**1. (b)**

$$\frac{-2GMm}{L}+\frac{1}{2}mv^2=0$$

$$\Rightarrow v=2\sqrt{\frac{GM}{L}}$$

Note: The energy of mass 'm' means its kinetic energy (KE) only and not the potential energy of interaction between m and the two bodies (of mass M each) – which is the potential energy of the system.

**2. (a, d)**

v = $u_0$ sin $\omega$t (suppose $t_1$ is the time of collision) $\frac{u_0}{2}=u_0\cos\omega t_1 \Rightarrow t_1=\frac{\pi}{3\omega}$

Now the particle returns to equilibrium position at time $t_2$ = $2t_1$ i.e. $\frac{2\pi}{3\omega}$ with the same mechanical energy i.e. its speed will $u_0$.

Let $t_3$ is the time at which the particle passes through the equilibrium position for the second time.

$$\therefore t_3=\frac{T}{2}+2t_1$$

$$=\frac{\pi}{\omega}+\frac{2\pi}{3\omega}=\frac{5\pi}{3\omega}$$

$$=\frac{5\pi}{3}\sqrt{\frac{m}{k}}$$

Energy of particle and spring remains conserved.

**3. (a d)**

Due to field of solenoid is non zero in region 0 < r < R and non-zero in region r>2R due to conductor.

**4. (a, b)**

If wind blows from source to observer

$$f_2=f_1\left(\frac{V+w+u}{V+w-u}\right)$$

When wind blows from observer towards source

$$f_2=f_1\left(\frac{V-w+u}{V-w-u}\right)$$

In both cases, $f_2 > f_1$.

**5. (d)**

$$d=\frac{\lambda}{2\sin\theta}$$

$$\ln\ d=\ln\left(\frac{\lambda}{2}\right)-\ln\sin\theta$$

$$\frac{\Delta d}{d}=0-\frac{\cos\theta d\theta}{\sin\theta}$$

$$\left(\frac{\Delta d}{d}\right)_{\max}=\pm\cot\theta\Delta\theta$$

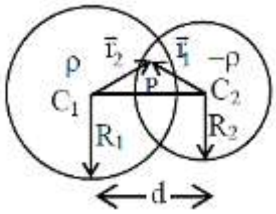
Also $(\Delta d)_{\max}=d\cos\theta\Delta\theta$

$$\frac{\lambda}{2\sin\theta}\cos\theta\Delta\theta$$

$$=\frac{\lambda}{2}\frac{\cos\theta}{\sin^2\theta}\Delta\theta$$

As $\theta$ increases $\cot\theta$ decreases and $\frac{\cos\theta}{\sin^2\theta}$ also decreases.

**6. (c, d)**




In triangle $PC_1C_2$

$\vec{r}_2 = \vec{d} + \vec{r}_1$

The electrostatic field at point P is

$$\vec{E} = \frac{K\left(\rho\frac{4}{3}\pi R_1^3\right)\vec{r}_2}{R_1^3} + \frac{K\left(\rho\frac{4}{3}\pi R_2^3\right)\left(-\vec{r}_1\right)}{R_2^3}$$

$$\vec{E} = K\rho\frac{4}{3}\pi\left(\vec{r}_2 - \vec{r}_1\right)$$

$$\vec{E} = \frac{\rho}{3\varepsilon_0}\vec{d}$$

**7. (a, b, c, d)**

Option (a) is correct because the graph between (0 – 100 K) appears to be a straight line up to a reasonable approximation.

Option (b) is correct because area under the curve in the temperature range (0 □ 100 K) is less than in range (400 - 500 K.)

Option (c) is correct because the graph of C versus T is constant in the temperature range (400 - 500 K)

Option (d) is correct because in the temperature range (200 – 300 K) specific heat capacity increases with temperature.

**8. (a, c)**

Given data

$$4.5a_0 = a_0\frac{n^2}{Z} \ldots\text{(i)}$$

$$\frac{nh}{2\pi} = \frac{3h}{2\pi} \ldots\text{(ii)}$$

So n = 3 and z = 2

So possible wavelength are

$$\frac{1}{\lambda_1} = RZ^2\left[\frac{1}{1^2} - \frac{1}{3^2}\right] \Rightarrow \lambda_1 = \frac{9}{32R}$$

$$\frac{1}{\lambda_2} = RZ^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right] \Rightarrow \lambda_2 = \frac{1}{3R}$$

$$\frac{1}{\lambda_3} = RZ^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right] \Rightarrow \lambda_3 = \frac{9}{5R}$$

