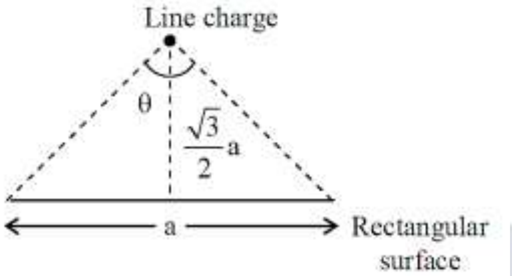
1. (7) Image by mirror is formed at 30 cm from mirror at its right and finally by the combination it is formed at 20 cm on right of the lens. So in air medium, magnification by lens is unity. In second medium, $\mu = \frac{7}{6}$, focal length of the lens is given by, $\frac{\frac{1}{10}}{\frac{1}{f}} = \frac{(1.5-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}{\left(\frac{1.5}{7/6}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)} \Rightarrow f = \frac{35}{2}cm$

So in second medium, final image is formed at 140 cm to the right of the lens. Second medium does not change the magnification by mirror. So $\left|\frac{M_2}{M_1}\right| = \left|\frac{M_{m_2}M_{l_2}}{M_{m_1}M_{l_1}}\right| = 7$

2. (6) From the figure θ = 60°



So No. of rectangular surfaces used to form a

$m = \frac{360}{60} = 6$

So, $\phi = \frac{(\lambda L)}{6\varepsilon_0}$, Hence 'n' = 6

3. (2) $E_{photon} = E_{ionize\ atom} + E_{kinetic\ energy}$

$\frac{1242}{90} = \frac{13.6}{n^2} + 10.4$

from this, n = 2

4. (2) At height R from the surface of planet acceleration due to planet's gravity is $\frac{1}{4}$th in comparison to the value at the surface

So, $-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+R}$ and $-\frac{GMm}{R} + \frac{1}{2}mv_{esc}^2 = 0$

$\therefore v_{esc} = v\sqrt{2}$

5. (7) Kinetic energy of a pure rolling disc having velocity of center of mass $v = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\frac{v^2}{R^2} = \frac{3}{4}mv^2$

So, $\frac{3}{4}m(3)^2 + mg(30) = \frac{3}{4}m(v_2)^2 + mg(27) \therefore v_2 = 7m/s$

6. (2) $\left(\frac{dQ}{dt}\right)_A = 10^4\left(\frac{dQ}{dt}\right)_B$

$(400R)^2 T_A^4 = 10^4\left(R^2 T_B^4\right)$

So, $2T_A = T_B$ and $\frac{\lambda_A}{\lambda_B} = \frac{T_B}{T_A} = 2$

7. (3) $\frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{T}}$

Where, A_0 is the initial activity of the radioactive material and A is the activity at t.

So, $\frac{12.5}{100}=\left(\frac{1}{2}\right)^{\frac{t}{T}}$

$\therefore t=3T$

8. **(3)** For maxima,

$\frac{4}{3}\sqrt{d^2+x^2}-\sqrt{d^2+x^2}=m\lambda$, m is an integer

So, $x^2=9m^2\lambda^2-d^2$

$\therefore$ p = 3

9. **(a, c)** For photoelectric emission

$V_0=\left(\frac{hc}{e}\right)\frac{1}{\lambda}-\frac{\phi}{e}$

10. **(b, c)** For vernier callipers,

1 main scale division $=\frac{1}{8}$ cm

1 vernier scale division $=\frac{1}{10}$ cm

So least count $=\frac{1}{40}$ cm

For screw gauge,
pitch (p) = 2 main scale division

So least count $=\frac{p}{100}$

11. **(a, c, d)** $h\equiv\left[ML^2T^{-1}\right], c\equiv\left[LT^{-1}\right], G\equiv\left[M^{-1}L^3T^{-2}\right]$

$M\propto\sqrt{\frac{hc}{G}}, L\propto\sqrt{\frac{hG}{c^3}}$

12. **(b, d)** For first oscillator

$E_1=\frac{1}{2}m\omega_1^2a^2$, and $p=mv=m\omega_1 a=b\Rightarrow\frac{a}{b}=\frac{1}{m\omega_1}\ldots$(i)

For second oscillator

$E_2=\frac{1}{2}m\omega_2^2R^2$, and $m\omega_2=1\ldots$(ii)

$\left(\frac{a}{b}\right)=\frac{\omega_2}{\omega_1}=n^2$

$\frac{E_1}{\omega_1^2a^2}=\frac{E_2}{\omega_2^2R^2}\Rightarrow\frac{E_1}{\omega_1}=\frac{E_2}{\omega_2}$

13. **(d)** Using conservation of angular momentum

$$mR^2\omega=\left(mR^2\times\frac{8\omega}{9}\right)+\left(\frac{m}{8}\times\frac{9R^2}{25}\times\frac{8\omega}{9}\right)+\left(\frac{m}{8}\times x^2\times\frac{8\omega}{9}\right)\Rightarrow x=\frac{4R}{5}$$

