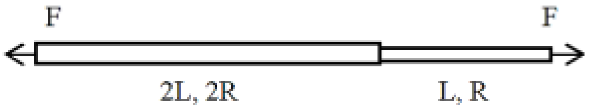
1. (c)



$$k_1 = \frac{\pi 4R^2 x}{2L}, k_2 = \frac{\pi R^2 y}{L}$$

$$F = k_1 x = k_2 y \Rightarrow \frac{y}{x} = \frac{k_1}{k_2} = 2$$

2. (d)

$$dw = \vec{F}.d\vec{r} = \vec{F}.\left(dx\hat{i} + dy\,j\right) = K\int \frac{xdx}{\left(x^2+y^2\right)^{3/2}} + \frac{ydy}{\left(x^2+y^2\right)^{3/2}}$$

$$x^2 + y^2 = a^2$$

$$w = \frac{K}{a^3}\int_a^0 xdx + \int_0^a ydx = \frac{K}{a^3}\left(\frac{-a^2}{2} + \frac{a^2}{2}\right) = 0$$

3. (a)

$$R_1 = \frac{L}{\kappa A} + \frac{L}{2\kappa A} = \frac{3L}{2\kappa A}$$

$$\frac{1}{R_2} = \frac{1}{\left(\frac{L}{\kappa A}\right)} + \frac{1}{\left(\frac{L}{2\kappa A}\right)} = \frac{3\kappa A}{L}$$

$$R_2 = \frac{L}{3\kappa A}$$

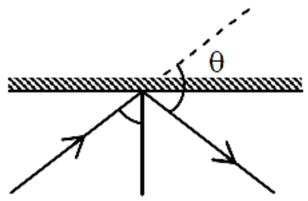
$$\Delta Q_1 = \Delta Q_2$$

$$\frac{\Delta T}{R_1}t_1 = \frac{\Delta T}{R_2}t_2$$

$$\Rightarrow t_2 = \frac{R_2}{R_1}t_1 = 2\sec$$

4. (a)

Let angle between the directions of incident ray and reflected ray be θ



$$\cos\theta = \frac{1}{2}\left(\hat{i} + \sqrt{3}\,j\right).\frac{1}{2}\left(\hat{i} - \sqrt{3}\,j\right)$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = 120^0$$

5. (b)

Main scale division (s) = .05 cm

Vernier scale division (v) = $\frac{49}{100}$ = .049

Least count = .05 – .049 = .001 cm

Diameter: 5.10 + 24 × .001

= 5.124 cm

6. (d)

$PV = nRT = \frac{m}{M} RT$

$\Rightarrow PM = \rho RT$

$$\frac{\rho_1}{\rho_2} = \frac{P_1 M_1}{P_2 M_2} = \left(\frac{P_1}{P_2}\right) \times \left(\frac{M_1}{M_2}\right) = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$$

Here ρ_1 and ρ_2 are the densities of gases in the vessel containing the mixture.

7. (b)

$$\frac{I_{\max}}{2} = I_m \cos^2\left(\frac{\phi}{2}\right)$$

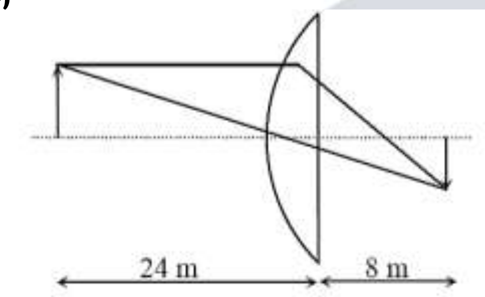
$$\Rightarrow \cos\left(\frac{\phi}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\phi}{2} = \frac{\pi}{4}$$

$$\Rightarrow \phi = \frac{\pi}{2}(2n+1)$$

$$\Rightarrow \Delta x = \frac{\lambda}{2\pi}\phi = \frac{\lambda}{2\pi} \times \frac{\pi}{2}(2n+1) = \frac{\lambda}{4}(2n+1)$$

8. (c)



$$\mu = \frac{\lambda_a}{\lambda_m} = \frac{3}{2}$$

$$\Rightarrow \frac{1}{f} = \frac{\mu - 1}{R} = \frac{1}{2R}$$

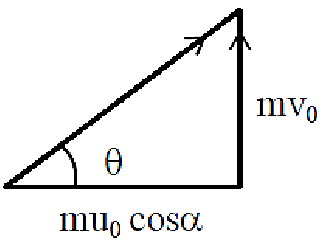
$$\Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{8} - \frac{1}{-24} = \frac{1}{2R}$$

$$\Rightarrow \frac{3+1}{24} = \frac{1}{2R}$$

$$\Rightarrow R = 3m$$

9. (a)



