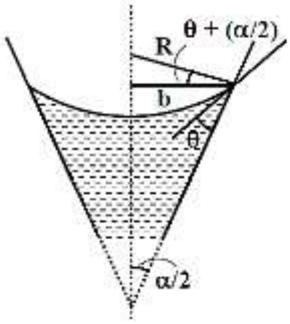
1. (c) For point outside dielectric sphere $E = \frac{Q}{4\pi\varepsilon_0 r^2}$

For point inside dielectric sphere $E = E_s \frac{r}{R}$

Exact Ratio $E_1 : E_2 : E_3 = 2 : 4 : 1$

2. (d) If R be the meniscus radius



R cos (θ + α /2) = b

Excess pressure on concave side of meniscus = $= \frac{2S}{R}$

$$h\rho g = \frac{2S}{R} = \frac{2S}{b}\cos\left(\theta + \frac{\alpha}{2}\right)$$

$$\Rightarrow h = \frac{2S}{b\rho g}\cos\left(\theta + \frac{\alpha}{2}\right)$$

3. (b) $\frac{\lambda_{Cu}}{\lambda_{Mo}} = \left(\frac{Z_{Mo} - 1}{Z_{Cu} - 1}\right)^2$

4. (b) Inside planet

$$g_i = g_s \frac{r}{R} = \frac{4}{3}G\pi r\rho$$

Force to keep the wire at rest (F)

= weight of wire

$$= \int_{4R/5}^{R} (\lambda dr)\left(\frac{4}{3}G\pi r\rho\right) = \left(\frac{4}{3}G\pi\rho\right)\left(\frac{9\lambda}{50}\right)R^2$$

Here, ρ = density of earth = $\frac{M_e}{\frac{4}{3}\pi R_e^2}$

Also, $R = \frac{R_e}{10}$; putting all values, F = 108 N

5. (b) $\frac{d(KE)}{dt} = mv\frac{dv}{dt}$

6. (a) $\frac{\frac{hc}{\lambda_1} - \phi}{\frac{hc}{\lambda_2} - \phi} = \frac{u_1^2}{u_2^2}$

$\phi = 3.7eV$

7. (d) Initially bead is applying radially inward normal force.

During motion at an instant, N = 0, after that N will act radially outward.

8. (c) $R = \frac{x}{100 - x}90$

$\therefore R = 60\Omega$

$\frac{dR}{R} = \frac{100}{(x)(100-x)}dx$

$\therefore dR = \frac{100}{(40)(60)}0.1\times 60$

$= 0.25\Omega$

Alternatively

$\frac{dR}{R} = \frac{0.1}{40} + \frac{0.1}{60}$

$\therefore dR = 0.25\Omega$

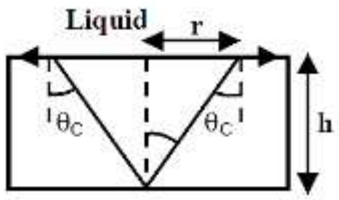
9. **(a)** Rate of radiation energy lost by the sphere

= Rate of radiation energy incident on it

$\Rightarrow \sigma \times 4\pi r^2\left[T^4 - (300)^4\right] = 912\times \pi r^2$

$\Rightarrow T = \sqrt{11}\times 10^2 \approx 330K$

10. **(c)**



$\tan\theta_C = \frac{r}{h} = \frac{5.77}{10} \approx \sqrt{3}$

$\Rightarrow \theta_C = 30^0$

$\Rightarrow \sin\theta_C = \frac{\mu_l}{\mu_b}$

$\Rightarrow \mu_l = 2.72\times\frac{1}{2} = 1.36$

11. **(i) (c)** The net magnetic field at the given point will be zero if.

$\left|\vec{B}_{wires}\right| = \left|\vec{B}_{loop}\right|$

$\Rightarrow 2\frac{\mu_0 I}{2\pi\sqrt{a^2+h^2}}\times\frac{a}{\sqrt{a^2+h^2}} = \frac{\mu_0 I a^2}{2\left(a^2+h^2\right)^{3/2}}$

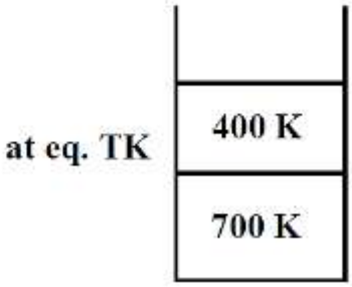
$\Rightarrow h \approx 1.2a$

The direction of magnetic field at the given point due to the loop is normally out of the plane. Therefore, the net magnetic field due the both wires should be into the plane. For this current in wire I should be along PQ and that in wire RS should be along SR.

