

$$1. (d) \quad l = \frac{1}{4\nu} \sqrt{\frac{\gamma RT}{M}}$$

Calculations for $l = \frac{1}{4\nu} \sqrt{\frac{\gamma RT}{M}}$ for gases mentioned in options A, B, C and D, work out to be 0.459 m, 0.363 m, 0.340 m & 0.348 m respectively. As $l = (0.350 \pm 0.005) \text{ m}$; Hence correct option is D.

2. (c) As current leads voltage by $\pi/2$ in the given circuit initially, then ac voltage can be represented as

$$V = V_0 \sin \omega t$$

$$\therefore q = CV_0 \sin \omega t = Q \sin \omega t$$

$$\text{where, } Q = 2 \times 10^{-3} \text{ C}$$

$$\text{At } t = 7\pi/6\omega; I = -\frac{\sqrt{3}}{2} I_0 \text{ and hence current is anticlockwise.}$$

$$\text{Current 'i' immediately after } t = \frac{7\pi}{6\omega} \text{ is}$$

$$i = \frac{V_c + 50}{R} = 10 \text{ A}$$

$$\text{Charge flow} = Q_{\text{final}} - Q_{(7\pi/6\omega)} = 2 \times 10^{-6}$$

3. (a) As $E = V/d$

$$E_1/E_2 = 1 \text{ (both parts have common potential difference)}$$

Assume C_0 be the capacitance without dielectric for whole capacitor.

$$k \frac{C_0}{3} + \frac{2C_0}{3} = C$$

$$\frac{C}{C_1} = \frac{2+k}{k}$$

$$\frac{Q_1}{Q_2} = \frac{k}{2}$$

4. (a, c, d) Taking $y(t) = A f(x) g(t)$ & Applying the conditions:

1; here $x = 3\text{m}$ is antinode & $x = 0$ is node

2; possible frequencies are odd multiple of fundamental frequency. where,

$$\nu_{\text{fundamental}} = \frac{v}{4l} = \frac{25}{3} \text{ Hz}$$

5. (a, c) For air to glass

$$\frac{1.5}{f_1} = \frac{1.4-1}{R} + \frac{1.5-1.4}{R}$$

$$\therefore f_1 = 3R$$

For glass to air.

$$\frac{1}{f_2} = \frac{1.4-1.5}{-R} + \frac{1-1.4}{-R}$$

$$\therefore f_2 = 2R$$

$$H = \frac{V^2}{R} \cdot 4 = \frac{V^2}{R/2} t_1 = \frac{V^2}{R/8} t_2$$

6. (b, d)

$$t_1 = 2 \text{ min}$$

$$t_2 = 0.5 \text{ min}$$

$$V_1 = \frac{R_1(V_1 + V_2)}{R_1 + R_3} \Rightarrow V_1 R_3 = V_2 R_1$$

7. (a, b, d)

$$V_2 = \frac{R_3(V_1 + V_2)}{R_1 + R_3} \Rightarrow V_2 R_1 = V_2 R_3$$

$$8. (c) \frac{Q}{4\pi\epsilon_0 r_0^2} = \frac{\lambda}{2\pi\epsilon_0 r_0} = \frac{\sigma}{2\epsilon_0}$$

$$E_1\left(\frac{r_0}{2}\right) = \frac{Q}{\pi\epsilon_0 r_0^2}, E_2\left(\frac{r_0}{2}\right) = \frac{\lambda}{\pi\epsilon_0 r_0}, E_3\left(\frac{r_0}{2}\right) = \frac{\sigma}{2\epsilon_0}$$

$$\therefore E_1\left(\frac{r_0}{2}\right) = 2E_2\left(\frac{r_0}{2}\right)$$

$$9. (a, b, c) \quad \beta = \frac{D\lambda}{d}$$

$$\because \lambda_2 > \lambda_1 \Rightarrow \beta_2 > \beta_1$$

$$\text{Also } m_1\beta_1 = m_2\beta_2 \Rightarrow m_1 > m_2$$

$$\text{Also } 3\left(\frac{D}{d}\right)(600nm) = (2 \times 5 - 1)\left(\frac{D}{2d}\right)400nm$$

$$\text{Angular width } \theta = \frac{\lambda}{d}$$

10. (c, d) Condition of translational equilibrium

$$N_1 = \mu_2 N_2$$

$$N_2 + \mu_1 N_1 = Mg$$

$$\text{Solving } N_2 = \frac{mg}{1 + \mu_1\mu_2}$$

$$N_1 = \frac{\mu_2 mg}{1 + \mu_1\mu_2}$$

Applying torque equation about corner (left) point on the floor

$$mg \frac{l}{2} \cos \theta = N_1 l \sin \theta + \mu_1 N_1 l \cos \theta$$

$$\text{Solving } \tan \theta = \frac{1 - \mu_1\mu_2}{2\mu_2}$$

11. (4) $Y = \frac{FL}{lA}$ since the experiment measures only change in the length of wire