Velocity of particle performing projectile motion at highest point

$= v_1 = v_0 \cos\alpha$

Velocity of particle thrown vertically upwards at the position of collision

$$= v_2^2 = u_0^2 - 2g\frac{u^2\sin^2\alpha}{2g} = v_0\cos\alpha$$

So, from conservation of momentum

$$\tan\theta = \frac{mv_0\cos\alpha}{mu_0\cos\alpha} = 1$$

$$\Rightarrow \theta = \pi/4$$

10. (b)

$t = 100\times10^{-9}\sec, P = 30\times10^{-3}Watt, C = C\times10^{8}m/s$

Momentum = $= \frac{Pt}{C} = \frac{30\times10^{-3}\times100\times10^{-9}}{3\times10^{8}} = 1.0\times10^{-17}kg\,ms^{-1}$

11. (b, d)

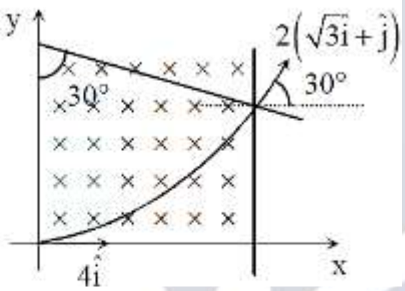
After switch S_1 is closed, C_1 is charged by $2CV_0$, when switch S_2 is closed, C_1 and C_2 both have upper plate charge CV_0.

When S_3 is closed, then upper plate of C_2 becomes charged by $-CV_0$ and lower plate by $+CV_0$.

12. (a, c)

So magnetic field is along -ve, z-direction.

Time taken in the magnetic field = $10\times10^{-3} = \frac{\pi}{M}6QB$

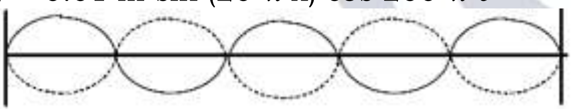


$$B = \frac{\pi M}{60\times10^{-3}Q} = \frac{1000\pi M}{60Q}$$

$$= \frac{50\pi M}{3Q}$$

13. (b,c)

y = 0.01 m sin (20 π x) cos 200 π t



no. of nodes is 6

$$20\pi = \frac{2\pi}{\lambda}$$

$$\therefore \lambda = \frac{1}{10}m = 0.1m$$

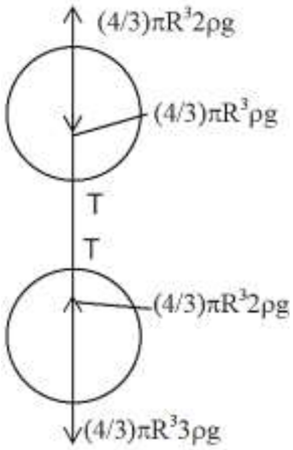
length of the spring = 0.5 x $\frac{1}{2}$ = 0.25

Mid-point is the antinode

Frequency at this mode is f = $\frac{200\pi}{2\pi} = 100Hz$

$\therefore$Fundamental frequency = $\frac{100}{5}$ = 20Hz.

14. (a, d)



At equilibrium,

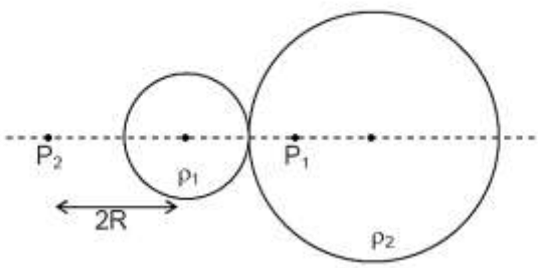
$$\frac{4}{3}\pi R^2 2\rho g = \frac{4}{3}\pi R^2 \rho g + T$$

$$T = \frac{4}{3}\pi R^2 \rho g$$

$$\therefore \Delta l = \frac{4}{3k}\pi R^2 \rho g$$

For equilibrium of the complete system, net force of buoyancy must be equal to the total weight of the sphere which holds true in the given problem. So both the spheres are completely submerged.

15. **(b, d)**



At point P_1, $\frac{1}{4\pi\varepsilon_0}\frac{\rho_1(4/3)\pi R^3}{4R^2} = \frac{\rho_2 R}{3\varepsilon_0}$

$$\frac{\rho_1 R}{12} = \frac{\rho_2 R}{3}$$

$$\frac{\rho_1}{\rho_2} = 4$$

At point P_2,

$$\frac{\rho_1(4/3)\pi R^3}{(2R)^2} + \frac{\rho_2(4/3)\pi 8R^3}{(5R)^2} = 0$$

$$\therefore \frac{\rho_1}{\rho_2} = -\frac{32}{25}$$

16. **(5)**

The initial speed of 1st bob (suspended by a string of length l_1)is $\sqrt{5gl_1}$.