(ii)(b) $\tau = MB\sin\theta = I\pi a^2\times 2\times\frac{\mu_0 I}{2\pi d}\sin 30^0 = \frac{\mu_0 I^2 a^2}{2d}$

12. **(i) (d)** Heat given by lower compartment = $2\times\frac{3}{2}R\times(700-T)$ ...(i)

Heat obtained by upper compartment $= 2\times\frac{7}{2}R\times(T-400)$ ...(ii)



equating (i) and (ii)

3 (700 – T) = 7 (T – 400)

2100 – 3T = 7 T – 2800

4900 = 10 T $\Rightarrow$ T = 490 K

(ii) **(d)** Heat given by lower compartment = $= 2\times\frac{5}{2}R\times(700-T)\ldots$(i)

Heat obtained by upper compartment = $= 2\times\frac{7}{2}R\times(T-400)$ ...(ii)

By equating (i) and (ii)

5(700 -T) = 7(T - 400)

3000 – 5T = 7T– 2800

6300 = 12 T

T = 525K

$\therefore$ Work done by lower gas = nRΔT = – 350 R

Work done by upper gas = nRΔT = +250 R

Net work done - 100 R

13. **(i) (c)** By $A_1V_1 = A_2V_2$

$\Rightarrow \pi(20)^2\times 5 = \pi(1)^2 V_2 \Rightarrow V_2 = 2m/s^2$

(i) (a) $\frac{1}{2}\rho_a V_a^2 = \frac{1}{2}\rho_l V_l^2$

For given V_a

$V_l \propto \sqrt{\frac{\rho_a}{\rho_l}}$

14. **(c)** In P, Q, R no horizontal velocity is imparted to falling water, so d remains same.

In S, since its free fall, $a_{eff} = 0$

$\therefore$ Liquid won't fall with respect to lift.

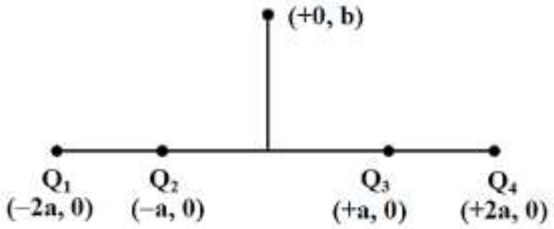
15. **(a)** P: By Q_1 and Q_4, Q_3 and Q_2 F is in +y

Q: By Q_1 and Q_4, Q_2 and Q_3 F is in +ve x.

R: By Q_1 and Q_4, F is in +ve y

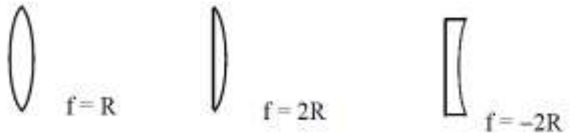
By Q_2 and Q_3, F is in –ve y

But later has more magnitude, since its closer to (0, b). Therefore net force is in – y



S: By Q_1 and Q_4, F is in +ve x and by Q_2 and Q_3, F is in –x, but later is more in magnitude, since its closer to (0, b). Therefore net force is in –ve x.

16. **(b)** $\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$



Use $\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}$

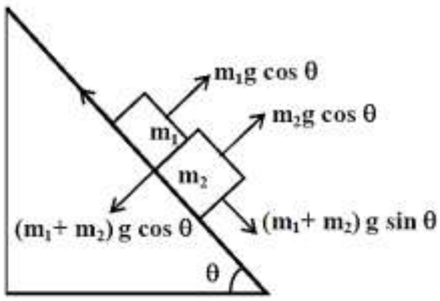
(P) $\frac{1}{f_{eq}}=\frac{1}{R}+\frac{1}{R}=\frac{2}{R}; f_{eq}=\frac{R}{2}$

(Q) $\frac{1}{f_{eq}}=\frac{1}{2R}+\frac{1}{2R}=\frac{2}{R}; f_{eq}=R$

(R) $\frac{1}{f_{eq}}=-\frac{1}{2R}-\frac{1}{2R}=-\frac{1}{R}; f_{eq}=-R$

(S) $\frac{1}{f_{eq}}=\frac{1}{R}-\frac{1}{2R}=\frac{1}{2R}; f_{eq}=2R$

17. **(d)** Condition for not sliding,



f_{max} > $(m_1 + m_2)$ g sin θ
μN > $(m_1 + m_2)$ g sin θ
0.3 m_2 g cos $\theta \geq 30\sin\theta$
$6 \geq 30\tan\theta$
$1/5 \geq \tan\theta$
$0.2 \geq \tan\theta$
$\therefore$ for P, Q
f = $(m_1 + m_2)$ g sin θ
For R and S
F = f_{max} = μm_2g sin θ