$$\therefore \frac{\Delta Y}{Y} \times 100 = \frac{\Delta l}{l} \times 100$$

$$\text{From the observation } l_1 = \text{MSR} + 20 \text{ (LC)}$$

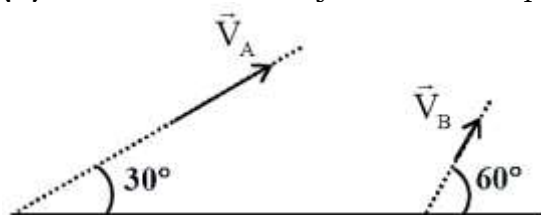
$$l_2 = \text{MSR} + 45 \text{ (LC)}$$

$$\Rightarrow \text{change in lengths} = 25 \text{ (LC)}$$

and the maximum permissible error in elongation is one LC

$$\therefore \frac{\Delta Y}{Y} \times 100 = \frac{(LC)}{25(LC)} \times 100 = 4$$

12. (5) The relative velocity of B with respect to A is perpendicular to line of motion of A.



$$\therefore V_B \cos 30^\circ = V_A$$

$$\Rightarrow V_B = 200 \text{ m/s}$$

And time $t_0 = (\text{Relative distance}) / (\text{Relative velocity})$

$$= \frac{500}{V_B \sin 30^\circ} = 5 \text{ sec}$$

13. (2) $U_b = 200 \text{ J}$, $U_i = 100 \text{ J}$
Process iaf

Process	W(in Joule)	ΔU (in Joule)	Q(in Joule)
ia		0	
af		200	
Net	300	200	500

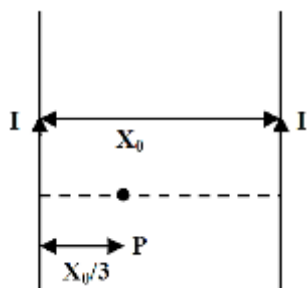
$$\Rightarrow U_f = 400 \text{ Joule}$$

Process ibf

Process	W(in Joule)	ΔU (in Joule)	Q(in Joule)
ib	100	50	150
bf	200	100	300
Net	300	150	450

$$\Rightarrow \frac{Q_{bf}}{Q_{ib}} = \frac{300}{150} = 2$$

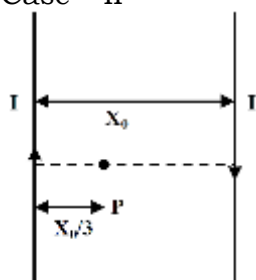
14. (3) Case - I



$$B_1 = \frac{1}{2} \left(\frac{\mu_0}{2\pi} \right) \left(\frac{3I}{x_0} \right)$$

$$R_1 = \frac{mv}{qB_1}$$

Case - II



$$R_2 = \frac{mv}{qB_2}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{1/3}{1/9} = 3$$

15. (3) $d \propto \rho^x S^y F^z$

$$\Rightarrow [L] = [ML^{-3}]^x [ML^{-3}]^y [T^{-1}]^z$$

$$\Rightarrow x + y = 0, -3x = 1, -3y - z = 0$$

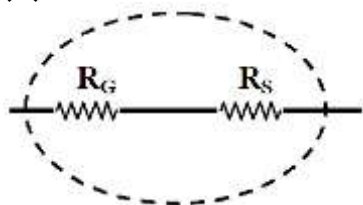
$$\Rightarrow x = -\frac{1}{3}, y = \frac{1}{3}, z = -1$$

$$\Rightarrow y = \frac{1}{n}$$

$$\Rightarrow n=3$$

16. (2) Maximum displacement of the left ball from the left wall of the chamber is 2.25 cm, so the right ball has to travel almost the whole length of the chamber (4m) to hit the left ball. So the time taken by the right ball is 1.9 sec (approximately 2 sec)