**9. (i) (b)**

Using work energy theorem

$$mgR\sin 30^0 + W_f = \frac{1}{2}mv^2$$

$$200 - 150 = \frac{v^2}{2}$$

v = 10 m/s

**(ii) (a)**

$$N - mg\cos 60^0 = \frac{mv^2}{R}$$

$$N = 5 + \frac{5}{2} = 7.5\,\text{Newton.}$$

**10. (i) (b)**

For direct transmission

P = $i^2R$ = $(150)^2$ (0.4x20) = 1.8x$10^5$ w

Fraction (in %) $= \frac{1.8\times10^5}{6\times10^5}\times100 = 30\%$

**(ii) (a)**

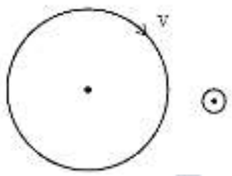
$$\frac{40000}{200} = 200$$

**11. (i) (b)**

$$E(2\pi R) = \pi R^2 \frac{dB}{dt}$$

$$E = \frac{RB}{2}$$

**(ii) (b)**




$$\Delta L = \int \tau dt$$

$$= Q\left(\frac{R}{2}B\right)R(1)$$

$= \frac{QR^2B}{2}$, in magnitude

$$\Delta\mu = \gamma\Delta L$$

$= -\gamma\frac{BQR^2}{2}$ (taking into account the direction)

**12. (i) (c)**

$${}^6_3Li \to {}^4_2He + {}^2_1H$$

$$\frac{Q}{C^2} = 6.015123 - 4.002603 - 2.014102$$

$$= 0 = -0.001582 < 0$$

So no $\alpha$-decay is possible

$${}^{210}_{84}P_0 \to {}^2_1H + {}^{209}_{83}Bi$$

$$\frac{Q}{C^2} = 209.9828766 - 1.007825 - 208.980388 = -0.005337 < 0$$

So, this reaction is not possible

$${}^2_1H + {}^4_2He \to {}^6_3Li$$

$$\frac{Q}{C^2} = 2.014102 + 4.002603 - 6.015123 = 0.001582 > 0$$

So, this reaction is possible

$${}^{70}_{30}Zn + {}^{82}_{34}Se \to {}^{152}_{64}Gd$$

$$\frac{Q}{C^2} = 69.925325 + 81.916709 - 151.919803 = -0.077769 < 0$$

So this reaction is not possible

**(ii) (a)**

$$^{210}_{84}Po \rightarrow ^{4}_{2}He + ^{206}_{82}Pb$$

Q=(209.982876-4.002603-205.97455)$C^2$

=5.422MeV

from conservation of momentum

$$\sqrt{2K_1(4)} = \sqrt{2K_2(206)}$$

$4K_1=206K_2$

$$\therefore K_1 = \frac{103}{2}K_2$$

$K_1 + K_2 = 5.422$

$$K_1 + \frac{2}{103}K_1 = 5.422$$

$$\Rightarrow \frac{105}{103}K_1 = 5.422$$

$\therefore K_1$ = 5.319 MeV = 5319 KeV

**13. (d)**

P. → (2) ; Q. → (3); R. → (4); S. → (1)

P. $\mu_2 > \mu_1 \ldots$ (towards normal)

$\mu_2 > \mu_3$ … (away from normal)

Q. $\mu_1 = \mu_2 \ldots$ (No change in path)

$\angle i = 0 \Rightarrow \angle r = 0$ on the block.

R. $\mu_1 > \mu_2$ … (Away from the normal)

$\mu_2 > \mu_3$ … (Away from the normal)

$\mu_1 \times \frac{1}{\sqrt{2}} = \mu_2 \sin r \Rightarrow \sin r = \frac{\mu_1}{\sqrt{2}\mu_2}$. Since sin $r < 1 \Rightarrow \mu_1 < \sqrt{2}\mu_2$

S. For TIR : $45^0$ > C ⇒ sin $45^0$ > sin C $\Rightarrow \frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1} \Rightarrow \mu_1 > \sqrt{2}\mu_2$

**14. (c)**

P. → (4) ; Q. → (2); R. → (1); S. → (3)

P. KE = $\frac{3}{2}$ K'T ⇒ $[ML^2T^{-2}]$ = K'[K] ⇒ K' = $[ML^2T^{-2}K^{-1}]$

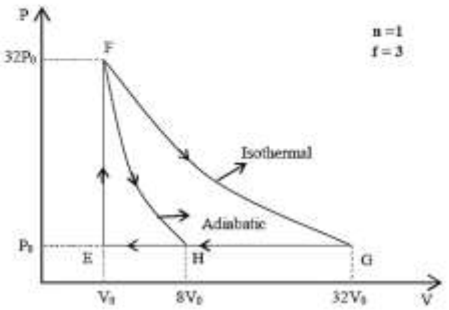
Q. F = $6\pi\eta rv \Rightarrow$ $[MLT^{-2}]$ = $\eta$ [L]$[LT^{-1}]$ $\Rightarrow \eta$ = $[ML^{-1}T^{-1}]$

R. E = hf ⇒ $[ML^2T^{-2}]$ = $\frac{h}{[T]} \Rightarrow$ h = $[ML^2T^{-1}]$

S. $\frac{dQ}{dt} = \frac{K'A(\Delta T)}{\Delta x} \Rightarrow \frac{[ML^2T^{-2}]}{[T]} = \frac{k[L^2][K']}{[L]}$

$K' = [MLT^{-3}K^{-1}]$

**15. (a)**



P. → (4) ; Q. → (3); R. → (2); S. → (1)

Apply $PV^{1+2/3}$ = constant for F to H.