18. **(c)** Assuming that no 2s-2p mixing takes place

(a) $Be_2 \rightarrow \sigma1s^2, \sigma*1s^2, \sigma2s^2, \sigma*2s^2$ (diamagnetic)

(b) $B_2 \rightarrow \sigma1s^2, \sigma*1s^2, \sigma2s^2, \sigma*2s^2, \sigma2p_z^2, {}^{\pi2p_x^0}_{\pi2p_y^0}$ (diamagnetic)

(c) $C_2 \rightarrow \sigma1s^2, \sigma*1s^2, \sigma2s^2, \sigma*2s^2, \sigma2p_z^2, {}^{\pi2p_x^1}_{\pi2p_y^1}, {}^{\pi*2p_x^0}_{\pi*2p_y^0}, \sigma*2p_z^0$ (paramagnetic)

(d) $N_2 \rightarrow \sigma1s^2, \sigma*1s^2, \sigma2s^2, \sigma*2s^2, \sigma2p_z^2, {}^{\pi2p_x^2}_{\pi2p_y^2}, {}^{\pi*2p_x^0}_{\pi*2p_y^0}, \sigma*2p_z^0$ (diamagnetic)

19. **(b)** At 100°C and 1 atmosphere pressure $H_2O(l) \rightleftharpoons H_2O(g)$ is at equilibrium. For equilibrium

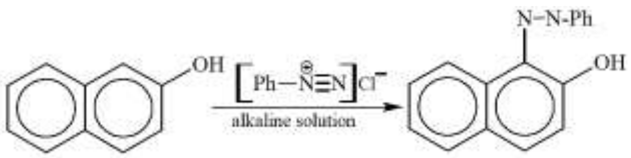
$\Delta S_{total}=0$ and $\Delta S_{system}+\Delta S_{surrounding}=0$

$\therefore \Delta S_{system}>0\, and\, \Delta S_{surrounding}<0$

$$\frac{r_1}{r_2}=\frac{1}{8}=\frac{[M]^n}{[2M]^n}\Rightarrow n=3$$

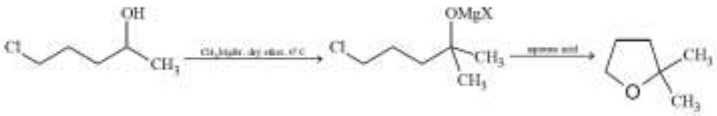
20. **(b)**

21. **(d)**



22. **(b)** III > II > I
More the branching in an alkane, lesser will be the surface area, lesser will be the boiling point.

23. **(d)**



24. **(c)**

$$XeF_6 + 3H_2O \longrightarrow XeO_3 + 3H_2F_2$$

$$\downarrow OH^-$$

$$HXeO_4^-$$

$$\downarrow OH^- / H_2O \text{ (disproportionation)}$$

$$XeO_{6\,(s)}^{4-} + Xe_{(g)} + H_2O_{(l)} + O_{2(g)}$$

25. **(a)** $P_{4(s)} + 8SOCl_{2(l)} \rightarrow 4PCl_{3(l)} + 4SO_{2(g)} + 2S_2Cl_{2(g)}$

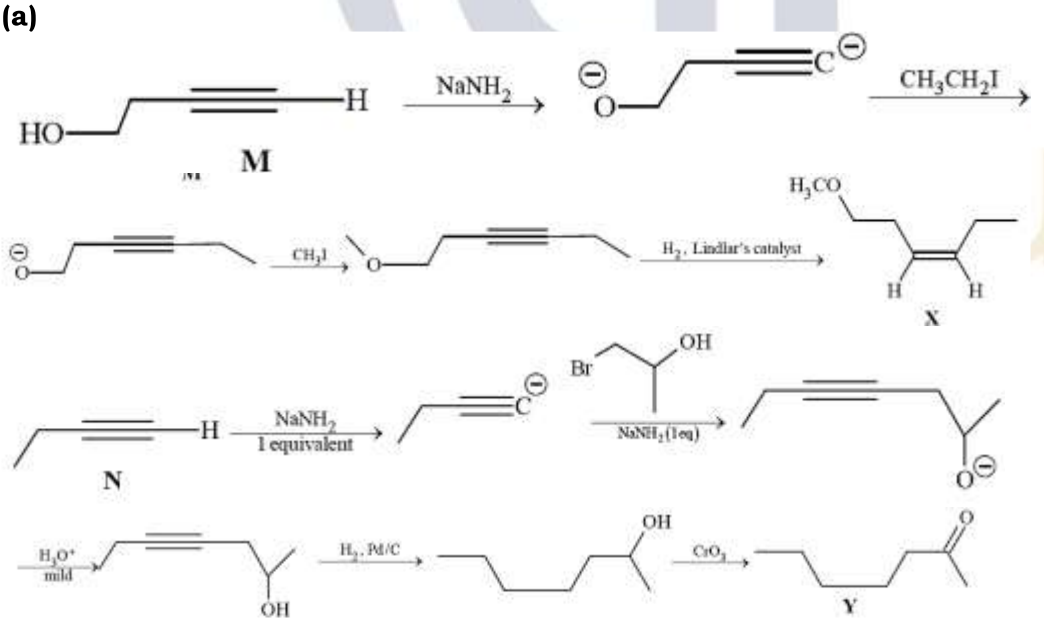
26. **(a)** $KIO_4 + H_2O_2 \rightarrow KIO_3 + H_2O + O_2$
$NH_2OH + 3H_2O_2 \rightarrow HNO_3 + 4H_2O$