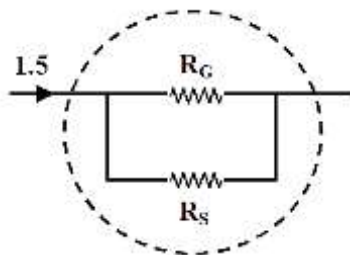
17. (5)



$$i = \frac{V}{R}$$

$$0.006 = \frac{30}{4990 + R}$$

$$R=10$$



$$i_{RG} = 0.006$$

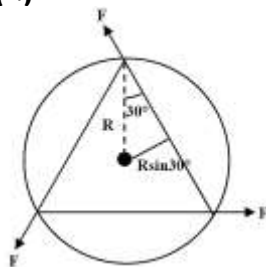
$$i_{RS} = 1.494$$

Since R_G and R_S are in parallel, $i_G R_G = i_S R_S$

$$0.006R = 1.494 \left(\frac{2n}{249} \right)$$

$$\therefore n = 5$$

18. (2) $\tau = I\alpha$



$$3 FR \sin 30^\circ = I \alpha$$

$$I = \frac{MR^2}{2}$$

$$\alpha = 2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega = 2 \text{ rad/s}$$

19. (4) Since net torque about center of rotation is zero, so we can apply conservation of angular momentum of the system about center of disc

$$L_i = L_f$$

$$0 = I \omega + 2mv(r/2); \text{ comparing magnitude}$$

$$\therefore \left(\frac{0.45 \times 0.5 \times 0.5}{2} \right) \omega = 0.05 \times 9 \times \frac{0.5}{2} \times 2$$

$$\therefore \omega = 4$$

20. (5) Using work energy theorem

$$W_{mg} + W_F = \Delta KE$$

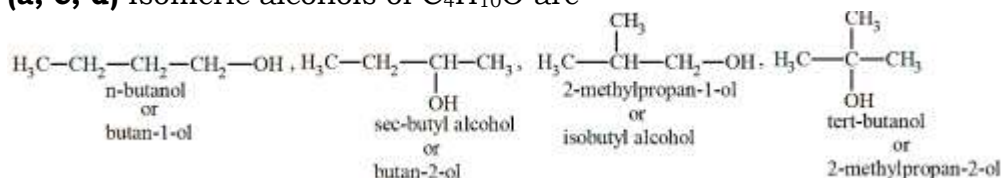
$$-mgh + Fd = \Delta KE$$

$$-1 \times 10 \times 4 + 18(5) = \Delta KE$$

$$\Delta KE = 50$$

$$\therefore n = 5$$

21. (a, c, d) Isomeric alcohols of $C_4H_{10}O$ are



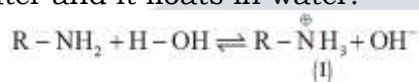
22. (a, b, c) Since container is thermally insulated. So, $q = 0$, and it is a case of free expansion therefore $W = 0$ and $\Delta E = 0$

$$\text{So, } T_1 = T_2$$

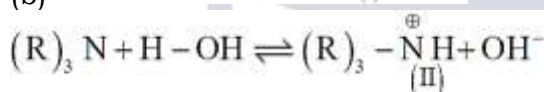
$$\text{Also, } P_1 V_1 = P_2 V_2$$

23. (a, b, d)

(a) Ice has cage-like structure in which each water molecule is surrounded by four other water molecules tetrahedrally through hydrogen bonding, due to this density of ice is less than water and it floats in water.



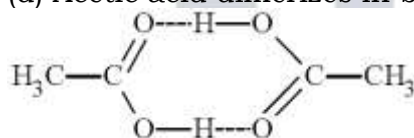
(b)



The cation (I) more stabilized through hydrogen bonding than cation (II). So, $\text{R}-\text{NH}_2$ is better base than $(\text{R})_3\text{N}$ in aqueous solution.

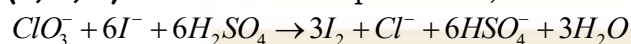
(c) HCOOH is stronger acid than CH_3COOH due to inductive effect and not due to hydrogen bonding.

(d) Acetic acid dimerizes in benzene through intermolecular hydrogen bonding.

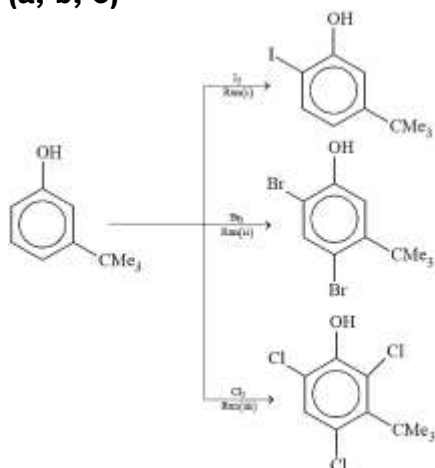


24. (a)

25. (a, b, d) The balanced equation is,



26. (a, b, c)

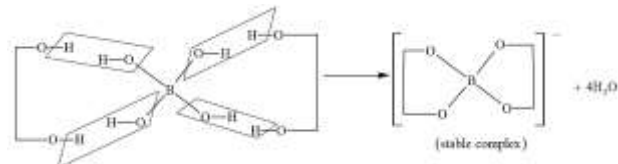


27. (b, d)

(a) H_3BO_3 is a weak monobasic Lewis acid.