$(32P_0)V_0^{5/3} = P_0V_H^{5/3} \Rightarrow V_H = 8V_0$

For path FG PV = constant

⇒ $(32P_0)V_0 = P_0V_G$ ⇒ V⇒ = $32V_0$

Work done in GE = 31 $P_0V_0$

Work done in GH = 24 $P_0V_0$

Work done in FH = $\dfrac{P_HV_H - P_FV_F}{(-2/f)} = 36P_0V_0$

Work done in FG = RT ln $\left(\dfrac{V_G}{V_F}\right)$

= $160P_0V_0\ln 2$.

**16. (c)**

P. → (2) ; Q. → (1); R. → (4); S. → (3)

${}^{15}_{8}O \rightarrow {}^{15}_{7}N + {}^{0}_{1}\beta$ (Beta decay)

${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$ (Alpha decay)

${}^{185}_{83}Bi \rightarrow {}^{184}_{82}Pb + {}^{1}_{1}H$ (Proton emission)

${}^{239}_{94}Ph \rightarrow {}^{140}_{57}La + {}^{99}_{37}Rb$ (fission)

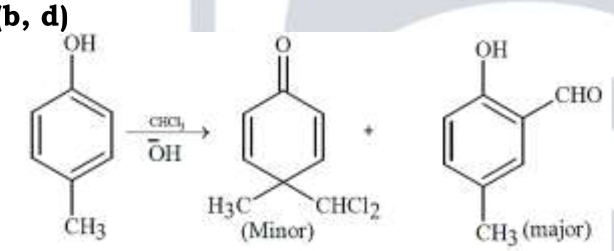
**17. (b)**

$K_{sp} = 1.1\times10^{-12} = \left[Ag^+\right]^2\left[CrO_4^{-2}\right]$

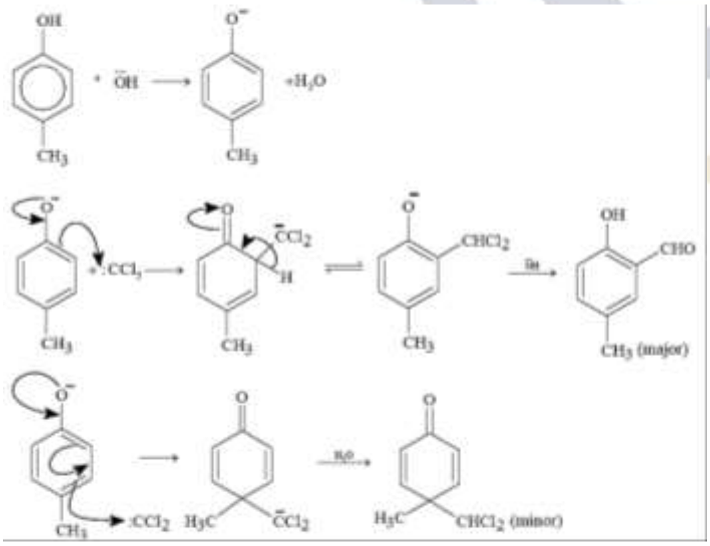
$1.1\times10^{-12} = [0.1]^2[s]$

$s = 1.1\times10^{-10}$

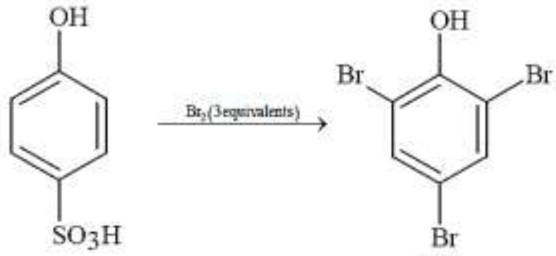
**18. (b, d)**




$CHCl_3 + \overline{O}H \rightarrow :CCl_2 + H_2O + Cl^-$



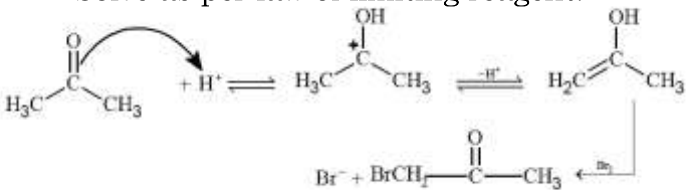
**19. (b)**



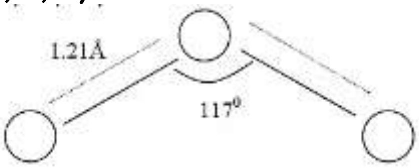

(Q)

**20.** **(c)**

Solve as per law of limiting reagent.