27. **(c)** When two phenyl groups are replaced by two para methoxy group, carbocation formed will be more stable

28. **(c)** $\frac{r_x}{r_y} = \frac{d}{24-d} = \sqrt{\frac{40}{10}} = 2$
d = 48 – 2d
3d = 48
d = 16cm

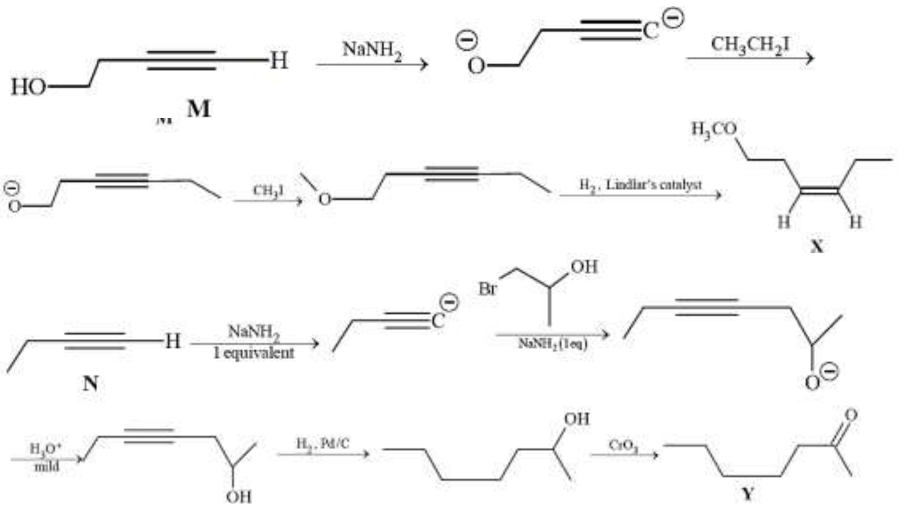
29. **(d)** As the collision frequency increases then molecular speed decreases than the expected.

30. **(a)**



X and Y are functional isomers of each other and Y gives iodoform test.

31. **(c)**



X and Y are functional isomers of each other and Y gives iodoform test.

32. (b)

$$\underset{\text{Tetrahedral}}{[NiCl_4]^{2-}} \xleftarrow[\text{excess}]{HCl(Q)} \underset{(M_1)}{Ni^{2+}} \xrightarrow[\text{excess}]{KCN(R)} \underset{\text{square planar}}{[Ni(CN)_4]^{2-}}$$

$$\underset{\text{Tetrahedral}}{[ZnCl_4]^{2-}} \xleftarrow[\text{excess}]{HCl(Q)} \underset{(M_2)}{Zn^{2+}} \xrightarrow[\text{excess}]{KCN(R)} \underset{\text{Tetrahedral}}{[Zn(CN)_4]^{2-}}$$

$$Zn^{2+} \xrightarrow[(S)]{KOH} \underset{\text{White ppt.}}{Zn(OH)_2} \xrightarrow[\text{excess}]{KOH} \underset{\text{Soluble}}{[Zn(OH)_4]^{2-}}$$

33. (d)

$$\underset{\text{Tetrahedral}}{[NiCl_4]^{2-}} \xleftarrow[\text{excess}]{HCl(Q)} \underset{(M_1)}{Ni^{2+}} \xrightarrow[\text{excess}]{KCN(R)} \underset{\text{square planar}}{[Ni(CN)_4]^{2-}}$$