The speed of this bob at highest point will be $\sqrt{gl_1}$.

When this bob collides with the other bob there speeds will be interchanged.

$$\sqrt{gl_1} = \sqrt{5gl_1} \Rightarrow \frac{l_1}{l_2} = 5$$

17. **(5)**

Power = $\frac{dW}{dt} \Rightarrow W = 0.5 \times 5 = 2.5 = KE_f - KE_i$

$2.5 = \frac{M}{2}\left(v_f^2 - v_i^2\right)$

$\Rightarrow v_f = 5$

18. (1)

Slope of graph is h/e = constant

$\Rightarrow$ 1

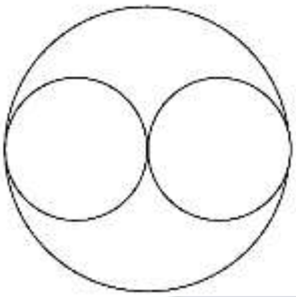
19. (4)

$f = \left(1 - e^{-\lambda t}\right) = 1 - e^{-\lambda t} \approx 1 - \left(1 - \lambda t\right) = \lambda t$

f = 0.04

Hence % decay ≈ 4 %

20. (8)



Conservation of angular momentum about vertical axis of disc

$\frac{50(0.4)^2}{2} \times 10 = \left[\frac{50(0.4)^2}{2} + 4(6.25)(0.2)^2\right]\omega$

ω = 8 rad/sec

21. (d) Overall order of reaction can be decided by the data given $t_{75\%} = 2t_{50\%}$

$\therefore$ It is a first order reaction with respect to P.

From graph [Q] is linearly decreasing with time, i.e. order of reaction with respect to Q is zero and the rate expression is r = k $[P]^1[Q]^0$.

Hence (d) is correct.

22. (b)

P = Fe^{+3} (no. of unpaired e^- = 5)

Q = V^{+2} (no. of unpaired e^- = 3)

R = Fe^{+2} (no. of unpaired e^- = 4)

As all ligands are weak field, hence the no. of unpaired electrons remains same in the complex ion.

$\mu = \sqrt{n(n+2)}$ B.M.

Hence (b) is correct.

23. (a)

According to the given figure, A^+ is present in the octahedral void of X^-. The limiting radius in octahedral void is related to the radius of sphere as

$r_{void} = 0.414\, r_{sphere}$

$r_{A^+} = 0.414 r_{X^-}$

$= 0.414 \times 250\, pm = 103.5$

$\approx 104\, pm$

24. (b)

$4HNO_3 \rightarrow 2H_2O + 4NO_2 + O_2$

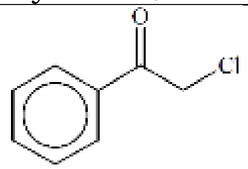
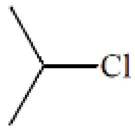
NO_2 remains dissolved in nitric acid colouring it yellow or even red at higher temperature.

25. **(d)** pKa of PhOH (carbolic acid) is 9.98 and that of carbonic acid (H_2CO_3) is 6.63 thus phenol does not give effervescence with HCO_3^- ion.

26. **(a)**
Sulfide ore of Ag → Argentite (Ag_2S), Pb → Galena (PbS), Cu → Copper glance (Cu_2S)

27. **(b)**
Adsorption of methylene blue on activated charcoal is physical adsorption hence it is characterized by decrease in enthalpy. Hence (b) is correct.

28. **(b)**
Relative reactivity for SN_2 reaction in the given structures is

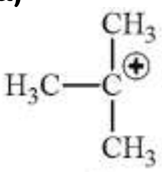
Substrate	 (S)	CH_3Cl (P)	$H_2C=CH-CH_2Cl$ (R)	
Relative Rates Towards S_N2	100000	200	79	0.02

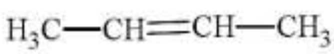
29. **(c)**
Combustion of glucose
$C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O$
$\Delta H_{combustion} = \left(6 \times \Delta H_f CO_2 + 6 \times \Delta H_f H_2O\right) - \Delta H_f C_6H_{12}O_6$
= (6 × - 400 + 6 × - 300) – (- 1300)
= - 2900 kJ/mol
= - 2900/180 kJ/g
= - 16.11 kJ/g

30. **(d)**
Among Fe^{3+}, Al^{3+}, Mg^{2+}, Zn^{2+} only Zn^{2+} is precipitated with ammonical H_2S as ZnS.