14. **(c)** In Case I :

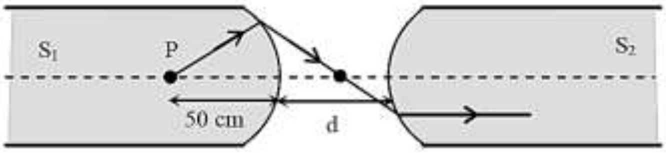
$\vec{F}=\frac{\lambda q}{2\pi\varepsilon_0(r+x)}\hat{i}+\frac{\lambda q}{2\pi\varepsilon_0(r-x)}\left(-\hat{i}\right)$

$=\frac{\lambda q}{\pi\varepsilon_0 r^2}x\left(-\hat{i}\right)$

Hence +q, charge will performs SHM with time period $T=2\pi\sqrt{\frac{\pi r^2\varepsilon_0 m}{\lambda q}}$

In case II: Resultant force will act along the direction of displacement.

15. **(b)** For Ist refraction



$$\frac{1}{v}-\frac{1.5}{-50}=\frac{1-1.5}{-10}$$

$\Rightarrow$ v = 50 cm

For IInd refraction

$$\frac{1.5}{\infty}-\frac{1}{-x}=\frac{1.5-1}{+10}$$

$\Rightarrow$ x = 20 cm

$\Rightarrow$ d = 70 cm

16. **(a, b, c)** 2(L + R)

$$\vec{F}=2I(L+R)\left[\hat{i}\times\vec{B}\right]$$

17. **(a, b, d)** $U=nC_{v_1}T+nC_{V_2}T$

$$=1\times\frac{5}{2}RT+1\times\frac{3}{2}RT=4RT$$

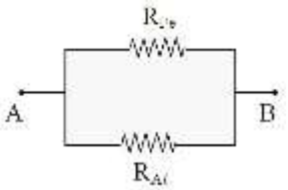
$$\Rightarrow 2C_{V_{mix}}T=4RT$$

Average energy per mole $=2RT\Rightarrow C_{V_{mix}}=2R$

$$\frac{C_{mix}}{C_{He}}=\sqrt{\left(\frac{\gamma_{mix}}{\gamma_{He}}\right)\left(\frac{M_{He}}{M_{mix}}\right)}=\sqrt{\frac{3}{2}\times\frac{3}{5}\times\frac{4}{3}}=\sqrt{\frac{6}{5}}$$

$$\frac{V_{rms\,He}}{V_{rms\,H_2}}=\sqrt{\frac{M_{H_2}}{M_{He}}}=\frac{1}{\sqrt{2}}$$

18. **(b)**



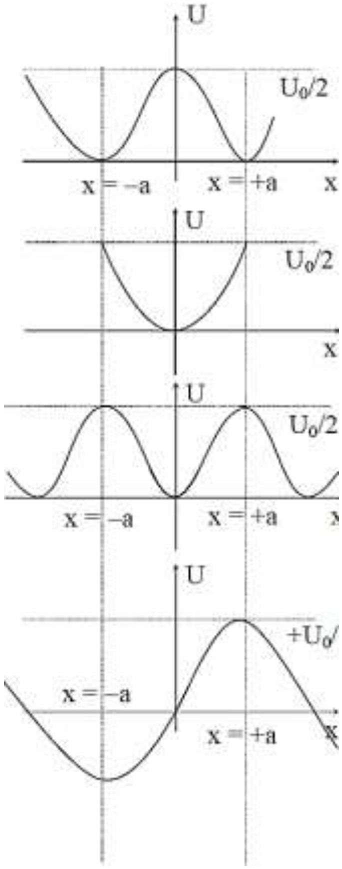
$$R_{Fe}=\frac{\rho_{Fe}\times 50\times 10^{-3}}{\left(2\times 10^{-3}\right)^2}=1250\mu\Omega$$

$$R_{Al}=\frac{\rho_{Al}\times 50\times 10^{-3}}{(49-4)\times 10^{-6}}=30\mu\Omega$$

$$R_{eq}=\frac{1250\times 30}{1280}=\frac{1875}{64}\mu\Omega$$

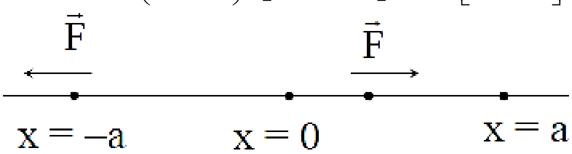
19. (a) $\rightarrow$ (R, T) ; (b) $\rightarrow$ (P, S) ; (c) $\rightarrow$ (P, Q, R, T); (d) $\rightarrow$ (P, Q, R, T)

20. (a) $\rightarrow$ (P, Q, R, T) ; (b) $\rightarrow$ (Q, S) ; (c) $\rightarrow$ (P, Q, R, S); (d) $\rightarrow$ (P, R, T)