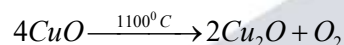
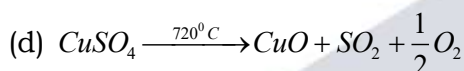
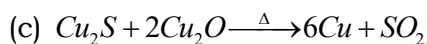
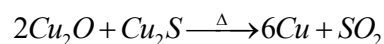
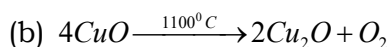
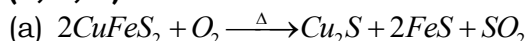
(b) Equilibrium (i) is shifted in forward direction by the addition of syn-diols like ethylene glycol which forms a stable complex with $B(OH)_4^-$



(c) It has a planar sheet like structure due to hydrogen bonding.

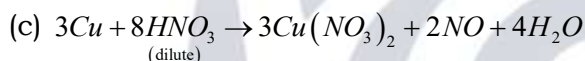
(d) H_3BO_3 is a weak electrolyte in water.

28. (b, c, d)



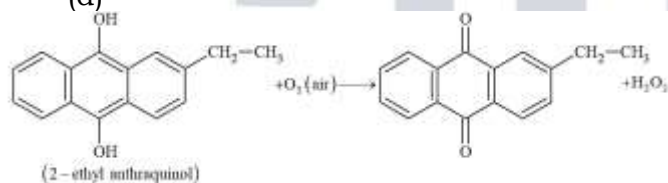
29. (a, b, c)

(a) sodium (Na) when dissolved in excess liquid ammonia, forms a blue coloured paramagnetic solution.



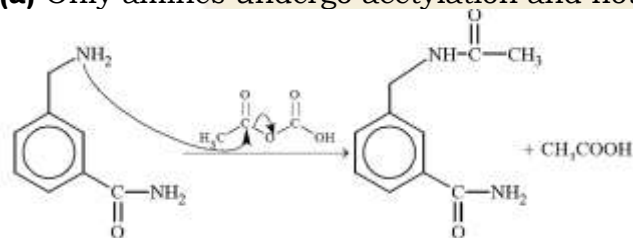
Where "NO" is paramagnetic.

(d)



Where " H_2O_2 " is diamagnetic.

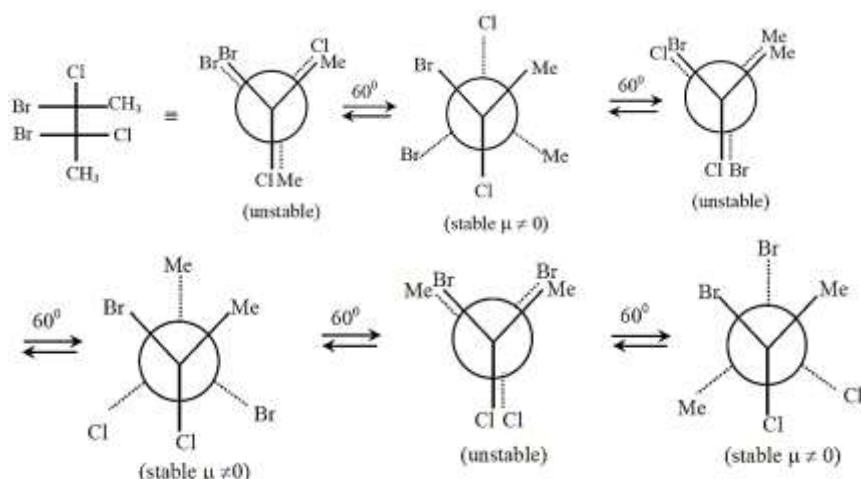
30. (a) Only amines undergo acetylation and not acid amides.



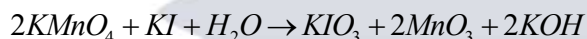
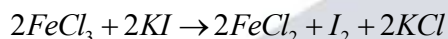
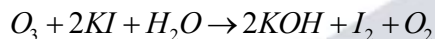
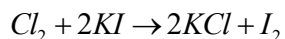
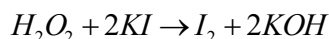
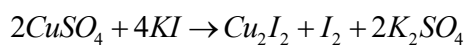
31. (6) Black coloured sulphides { PbS , CuS , HgS , Ag_2S , NiS , CoS }

* Bi_2S_3 in its crystalline form is dark brown but Bi_2S_3 precipitate obtained is black in colour.