31. **(a)**
Rate in weak acid = $\frac{1}{100}$ (rate in strong acid)
$\therefore \left[H^+\right]_{weak\ acid} = \frac{1}{100}\left[H^+\right]_{strong\ acid}$
$\therefore \left[H^+\right]_{weak\ acid} = \frac{1}{100} M = 10^{-2} M$
$\therefore C\alpha = 10^{-2}$
$\therefore K_a = 10^{-4}$
weak acid strongacid

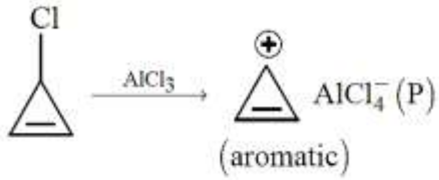
32. **(a)**

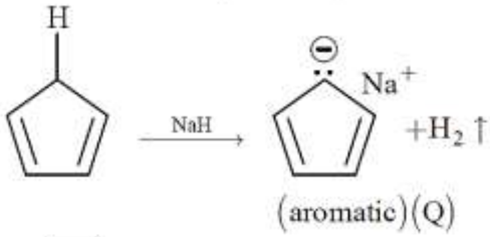

$\sigma - p$(empty)

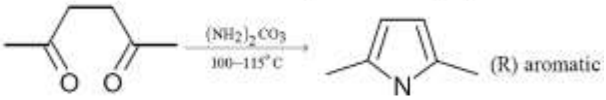

$\sigma - \pi^*$(antibonding)

33. **(b, d)**
$\left[Co\left(NH_3\right)_4 Cl_2\right]^+$ (an octahedral complex) and $\left[Pt\left(NH_3\right)_2\left(H_2O\right)Cl\right]^+$ (a square planar complex) will show geometrical isomerism.
$\left[Pt\left(NH_3\right)_3\left(NO_3\right)\right]Cl$ and $\left[Pt\left(NH_3\right)_3 Cl\right]Br$ will show ionization isomerism.

34. (a, b, c, d)

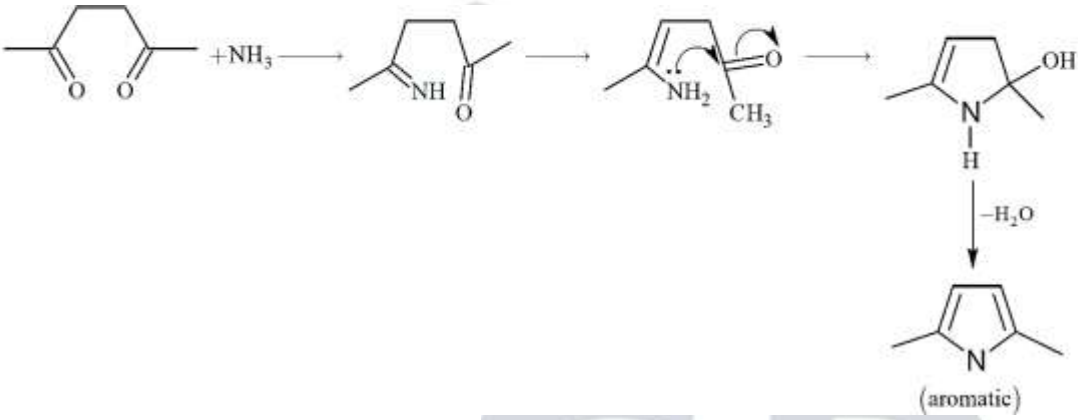


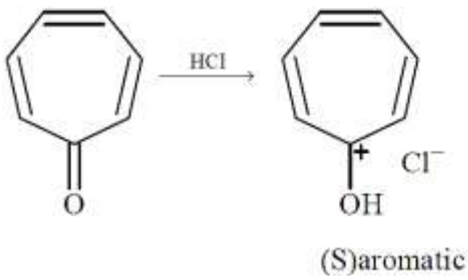




Mechanism

$(NH_4)_2CO_3 \xrightarrow{\Delta} 2NH_3 + CO_2 + H_2O$





35. (b, c, d)

For ideal solution, $\Delta S_{system} > 0$

$\Delta S_{surrounding} = 0$

$\Delta H_{mixing} = 0$

36. (5)

