\mathrm{B}$
where $n =$ number of unpaired electrons.
Given:
$\sqrt{n(n+2)} = \sqrt{35} \Longrightarrow n(n+2) = 35$
$n^2 + 2n - 35 = 0$
Solve quadratic:
$$n = \frac{-2 \pm \sqrt{4+140}}{2} = \frac{-2 \pm \sqrt{144}}{2} = \frac{-2 \pm 12}{2}$$
$n = 5$(ignore negative solution)
Step 2: Identify metal with 5 unpaired electrons in its divalent state
$\mathrm{Cr^{2+}} \rightarrow$ [Ar] 3 $\mathrm{d^4}$ $\rightarrow$ 4 unpaired electrons $\rightarrow \mu = \sqrt{(4 \times 6)} = \sqrt{24} \approx 4.90$ B
$\mathrm{Mn^{2+}} \rightarrow$ [Ar]3 $\mathrm{d^5}$ $\rightarrow$ 5 unpaired electrons $\rightarrow \mu = \sqrt{(5 \times 7)} = \sqrt{35} \approx 5.92$ B
$\mathrm{Fe^{2+}} \rightarrow$ [Ar]3 $\mathrm{d^6}$ $\rightarrow$ 4 unpaired electrons $\rightarrow \mu = \sqrt{24} \approx 4.90$ B
$\mathrm{Co^{2+}} \rightarrow$ [Ar]3 $\mathrm{d^7}$ $\rightarrow$ 3 unpaired electrons $\rightarrow \mu = \sqrt{15} \approx 3.87$ B

47. **(a)** We are asked about the behavior of $\mathrm{MnO_4^{2-}}$ (manganate ion) in acidic solution.
Reaction: Disproportionation of manganate in acidic medium
$\mathrm{3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O}$
Products:
$\mathrm{MnO_4^-}$(permanganate, +7 oxidation state)
$\mathrm{MnO_2}$ (manganese dioxide, +4 oxidation state)

48. **(a)** Ziegler-Natta catalyst is usually $\mathrm{TiCl_3}$ supported on $\mathrm{Al(C_2H_5)_3}$.
In $\mathrm{TiCl_3}$, the oxidation state of Ti :
$\mathrm{Ti} + 3(-1) = 0 \Longrightarrow \mathrm{Ti} = +3$

49. **(d)** We are asked about facial (fac) and meridional (mer) isomers in octahedral complexes.
Step 1: Understand fac and mer
Fac-isomer: Three identical ligands occupy one face of the octahedron.
Mer-isomer: Three identical ligands lie in a meridian plane.
Condition: Only possible when there are three identical ligands and the remaining three ligands are another type.
So the complex must have 3 of one type and 3 of another type:
$\mathrm{[Co(NH_3)_3(NO_2)_3]}$
$x = 3, y = 3$

50. **(b)** We are comparing $\mathrm{[NiCl_4]^{2-}}$ and $\mathrm{[Ni(CN)_4]^{2-}}$.
Step 1: Determine geometry and type
(1) $\mathrm{[NiCl_4]^{2-}}$

Ligand = Cl^-(weak field, from spectrochemical series)
Geometry = tetrahedral → uses outer d-orbitals
Outer orbital complex
(2) $[Ni(CN)_4]^{2-}$
Ligand = CN^-(strong field, from spectrochemical series)
Geometry = square planar → uses inner d-orbitals (dsp^2 hybridization)
Inner orbital complex

51. (a) We are asked about potassium trioxalatoaluminate(III):
$K_3[Al(C_2O_4)_3]$
Step 1: Identify ions on dissociation
This coordination compound dissociates completely in water:
$K_3[Al(C_2O_4)_3] \rightarrow 3K^+ + [Al(C_2O_4)_3]^{3-}$
Potassium ions = 3
Complex ion = 1
Total particles = $3 + 1 = 4$
Step 2: Van't Hoff factor
$i =$ number of particles in solution $= 4$

52. (c) According to Crystal Field Theory (CFT):
$\boldsymbol{\Delta_0}$ (octahedral crystal field splitting) increases with:
Charge on the metal ion → higher positive charge → stronger splitting
Strength of ligands → stronger field ligands (from spectrochemical series) → larger Δ_0
Step 1: Compare given complexes
(a) $[CoCl(NH_3)_5]^{2+}$
Ligands: NH_3 (stronger) $+Cl^-$(weaker) → mixed field
Metal charge = +2
(b) $[Co(NH_3)_6]^{3+}$
All NH_3 ligands → strong field
Metal charge $= +3 \rightarrow$ larger Δ_0 than $[CoCl(NH_3)_5]^{2+}$
(c) $[Co(CN)_6]^{3-}$
Ligand = CN^-(strongest field ligand, π-acceptor)
Metal charge $= +3 \rightarrow$ maximum Δ_0
(d) $[Co(NH_3)_5(H_2O)]^{3+}$
H_2O is weaker ligand than NH_3 → smaller Δ_0 than $[Co(NH_3)_6]^{3+}$

53. (d)

54. (c) Optical isomerism occurs when a molecule has a chiral carbon: a carbon atom bonded to four different groups.
For monohaloalkanes (R-X):
The halogen (X) counts as one group.
The other three substituents on the carbon must all be different.
Step 1: Determine minimum carbon atoms
A CHX-CH3 (2 carbons) → CHX bonded to H , X , and CH_3 → 3 groups → not chiral
A CHX-CH2-CH3 (3 carbons) → CHX bonded to H, X, CH_2CH_3 → 4 different groups → chiral
So minimum number of carbon atoms = 3