Second Method

(a) $\vec{F} = -\frac{dU}{dx}\hat{i} = -\frac{U_0}{2}2\left(1-\left(\frac{x}{a}\right)^2\right)\times\left[-2\left(\frac{x}{a}\right)\times\frac{1}{a}\right]\hat{i} = 2U_0\left[1-\left(\frac{x}{a}\right)^2\right]\left[\frac{x}{a^2}\right]\hat{i}$

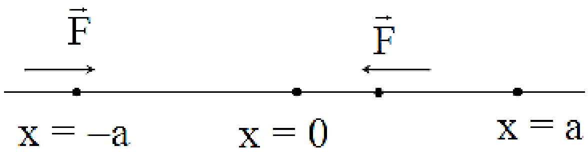


If $x = 0 \Rightarrow \vec{F} = \frac{U_0}{2}[2(1)\times 0] = \vec{0}, U = \frac{U_0}{2}$

If $x = a \Rightarrow \vec{F} = \vec{0}, \& U = 0$

If $x = -a \Rightarrow \vec{F} = \vec{0}, \& U = 0$

(b) $\vec{F} = -\frac{U_0}{2}\times 2\left(\frac{x}{a}\right)\times\frac{1}{a}\hat{i} = -\frac{U_0 x}{a^2}\hat{i}$



If $x = 0 \Rightarrow \vec{F} = 0$ and U=0

If $x = a \Rightarrow \vec{F} = -\frac{U_0}{a}\hat{i}$ and $U = \frac{U_0}{2}$

If $x = -a \Rightarrow \vec{F} = +\frac{U_0}{a}\hat{i}$ and $U = \frac{U_0}{2}$

For (c) and (d), similarly we can solve

21. (1) $\Delta T_f = iK_f m$

0.0558 = i x1.86 x 0.01

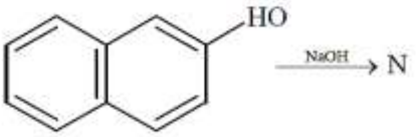
i = 3

$\therefore$ Complex is $[Co(NH_3)_5Cl]Cl_2$

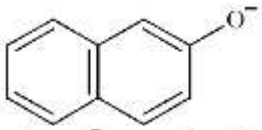
22. (2)

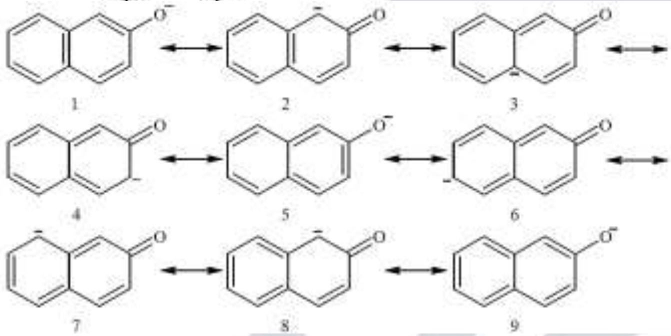
Bridging does not allow the other 2 variants to exist.
Total no. of stereoisomers of M = 2

23. (9)

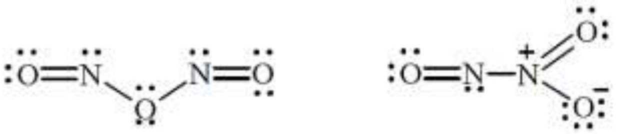


N is





24. (8)

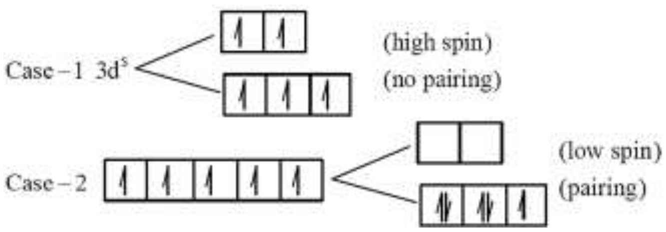


Total no. of lone pairs = 8

25. (4) $[Fe(SCN)_6]^{3-}$ and $[Fe(CN)_6]^{3-}$

In both the cases the electronic configuration of Fe^{3+} will be
$1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^5$

Since SCN is a weak field ligand and $\overline{C}N$ is a strong field ligand, the pairing will occur only in case of $\left[Fe(CN)_6\right]^{3-}$



Case – 1 $\mu = \sqrt{n(n+2)} = \sqrt{5(5+2)} = \sqrt{35} = 5.91BM$

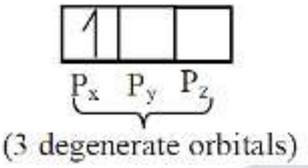
Case – 2 $\mu = \sqrt{n(n+2)} = \sqrt{1(1+2)} = \sqrt{3} = 1.73BM$

Difference in spin only magnetic moment = 5.91 – 1.73 = 4.18
$\approx$ 4

26. (4) $BeCl_2, N_3^-, N_2O, NO_2^+, O_3, SCl_2, ICl_2^-, I_3^-, XeF_2$

$BeCl_2 \rightarrow sp \rightarrow$ linear
$N_3^- \rightarrow sp \rightarrow$ linear
$N_2O \rightarrow sp \rightarrow$ linear
$\overset{\oplus}{N}O_2 \rightarrow$ sp $\rightarrow$ linear
$O_3 \rightarrow sp \rightarrow$ bent
$SCl_2 \rightarrow sp \rightarrow$ bent
$I_3^- \rightarrow sp^3d \rightarrow$ linear
$ICl_2^- \rightarrow sp^3d \rightarrow$ linear
$XeF_2 \rightarrow sp^3d \rightarrow$ linear
So among the following only four (4) has linear shape and no d-orbital is involved in hybridization.