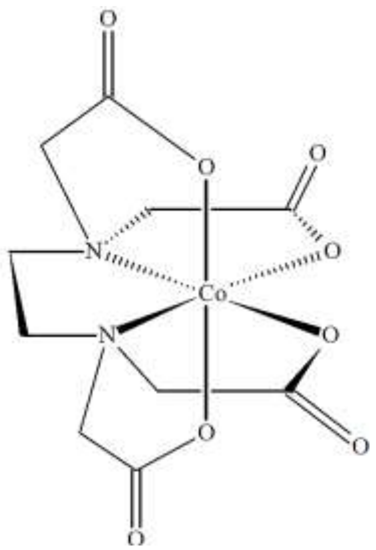
Since, $\lambda = \dfrac{h}{mV} = \dfrac{h}{\sqrt{2M\,K.E}}$ (since K.E. $\propto$ T)

$\Rightarrow \lambda \propto \dfrac{1}{\sqrt{MT}}$

For two gases,

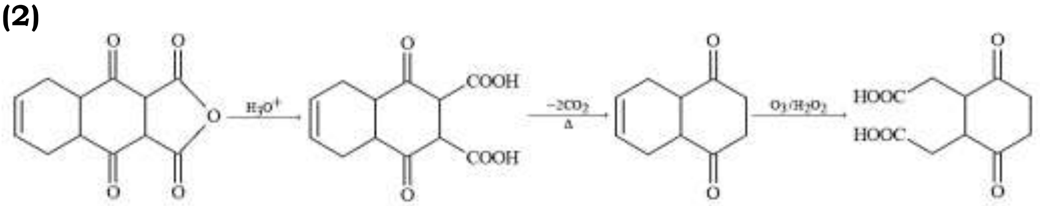
$$\frac{\lambda_{He}}{\lambda_{Ne}} = \sqrt{\frac{M_{Ne}T_{Ne}}{M_{He}T_{He}}} = \sqrt{\frac{20}{4} \times \frac{1000}{200}}$$

37. (8)



Total no. of N – Co – O bond angles is 8.

38. **(2)**



39. **(4)**

Because –COOH group of tetrapeptide is intact on alanine, its NH2 must be participating in condensation.

$\therefore$ Alanine is at one terminus, – – – A.

To fill the 3 blanks, possible options are:

When NH_2 group attached to non- chiral carbon

G	V	P
G	P	V

(ii) When NH_2 group attached to chiral carbon

V	G	P	P	V	G
V	P	G	P	G	V

where, Glycine (G)

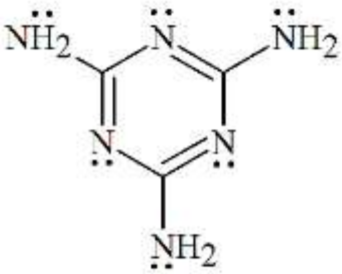
Valine (V)

Phenyl alanine (P)

Alanine (a)

So the number of possible sequence are 4.

40. **(6) lone pairs**



Melamine

41. **(d)**

Any point B on line is $(2\lambda - 2, -\lambda - 1, 3\lambda)$

Point B lies on the plane for some λ

$\Rightarrow (2\lambda - 2) + (-\lambda - 1) + 3\lambda = 3$

$\Rightarrow 4\lambda = 6 \Rightarrow \lambda = \frac{3}{2} \Rightarrow B \equiv \left(1, \frac{-5}{2}, \frac{9}{2}\right)$

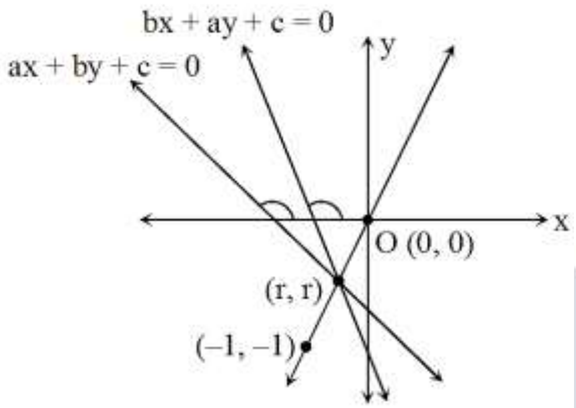
The foot of the perpendicular from point (– 2, – 1, 0) on the plane is the point A (0, 1, 2)

$\Rightarrow$ D.R. of AB = $\left(1, \frac{-7}{2}, \frac{5}{2}\right) \equiv (2, -7, 5)$

Hence $\frac{x}{2} = \frac{y-1}{-7} = \frac{z-2}{5}$

42. (a)

For point of intersection $(a - b)x_1 = (a - b)y_1$



$\Rightarrow$ point lie on line y = x ….. (1)

Let point is (r, r)

$\sqrt{(r-1)^2 + (r-1)^2} < 2\sqrt{2}$

$\sqrt{2}|(r-1)| < 2\sqrt{2}$

$\Rightarrow |r - 1| < 2$

$-1 < r < 3$

$\Rightarrow$ (–1, –1) lies on the opposite side of origin for both lines

$\Rightarrow -a - b + c < 0$

$\Rightarrow a + b - c > 0$

43. (b)