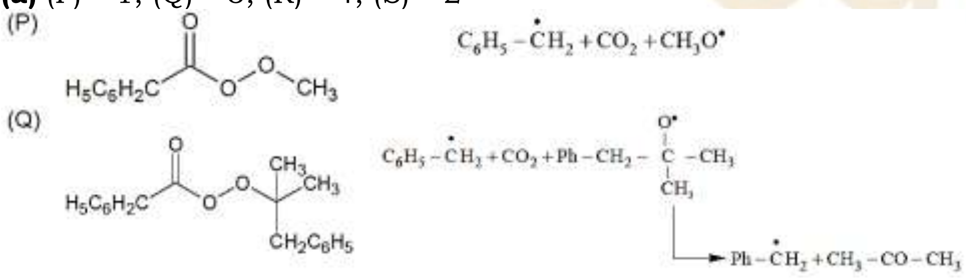
$$\underset{\text{Tetrahedral}}{[ZnCl_4]^{2-}} \xleftarrow[\text{excess}]{HCl(Q)} \underset{(M_2)}{Zn^{2+}} \xrightarrow[\text{excess}]{KCN(R)} \underset{\text{Tetrahedral}}{[Zn(CN)_4]^{2-}}$$

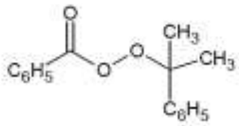
$$Zn^{2+} \xrightarrow[(S)]{KOH} \underset{\text{White ppt.}}{Zn(OH)_2} \xrightarrow[\text{excess}]{KOH} \underset{\text{Soluble}}{[Zn(OH)_4]^{2-}}$$

34. (b) (P) $[Cr(NH_3)_4Cl_2]Cl$ $\rightarrow$ Paramagnetic and exhibits cis-trans isomerism
(Q) $[Ti(H_2O)_5Cl](NO_3)_2$ $\rightarrow$ Paramagnetic and exhibits ionization isomerism
(R) $[Pt(en)(NH_3)Cl]NO_3$ $\rightarrow$ Diamagnetic and exhibits ionization isomerism
(S) $[Co(NH_3)_4(NO_3)_2]NO_3$ $\rightarrow$ Diamagnetic and exhibits cis-trans isomerism

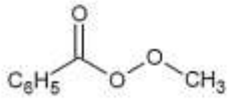
35. (c) Match the orbital overlap figures shown in List-I with the description given in List-II and select the correct answer using the code given below the lists.

36. (a) (P) – 1; (Q) – 3; (R) – 4; (S) – 2



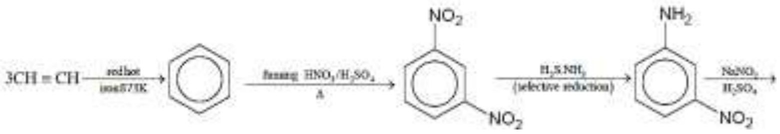
(R) 

$$C_6H_5-\dot{C}O_2 + CH_3-\underset{C_6H_5}{\overset{O^\bullet}{C}}-CH_3 \xrightarrow{-CO_2} \dot{P}h + CH_3-CO-Ph + \dot{C}H_3$$

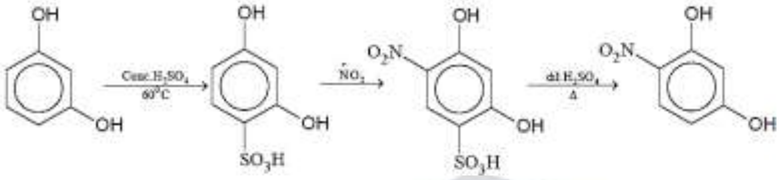
(S) 

$$C_6H_5-\dot{C}O_2 + CH_3O^\bullet$$
$$\longrightarrow C_6H_5^\bullet + CO_2$$

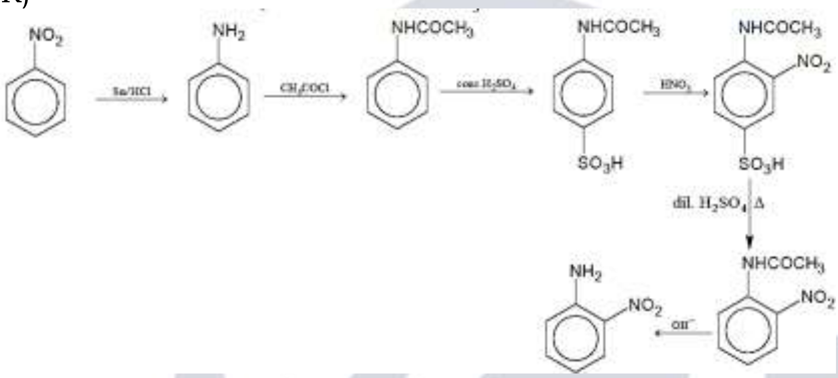
37. (c) (P)



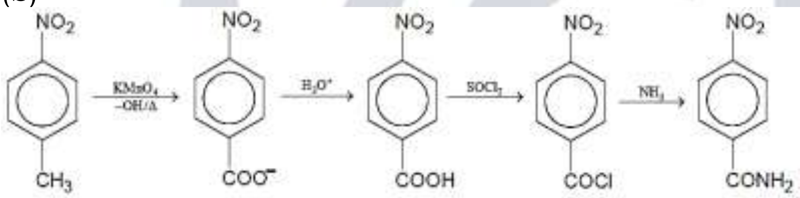
(Q)



(R)



(S)



38. (a) Either a girl will start the sequence or will be at second position and will not acquire the last position as well.