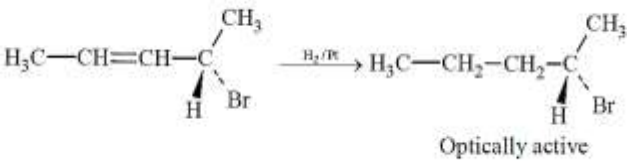
27. **(3)** Single electron species don't follow the $(n+l)$ rule but multi electron species do.
Ground state of H^- = $1s^2$
First excited state of H^- = $1s^1, 2s^1$
Second excited state of H^- = $1s^1, 2s^0, 2p^1$



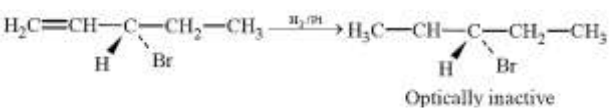
28. **(4)** $X \rightarrow Y \quad \Delta_r G^0 = -193 KJ/mol$
$M^+ \rightarrow M^{3+} + 2e^- \quad E^0 = -0.25V$
ΔG^0 for the this reaction is
$\Delta G^0 = -nFE^0 = -2\times(-0.25)\times 96500 = 48250$ J / mol
48.25 kJ/mole
So the number of moles of M^+ oxidized using $X \rightarrow Y$ will be
$= \frac{193}{48.25}$ = 4 moles

29. **(a)** In ccp lattice:
Number of O atoms $\rightarrow$ 4
Number of Octahedral voids $\rightarrow$ 4
Number of tatrahedral voids $\rightarrow$ 8
Number of Al^{3+}= 4xm
Number of Mg^{2+}= 8xn
Due to charge neutrality
4(-2)+4m(+3)+8n(+2)= 0
$\therefore m = \frac{1}{2}$ and $n = \frac{1}{8}$

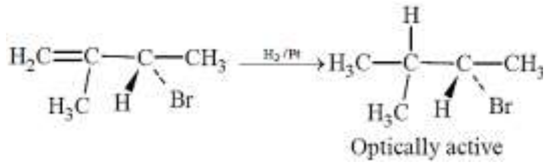
30. **(b, d)** (1)



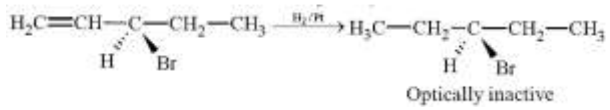
(2)



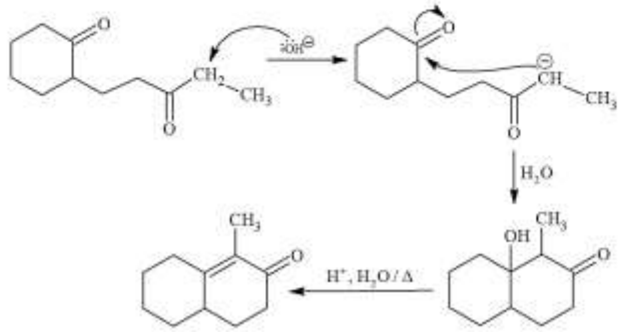
(3)



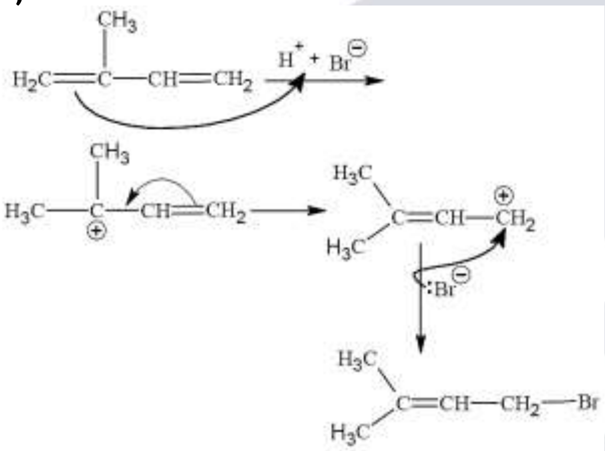
(4)



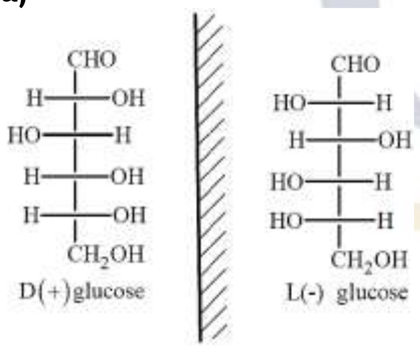
31. (a)