$y_1 = \sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$

$y_2 = \sqrt{2}\left|\sin\left(\frac{\pi}{4} - x\right)\right|$

$\Rightarrow Area = \int_0^{\frac{\pi}{4}} \left((\sin x + \cos x) - (\cos x - \sin x)\right)dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left((\sin x + \cos x) - (\sin x - \cos x)\right)dx$

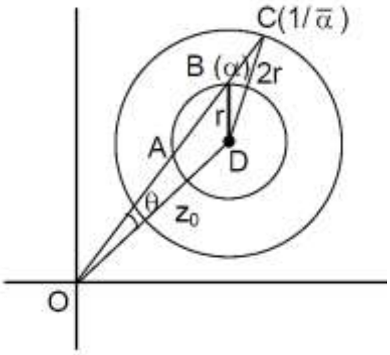
$= 4 - 2\sqrt{2}$

44. (a)

P(at least one of them solves correctly) = 1 – P(none of them solves correctly)

$= 1 - \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} \times \frac{7}{8}\right) = \frac{235}{256}$

45. (c)



$OB = |\alpha|$

$OC = \dfrac{1}{|\bar{\alpha}|} = \dfrac{1}{|\alpha|}$

In ΔOBD

$\cos\theta = \dfrac{|z_0|^2 + |\alpha|^2 - r^2}{2|z_0||\alpha|}$

In ΔOCD

$$\cos\theta = \frac{|z_0|^2 + \frac{1}{|\alpha|^2} - 4r^2}{2|z_0|\frac{1}{|\alpha|}}$$

$$\frac{|z_0|^2 + |\alpha|^2 - r^2}{2|z_0||\alpha|} = \frac{|z_0|^2 + \frac{1}{|\alpha|^2} - 4r^2}{2|z_0|\frac{1}{|\alpha|}}$$

$\Rightarrow |\alpha| = \dfrac{1}{\sqrt{7}}$

46. (c)

Let f(x) = x^2 – x sin x – cos x $\Rightarrow$ f'(x) = 2x – x cos x

$\lim\limits_{x\to\infty} f(x) \to \infty$

f(0) = –1

Hence 2 solutions.

47. (d)

Given f' (x) – 2f (x) < 0

$\Rightarrow$ f (x) < ce^{2x}

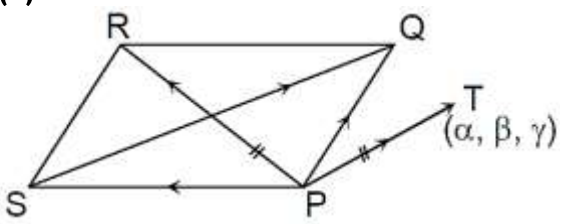
Put $x = \dfrac{1}{2} \Rightarrow c > \dfrac{1}{e}$

Hence f (x) < e^{2x-1}.

$\Rightarrow 0 < \int\limits_{1/2}^{1} f(x)dx < \int\limits_{1/2}^{1} e^{2x-1}dx$

$0 < \int\limits_{1/2}^{1} f(x)dx < \dfrac{e-1}{2}$

48. (c)



Area of base (PQRS) = $\frac{1}{2}\left|\overrightarrow{PR}\times\overrightarrow{SQ}\right| = \frac{1}{2}\begin{vmatrix}\hat{i} & j & k\\ 3 & 1 & -2\\ 1 & -3 & -4\end{vmatrix}$

$= \frac{1}{2}\left|-10\hat{i}+10j-10k\right| = 5\left|\hat{i}-j+k\right| = 5\sqrt{3}$

Height = proj. of PT on $\hat{i}-j+k = \left|\frac{1-2+3}{\sqrt{3}}\right| = \frac{2}{\sqrt{3}}$

Volume = $=\left(5\sqrt{3}\right)\left(\frac{2}{\sqrt{3}}\right) = 10$ cu. units

49. (b)

$\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(n^2+n+1\right)\right)$

$\cot\left(\sum_{n=1}^{23}\tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right)\right)$

$\Rightarrow \cot\left(\tan^{-1}\left(\frac{23}{25}\right)\right) = \frac{25}{23}$

50. (a)