**21.** **(a, c, d)**




**22.** **(a, b)**

$^{9}_{4}Be+\gamma\rightarrow{}^{8}_{4}Be+{}^{1}_{0}n$

$^{9}_{4}Be+{}^{1}_{1}P\rightarrow{}^{8}_{4}Be+{}^{2}_{1}H$

**23.** **(c, d)** $Fe_2O_3$ and $SnO_2$ undergoes C reduction.

**24.** **(a, b, d)**

For the equilibrium $CaCO_3(s)\rightleftharpoons CaO(S)+CO_2(g)$.

The equilibrium constant (K) is independent of initial amount of $CaCO_3$ where as at a given temperature is independent of pressure of $CO_2$. ΔH is independent of catalyst and it depends on temperature.

**25.** **(i) (a)**

**(ii) (d)**

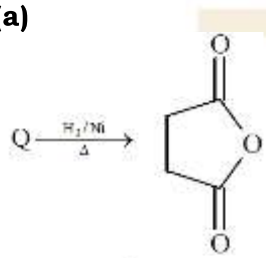
Solution for the Q. No. (i) to (ii)

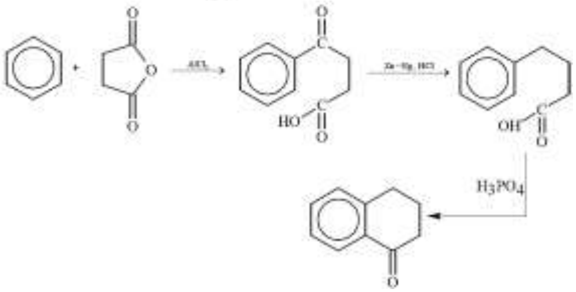
$Pb^{+2}+2HCl\rightarrow PbCl_2\xrightarrow[Water]{Hot}$ soluble.

$Cr^{+3}\xrightarrow[\text{ammoniacal}]{H_2S}Cr(OH)_3\downarrow$

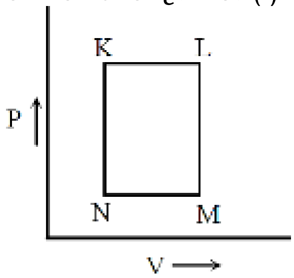
$Cr(OH)_3\xrightarrow[H_2O_2]{NaOH}\underset{\text{Yellow solution}}{Na_2CrO_4}$

**26.** **(a)**



**27. (i) (c)**
**(ii) (b)**
Solution for the Q. No. (i) to (ii)




K – L heating, isobaric
L – M cooling, isochoric
M – N cooling, isobaric
N – K heating, isochoric

**28. (i) (a)**
**(ii) (a)**
Solution for the Q. No. (i) to (ii)

$$\underset{(Cold+dil)}{2NaOH}+Cl_2 \rightarrow NaCl+\underset{P}{NaClO}+H_2O$$

$$\underset{(Hot+conc.)}{6NaOH}+3Cl_2 \rightarrow 5NaCl+\underset{Q}{NaClO_3}+3H_2O$$

$$SO_2+Cl_2 \xrightarrow{Charcoal} \underset{R}{SO_2Cl_2}$$

$$SO_2Cl_2+P_4 \rightarrow \underset{S}{PCl_5}+SO_2$$

$$PCl_5+H_2O \rightarrow \underset{T}{H_3PO_4}+HCl$$

**29. (d)**

(P) $PbO_2+H_2SO_4 \xrightarrow{\Delta} PbSO_4+H_2O+\frac{1}{2}O_2$

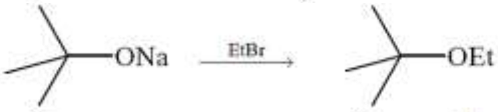
(Q) $2Na_2S_2O_3+Cl_2+2H_2O \rightarrow 2NaCl+2NaHSO_4+2S$

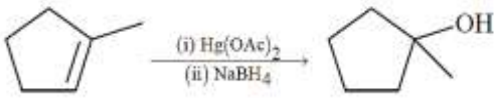
(R) $N_2H_4+2I_2 \rightarrow N_2+4HI$

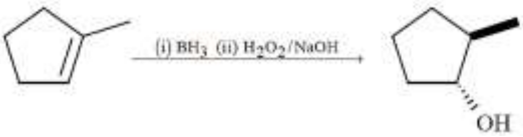
(S) $XeF_2+2NO \rightarrow Xe+2NOF$

**30. (a)**

(P) 


(Q) 


(R) 


(S) 


**31. (a)**

(P) $\underset{X}{(C_2H_5)_3N}+\underset{Y}{CH_3COOH} \rightarrow (C_2H_5)_3NH^+CH_3COO^-$

Initially conductivity increases due to ion formation after that it becomes practically constant because X alone can not form ions. Hence (3) is the correct match.