Required probability $= \dfrac{\left({}^3C_1 + {}^2C_1\right)}{{}^5C_2} = \dfrac{1}{2}$

39. (b) x = a + b

y = ab

$x^2 - c^2 = y$

$\Rightarrow \dfrac{a^2+b^2-c^2}{2ab} = -\dfrac{1}{2} = \cos\left(120^0\right)$

$\Rightarrow \angle C = \dfrac{2\pi}{3}$

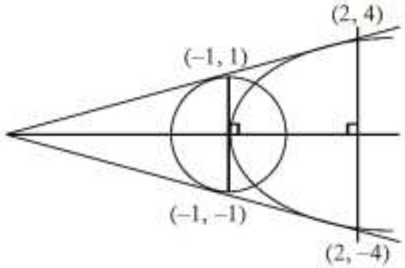
$\Rightarrow R = \dfrac{abc}{4\Delta}, r = \dfrac{\Delta}{s}$

$$\Rightarrow \frac{r}{R}=\frac{4\Delta^2}{s(abc)}=\frac{4\left[\frac{1}{2}ab\sin\left(\frac{2\pi}{3}\right)\right]^2}{\frac{x+c}{2}.y.c}$$

$$\frac{r}{R}=\frac{3y}{2c(x+c)}$$

40. **(c)** Number of required ways = $5!-\left\{4.4!-{}^4C_2.3!+{}^4C_3.!-1\right\}=53$

41. **(d)** Area = $\left(\frac{1}{2}(1+4)3\right)\times 2=15$



42. **(d)** P (x) = ax² + b with a, b of same sign.
P(P(x)) = a(ax² + b)² + b
If x ∈ R or ix ∈ R
$\Rightarrow x^2 \in R$
$\Rightarrow P(x)\in R$
$\Rightarrow P(P(x))\neq 0$
Hence real or purely imaginary number can not satisfy P(P(x)) = 0.

43. **(a)** $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(2\cos ecx)^{17}\,dx$

Let $e^u+e^{-u}=2\cos ecx, x=\frac{\pi}{4}\Rightarrow u=\ln\left(1+\sqrt{2}\right), x=\frac{\pi}{2}\Rightarrow u=0$

$\Rightarrow \cos ecx+\cot x=e^u$ and $\cos ecx-\cot x=e^{-u}\Rightarrow \cot x=\frac{e^u-e^{-u}}{2}$

$\left(e^u-e^{-u}\right)dx=-2\cos ecx\cot xdx$

$$\Rightarrow -\int\left(e^u+e^{-u}\right)^{17}\frac{\left(e^u-e^{-u}\right)}{2\cos ecx\cot x}du$$

$$=-2\int_{\ln\left(1+\sqrt{2}\right)}^{0}\left(e^u+e^{-u}\right)^{16}du$$

$$=\int_{0}^{\ln\left(1+\sqrt{2}\right)}2\left(e^u+e^{-u}\right)^{16}du$$

44. **(b)** $\frac{dy}{dx}+\frac{x}{x^2-1}y=\frac{x^4+2x}{\sqrt{1-x^2}}$

This is a linear differential equation

$I.F.=e^{\int\frac{x}{x^2-1}dx}=e^{\frac{1}{2}\ln\left|x^2-1\right|}=\sqrt{1-x^2}$

$\Rightarrow$solution is

$$y\sqrt{1-x^2} = \int \frac{x\left(x^3+2\right)}{\sqrt{1-x^2}}.\sqrt{1-x^2}\,dx$$

or $y\sqrt{1-x^2} = \int\left(x^4+2x\right)dx = \frac{x^5}{5}+x^2+c$

$f(0)=0 \Rightarrow c=0$

$\Rightarrow f(x)\sqrt{1-x^2} = \frac{x^5}{5}+x^2$

Now, $\int_{-\sqrt{3}/2}^{\sqrt{3}/2} f(x)dx = \int_{-\sqrt{3}/2}^{\sqrt{3}/2} \frac{x^2}{\sqrt{1-x^2}}dx$ (Using property)

$= 2\int_0^{\sqrt{3}/2} \frac{x^2}{\sqrt{1-x^2}}dx = 2\int_0^{\pi/3} \frac{\sin^2\theta}{\cos\theta}\cos\theta d\theta$ (Taking x = sin θ)

$= 2\int_0^{\pi/3}\sin^2\theta d\theta = 2\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{\pi/3} = 2\left(\frac{\pi}{6}\right)-2\left(\frac{\sqrt{3}}{8}\right) = \frac{\pi}{3}-\frac{\sqrt{3}}{4}$

45. **(b)** F(0) = 0

F'(x) = 2x f(x) = f'(x)

$f(x) = e^{x^2+c}$

$f(x) = e^{x^2}\left(\because f(0)=1\right)$

$F(x) = \int_0^{x^2} e^x dx$

$F(x) = e^{x^2}-1\left(\because F(0)=0\right)$

$\Rightarrow F(2) = e^4-1$

46. **(c)** $2x_1 + 3x_2 + 4x_3 = 11$

Possibilities are (0, 1, 2); (1, 3, 0); (2, 1, 1); (4, 1, 0).