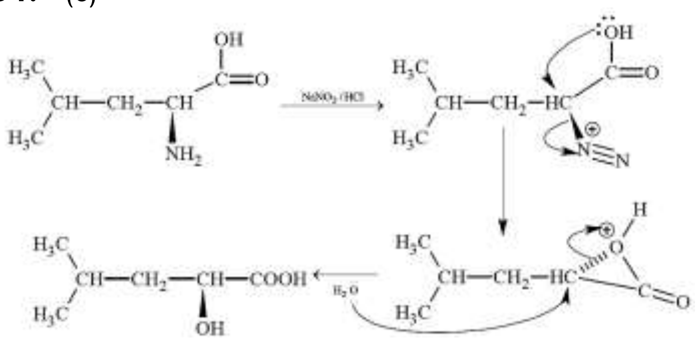
32. (d)



33. (a)



34. (c)



35. (a, b, c) (1) Cr^{2+} is a reducing agent because Cr^{3+} is more stable.

(2) Mn^{3+} is an oxidizing agent because Mn^{2+} is more stable.
(3) Cr^{2+} and Mn^{3+} exhibit d^4 electronic configuration.

36. **(b, c, d)** (1) Impure Cu strip is used as anode and impurities settle as anode mud.
(2) Pure Cu deposits at cathode.
(3) Acidified aqueous $CuSO_4$ is used as electrolyte.

37. **(a, b)** $2Fe^{+3} + H_2O_2 + 2OH^- \rightarrow 2Fe^{+2} + 2H_2O + O_2$
$Na_2O_2 + H_2O \rightarrow H_2O_2 + NaOH$

38. **(b)** $N_2(g) + 3H_2(g)$
$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g); \Delta H < 0$
Increasing the temperature lowers equilibrium yield of ammonia.
However, at higher temperature the initial rate of forward reaction would be greater than at lower temperature that is why the percentage yield of NH_3 too would be more initially.

39. (a) → (P, Q, S); (b) → (T); (c) → (Q, R); (d) → (R)
Siderite $FeCO_3$
Malachite $CuCO_3.Cu(OH)_2$
Bauxite $AlO_x(OH)_{3-2x}$; $0 < x < 1$
Calamine $ZnCO_3$
Argentite Ag_2S

40. (a) → (R, T); (b) → (P, Q, S); (c) → (P, Q, S); (d) → (P, Q, S, T)

41. **(3)** F'(a) + 2 = $\int_0^a f(x)dx$
Differentiating w.r.t. a
F"(a) = f(a)

$$F''(x) = 2\cos^2\left(x^2 + \frac{\pi}{6}\right).2x - 2\cos^2 x$$

$$F''(x) = 4\cos^2\left(x^2 + \frac{\pi}{6}\right) - 16x^2\cos\left(x^2 + \frac{\pi}{6}\right)\sin\left(x^2 + \frac{\pi}{6}\right) + 4\cos x \sin x$$

$$F''(0) = f(0) = 4\cos^2\frac{\pi}{6} = 3$$

42. **(8)** $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$

$$\Rightarrow \frac{5}{4}\cos^2 2x - 5\cos^2 x\sin^2 x = 0$$

$\Rightarrow \tan^2 2x = 1$, where $2x \in [0, 4\pi]$
Number of solutions = 8

43. **(4)** Image of y = –5 about the line x + y + 4 = 0 is x = 1
$\Rightarrow$ Distance AB = 4

44. **(8)** Let coin was tossed 'n' times
Probability of getting atleast two heads = $1 - \left[\frac{1}{2^n} + \frac{n}{2^n}\right]$

$$\Rightarrow 1 - \left[\frac{n+1}{2^n}\right] \geq 0.96$$

$$\Rightarrow \frac{2^n}{n+1} \geq 25$$

$\Rightarrow n \geq 8$

45. **(5)** n = 6!·5! (5 girls together arranged along with 5 boys)
$m = {}^5C_4.(7! - 2.6!).4!$
(4 out of 5 girls together arranged with others – number of cases all 5 girls are together)

$$\frac{m}{n} = \frac{5.5.6!.4!}{6!.5!} = 5$$

46. **(2)** If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x - 3)^2 + (y + 2)^2 = r^2$, then the value of r^2 is

47. **(0)** $I = \int_{-1}^{0} \frac{x.0}{2+0}dx + \int_{0}^{1} \frac{x.0}{2+1}dx = \int_{1}^{\sqrt{2}} \frac{x.1}{2+0}dx + 0 = \frac{1}{4}$