$\frac{dy}{dx} = \frac{y}{x} + \sec\frac{y}{x}$. Let y=vx

$\Rightarrow \frac{dv}{\sec v} = \frac{dx}{x}$

$\int \cos v dv = \int \frac{dx}{x}$

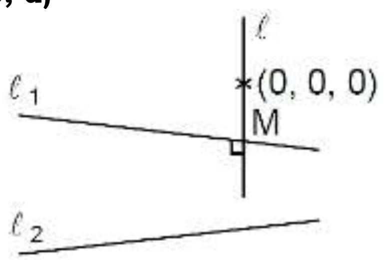
$\sin v = \ln x + c$

$\sin\left(\frac{y}{x}\right) = \ln x + c$

The curve passes through $\left(1, \frac{\pi}{6}\right)$

$\Rightarrow \sin\left(\frac{y}{x}\right) = \ln x + \frac{1}{2}$

51. (b, d)



The common perpendicular is along $\begin{vmatrix} \hat{i} & j & k \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -2\hat{i} + 3j - 2k$

Let $M \equiv (2\lambda, -3\lambda, 2\lambda)$

So, $\frac{2\lambda - 3}{1} = \frac{-3\lambda + 1}{2} = \frac{2\lambda - 4}{2} \Rightarrow \lambda = 1$

So, M ≡ (2, –3, 2)

Let the required point be P

Given that PM = $\sqrt{17}$

$\Rightarrow (3 + 2s - 2)^2 + (3 + 2s + 3)^2 + (2 + s - 2)^2 = 17$

$\Rightarrow 9s^2 + 28s + 20 = 0$

$\Rightarrow s = -2, -\frac{10}{9}$

So, $P \equiv (-1, -1, 0)$ or $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

52. (b, c)

We have $f'(x) = \sin \pi x + \pi x \cos \pi x = 0$

$\Rightarrow \tan \pi x = -\pi x$

$\Rightarrow \pi x \in \left(\frac{2n+1}{2}\pi, (n+1)\pi\right) \Rightarrow x \in \left(n + \frac{1}{2}, n+1\right) \in (n, n+1)$

53. (a, b)

$S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2 = \sum_{r=0}^{(n-1)} \left((4r+4)^2 + (4r+3)^2 - (4r+2)^2 - (4r+1)^2\right)$

$= \sum_{r=0}^{(n-1)} \left(2(8r+6) + 2(8r+4)\right)$

$= \sum_{r=0}^{(n-1)} (32r + 20)$

$= 16(n-1)n + 20n$

$= 4n(4n+1)$

$= \begin{cases} 1056 \; for \; n = 8 \\ 1332 \; for \; n = 9 \end{cases}$

54. (c, d)

(a) $(N^TMN)^T = N^TM^TN = N^TMN$ if M is symmetric and is $-N^TMN$ if M is skew symmetric

(b) $(MN - NM)^T = N^TM^T - M^TN^T = NM - MN = -(MN - NM)$. So, (MN – NM) is skew symmetric

(c) $(MN)^T = N^TM^T = NM \neq MN$ if M and N are symmetric. So, MN is not symmetric

(d) (adj. M) (adj. N) = adj(NM) ≠ adj (MN).

55. (a, c)

Let the sides of rectangle be 15k and 8k and side of square be x then (15k – 2x)(8k – 2x)x is volume.

$v = 2(2x^3 - 23kx^2 + 60k^2x)$

$\left.\frac{dv}{dx}\right|_{x=5} = 0$

$6x^2 - 46kx + 60k^2 |_{x=5} = 0$

$6k^2 - 23k + 15 = 0$

k = 3, $k = \frac{5}{6}$. Only k = 3 is permissible.

So, the sides are 45 and 24.

56. (5)

Let (1, 1, 1), (–1, 1, 1), (1, –1, 1), (–1, –1, 1) be vectors $\vec{a},\vec{b},\vec{c},\vec{d}$ rest of the vectors are $-\vec{a},-\vec{b},-\vec{c},-\vec{d}$ and let us find the number of ways of selecting co–planar vectors.
Observe that out of any 3 coplanar vectors two will be collinear (anti parallel)
Number of ways of selecting the anti-parallel pair = 4
Number of ways of selecting the third vector = 6
Total = 24
Number of non-co–planar selections = ${}^{8}C_3$ - 24 = 32 = 2^5 , p = 5
Alternate Solution:

Required value = $\frac{8\times6\times4}{3!}$

$\therefore$ p = 5

57. (6)