55. (d) For $\mathbf{S_n2}$ reactions, reactivity depends on steric hindrance:
Primary halides react fastest.
Tertiary halides react very slowly.
Analyzing the options:
(a) 1-Bromo-3-methylbutane → primary carbon, slightly branched → moderately reactive
(b) 1-Bromo-2,2-dimethylpropane → tertiary carbon → very slow
(c) 1-Bromo-2-methylbutane → primary carbon, slightly branched → moderately reactive
(d) 1-Bromobutane → primary carbon, unbranched → least steric hindrance → highest reactivity

56. (a) The compound that slowly oxidizes in air and light to produce phosgene ($COCl_2$) is chloroform (trichloromethane, $CHCl_3$).
Reaction:
$$2CHCl_3 + O_2 \xrightarrow{\text{light}} 2COCl_2 + 2HCl$$

57. (b) We are asked to identify R', R'', and R''' in the formation of 2-methylbutan-2-ol via a Grignard reaction.
Step 1: Structure of 2-methylbutan-2-ol
$CH_3 - C(OH) - CH_2 - CH_3$
The central carbon ($C - 2$) is bonded to $OH_1 CH_3, CH_2CH_3$, and the Grignard R group.
So the tertiary alcohol has the structure:
$(CH_3)(C_2H_5)C(OH) - CH_3$
or written as 2-methylbutan-2-ol:
$(CH_3)_2C(OH) - CH_2CH_3$
Step 2: Identify groups
Tertiary carbon attached to:
$R' = CH_3$
$R'' = C_2H_5$
$R''' = CH_3$

58. (c)

59. (d) The enzyme responsible for fermentation of glucose to ethanol and CO_2 in yeast is:
Zymase - a complex of enzymes in yeast that converts glucose → ethanol $+CO_2$.

60. (a) The reaction is:
$$2C_2H_5OH \xrightarrow{H_2SO_4, 413\text{ K}} C_2H_5 - O - C_2H_5 + H_2O$$
This is dehydration of alcohol to form diethyl ether.
Mechanism: acid-catalyzed nucleophilic substitution (SN2-like)
Ethanol acts as nucleophile attacking the protonated ethanol (electrophilic center).

61. (b) Vanillin has the structure:
4-hydroxy-3-methoxybenzaldehyde
Functional groups present:
-CHO → aldehyde
-OH → phenolic hydroxyl
$-OCH_3$ → methoxy group

62. (c)

63. (d) To distinguish acetophenone ($C_6H_5COCH_3$) and benzophenone ($C_6H_5COC_6H_5$) :
Acetophenone has a methyl group ($-CH_3$) next to the carbonyl → contains α-hydrogen.
Benzophenone has no α-hydrogen.
Reagent used:
Iodoform test ($NaOI/I_2 + NaOH$) → reacts with methyl ketones ($CH_3 - CO -$) to give yellow precipitate of CHI_3.

64. (b) pKa measures acid strength: higher pKa → weaker acid.
Factors affecting carboxylic acid acidity:
Electron-withdrawing groups → stabilize conjugate base → lower pKa (stronger acid)
Electron-donating groups → destabilize conjugate base → higher pKa (weaker acid)
Step 1: Analyze the compounds
(a) HCOOH (formic acid) → no alkyl group → moderately acidic
(b) CH_3CH_2COOH (propionic acid) → alkyl group is electron-donating → destabilizes conjugate base → weaker acid → higher pKa
(c) $C_6H_5CH_2COOH$ (phenylacetic acid) → benzyl group mildly electron-withdrawing by resonance → slightly stronger acid than propionic acid → lower pKa
(d) $ClCH_2CH_2COOH$ (chloroacetic acid derivative) → Cl is strongly electron-withdrawing → stabilizes conjugate base → strong acid → low pKa
Step 2: Compare
Highest pKa (weakest acid) → CH_3CH_2COOH

65. (d) We are dealing with the reaction:
$$R - NH_2 \xrightarrow{NaOBr} \text{Product X}$$
This is the Hofmann bromamide reaction (Hofmann rearrangement), where:
A primary amide reacts with NaOBr → forms a primary amine ($R - NH_2$) with one less carbon.
Product X is thus a primary amine.

66. (b)

67. (b)

68. (a) For azo coupling reaction to prepare orange dye:
The phenol must be in alkaline solution → phenoxide ion $(-O^-)$is more reactive toward the diazonium salt.

69. (d) In RNA, the nitrogenous bases are:
Adenine (A)
Guanine (G)
Cytosine (C)
Uracil (U)
Thymine (T) is present only in DNA, not in RNA.

70. (c) The sequence of amino acids in a polypeptide chain is called the primary structure of a protein.

71. (b) Scurvy is caused by deficiency of Vitamin C (Ascorbic acid).

72. (b) Glucose contains an aldehyde group (in open-chain form).
Aldehydes react with hydroxylamine (NH_2OH) to form oximes.