$\Rightarrow 4I - 1 = 0$

48. **(4)** Let inner radius be r and inner length be l

$\pi r^2 l = V$

Volume of material be M

$M = \pi(r+2)^2(l+2) - \pi r^2 l$

$\frac{dM}{dr} = -\frac{4V}{r^2} - \frac{8V}{r^3} + 8\pi + 0 + 4\pi r$

$\frac{dM}{dr} = 0$ when r = 10

$\Rightarrow V = 1000\pi \Rightarrow \frac{V}{250\pi} = 4$

49. **(a, c, d)** $|\vec{b} + \vec{c}| = |\vec{a}|$

$\Rightarrow |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b}.\vec{c} = |\vec{a}|^2$

$\Rightarrow 48 + |\vec{c}|^2 + 48 = 144$

$\Rightarrow |\vec{c}|^2 = 4\sqrt{3}$

$\therefore \frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$

Also, $|\vec{a} + \vec{b}| = |\vec{c}|$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = |\vec{c}|$

$\Rightarrow \vec{a}.\vec{b} = -72$

$\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \vec{a} \times \vec{b} = \vec{c} \times \vec{a}$

$\Rightarrow |\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 2|\vec{a} \times \vec{b}| = 48\sqrt{3}$

50. **(c, d)** $(Y^3Z^4 - Z^4Y^3)^T$

$= (Z^T)^4(Y^T)^3 - (Y^T)^3(Z^T)^4$

$= -Z^4Y^3 + Y^3Z^4 \Rightarrow$ symmetric

$X^{44} + Y^{44}$ is symmetric

$X^4 Z^3 - Z^3 X^4$ skew symmetric

$X^{23} + Y^{23}$skew symmetric.

51. **(b, c)** We get

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 5+2\alpha & 5+4\alpha & 5+6\alpha \end{vmatrix} = -648\alpha \left(R_3 \to R_3 - R_2; R_2 \to R_2 - R_1\right)$$

$$\Rightarrow \begin{vmatrix} \alpha^2 - 2 & 4\alpha^2 - 2 & 9\alpha^2 - 2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 2 & 2 & 2 \end{vmatrix} = -648\alpha \left(R_1 \to R_1 - R_2; R_3 \to R_3 - R_2\right)$$

$$\Rightarrow \begin{vmatrix} -3\alpha^2 & -5\alpha^2 & 9\alpha^2-3 \\ -2\alpha & -2\alpha & 3+6\alpha \\ 0 & 0 & 2 \end{vmatrix} = -648\alpha$$

$$\Rightarrow -8\alpha^2 = -648\alpha \Rightarrow \alpha = \pm 9$$

52. **(b, d)** Let the required plane be x + z + λy – 1 = 0

$$\Rightarrow \frac{|\lambda-1|}{\sqrt{\lambda^2+2}} = 1 \Rightarrow \lambda = -\frac{1}{2}$$

$$\Rightarrow P_3 \equiv 2x - y + 2z - 2 = 0$$

Distance of P_3 from (α, β, γ) is 2

$$\frac{|2\alpha - \beta + 2\gamma - 2|}{\sqrt{4 \times 1 + 4}} = 2$$

$$\Rightarrow 2\alpha - \beta + 2\lambda + 4 = 0 \text{ and } 2\alpha - \beta + 2\lambda - 8 = 0$$

53. **(a, b)** Line L will be parallel to the line of intersection of P_1 and P_2

Let a, b and c be the direction ratios of line L

$\Rightarrow$ a + 2b – c = 0 and 2a – b + c = 0

$\Rightarrow$ a : b : c :: 1 : –3 : –5

Equation of line L is $\frac{x-0}{1} = \frac{y-0}{-3} = \frac{z-0}{-5}$

Again, foot of perpendicular from origin to plane P_1 is $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$

$\therefore$ Equation of projection of line L on plane P_1 is

$$\frac{x+\frac{1}{6}}{1} = \frac{y+\frac{2}{6}}{-3} = \frac{z-\frac{1}{6}}{-5} = k$$

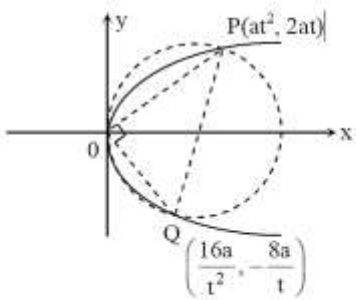
Clearly points $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$ and $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$ satisfy the line of projection i.e. M

Alternative solution

Direction ratio of plane can be found by $(\vec{n}_1 \times \vec{n}_2) \times \vec{n}_1 \equiv (13, -4, 5)$

So, equation of plane is 13x – 4y + 5z = 0 and point $\left(0, -\frac{5}{6}, -\frac{2}{3}\right) \& \left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$ satisfy

54. **(a, d)**



P(at², 2at)

$$Q\left(\frac{16a}{t^2}, -\frac{8a}{t}\right)$$

$$\Delta OPQ = \frac{1}{2} OP.OQ$$

$$\Rightarrow \frac{1}{2}\left| at\sqrt{t^2+4}.\frac{a(-4)}{t}\sqrt{\frac{16}{t^2}+4} \right| = 3\sqrt{2}$$