(Q) $\underset{X}{KI(0.1M)}+\underset{Y}{AgNO_3(0.01M)} \rightarrow AgI\downarrow+KNO_3$

Number of ions in the solution remains constant until all the $AgNO_3$ precipitated as AgI. Thereafter conductance increases due to increases in number of ions. Hence (4) is the correct match.

(R) Initially conductance decreases due to the decrease in the number of $\overline{O}H$ ions thereafter it slowly increases due to the increases in number of $H^+$ ions. Hence (2) is the correct match.

(S) Initially it decreases due to decrease in $H^+$ ions and then increases due to the increases in OH ions.

**32. (d)**

(P) $\Delta G^0_{Fe^{3+}/Fe} = \Delta G^0_{Fe^{3+}/Fe^{2+}} + \Delta G^0_{Fe^{2+}/Fe}$

$\Rightarrow -3\times FE^0_{\left(Fe^{+3}/Fe\right)} = -1\times FE^0_{\left(Fe^{+3}/Fe^{2+}\right)} + \left(-2\times FE^0_{Fe^{2+}/Fe}\right)$

$\Rightarrow E^0_{Fe^{+3}/Fe} = -0.04V$

(Q) $O_2(g) + 2H_2O + 4e^- \rightarrow 4\overline{O}H \qquad E^0 = 0.40V \quad \ldots$ (i)

$2H_2O \rightarrow O_2(g) + 4H^+ + 4e^- \qquad E^0 = -1.23V$ .. (ii)

So $4H_2O \rightleftharpoons 4H^+ + 4\overline{O}H$ ... (iii)

$E^0$ for IIIrd reduction = 0.40 – 1.23 = - 0.83 V.

(R) $\Delta G^0_{\left(Cu^{+2}/Cu\right)} = \Delta G^0_{\left(Cu^{+2}/Cu^{+}\right)} + \Delta G^0_{\left(Cu^{+}/Cu\right)}$

$-2\times FE^0_{Cu^{+2}/Cu} = -1\times FE^0_{Cu^{+2}/Cu^{+}} + \left(-1\times F\times E^0_{Cu^{+}/Cu}\right)$

$\Rightarrow E^0_{Cu^{+2}/Cu} = -0.18V$

(S) $\Delta G^0_{Cr^{+3}/Cr^{+2}} = \Delta G^0_{Cr^{+3}/Cr} + \Delta G^0_{Cr/Cr^{+2}}$

$-1\times F\times E^0_{Cr^{+3}/Cr^{+2}} = -3\times F\times E^0_{Cr^{+3}/Cr} + \left(-2\times F\times E^0_{Cr/Cr^{+2}}\right)$

$\Rightarrow E^0_{Cr^{+3}/Cr^{+2}} = -0.4V$

**33. (b, d)**

Required limit = $= \dfrac{\int_0^1 x^a dx}{\int_0^1 (a+x)dx} = \dfrac{2}{(2a+1)(a+1)} = \dfrac{2}{120}$

$\Rightarrow a = 7$ or $-\dfrac{17}{2}$

**34. (a), (c)**

Equation of circle can be written as

$(x - 3)^2 + y^2 + \lambda (y) = 0$

$\Rightarrow x^2 + y^2 - 6x + \lambda y + 9 = 0.$

Now, $(\text{radius})^2 = 7 + 9 = 16$

$\Rightarrow 9 + \dfrac{\lambda^2}{4} - 9 = 16$

$\Rightarrow \lambda^2 = 64 \Rightarrow \lambda = \pm 8$

$\therefore$ Equation is $x^2 + y^2 - 6x \pm 8y + 9 = 0.$

**35. (a, d)**

$\dfrac{x-5}{0} = \dfrac{y-0}{3-\alpha} = \dfrac{z-0}{-2}$

$\dfrac{x-\alpha}{0} = \dfrac{y}{-1} = \dfrac{z}{2-\alpha}$

will be coplanar if shortest distance is zero

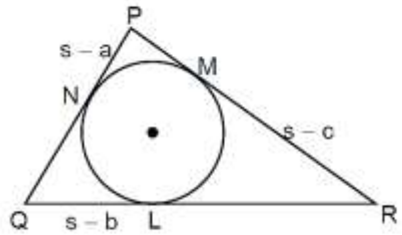
$(5-\alpha)(\alpha^2 - 5\alpha + 4) = 0, \alpha = 1,4,5$

so $\alpha$ = 1, 4

Alternate Solution:
As x = 5 and x = $\alpha$ are parallel planes so the remaining two planes must be coplanar.