73. (b) We are asked for mass % of NaOH when mole fraction of NaOH $= 0.2$.
Step 1: Assume 1 mole of solution
Moles of NaOH = 0.2
Moles of $H_2O = 0.8$
Step 2: Calculate masses
Mass of NaOH $= 0.2 \times 40 = 8$ g
Mass of $H_2O = 0.8 \times 18 = 14.4$ g
Total mass $= 8 + 14.4 = 22.4$ g
Step 3: Mass percent

$$\%\text{NaOH} = \frac{8}{22.4} \times 100 \approx 35.71\%$$

74. (c)

75. (a) Solubility in n-octane (non-polar solvent) depends on polarity:
Non-polar compounds → high solubility
Polar/ionic compounds → low solubility
Step 1: Analyze compounds
(I) Cyclohexane→ non-polar → highly soluble
(II) KCI → ionic → practically insoluble
(III) CH_3OH → polar, H -bonding → low solubility
(IV) CH_3CN → polar, dipole → moderate solubility
Step 2: Order of solubility in n-octane
Least soluble → Most soluble: KCl $<$ CH_3OH $<$ CH_3CN $<$ Cyclohexane

76. (d)

77. (b) We are asked about $\Lambda_m(H_2O)^\circ$, i.e., the molar conductivity of water, which can be calculated using Kohlrausch's law of independent migration of ions:
$\Lambda_m^\circ(\text{ salt }) = \lambda^\circ(\text{ cation }) + \lambda^\circ(\text{ anion })$
For water, the ions are $H^+$and OH^- → $\Lambda_m(H_2O)^\circ = \lambda^\circ(H^+) + \lambda^\circ(OH^-)$
Using conductivities of strong acid + base and their salt:
$\Lambda_m(H_2O)^\circ = \Lambda_m^\circ(\text{HCl}) + \Lambda_m^\circ(\text{NaOH}) - \Lambda_m^\circ(\text{NaCl})$

78. (a) For a Daniell cell, the relation between cell potential and equilibrium constant is given by the Nernst equation at standard conditions:

$$E^\circ_{\text{cell}} = \frac{0.0591}{n}\log K_c$$

Where:
$E^\circ_{\text{cell}} = 1.1$ V
$n = 2$ (number of electrons transferred in Daniell cell)
Step 1: Solve for K_c
$\log K_c = \frac{nE^\circ_{\text{cell}}}{0.059} = \frac{2\times1.1}{0.059} = \frac{2.2}{0.059}$
$\Rightarrow K_c = 10^{2.2/0.059}$

79. (d) The reaction is:

$\frac{2}{3}\text{Al}_2O_3 \rightarrow \frac{4}{3}\text{Al} + O_2$

Step 1: Calculate moles of electrons

$\text{Al}^{3+} \rightarrow$ Al requires 3 electrons per Al atom.

Moles of AI formed $= 4/3$ mol $\rightarrow$ electrons required $= 4/3 \times 3 = 4$ mole$^-$

Step 2: Convert moles of electrons to charge

$Q = n_e \cdot F = 4 \times 96500\text{C} = 3.86 \times 10^5\text{C}$

80. **(c)** For an electrochemical cell:

ΔG (Gibbs free energy change) ⟶ depends on amount of substance ⟶ extensive property

E (cell) (cell potential) ⟶ does not depend on amount of substance ⟶ intensive property

MATHEMATICS

81. **(a)** The Cartesian equation of a line passing through point (x_0, y_0, z_0) and parallel to vector $\vec{v} = ai + bj + ck$ is:

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Step 1: Given data

Point: $P(5, -2, 4) \rightarrow x_0 = 5, y_0 = -2, z_0 = 4$

Direction vector: $\vec{v} = 3i - 2j + 8k \rightarrow a = 3, b = -2, c = 8$

Step 2: Write equation

$$\frac{x - 5}{3} = \frac{y - (-2)}{-2} = \frac{z - 4}{8}$$

$$\frac{x - 5}{3} = \frac{y + 2}{-2} = \frac{z - 4}{8}$$

82. **(b)**

83. **(c)** The angle between two lines is the angle between their direction vectors.

Direction vectors:

$\overrightarrow{v_1} = 3\hat{\imath} + 5\hat{\jmath} + 4\hat{k}, \overrightarrow{v_2} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$

Formula:

$$\cos\theta = \frac{\overrightarrow{v_1} \cdot \overrightarrow{v_2}}{|\overrightarrow{v_1}||\overrightarrow{v_2}|}$$

(1) Dot product:

$\overrightarrow{v_1} \cdot \overrightarrow{v_2} = (3)(1) + (5)(1) + (4)(2) = 3 + 5 + 8 = 16$

(2) Magnitudes:

$|\overrightarrow{v_1}| = \sqrt{3^2 + 5^2 + 4^2} = \sqrt{9 + 25 + 16} = \sqrt{50} = 5\sqrt{2}$

$|\overrightarrow{v_2}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1 + 1 + 4} = \sqrt{6}$

(3) Cosine:

$$\cos\theta = \frac{16}{5\sqrt{2} \cdot \sqrt{6}} = \frac{16}{5\sqrt{12}} = \frac{16}{5 \cdot 2\sqrt{3}} = \frac{16}{10\sqrt{3}} = \frac{8}{5\sqrt{3}}$$

$$\theta = \cos^{-1}\frac{8\sqrt{3}}{15}$$

84. **(d)** We are asked to maximize:

$z = 40x + 30y$

Given corner points of feasible region:

$(0,0), (0,40), (20,40), (60,20), (60,0)$

Step 1: Calculate z at each corner

(1) $(0,0): z = 40(0) + 30(0) = 0$

(2) $(0,40): z = 40(0) + 30(40) = 1200$

(3) $(20,40): z = 40(20) + 30(40) = 800 + 1200 = 2000$

(4) $(60,20): z = 40(60) + 30(20) = 2400 + 600 = 3000$

(5) $(60,0): z = 40(60) + 30(0) = 2400$

Step 2: Maximum z

Maximum $z = 3000$ at $(60,20)$

85. **(b)** Check corner points of feasible region:

Constraints: $3x + 5y \leq 15, x \geq 0, y \geq 0$

(a) $(0,0): z = 5(0) + 3(0) = 0$

(b) $(0,3) \rightarrow 3(0)+5y=15 \Rightarrow y=3: z=5(0)+3(3)=9$
(d) $(5,0) \rightarrow 3x+5(0)=15 \Rightarrow x=5: z=5(5)+3(0)=25$

86. **(b)** We are asked:
$\mathrm{P}(\mathrm{E}'/\mathrm{F})+\mathrm{P}(\mathrm{F}'/\mathrm{E})$
where E and F are independent: $\mathrm{P}(\mathrm{E})=3/5, \mathrm{P}(\mathrm{F})=3/10$.
Step 1: Recall formulas
$\mathrm{P}(\mathrm{E}'/\mathrm{F})=1-\mathrm{P}(\mathrm{E} \mid \mathrm{F})$
For independent events: $\mathrm{P}(\mathrm{E} \mid \mathrm{F})=\mathrm{P}(\mathrm{E})$
$\Rightarrow \mathrm{P}(\mathrm{E}'/\mathrm{F})=1-\mathrm{P}(\mathrm{E})=1-\frac{3}{5}=\frac{2}{5}$
Similarly: $\mathrm{P}(\mathrm{F}'/\mathrm{E})=1-\mathrm{P}(\mathrm{F} \mid \mathrm{E})=1-\mathrm{P}(\mathrm{F})=1-\frac{3}{10}=\frac{7}{10}$
Step 2: Sum
$\mathrm{P}(\mathrm{E}'/\mathrm{F})+\mathrm{P}(\mathrm{F}'/\mathrm{E})=\frac{2}{5}+\frac{7}{10}=\frac{4}{10}+\frac{7}{10}=\frac{11}{10}$

87. **(a)** We are asked to calculate:
$P(A' \mid B)-P(A \mid B)$
Step 1: Recall formula
$P(A \mid B)=\frac{P(A \cap B)}{P(B)}, P(A' \mid B)=\frac{P(A' \cap B)}{P(B)}$
Also, $P(A' \cap B)=P(B)-P(A \cap B)$
$P(A' \mid B)-P(A \mid B)=\frac{P(B)-P(A \cap B)}{P(B)}-\frac{P(A \cap B)}{P(B)}=\frac{P(B)-2P(A \cap B)}{P(B)}$
Step 2: Find $P(A \cap B)$
$P(A \cup B)=P(A)+P(B)-P(A \cap B) \Rightarrow P(A \cap B)=P(A)+P(B)-P(A \cup B)$
$P(A \cap B)=\frac{3}{8}+\frac{5}{8}-\frac{3}{4}=\frac{8}{8}-\frac{6}{8}=\frac{2}{8}=\frac{1}{4}$
Step 3: Compute
$$P(A' \mid B)-P(A \mid B)=\frac{P(B)-2P(A \cap B)}{P(B)}=\frac{\frac{5}{8}-2 \cdot \frac{1}{4}}{\frac{5}{8}}=\frac{\frac{5}{8}-\frac{2}{4}}{\frac{5}{8}}=\frac{\frac{5}{8}-\frac{4}{8}}{\frac{5}{8}}=\frac{1/8}{5/8}=\frac{1}{5}$$

88. **(b)** We are asked: Probability that it was actually a six given he reports six.
This is a Bayes' theorem problem.
Step 1: Define events
$A=$ "Die shows 6 " $\rightarrow P(A)=1/6$
$B=$ "Reports 6"
He speaks truth 4/5 of the time $\rightarrow$
$P(B \mid A)=4/5, P(B \mid A')=1/5$
where $A'=$ "Die shows not 6", $P(A')=5/6$.
Step 2: Apply Bayes' theorem
$$P(A \mid B)=\frac{P(B \mid A)P(A)}{P(B \mid A)P(A)+P(B \mid A')P(A')}$$
$$P(A \mid B)=\frac{\frac{4}{5} \cdot \frac{1}{6}}{\frac{4}{5} \cdot \frac{1}{6}+\frac{1}{5} \cdot \frac{5}{6}}=\frac{4/30}{4/30+5/30}=\frac{4/30}{9/30}=\frac{4}{9}$$

89. **(d)** We are asked: number of relations on $\mathbf{A}=\{\mathbf{1},\mathbf{2},\mathbf{3}\}$ containing $(\mathbf{1},\mathbf{2})$ which are symmetric, transitive, but not reflexive.
Step 1: Symmetric property
Since $(1,2) \in R$ and R symmetric, we must also include $(2,1) \in R$.
Step 2: Transitive property
For transitivity: if $(1,2),(2,1) \in R \rightarrow$ must include $(1,1) \in R$ (because $1 \rightarrow 2 \rightarrow 1$ gives $1 \rightarrow 1$).
But relation must not be reflexive, so $(1,1) \notin R$.
Step 3: Conclusion
No such relation exists.

90. **(d)** We are given $f: \mathbb{R} \rightarrow \mathbb{R}, f(x)=x^3$.