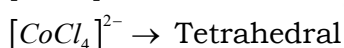
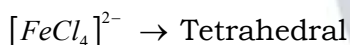
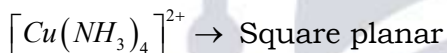
32. (3)



33. (7) $K_2Cr_2O_7$, $CuSO_4$, H_2O_2 , Cl_2 , O_3 , $FeCl_3$, HNO_3
 $K_2Cr_2O_7 + 7H_2SO_4 + 6KI \rightarrow 4K_2SO_4 + Cr_2(SO_4)_3 + 3I_2 + 7H_2O$

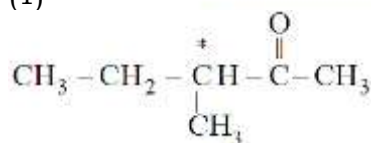


34. (4) $XeF_4 \rightarrow$ Square planar

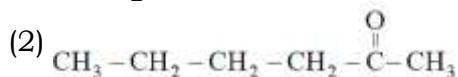


35. (5)

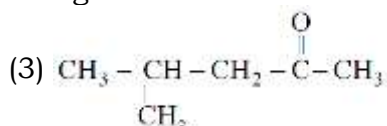
(1)



Will not give a racemic mixture on reduction with $NaBH_4$

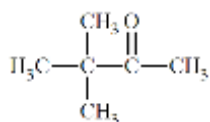


Will give a racemic mixture on reduction with $NaBH_4$

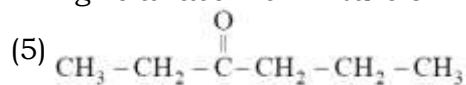


Will give a racemic mixture on reduction with $NaBH_4$

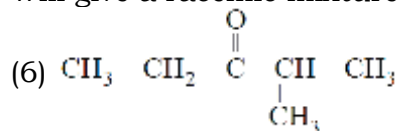
(4)



Will give a racemic mixture on reduction with NaBH_4



Will give a racemic mixture on reduction with NaBH_4



Will give a racemic mixture on reduction with NaBH_4

36. (3) $n = 4$
 $l = 0, 1, 2, 3$
 $|m_l| = 1 \Rightarrow \pm 1$

$$m_s = -\frac{1}{2}$$

For $l = 0, m_l = 0$

$l = 1, m_l = -1, 0, +1$

$l = 2, m_l = -2, -1, 0, +1, +2$

$l = 3, m_l = -3, -2, -1, 0, +1, +2, +3$

So, six electrons can have $|m_l| = 1$ & $m_s = -\frac{1}{2}$

37. (4)

$$k = \frac{R}{N_A}$$

$$R = kN_A$$

$$= 1.380 \times 10^{-23} \times 6.023 \times 10^{23}$$

$$= 8.31174$$

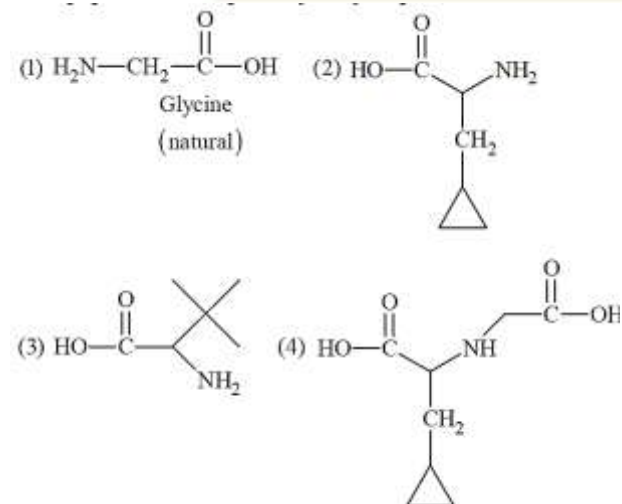
$$\approx 8.312$$

38. (2) $\text{MX}_2 \rightleftharpoons \text{M}^{2+} + 2\text{X}^-$

$$\begin{array}{ccc} 1-\alpha & \alpha & 2\alpha \\ i=1+2\alpha & & \{\alpha=0.5\} \end{array}$$

$$i=2$$

39. (1) This peptide on complete hydrolysis produced 4 distinct amino acids which are given below:



Only glycine is naturally occurring amino acid.