So, $\frac{3-\alpha}{-1} = \frac{-2}{2-\alpha} \Rightarrow \alpha^2 - 5\alpha + 4 = 0 \Rightarrow \alpha = 1, 4$

**36.** **(b), (d)**

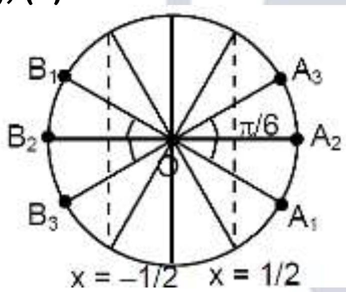



Let
s - a = 2k - 2, s - b = 2k, s - c = 2k + 2, k $\in$ I, k > 1
Adding we get,
s = 6k
So, a = 4k + 2, b = 4k, c = 4k - 2

Now, cos P = $\frac{1}{3}$

$\Rightarrow \frac{b^2+c^2-a^2}{2bc} = \frac{1}{3} \Rightarrow$ 3 [$(4k)^2 + (4k-2)^2 - (4k+2)^2$] = 2 x 4k (4k - 2)
$\Rightarrow$ 3 [$16k^2$ - 4 (4k) x 2] = 8k (4k - 2)
$\Rightarrow$ $48k^2 - 96k = 32k^2 - 16k$
$\Rightarrow$ $16k^2 = 80k \Rightarrow k = 5$
So, sides are 22, 20, 18

**37.** **(c), (d)**




$w = \frac{\sqrt{3}+i}{2} = e^{\frac{i\pi}{6}}$, so $w^n = e^{i\left(\frac{n\pi}{6}\right)}$

Now, for $z_1$, $\cos\frac{n\pi}{6} > \frac{1}{2}$ and for $z_2$, $\cos\frac{n\pi}{6} < -\frac{1}{2}$

Possible position of $z_1$ are $A_1, A_2, A_3$ whereas of $z_2$ are $B_1, B_2, B_3$ (as shown in the figure)

So, possible value of $\angle z_1Oz_2$ according to the given options is $\frac{2\pi}{3}$ or $\frac{5\pi}{6}$

**38.** **(a, b, c)**

$\log 2^{3x} = (x-1)\log_2 4 = 2(x-1)$
$\Rightarrow x \log_2 3 = 2x - 2$

$\Rightarrow x = \frac{2}{2-\log_2 3}$

Rearranging, we get

$x = \frac{2}{2-\frac{1}{\log_3 2}} = \frac{2\log_3 2}{2\log_3 2 - 1}$

Rearranging again,

$$x = \frac{\log_3 4}{\log_3 4 - 1} = \frac{\frac{1}{\log_4 3}}{\frac{1}{\log_4 3} - 1} = \frac{1}{1 - \log_4 3}$$

**39. (a, c, d)**

$$P = \begin{bmatrix} \omega^2 & \omega^3 & \omega^4 \ldots\ldots & \omega^{n+2} \\ \omega^3 & \omega^4 & \omega^5 \ldots\ldots & \omega^{n+3} \\ \vdots & \vdots & \vdots \quad \vdots & \vdots \\ \omega^{n+2} & \omega^{n+3} & \ldots\ldots\ \ldots\ldots & \omega^{2n+4} \end{bmatrix}$$

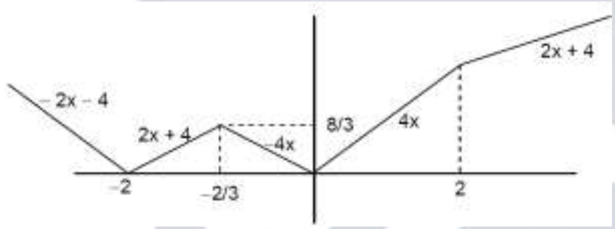
$$P^2 = \begin{bmatrix} \omega^4 + \omega^6 \ldots & \omega^5 + \omega^7 + \omega^9 & \ldots. & \ldots. \\ \omega^5 + \omega^7 + \omega^9 \ldots & \ldots & \ldots. & \ldots. \\ \vdots & \vdots & \vdots & \ldots.. \\ \omega^{n+4} + \omega^{n+6} \ldots. & \ldots.. & \ldots. & \omega^{2n+4} + \omega^{2n+6} \ldots. \end{bmatrix}$$

$P^2$ = Null matrix if n is a multiple of 3

**40. (a), (b)**

As, $\frac{f(x)+g(x)-|f(x)-g(x)|}{2} = Min(f(x), g(x))$

$\Rightarrow \frac{2|x|+|x+2|-||x+2|-2|x||}{2} = Min(|2x|, |x+2|)$




According to the figure shown, points of local minima/maxima are x = -2, $\frac{-2}{3}$, 0

**41. (i) (d)**

Let g (x) = $e^{-x}$ f (x)

and g”(x) > 1 > 0

So, g (x) is concave upward and g (0) = g (1) = 0

Hence, g (x) < 0 $\forall$ x $\in$ (0, 1)

$\Rightarrow$ $e^{-x}$ f (x) < 0

f (x) < 0 $\forall$ x $\in$ (0, 1)