Step 1: Check one-one (injective)
Suppose $f(x_1) = f(x_2) \rightarrow x_1^3 = x_2^3 \rightarrow x_1 = x_2$
f is one-one
Step 2: Check onto (surjective)
For any $y \in \mathbb{R}, x = \sqrt[3]{y} \in \mathbb{R} \rightarrow f(x) = y$
f is onto
Step 3: Conclusion
f is one-one and onto

91. **(c)** Step 1: Simplify $\sin^{-1}(1/\sqrt{2})$
$\sin^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$
So we have:
$\tan^{-1}\left[\frac{\sqrt{2}}{\sqrt{3}}\cos(5\cdot\pi/4)\right] = \tan^{-1}\left[\frac{\sqrt{2}}{\sqrt{3}}\cos(5\pi/4)\right]$
Step 2: Evaluate $\cos(5\pi/4)$
$\cos(5\pi/4) = -\frac{1}{\sqrt{2}}$
Step 3: Multiply by $\sqrt{2}/\sqrt{3}$
$\frac{\sqrt{2}}{\sqrt{3}}\cdot\left(-\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{3}}$
Step 4: Final step
$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

92. **(a)** We are given:
$y = 3\sin^{-1}x + \sin^{-1}(3x - 4x^3), x \in [-1/2, 1/2]$
Step 1: Recall identity
$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
Here, $3x - 4x^3 = \sin 3(\sin^{-1}x)$
$\sin^{-1}(3x - 4x^3) = \sin^{-1}(\sin(3\sin^{-1}x))$
For $x \in [-1/2, 1/2], \theta = \sin^{-1}x \in [-\pi/6, \pi/6]$
Then $3\theta \in [-\pi/2, \pi/2]$
In this interval, $\sin^{-1}(\sin(3\theta)) = 3\theta = 3\sin^{-1}x$
Step 2: Substitute back
$y = 3\sin^{-1}x + \sin^{-1}(3x - 4x^3) = 3\sin^{-1}x + 3\sin^{-1}x = 6\sin^{-1}x$
Maximum of $6\sin^{-1}x$ when $x = 1/2$:
$6\cdot\sin^{-1}(1/2) = 6\cdot\pi/6 = \pi$
Minimum when $x = -1/2$:
$6\cdot(-\pi/6) = -\pi$

93. **(c)**

94. **(a)** We are asked to find the determinant:
$\begin{vmatrix} \cos^2\theta & -\sin^2\theta \\ \sin^2\theta & \cos^2\theta \end{vmatrix}$
Step 1: Use formula for 2×2 determinant
$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
Here:
$a = \cos^2\theta, b = -\sin^2\theta, c = \sin^2\theta, d = \cos^2\theta$
Determinant $= (\cos^2\theta)(\cos^2\theta) - (-\sin^2\theta)(\sin^2\theta)$
$= \cos^4\theta + \sin^4\theta$
Step 2: Simplify using identity
$\cos^4\theta + \sin^4\theta = (\cos^2\theta + \sin^2\theta)^2 - 2\cos^2\theta\sin^2\theta = 1 - 2\sin^2\theta\cos^2\theta$
$\sin^2 2\theta = 4\sin^2\theta\cos^2\theta \Longrightarrow \sin^2\theta\cos^2\theta = \frac{1}{4}\sin^2 2\theta$

Determinant $= 1 - 2 \cdot \frac{1}{4}\sin^2 2\theta = 1 - \frac{1}{2}\sin^2 2\theta$

95. **(c)** We are asked: For an invertible 3×3 matrix A, find $|(\text{adj}A) \cdot A|$.

Step 1: Recall formula

$A \cdot \text{adj}A = \text{adj}A \cdot A = |A|I_3$

Where I_3 is the 3×3 identity matrix.

Step 2: Take determinant

$|\text{adj}A \cdot A| = ||A|I_3| = |A|^3$

Because for scalar k and $n \times n$ identity: $|kI_n| = k^n$

Here, $n = 3$.

96. **(c)** Let the midpoints of the sides be $M_1 = (2,7), M_2 = (1,1), M_3 = (10,8)$.

Area of triangle $= 4 \times$ area of triangle formed by midpoints

Area of midpoints triangle $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= \frac{1}{2}|2(1-8) + 1(8-7) + 10(7-1)| = \frac{1}{2}|2(-7) + 1(1) + 10(6)|$

$= \frac{1}{2}|-14 + 1 + 60| = \frac{1}{2}|47| = 47/2$

So area of original triangle $= 4 \times 47/2 = 94$

97. **(a)** We are given the matrix:

$$A = \begin{bmatrix} x & x^2+3x & 5 \\ -2x-6 & x^2 & -4x-2 \\ 5 & x^2+2 & x^3 \end{bmatrix}$$

and it is symmetric, i.e., $A = A^T$.

Step 1: Equate symmetric entries

(1) $a_{12} = a_{21} \to x^2 + 3x = -2x - 6$

$x^2 + 3x + 2x + 6 = 0 \Longrightarrow x^2 + 5x + 6 = 0 \Longrightarrow (x+2)(x+3) = 0$

$x = -2, -3$

(2) Check $a_{13} = a_{31} \to 5 = 5$ OK

(3) Check $a_{23} = a_{32} \to -4x - 2 = x^2 + 2$

$x^2 + 4x + 4 = 0 \Longrightarrow (x+2)^2 = 0 \Longrightarrow x = -2$

Only $x = -2$ satisfies all conditions.

98. **(a)** We are asked to compute:

$(A+I)^3 + (A-I)^3$ where $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Step 1: Compute $A + I$ and $A - I$

$A + I = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A - I = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

Step 2: Use binomial identity

$(X+Y)^3 + (X-Y)^3 = 2X^3 + 6XY^2$ (for matrices, if XY = YX)

Let $X = A, Y = I \to I$ commutes with A

$(A+I)^3 + (A-I)^3 = 2A^3 + 6AI^2 = 2A^3 + 6A$

But $A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^2 = I \to A^3 = A^2 \cdot A = I \cdot A = A$

$\Longrightarrow 2A^3 + 6A = 2A + 6A = 8A$

99. We are asked: For

$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, if $A^2 - 2I = KA$, find K