40. (8) Here, $V_{\text{solution}} \approx V_{\text{solvent}}$

Since, in 1 l solution, 3.2 moles of solute are present,

So, 1 l solution $\approx$ 1 l solvent ($d = 0.4\text{g/ml}$) $\approx$ 0.4 kg

$$\text{So, molality (m)} = \frac{\text{moles of solute}}{\text{mass of solvent (kg)}} = \frac{3.2}{0.4} = 8$$

41. (a, c, d) $f'(x) = \frac{2e^{-\left(x+\frac{1}{x}\right)}}{x}$

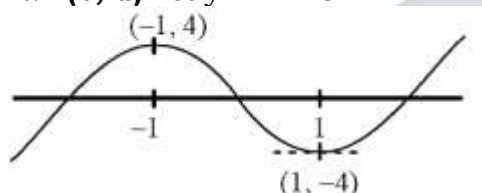
Which is increasing in $[1, \infty)$

$$\text{Also, } f(x) + f\left(\frac{1}{x}\right) = 0$$

$$g(x) = f(2^x) = \int_{2^{-x}}^{2^x} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt$$

$$g(-x) = f(2^{-x}) = \int_{2^x}^{2^{-x}} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt = -g(x)$$

42. (b, d) Let $y = x^5 - 5x$



43. (a, d) Let $f(x)$ and $g(x)$ achieve their maximum value at x_1 and x_2 respectively

$$h(x) = f(x) - g(x)$$

$$h(x_1) = f(x_1) - g(x_1) \geq 0$$

$$h(x_2) = f(x_2) - g(x_2) \leq 0$$

$$\Rightarrow h(c) = 0 \text{ where } c \in [0, 1] \Rightarrow f(c) = g(c).$$

44. (b, c) Given circles

$$x^2 + y^2 - 2x - 15 = 0$$

$$x^2 + y^2 - 1 = 0$$

$$\text{Radical axis } x + 7 = 0 \dots (1)$$

Centre of circle lies on (1)

Let the center be $(-7, k)$

$$\text{Let equation be } x^2 + y^2 + 14x - 2ky + c = 0$$

Orthogonality gives

$$-14 = c - 15 \Rightarrow c = 1 \dots (2)$$

$$(0, 1) \rightarrow 1 - 2k + 1 = 0 \Rightarrow k = 1$$

$$\text{Hence radius} = \sqrt{7^2 + k^2 - c} = \sqrt{49 + 1 - 1} = 7$$

Alternate solution

$$\text{Given circles } x^2 + y^2 - 2x - 15 = 0$$

$$x^2 + y^2 - 1 = 0$$

$$\text{Let equation of circle } x^2 + y^2 + 2gx + 2fy + c = 0$$

Circle passes through $(0, 1)$

$$\Rightarrow 1 + 2f + c = 0$$

Applying condition of orthogonality

$$-2g = c - 15, 0 = c - 1$$

$$\Rightarrow c = 1, g = 7, f = -1$$

$$r = \sqrt{49 + 1 - 1} = 7; \text{ center } (-7, 1)$$

45. (a, b, c) $\vec{a}$ is in direction of $\vec{x} \times (\vec{y} \times \vec{z})$

i.e. $(\vec{x} \cdot \vec{z})\vec{y} - (\vec{x} \cdot \vec{y})\vec{z}$

$$\Rightarrow \vec{a} = \lambda_1 \left[2 \times \frac{1}{2} (\vec{y} - \vec{z}) \right]$$

$$\vec{a} = \lambda_1 (\vec{y} - \vec{z}) \dots (1)$$

Now $\vec{a} \cdot \vec{y} = \lambda_1 (\vec{y} \cdot \vec{y} - \vec{y} \cdot \vec{z})$

$$= \lambda_1 (2 - 1) \Rightarrow \lambda_1 = \vec{a} \cdot \vec{y} \dots (2)$$

From (1) and (2), $\vec{a} = \vec{a} \cdot \vec{y} (\vec{y} - \vec{z})$

Similarly, $\vec{b} = (\vec{b} \cdot \vec{z}) (\vec{z} - \vec{x})$

Now, $\vec{a} \cdot \vec{b} = (\vec{a} \cdot \vec{y}) (\vec{b} \cdot \vec{z}) [(\vec{y} - \vec{z}) \cdot (\vec{z} - \vec{x})]$

$$= (\vec{a} \cdot \vec{y}) (\vec{b} \cdot \vec{z}) [1 - 1 - 2 + 1]$$

$$= -(\vec{a} \cdot \vec{y}) (\vec{b} \cdot \vec{z})$$

46. (c) Line 1: $\frac{x}{1} = \frac{y}{1} = \frac{z-1}{0} = r$, Q (r, r, 1)