Alternate Solution

f"(x) – 2f’(x) + f (x) $\geq$ $e^x$

$\Rightarrow \left(f(x)e^{-x} - \frac{x^2}{2}\right)'' \geq 0$

Let g (x) = f (x) $e^{-x} - \frac{x^2}{2}$

g (0) = 0, g (1) = $\frac{1}{2}$

Since g is concave up so it will always lie below the chord joining the extremities which is y = $-\frac{x}{2}$

$\Rightarrow f(x)e^{-x} - \frac{x^2}{2} < -\frac{x}{2}$

$$\Rightarrow f(x) < \frac{(x^2 - x)e^x}{2} < 0 \forall x \in (0,1)$$

**(ii) (c)**

Let, g (x) = $e^{-x}$ f (x)
As g"(x) > 0 so g' (x) is increasing.
So, for x < 1/4, g'(x) < g'(1/4) = 0
⇒ (f' (x) – f (x))$e^{-x}$ < 0
⇒ f' (x) < f (x) in (0, 1/4).

**42. (b)**

Let P($at^2$ , 2at) , $Q\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$ as PQ is focal chord

Point of intersection of tangents at P and Q

$$\left(-a, a\left(t - \frac{1}{t}\right)\right)$$

as point of intersection lies on y = 2x + a

$$\Rightarrow a\left(t - \frac{1}{t}\right) = -2a + a$$

$$t - \frac{1}{t} = -1 \Rightarrow \left(t + \frac{1}{t}\right)^2 = 5$$

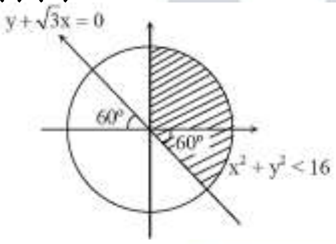
Length of focal chord = $a\left(t + \frac{1}{t}\right)^2 = 5a$

**(ii) (d)**

Angle made by chord PQ at vertex (0, 0) is given by

$$\tan\theta = \left(\frac{\frac{2}{t} + 2t}{1 - 4}\right) = \frac{2\left(\frac{1}{t} + t\right)}{-3} = \frac{-2}{3}\sqrt{5}$$

**43. (i) (b)**




Area of region $S_1 \cap S_2 \cap S_3$ = shaded area

$$= \frac{\pi \times 4^2}{4} + \frac{4^2 \times \pi}{6}$$

$$= 4^2 \pi \left\{\frac{1}{4} + \frac{1}{6}\right\}$$

$$= \frac{20\pi}{3}$$

**(ii) (c)**

Distance of (1, –3) from $y + \sqrt{3}x = 0$

$$> \left|\frac{-3 + \sqrt{3} \times 1}{2}\right|$$

$$> \frac{3 - \sqrt{3}}{2}$$

**44. (d)**

Let A: one ball is white and other is red

$E_1$: both balls are from box $B_1$

$E_2$: both balls are from box $B_2$

$E_3$: both balls are from box $B_3$

Here, P (required) = $P\left(\frac{E_2}{A}\right)$

$$=\frac{P\left(\frac{A}{E_2}\right).P(E_2)}{P\left(\frac{A}{E_1}\right).P(E_1)+P\left(\frac{A}{E_2}\right).P(E_2)+P\left(\frac{A}{E_3}\right).P(E_3)}$$

$$=\frac{\frac{{}^2C_1\times{}^3C_1}{{}^9C_2}\times\frac{1}{3}}{\frac{{}^1C_1\times{}^3C_1}{{}^6C_2}\times\frac{1}{3}+\frac{{}^2C_1\times{}^3C_1}{{}^9C_2}\times\frac{1}{3}+\frac{{}^3C_1\times{}^4C_1}{{}^{12}C_2}\times\frac{1}{3}}=\frac{\frac{1}{6}}{\frac{1}{5}+\frac{1}{6}+\frac{2}{11}}=\frac{55}{181}$$

**45. (b)**

$$P\to\frac{\cos\left(\tan^{-1}y\right)+y\sin\left(\tan^{-1}y\right)}{\cot\left(\sin^{-1}y\right)+\tan\left(\sin^{-1}y\right)}$$

$$=\frac{\frac{1}{\sqrt{1+y^2}}+\frac{y^2}{\sqrt{1+y^2}}}{\frac{\sqrt{1-y^2}}{y}+\frac{y}{\sqrt{1-y^2}}}=\frac{\sqrt{1+y^2}}{\frac{1}{y\sqrt{1-y^2}}}=y\sqrt{1-y^4}$$

$$\Rightarrow\frac{1}{y^2}\left(\frac{\cos\left(\tan^{-1}y\right)+y\sin\left(\tan^{-1}y\right)}{\cot\left(\sin^{-1}y\right)+\tan\left(\sin^{-1}y\right)}\right)+y^4$$

$$=\frac{1}{y^2}\left(y^2\left(1-y^4\right)\right)+y^4=1-y^4+y^4=1$$

Q $\to$ cos x + cos y + cos z = 0

sin x + sin y + sin z = 0

cos x + cos y = – cos z ..... (1)

sin x + sin y = – sin z ..... (2)