∴ Required coefficients

= $({}^4C_0 \times {}^7C_1 \times {}^{12}C_2) + ({}^4C_1 \times {}^7C_3 \times {}^{12}C_0) + ({}^4C_2 \times {}^7C_1 \times {}^{12}C_1) + ({}^4C_4 \times {}^7C_1 \times 1)$

= (1 x 7 x 66) + (4 x 35 x 1) + (6 x 7 x 12) + (1 x 7)

= 462 + 140 + 504 + 7 = 1113.

47. **(d)** sin x + 2 sin 2x – sin 3x = 3

sin x + 4 sin x cos x – 3 sin x + 4 sin3 x = 3

sin x [–2 + 4 cos x + 4(1 – cos2 x)] = 3

sin x [2 – (4 cos2 x – 4 cos x + 1) + 1] = 3

sin x [3 – (2 cos x – 1)2] = 3

⇒ sin x = 1 and 2 cos x – 1 = 0

$\Rightarrow x = \frac{\pi}{2}$ and $x = \frac{\pi}{3}$

which is not possible at same time

Hence, no solution

48. **(i) (b)** Case I : One odd, 2 even

Total number of ways = 2 x 2 x 3 + 1 x 3 x 3 + 1 x 2 x 4 = 29.

Case II: All 3 odd

Number of ways = 2 x 3 x 4 = 24

Favourable ways = 53

Required probability = $\frac{53}{3\times5\times7} = \frac{53}{105}$.

(ii) **(c)** Here $2x_2 = x_1 + x_3$

$\Rightarrow x_1 + x_3$ = even

Hence number of favourable ways = ${}^2C_1 \cdot {}^4C_2 + {}^1C_1 \cdot {}^3C_1 = 11$.

49. (i) (d) Slope (QR) = Slope (PK)

$$\frac{2at-0}{at^2-2a}=\frac{-\frac{2a}{t}-2ar}{\frac{a}{t^2}-ar^2}$$

$$\Rightarrow \frac{t}{t^2-2}=-\left(\frac{\frac{1}{t}+r}{\frac{1}{t^2}-r^2}\right)\Rightarrow r=\frac{t^2-1}{t}$$

(ii) (b) Tangent at P: ty = x + at2 or y = $\frac{x}{t}$+ at

Normal at $S: y+\frac{x}{t}=\frac{2a}{t}+\frac{a}{t^3}$

Solving, $2y=at+\frac{2a}{t}+\frac{a}{t^3}$

$$y=\frac{a\left(t^2+1\right)^2}{2t^3}$$

50. (i) (a) $g\left(\frac{1}{2}\right)=\lim_{h\to 0^+}\int_h^{1-h} t^{-1/2}\left(1-t\right)^{-1/2}dt$

$$=\int_0^1\frac{dt}{\sqrt{t-t^2}}=\int_0^1\frac{dt}{\sqrt{\frac{1}{4}-\left(t-\frac{1}{2}\right)^2}}=\sin^{-1}\left.\left(\frac{t-\frac{1}{2}}{\frac{1}{2}}\right)\right|_0^1$$

$$=\sin^{-1}1-\sin^{-1}\left(-1\right)=\pi$$

(ii) (d) We have g (a) = g (1 - a) and g is differentiable

Hence $g'\left(\frac{1}{2}\right)=0$

51. (d) (P) $f\left(x\right)=ax^2+bx,\int_0^1 f\left(x\right)dx=1$

$\Rightarrow$2a + 3b = 6

$\Rightarrow$ (a, b) $\equiv$ (0, 2) and (3, 0).

(Q) $f\left(x\right)=\sqrt{2}\cos\left(x^2-\frac{\pi}{4}\right)$

For maximum value, $x^2-\frac{\pi}{4}=2n\pi$

$$\Rightarrow x^2=2n\pi+\frac{\pi}{4}$$

$\Rightarrow x=\pm\sqrt{\frac{\pi}{4}},\pm\sqrt{\frac{9\pi}{4}}$ as $x\in\left[-\sqrt{3},\sqrt{13}\right]$

(R) $\int_0^2\left(\frac{3x^2}{1+e^x}+\frac{3x^2}{1+e^{-x}}\right)dx=\int_0^2 3x^2dx=8$

(S) $\int_{-1/2}^{1/2}\cos 2x\ln\left(\frac{1+x}{1-x}\right)dx=0$ as it is an odd function.