Line 2: $\frac{x}{1} = \frac{y}{-1} = \frac{z-1}{0} = k$, R (k, -k, -1)

$$\overrightarrow{PQ} = (\lambda - r)\hat{i} + (\lambda - r)\hat{j} + (\lambda - 1)\hat{k}$$

and $\lambda - r + \lambda - r = 0$ as $\overrightarrow{PQ}$ is $\perp$ to L_1

$$\Rightarrow k = 0$$

so $PQ \perp PR$

$$(\lambda - r)(\lambda - k) + (\lambda - r)(\lambda + k) + (\lambda - 1)(\lambda + 1) = 0$$

$$\Rightarrow \lambda = 1, -1$$

For $\lambda = 1$ as points P and Q coincide

$$\Rightarrow \lambda = -1$$

47. (c, d) Let $M = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$ (where a, b, c $\in \mathbb{I}$)

then $\text{Det } M = ab - c^2$

if $a = b = c$, $\text{Det}(M) = 0$

if $c = 0$, $a, b \neq 0$, $\text{Det}(M) \neq 0$

if $ab \neq \text{square of integer}$, $\text{Det}(M) \neq 0$

48. (a, b) $M^2 = N^4 \Rightarrow M^2 - N^4 = O \Rightarrow (M - N^2)(M + N^2) = O$ (As M, N commute.)

Also, $M \neq N^2$, $\text{Det}((M - N^2)(M + N^2)) = 0$

As $M + N^2$ is not null $\Rightarrow \text{Det}(M - N^2) = 0$

Also $\text{Det}(M^2 + MN^2) = (\text{Det } M)(\text{Det}(M + N^2)) = 0$

$$\Rightarrow \text{There exist non-null } U \text{ such that } (M^2 + MN^2)U = O$$

49. (a, c) Since $f(x) \geq 1 \quad \forall x \in [a, b]$

for $g(x)$

LHD at $x = a$ is zero

$$\int_a^x f(t) dt - 0$$

and RHD at $(x = a) = \lim_{x \rightarrow a^+} \frac{a}{x - a} = \lim_{x \rightarrow a^+} f(x) \geq 1$

Hence $g(x)$ is not differentiable at $x = a$

Similarly, LHD at $x = b$ is greater than 1

$g(x)$ is not differentiable at $x = b$

50. (a, b, c) $f(x) = (\log(\sec x + \tan x))^3 \quad \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$f(-x) = -f(x)$, hence $f(x)$ is odd function

Let $g(x) = \sec x + \tan x \quad \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\Rightarrow g'(x) = \sec x (\sec x + \tan x) > 0 \quad \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$\Rightarrow g'(x)$ is one-one function

Hence $(\log(g(x)))^3$ is one-one function.

and $g(x) \in (0, \infty) \quad \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\Rightarrow \log(g(x)) \in \mathbb{R}$. Hence $f(x)$ is an onto function.

51. (7) When n_5 takes value from 10 to 6 the carry forward moves from 0 to 4 which can be arranged in

$${}^4C_0 + \frac{{}^4C_1}{4} + \frac{{}^4C_2}{3} + \frac{{}^4C_3}{2} + \frac{{}^4C_4}{1} = 7$$

Alternate solution

Possible solutions are

1, 2, 3, 4, 10

1, 2, 3, 5, 9

1, 2, 3, 6, 8

1, 2, 4, 5, 8

1, 2, 4, 6, 7

1, 3, 4, 5, 7

2, 3, 4, 5, 6

52. (5) Number of red lines = ${}^nC_2 - n$

Number of blue lines = n

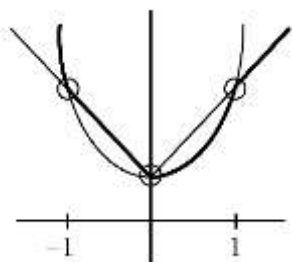
Hence, ${}^nC_2 - n = n$

$${}^nC_2 = 2n$$

$$\frac{n(n-1)}{2} = 2n$$

$$n-4=4 \Rightarrow n=5$$

53. (3)



$$h(x) = \begin{cases} x^2 + 1, & x \in (-\infty, -1] \\ -x + 1, & x \in [-1, 0] \\ x^2 + 1, & x \in [0, 1] \\ x + 1, & x \in [1, \infty) \end{cases}$$