Let P (E_1) = x, P (E_2) = y and P (E_3) = z
then (1 - x) (1 - y) (1 - z) = p
x (1 - y) (1 - z) = α
(1 - x) y(1 - z) = β
(1 - x) (1 - y) (1 - z) = γ

so $\frac{1-x}{x}=\frac{p}{\alpha}$ $\qquad x=\frac{\alpha}{\alpha+p}$

similarly, z= $\frac{\gamma}{\gamma+p}$

so $\frac{P(E_1)}{P(E_3)}=\frac{\frac{\alpha}{\alpha+p}}{\frac{\gamma}{\gamma+p}}=\frac{\frac{\gamma+p}{\gamma}}{\frac{\alpha+p}{\alpha}}=\frac{1+\frac{p}{\gamma}}{1+\frac{p}{\alpha}}$

also given $\frac{\alpha\beta}{\alpha-2\beta}=p=\frac{2\beta\gamma}{\beta-3\gamma}\Rightarrow\beta=\frac{5\alpha\gamma}{\alpha+4\gamma}$

Substituting back $\left(\alpha-2\left(\frac{5\alpha\gamma}{\alpha+4\gamma}\right)\right)p=\frac{\alpha.5\alpha\gamma}{\alpha+4\gamma}$

$\Rightarrow \alpha p-6p\gamma=5\alpha\gamma$

$\Rightarrow\left(\frac{p}{\gamma}+1\right)=6\left(\frac{p}{\alpha}+1\right)\Rightarrow\frac{\frac{p}{\gamma}+1}{\frac{p}{\alpha}+1}=6$

58. (6)

Let T_{r-1}, T_r, T_{r+1} are three consecutive terms of $(1 + x)^{n+5}$

$T_{r-1}={}^{n+5}C_r\,(x)^{r-2},T_r={}^{n+5}C_{r-1}x^{r-1},T_{r+1}={}^{n+5}C_rx^r$

Where, ${}^{n+5}C_{r-2}$: ${}^{n+5}C_{r-1}$: ${}^{n+5}C_r$ = 5 : 10 : 14.

So $\frac{{}^{n+5}C_{r-2}}{5}=\frac{{}^{n+5}C_{r-1}}{10}=\frac{{}^{n+5}C_r}{14}$

So from $\frac{{}^{n+5}C_{r-2}}{5}=\frac{{}^{n+5}C_{r-1}}{10}\Rightarrow n-3r=-3$..... (1)

$\frac{{}^{n+5}C_{r-1}}{10}=\frac{{}^{n+5}C_r}{14}\Rightarrow 5n-12r=-30$ ..... (2)

From equation (1) and (2) n = 6

59. (5)

Clearly, 1 + 2 + 3 + ... + n – 2 ≤ 1224 ≤ 3 + 4 + ... n

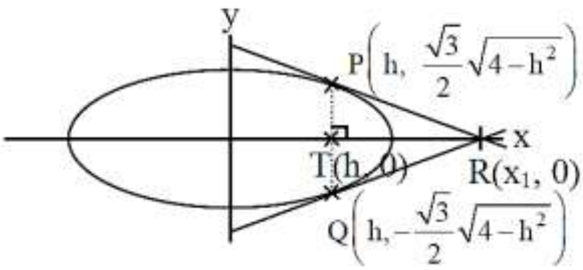
$\Rightarrow\frac{(n-2)(n-1)}{2}\le1224\le\frac{(n-2)}{2}(3+n)$

$\Rightarrow n^2 - 3n - 2446 \le 0$ and $n^2 + n - 2454 \ge 0$

$\Rightarrow 49 < n < 51 \Rightarrow n = 50$

$\therefore \frac{n(n+1)}{2} - (2k+1) = 1224 \Rightarrow k = 25 \Rightarrow k - 20 = 5$

60. (9)



$\frac{x^2}{4} + \frac{y^2}{3} = 1$

$y = \frac{\sqrt{3}}{2}\sqrt{4-h^2}$ at $x = h$

Let $R(x_1, 0)$

PQ is chord of contact, so $\frac{xx_1}{4} = 1 \Rightarrow x = \frac{4}{x_1}$

which is equation of PQ, x = h

so $\frac{4}{x_1} = h \Rightarrow x_1 = \frac{4}{h}$

$\Delta(h)$ = area of Δ PQR = $\frac{1}{2} \times PQ \times RT$

$= \frac{1}{2} \times \frac{2\sqrt{3}}{2}\sqrt{4-h^2} \times (x_1 - h) = \frac{\sqrt{3}}{2h}(4-h^2)^{3/2}$

$\Delta'(h) = \frac{-\sqrt{3}(4+2h^2)}{2h^2}\sqrt{4-h^2}$ which is always decreasing

so Δ_1 = maximum of Δ (h) = $\frac{45\sqrt{5}}{8}$ at $h = \frac{1}{2}$

Δ_2 = minimum of Δ (h) = $\frac{9}{2}$ at h = 1

so $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 = \frac{8}{\sqrt{5}} \times \frac{45\sqrt{5}}{8} - 8.\frac{9}{2} = 45 - 36 = 9$