$t^2 - 3\sqrt{2}t + 4 = 0$
$t = \sqrt{2}, 2\sqrt{2}$

Hence, P(at², 2at) $= P\left(\frac{t^2}{2}, t\right)$

$t = \sqrt{2} \Rightarrow P\left(1, \sqrt{2}\right)$

$t = 2\sqrt{2} \Rightarrow P\left(4, 2\sqrt{2}\right)$

55. **(a, c)** $\frac{dy}{dx} + \frac{ye^x}{1+e^x} = \frac{1}{e^x+1}$

$I.F. = e^{\int \frac{e^x}{1+e^x}dx} = e^{\ln\left(1+e^x\right)} = 1+e^x$

$\Rightarrow y\left(1+e^x\right) = \int 1dx$

$y\left(1+e^x\right) = x + c$

$y = \frac{x+c}{1+e^x}$

$y(0) = 2 \Rightarrow c = 1$

$\Rightarrow y = \frac{x+4}{1+e^x}$

$y(-4) = 0$

$\Rightarrow y' = \frac{\left(1+e^x\right) - (x+4)e^x}{\left(1+e^x\right)^2} = 0$

Let $g(x) = \frac{\left(1+e^x\right) - (x+4)e^x}{\left(1+e^x\right)^2}$

$g(0) = \frac{2-2}{2^2} < 0$

$g(-1) = \frac{\left(1+\frac{1}{e}\right) - \frac{3}{e}}{\left(1+\frac{1}{e}\right)^2} = \frac{1-\frac{2}{e}}{\left(1+\frac{1}{e}\right)^2} > 0$

g(0)·g(–1) < 0. Hence g(x) has a root in between (–1, 0)

56. **(b, c)** Let the family of circles be $x^2 + y^2 - \alpha x - \alpha y + c = 0$

On differentiation $2x + 2yy' - \alpha - \alpha y' = 0$

Again, on differentiation and substituting 'α' we get $2 + 2y'^2 + 2yy'' - \left(\frac{2x+2yy'}{1+y'}\right)y'' = 0$

$\Rightarrow (y-x)y'' + y'\left(1+y'+y'^2\right) + 1 = 0$

57. **(a, d)** Differentiability of f(x) at x = 0

LHD $f'\left(0^-\right) = \lim_{\delta \to 0}\left(\frac{f(0) - f(0-\delta)}{\delta}\right) = \lim_{\delta \to 0}\frac{0 + g(-\delta)}{\delta} = 0$

RHD $f'\left(0^+\right) = \lim_{\delta \to 0}\left(\frac{f(0+\delta) - f(0)}{\delta}\right) = \lim_{\delta \to 0}\frac{g(\delta)}{\delta} = 0$

$\Rightarrow$ f(x) is differentiable at x = 0.

Differentiability of h(x) at x = 0

$h'(0^+) = 1$, h(x) is an even function

hence non diff. at x = 0

Differentiability of f(h(x)) at x = 0

$f(h(x)) = g(e^{|x|}) \forall \in R$

LHD $f'(h(0^-)) = \lim_{\delta \to 0} \frac{f(h(0)) - f(h(0-\delta))}{\delta} = \lim_{\delta \to 0} \frac{g(1) - g(e^{\delta})}{\delta} = -g'(1)$

RHD $f'(h(0^+)) = \lim_{\delta \to 0} \frac{f(h(0+\delta)) - f(h(0))}{\delta} = \lim_{\delta \to 0} \frac{g(e^{\delta}) - g(1)}{\delta} = g'(1)$

Since g'(1) ≠ 0 ⇒ f(h(x)) is non diff. at x = 0

Differentiability of h(f(x)) at x = 0

$$h(f(x)) = \begin{cases} e^{|f(x)|}, x \neq 0 \\ 1, \quad x = 0 \end{cases}$$

LHD. $h'(f(0-\delta)) = \lim_{\delta \to 0} \frac{h(f(0)) - h(f(0-\delta))}{\delta} = \lim_{\delta \to 0} \frac{1 - e^{|g(-\delta)|}}{|g(-\delta)|} \cdot \frac{|g(-\delta)|}{\delta} = 0$

RHD $h'(f(0+\delta)) = \lim_{\delta \to 0} \frac{h(f(0+\delta)) - h(f(0))}{\delta} = \lim_{\delta \to 0} \frac{e^{|g(-\delta)|} - 1}{|g(\delta)|} \cdot \frac{|g(\delta)|}{\delta} = 0$

58. **(a, b, c)** Given $g(x) = \frac{\pi}{2} \sin x \forall x \in R$

$f(x) = \sin\left(\frac{1}{3} g(g(x))\right)$

$\Rightarrow g(g(x)) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \forall x \in R$

Also, g(g(g(x))) $\in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \forall x \in R$

Hence, f(x) and f(g(x)) $\in \left[-\frac{1}{2}, \frac{1}{2}\right]$

$$\lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{\sin\left(\frac{1}{3} g(g(x))\right)}{\frac{1}{3} g(g(x))} \cdot \frac{\frac{1}{3} g(g(x))}{g(x)}$$

$$\Rightarrow \lim_{x \to 0} \frac{\pi}{6} \frac{\sin\left(\frac{\pi}{2} \sin x\right)}{\frac{\pi}{2} \sin x} = \frac{\pi}{6}$$