Hence, not differentiable at $x = -1, 0, 1$

$$\frac{b}{a} = \frac{c}{b} =$$

54. (4) (integer)

$$b^2 = ac \Rightarrow c = \frac{b^2}{a}$$

$$\frac{a+b+c}{3} = b+2$$

$$a+b+c = 3b+6 \Rightarrow a-2b+c = 6$$

$$a-2b+\frac{b^2}{a} = 6 \Rightarrow 1-\frac{2b}{a}+\frac{b^2}{a^2} = \frac{6}{a}$$

$$\left(\frac{b}{a}-1\right)^2 = \frac{6}{a} \Rightarrow a=6 \text{ only}$$

55. (4) $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$$\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = p + q(\vec{a} \cdot \vec{b}) + r(\vec{a} \cdot \vec{c})$$

$$\text{And } [\vec{a} \vec{b} \vec{c}] = \frac{1}{\sqrt{2}}$$

$$p + \frac{q}{2} + \frac{r}{2} = [\vec{a} \vec{b} \vec{c}] \dots (1)$$

$$\frac{p}{2} + q + \frac{r}{2} = 0 \dots (2)$$

$$\frac{p}{2} + \frac{q}{2} + r = [\vec{a} \vec{b} \vec{c}] \dots (3)$$

$$\Rightarrow p = r = -q$$

$$\frac{p^2 + 2q^2 + r^2}{q^2} = 4$$

56. (8) $2(y-x^5)\left(\frac{dy}{dx} - 5x^4\right)$

$$= 1(1+x^2)^2 + (x)(2(1+x^2)(2x))$$

$$\text{Now put } x = 1, y = 3 \text{ and } \frac{dy}{dx} = m$$

$$2(3-1)(m-5) = 1(4) + (1)(4)(2)$$

$$m-5 = \frac{12}{4}$$

$$m = 5 + 3 = 8$$

$$\frac{dy}{dx} = m = 8$$

57. (2) $\int_0^1 4x^3 \frac{d}{dx^2} (1-x^2)^5 dx$

$$= \left[4x^3 \frac{d}{dx} (1-x^2)^5 \right]_0^1 - \int_0^1 12x^2 \frac{d}{dx} (1-x^2)^5 dx$$

$$= \left[4x^2 \times 5(1-x^2)^4 (-2x) \right]_0^1 - 12 \left[x^2 (1-x^2)^5 \right]_0^1 - \int_0^1 2x (1-x^2)^5 dx$$

$$= 0 - 0 - 12[0-0] + 12 \int_0^1 2x (1-x^2)^5 dx$$

$$= 12 \times \left[-\frac{(1-x^2)^6}{6} \right]_0^1$$

$$= 12 \left[0 + \frac{1}{6} \right] = 2$$

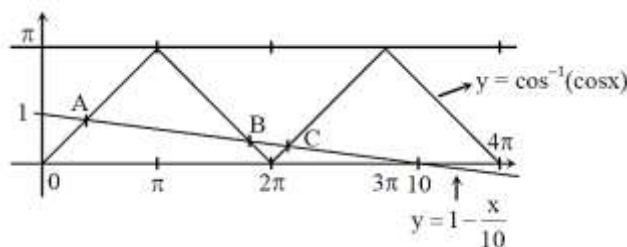
58. (2) $\lim_{x \rightarrow 1} \left(\frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right)^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$

$$\lim_{x \rightarrow 1} \left(\frac{\frac{\sin(x-1)}{(x-1)} - a}{\frac{\sin(x-1)}{(x-1)} + 1} \right)^{(1+\sqrt{x})} = \frac{1}{4} \Rightarrow \left(\frac{1-a}{2} \right)^2 = \frac{1}{4}$$

$$\Rightarrow a = 0, a = 2$$

$$\Rightarrow a = 2$$

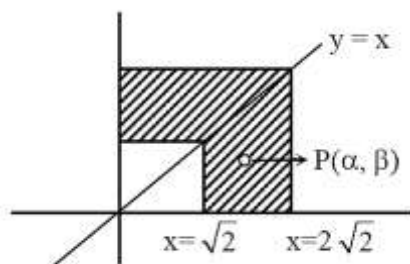
59. (3)



$$f: [0, 4\pi] \rightarrow [0, \pi], f(x) = \cos^{-1}(\cos x)$$

$$\Rightarrow \text{point A, B, C satisfy } f(x) = \frac{10-x}{10} \cdot 10$$

60. (6)



$$2 \leq d_1(p) + d_2(p) \leq 4$$

$$\text{For } P(\alpha, \beta), \alpha > \beta$$

$$\Rightarrow 2\sqrt{2} \leq 2\alpha \leq 4\sqrt{2}$$

$$\sqrt{2} \leq \alpha \leq 2\sqrt{2}$$

$$\Rightarrow \text{Area of region} = \left((2\sqrt{2})^2 - (\sqrt{2})^2 \right)$$

$$= 8 - 2 = 6 \text{ sq. units}$$