$(1)^2+(2)^2$

1 + 1 + 2(cos x cos y + sin x sin y) = 1

2 + 2 cos (x – y) = 1

2 cos (x – y) = –1

cos (x – y) = $-\frac{1}{2}$

$$2\cos^2\left(\frac{x-y}{2}\right)-1=-\frac{1}{2}$$

$$2\cos^2\left(\frac{x-y}{2}\right)=\frac{1}{2}$$

$$\cos^2\left(\frac{x-y}{2}\right)=\frac{1}{4}$$

$$\cos\left(\frac{x-y}{2}\right)=\frac{1}{2}$$

$$R\to\cos\left(\frac{\pi}{4}-x\right)\cos 2x+\sin x\sin 2x\sec x$$

$$= \cos x \sin 2x \sec x + \cos\left(\frac{\pi}{4} + x\right)\cos 2x$$

$$\left[\cos\left(\frac{\pi}{4} - 4\right) - \cos\left(\frac{\pi}{4} + x\right)\right]\cos 2x = (\cos x \sin 2x - \sin x \sin 2x)\sec x$$

$$\frac{2}{\sqrt{2}}\sin x \cos 2x = (\cos x - \sin x)\sin 2x \sec x$$

$$\sqrt{2}\sin x \cos 2x = (\cos x - \sin x)2\sin x$$

$$\sec x = \sec\frac{\pi}{4} = \sqrt{2}$$

$$S \to \cot\left(\sin^{-1}\sqrt{1-x^2}\right)$$

$$\cot\alpha = \frac{x}{\sqrt{1-x^2}}$$

$$\tan^{-1}\left(x\sqrt{6}\right) = \phi$$

$$\sin\phi = \frac{x\sqrt{6}}{\sqrt{6x^2+1}}$$

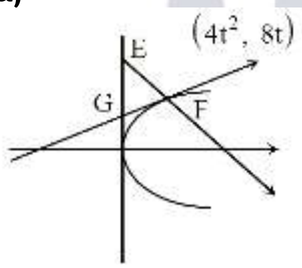
$$\Rightarrow \frac{x}{\sqrt{1-x^2}} = \frac{x\sqrt{6}}{\sqrt{6x^2+1}}$$

$$6x^2 + 1 = 6 - 6x^2$$

$$12x^2 = 5$$

$$x = \sqrt{\frac{5}{12}} = \frac{1}{2}\sqrt{\frac{5}{3}}$$

**46. (a)**




A(t) = $2t^2(3 - 4t)$

For max. A(t), t = $\frac{1}{2}$

$\Rightarrow$ m = 1

$\Rightarrow A(t)\big|_{\max.} = \frac{1}{2}$ sq. units

$y_0 = 4$ and $y_1 = 2$

**47. (c)**

$$P \to \left[\vec{a}\ \vec{b}\ \vec{c}\right] = 2$$

$$\left[2\vec{a}\times\vec{b}\ \ 3\vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right] = 6\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2 = 6\times 4 = 24$$

$$Q \to \left[\vec{a}\ \vec{b}\ \vec{c}\right] = 5$$

$$6\left[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}\right] = 12\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 60$$

$$R \to \frac{1}{2}\left|\vec{a}\times\vec{b}\right| = 20$$

$\frac{1}{2}\left|\left(2\vec{a}+3\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)\right|$

$\frac{1}{2}\left|-2\left(\vec{a}\times\vec{b}\right)-\left(\vec{a}\times\vec{b}\right)\right|$

$\frac{5}{2}\times 40=100$

$S\rightarrow\left|\vec{a}\times\vec{b}\right|=30$

$\Rightarrow\left|\left(\vec{a}+\vec{b}\right)\times\vec{a}\right|=\left|\vec{b}\times\vec{a}\right|=30$

**48.** **(a)**

Plane perpendicular to P1 and P2 has Direction Ratios of normal

$\begin{vmatrix}\hat{i} & j & k\\ 7 & 1 & 2\\ 3 & 5 & -6\end{vmatrix}=-16\hat{i}+48j+32k\ldots(1)$

For point of intersection of lines

$\left(2\lambda_1+1,-\lambda_1,\lambda_1-3\right)\equiv\left(\lambda_2+4,\lambda_2-3,2\lambda_2-3\right)$

$\Rightarrow 2\lambda_1+1=\lambda_2+4$ or $2\lambda_1-\lambda_2=3$

$-\lambda_1=\lambda_2-3$ or $\lambda_1+\lambda_2=3$

$\Rightarrow\lambda_1=2,\lambda_2=1$

$\therefore$ Point is (5, -2, -1) … (2)

From (1) and (2), required plane is

-1 (x - 5) + 3 (y + 2) + 2 (z + 1) = 0

or -x + 3y + 2z = -13

x - 3y - 2z = 13

$\Rightarrow$ a = 1, b = - 3, c = - 2, d = 13.