Step 1: Compute $\boldsymbol{A}^2$

$A^2 = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 2\cdot2+3\cdot4 & 2\cdot3+3\cdot5 \\ 4\cdot2+5\cdot4 & 4\cdot3+5\cdot5 \end{bmatrix} = \begin{bmatrix} 16 & 21 \\ 28 & 37 \end{bmatrix}$

Step 2: Compute $A^2 - 2I$

$2I = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \Longrightarrow A^2 - 2I = \begin{bmatrix} 16-2 & 21-0 \\ 28-0 & 37-2 \end{bmatrix} = \begin{bmatrix} 14 & 21 \\ 28 & 35 \end{bmatrix}$

Step 3: Solve $A^2 - 2I = KA$

$K\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}=\begin{bmatrix}14 & 21\\28 & 35\end{bmatrix}$

Compare elements:

$2K=14\Longrightarrow K=7$

100. **(c)** We are asked to differentiate:

$y=5^{\log x}$(log is natural logarithm, ln)

Step 1: Use exponential identity

$a^u=e^{u\ln a}\Rightarrow 5^{\log x}=e^{(\log x)\ln 5}$

Step 2: Differentiate using chain rule

$\frac{d}{dx}e^{(\log x)\ln 5}=e^{(\log x)\ln 5}\cdot\frac{d}{dx}[(\log x)\ln 5]=5^{\log x}\cdot\ln 5\cdot\frac{1}{x}$

$\frac{d}{dx}5^{\log x}=\frac{\ln 5}{x}\cdot 5^{\log x}$

Step 3: Rewrite in the form given in options

$5^{\log x}/x=x^{\log 5-1}\left(\text{ using } 5^{\log x}=x^{\log 5}\right)$

$\Rightarrow\frac{d}{dx}5^{\log x}=\ln 5\cdot x^{\log 5-1}=\log 5\cdot x^{\log(5/e)}$

101. **(a)** We are asked:

$x=a\cos\theta, y=a\sin\theta,\text{ find }\frac{d^2y}{dx^2}.$

Step 1: Compute $\frac{dy}{dx}$

$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{a\cos\theta}{-a\sin\theta}=-\cot\theta$

Step 2: Compute $\frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{d\theta}(-\cot\theta)\cdot\frac{d\theta}{dx}$

First, $\frac{d}{d\theta}(-\cot\theta)=\csc^2\theta$

Second, $\frac{dx}{d\theta}=-a\sin\theta\Longrightarrow\frac{d\theta}{dx}=-\frac{1}{a\sin\theta}$

$\frac{d^2y}{dx^2}=\csc^2\theta\cdot\left(-\frac{1}{a\sin\theta}\right)=-\frac{\csc^2\theta}{a\sin\theta}=-\frac{1}{a\sin^3\theta}=-\frac{1}{a}\csc^3\theta$

102. **(b)** We are asked to differentiate:

$f(x)=3\sin(60^\circ-x^\circ)-4\cos^3(30^\circ+x^\circ)$

Step 1: Use chain rule and convert degrees to radians

Recall: derivative w.r.t x in degrees:

$\frac{d}{dx}(\sin\theta^\circ)=\cos\theta\cdot\frac{\pi}{180},\frac{d}{dx}(\cos\theta^\circ)=-\sin\theta\cdot\frac{\pi}{180}$

Step 2: Differentiate first term

$\frac{d}{dx}[3\sin(60-x)]=3\cos(60-x)\cdot\left(-\frac{\pi}{180}\right)=-\frac{3\pi}{180}\cos(60-x)$

Step 3: Differentiate second term

$\frac{d}{dx}[-4\cos^3(30+x)]=-4\cdot 3\cos^2(30+x)\cdot(-\sin(30+x))\cdot\frac{\pi}{180}$

$=12\frac{\pi}{180}\cos^2(30+x)\sin(30+x)$

Step 4: Combine and simplify

$f'(x)=-\frac{3\pi}{180}\cos(60-x)+\frac{12\pi}{180}\cos^2(30+x)\sin(30+x)$

This simplifies to a standard form:

$f'(x)=\frac{\pi}{60}\sin(3x)$

103. **(b)**

104. **(a)** Marginal cost = derivative of total cost $C(x)$

$C(x) = 0.05x^3 - 0.2x^2 + 3x + 500$
Step 1: Differentiate
$C'(x) = 0.15x^2 - 0.4x + 3$
Step 2: Evaluate at $x = 3$
$C'(3) = 0.15(9) - 0.4(3) + 3$
$= 1.35 - 1.2 + 3 = 3.15$

105. **(c)** Given: $f(x) = \tan x - 4x$
Step 1: Find derivative
$f'(x) = \sec^2 x - 4$
Step 2: For strictly decreasing
$f'(x) < 0 \Rightarrow \sec^2 x - 4 < 0 \Rightarrow \sec^2 x < 4$
$\Rightarrow \frac{1}{\cos^2 x} < 4 \Rightarrow \cos^2 x > \frac{1}{4} \Rightarrow |\cos x| > \frac{1}{2}$
Step 3: Solve inequality
$|\cos x| > \frac{1}{2} \Rightarrow x \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

106. **(b)** Given: $f(x) = x^3 - 18x^2 + 96x, x \in [0,9]$
Step 1: Find critical points
$f'(x) = 3x^2 - 36x + 96 = 3(x^2 - 12x + 32)$
$= 3(x-4)(x-8)$
So critical points: $x = 4,8$
Step 2: Check values at endpoints and critical points
Evaluate $f(x)$ at $x = 0,4,8,9$
$f(0) = 0$
$f(4) = 64 - 288 + 384 = 160$
$f(8) = 512 - 1152 + 768 = 128$
$f(9) = 729 - 1458 + 864 = 135$
Step 3: Find minimum
Minimum value $= 0$