Range of $g(f(x)) \in \left[-\frac{\pi}{2} \sin\left(\frac{1}{2}\right), \frac{\pi}{2} \sin\left(\frac{1}{2}\right)\right]$

$\Rightarrow g(f(x)) \neq 1$

59. (a) → (P, Q), (b) → (P, Q), (c) → (P, Q, S, T), (d) → (Q, T)

(a) $\left|\frac{\sqrt{3}\alpha + \beta}{2}\right| = \sqrt{3}$

$\sqrt{3}\alpha + \beta = \pm 2\sqrt{3}$ ….. (1)

Given $\alpha = 2 + \sqrt{3}\beta$ ….. (2)

From equation (1) and (2), we get α = 2 or –1

So | α | = 1 or 2

(b) $f(x)=\begin{cases}-3ax^2-2, x<1\\ bx+a^2 \quad ,x\geq 1\end{cases}$

For continuity –3a – 2 = b + a^2
a^2 + 3a + 2 = –b ..... (1)
For differentiability –6a = b
6a = –b
a^2 – 3a + 2 = 0
a = 1, 2
(c) $(3-3\omega+2\omega^2)^{4n+3}+(2+3\omega-3\omega^2)^{4n+3}+(-3+2\omega+3\omega^2)^{4n+3}=0$
$(3-3\omega+2\omega^2)^{4n+3}+(\omega(2\omega^2+3-3\omega))^{4n+3}+(\omega^2(-3\omega+2\omega^2+3))^{4n+3}=0$
$\Rightarrow (3-3\omega+2\omega^2)^{4n+3}(1+\omega^{4n}+\omega^{8n})=0$
$\Rightarrow$ n $\neq$ 3k, k $\in$ N
(d) Let a = 5 – d
q = 5 + d
b = 5 + 2d
|q – a| = |2d|
Given $\frac{2ab}{a+b}=4$

$\frac{ab}{a+b}=2$

(5 – d)(5 + 2d) = 2(5 – d + 5 + 2d) = 2(10 + d)
25 + 10d – 5d – 2d2 = 20 + 2d
$2d^2$ – 3d – 5 = 0
d = –1, d = $\frac{5}{2}$
|2d| = 2, 5

60. (a) $\rightarrow$ (P, Q), (b) $\rightarrow$ (P, Q), (c) $\rightarrow$ (P, Q, S, T), (d) $\rightarrow$ (Q, T)

(a) $a^2-b^2=\frac{c^2}{2}$ (given)

$4R^2\left(\sin^2 X-\sin^2 Y\right)=\frac{4R^2}{2}\sin^2(Z)$

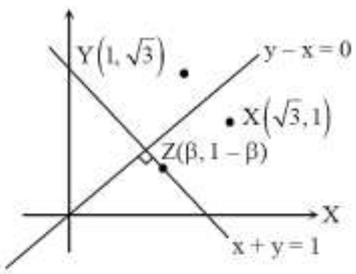
$\Rightarrow 2\sin(X-Y).\sin(X+Y)=\sin^2(Z)$

$\Rightarrow 2.\sin(X-Y).\sin(Z)=\sin^2(Z)$

$\Rightarrow \frac{\sin(x-Y)}{\sin Z}=\frac{1}{2}=\lambda \Rightarrow \cos\left(\frac{n\pi}{2}\right)=0$ for n = odd integer.

(b) 1 + cos2X – 2cos2Y = 2 sin X sin Y
$\sin^2$ X + sin X sin Y – 2 $\sin^2$ Y = 0
(sin X – sin Y)(sin X + 2sin Y) = 0
$\Rightarrow$ sin X = sin Y
$\Rightarrow \frac{\sin X}{\sin Y}=\frac{a}{b}=1$

(c) Here, distance of Z from bisector of $\overrightarrow{OX}$ and $\overrightarrow{OY}=\frac{3}{\sqrt{2}}$



$$\Rightarrow\left(\beta-\frac{1}{2}\right)^2+\left(\beta-\frac{1}{2}\right)^2=\frac{9}{2}$$

$$\Rightarrow\beta=2,-1$$

$$\Rightarrow|\beta|=2,1$$

(d) When α = 0

Area = $6-\int_0^2 2\sqrt{x}dx$

$$=6-\frac{8\sqrt{2}}{3}$$

When α = 1

Area = $\int_0^1\left(3-x-2\sqrt{x}\right)dx+\int_1^2\left(x+1-2\sqrt{x}\right)dx$

$$=\left|3x-\frac{x^2}{2}-\frac{4}{3}x^{3/2}\right|_0^1+\left|\frac{x^2}{2}+x-\frac{4}{3}x^{3/2}\right|_1^2$$

$$=5-\frac{8}{3}\sqrt{2}$$