52. (a) (P) $y=\cos\left(3\cos^{-1}x\right)$

$$y' = \frac{3\sin\left(3\cos^{-1}x\right)}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\,y' = 3\sin\left(3\cos^{-1}x\right)$$

$$\Rightarrow \frac{-x}{\sqrt{1-x^2}}y' + \sqrt{1-x^2}\,y'' = 3\cos\left(3\cos^{-1}x\right).\frac{-3}{\sqrt{1-x^2}}$$

$$\Rightarrow -xy' + \left(1-x^2\right)y'' = -9y$$

$$\Rightarrow \frac{1}{y}\left[\left(x^2-1\right)y'' + xy'\right] = 9$$

(Q) $\left(a_k \times a_{k+1}\right) = r^2 \sin\frac{2\pi}{n}$

$$a_k.a_{k+1} = r^2\cos\frac{2\pi}{n}$$

$$\Rightarrow \left|\sum_{k=1}^{n-1}\vec{a}_k \times \vec{a}_{k+1}\right| = \left|\sum_{k=1}^{n-1}a_k.a_{k+1}\right|$$

$$\Rightarrow r^2(n-1)\sin\frac{2\pi}{n} = r^2(n-1)\cos\frac{2\pi}{n}$$

$$\tan\frac{2\pi}{n} = 1 \Rightarrow n = \frac{8}{4k+1}$$

$$\Rightarrow n = 8$$

(R) $\frac{h^2}{6} + \frac{1^2}{3} = 1, h = \pm 2$

Tangent at (2, 1) is $\frac{2x}{6} + \frac{y}{3} = 1\ x + y = 3$

(S) $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\frac{1}{4x+1} = \tan^{-1}\frac{2}{x^2}$

$$\tan^{-1}\left(\frac{3x+1}{4x^2+3x}\right) = \tan^{-1}\frac{2}{x^2}$$

$$\Rightarrow 3x^2 - 7x - 6 = 0$$

$$x = -\frac{2}{3}, 3$$

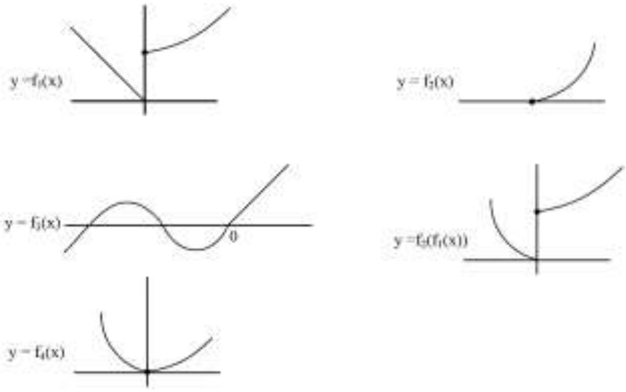
53. (d)

$$f_2(f_1) = \begin{cases} x^2, & x<0 \\ e^{2x}, & x \ge 0 \end{cases}$$

$$f_4 : R \to [0,\infty)$$

$$f_4(x) = \begin{cases} f_2(f_1(x)), & x<0 \\ f_2(f_1(x)) - 1, & x \ge 0 \end{cases}$$

$$= \begin{cases} x^2, & x<0 \\ e^{2x} - 1, & x \ge 0 \end{cases}$$



54. (c)

(P) z_k is 10^{th} root of unity $\Rightarrow \overline{Z}_k$ will also be 10^{th} root of unity. Take z_j as $\overline{Z}_k$.

(Q) $z_1 \neq 0$ take $z = \frac{z_k}{z_1}$, we can always find z.

(R) $z^{10} - 1 = (z - 1)(z - z_1) \ldots (z - z_9)$

$\Rightarrow (z - z_1)(z - z_2) \ldots (z - z_9) = 1 + z + z^2 + \ldots + z^9 \ \forall \ z \in$ complex number.

Put $z = 1$

$(1 - z_1)(1 - z_2) \ldots (1 - z_9) = 10.$

(S) $1 + z_1 + z_2 + \ldots + z_9 = 0$

$\Rightarrow \text{Re}(1) + \text{Re}(z_1) + \ldots + \text{Re}(z_9) = 0$

$\Rightarrow \text{Re}(z_1) + \text{Re}(z_2) + \ldots + \text{Re}(z_9) = -1.$

$$\Rightarrow 1 - \sum_{k=1}^{9} \cos\frac{2k\pi}{10} = 2$$