107. **(b)** Evaluate
$\int \frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} dx$
Step 1: Write numerator in terms of denominator derivative
Let
$D = 4e^x + 5e^{-x} \Rightarrow D' = 4e^x - 5e^{-x}$
Now express numerator:
$3e^x - 5e^{-x} = A(4e^x - 5e^{-x}) + B(4e^x + 5e^{-x})$
Equate coefficients:
For e^x :
$4A + 4B = 3 \Rightarrow A + B = \frac{3}{4}$
For e^{-x} :
$-5A + 5B = -5 \Rightarrow -A + B = -1$
Solve:
$A = \frac{7}{8}, B = \frac{1}{8}$
Step 2: Split integral
$\int \frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} dx = \int \left[\frac{7}{8}\frac{D'}{D} + \frac{1}{8}\right] dx$
Step 3: Integrate
$= \frac{7}{8}\ln|D| + \frac{1}{8}x + C$
$= \frac{1}{8}x + \frac{7}{8}\ln|4e^x + 5e^{-x}| + C$
Compare with given form:

$px+q\log|4e^{x}+5e^{-x}|$

$p=\frac{1}{8}, q=\frac{7}{8}$

108. **(b)** Evaluate

$$\int_0^{\pi/4} \sqrt{1+\sin 2x}dx$$

Step 1: Use identity

$1+\sin 2x=(\sin x+\cos x)^2$

So,

$\sqrt{1+\sin 2x}=|\sin x+\cos x|$

On $[0,\pi/4]$, both $\sin x, \cos x>0$, so:

$|\sin x+\cos x|=\sin x+\cos x$

Step 2: Integrate

$$\int_0^{\pi/4} (\sin x+\cos x)dx$$

$=[-\cos x+\sin x]_0^{\pi/4}$

Step 3: Evaluate

At $x=\pi/4$:

$-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}=0$

At $x=0$:

$-1+0=-1$

$=0-(-1)=1$

109. **(c)** Interpret the integral as:

$$\int e^{\tan^{-1}x}\frac{1+x+x^2}{1+x^2}dx$$

Step 1: Try substitution

Let

$y=xe^{\tan^{-1}x}$

Differentiate:

$\frac{dy}{dx}=e^{\tan^{-1}x}+x\cdot e^{\tan^{-1}x}\cdot\frac{1}{1+x^2}$

$=e^{\tan^{-1}x}\left(1+\frac{x}{1+x^2}\right)$

$=e^{\tan^{-1}x}\cdot\frac{1+x+x^2}{1+x^2}$

Step 2: Match with integrand

$\frac{dy}{dx}=e^{\tan^{-1}x}\frac{1+x+x^2}{1+x^2}$

So,

$$\int e^{\tan^{-1}x}\frac{1+x+x^2}{1+x^2}dx=y+C$$

$=xe^{\tan^{-1}x}+C$

110. **(a)** Evaluate

$$\int\frac{dx}{\sqrt{4x-9x^2}}$$

Step 1: Rewrite expression

$4x-9x^2=-9\left(x^2-\frac{4}{9}x\right)$

Complete the square:

$=-9\left[x^2-\frac{4}{9}x+\frac{4}{81}-\frac{4}{81}\right]$

$=-9\left(\left(x-\frac{2}{9}\right)^2-\frac{4}{81}\right)$

$= 9\left(\frac{4}{81} - \left(x - \frac{2}{9}\right)^2\right)$

Step 2: Simplify integral

$$\int \frac{dx}{\sqrt{9\left(\frac{4}{81} - \left(x - \frac{2}{9}\right)^2\right)}} = \frac{1}{3}\int \frac{dx}{\sqrt{\frac{4}{81} - \left(x - \frac{2}{9}\right)^2}}$$

Step 3: Standard form

$$\int \frac{dx}{\sqrt{a^2 - (x-h)^2}} = \sin^{-1}\left(\frac{x-h}{a}\right)$$

Here $a = \frac{2}{9}, h = \frac{2}{9}$

$$= \frac{1}{3}\sin^{-1}\left(\frac{x - \frac{2}{9}}{\frac{2}{9}}\right) + C$$

$$= \frac{1}{3}\sin^{-1}\left(\frac{9x-2}{2}\right) + C$$

111. **(b)** We know:

$$\int \tan^{-1} x dx$$

Use integration by parts:

Let

$$u = \tan^{-1} x \Rightarrow du = \frac{1}{1+x^2} dx$$

$$dv = dx \Rightarrow v = x$$

Step 1:

$$\int \tan^{-1} x dx = x\tan^{-1} x - \int \frac{x}{1+x^2} dx$$

Step 2:

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2)$$

Step 3:

$$\int \tan^{-1} x dx = x\tan^{-1} x - \frac{1}{2}\ln(1+x^2) + C$$

Step 4: Compare with given form

$$Ax\tan^{-1} x + B\log(1+x^2)$$

$$A = 1, B = -\frac{1}{2}$$

$$A + B = 1 - \frac{1}{2} = \frac{1}{2}$$

112. **(b)** "Area bounded" means total area, so take absolute value:

$$\text{Area } = \int_{-\pi/2}^{\pi/2} |\sin x| dx$$

Since $\sin x$ is negative on $[-\pi/2, 0]$ and positive on $[0, \pi/2]$:

$$= \int_{-\pi/2}^{0} (-\sin x) dx + \int_{0}^{\pi/2} \sin x dx$$

Evaluate:

$$\int_{-\pi/2}^{0} (-\sin x) dx = [\cos x]_{-\pi/2}^{0} = 1 - 0 = 1$$

$$\int_{0}^{\pi/2} \sin x dx = [-\cos x]_{0}^{\pi/2} = 0 - (-1) = 1$$

Total area:

$$1 + 1 = 2$$

113. **(a)**

114. **(a)** We need total area, so take absolute value:

$$\text{Area } = \int_{-1}^{2} |x^3| dx$$

Since x^3 is negative on $[-1,0]$ and positive on $[0,2]$:

$$= \int_{-1}^{0} (-x^3) dx + \int_{0}^{2} x^3 dx$$

Step 1: Evaluate

$$\int_{-1}^{0} (-x^3) dx = \left[-\frac{x^4}{4}\right]_{-1}^{0} = 0 - \left(-\frac{1}{4}\right) = \frac{1}{4}$$

$$\int_{0}^{2} x^3 dx = \left[\frac{x^4}{4}\right]_{0}^{2} = \frac{16}{4} = 4$$

Step 2: Total area

$$\frac{1}{4} + 4 = \frac{17}{4}$$

115. **(b)** Given:

$$\left(1 + \frac{dy}{dx}\right)^{1/2} = \left(\frac{d^2y}{dx^2}\right)^{1/3}$$

Step 1: Remove fractional powers (make polynomial form)
LCM of powers 2 and 3 is 6. Raise both sides to power 6:

$$\left(1 + \frac{dy}{dx}\right)^{3} = \left(\frac{d^2y}{dx^2}\right)^{2}$$

Step 2: Identify degree
Now the equation is polynomial in derivatives.
Highest order derivative is $\frac{d^2y}{dx^2}$, and its power is 2.

116. **(a)** Given:

$$\frac{dy}{dx} = e^{y-x} = e^{y} \cdot e^{-x}$$

Step 1: Separate variables

$$e^{-y} dy = e^{-x} dx$$

Step 2: Integrate both sides

$$\int e^{-y} dy = \int e^{-x} dx$$

$$-e^{-y} = -e^{-x} + C$$

Step 3: Simplify

$$e^{-x} - e^{-y} = C$$

117. **(d)** Given:

$$x \frac{dy}{dx} + 2y = x^2, (x \neq 0)$$

Step 1: Convert to standard linear form
Divide by x :

$$\frac{dy}{dx} + \frac{2}{x} y = x$$

So, $P(x) = \frac{2}{x}$

Step 2: Integrating Factor (I.F.)

$$\text{I.F. } = e^{\int P(x) dx} = e^{\int \frac{2}{x} dx} = e^{2\ln x} = x^2$$

118. **(c)** Evaluate:

$$\mathrm{i} \cdot (\mathrm{k} \times \mathrm{j}) + \mathrm{j} \cdot (\mathrm{i} \times \mathrm{k}) + \mathrm{k} \cdot (\mathrm{i} \times \mathrm{j})$$

Step 1: Use cross product identities

$$\mathrm{i} \times \mathrm{j} = \mathrm{k}, \mathrm{j} \times \mathrm{k} = \mathrm{i}, \mathrm{k} \times \mathrm{i} = \mathrm{j}$$

Also:

$$\mathrm{k} \times \mathrm{j} = -\mathrm{i}, \mathrm{i} \times \mathrm{k} = -\mathrm{j}$$

Step 2: Compute each term

(1) $i \cdot (k \times j) = i \cdot (-i) = -1$

(2) $j \cdot (i \times k) = j \cdot (-j) = -1$

(3) $k \cdot (i \times j) = k \cdot k = 1$

Step 3: Add

$-1 - 1 + 1 = -1$

119. **(a)** We need a unit vector perpendicular to both $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$.

Such a vector is along:

$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$

Step 1: Compute vectors

$\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\vec{a} + \vec{b} = (2,3,4), \vec{a} - \vec{b} = (0,-1,-2)$

Step 2: Cross product

$(2,3,4) \times (0,-1,-2)$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix}$$

$= \hat{\imath}(3 \cdot -2 - 4 \cdot -1) - \hat{\jmath}(2 \cdot -2 - 4 \cdot 0) + \hat{k}(2 \cdot -1 - 3 \cdot 0)$

$= \hat{\imath}(-6 + 4) - \hat{\jmath}(-4) + \hat{k}(-2)$

$= (-2)\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$

Step 3: Unit vector

Magnitude:

$\sqrt{(-2)^2 + 4^2 + (-2)^2} = \sqrt{4 + 16 + 4} = \sqrt{24} = 2\sqrt{6}$

Unit vector:

$$\frac{-2\hat{\imath} + 4\hat{\jmath} - 2\hat{k}}{2\sqrt{6}} = \frac{-1}{\sqrt{6}}\hat{\imath} + \frac{2}{\sqrt{6}}\hat{\jmath} - \frac{1}{\sqrt{6}}\hat{k}$$

120. Vertices:

$A\left(-1, \frac{1}{2}, 4\right), B\left(1, \frac{1}{2}, 4\right), C\left(1, -\frac{1}{2}, 4\right), D\left(-1, -\frac{1}{2}, 4\right)$

Step 1: Find adjacent sides

$\overrightarrow{AB} = B - A = (2,0,0)$

$\overrightarrow{BC} = C - B = (0,-1,0)$

Step 2: Lengths

$|\overrightarrow{AB}| = 2, |\overrightarrow{BC}| = 1$

Step 3: Area of rectangle

Area $= |\overrightarrow{AB}| \times |\overrightarrow{BC}| = 2 \times 1 = 2$