

$$1. (2) \quad mvr = \frac{nh}{2\pi} = \frac{3h}{2\pi}$$

$$\text{de-Broglie Wavelength } \lambda = \frac{h}{mv} = \frac{2\pi r}{3} = \frac{2\pi a_0 (3)^2}{z_{Li}} = 2\pi a_0$$

2. (7) Form closer to M

$$\frac{GMm}{9l^2} - \frac{Gm^2}{l^2} = ma \quad \dots(i)$$

and for the other m:

$$\frac{Gm^2}{l^2} + \frac{GMm}{16l^2} = ma \quad \dots(ii)$$

From both the equations,

$$k = 7$$

$$3. (4) \quad E(t) = A^2 e^{-\alpha t}$$

$$\Rightarrow dE = -\alpha A^2 e^{-\alpha t} dt + 2AdAe^{-\alpha t}$$

Putting the values for maximum error,

$$\Rightarrow \frac{dE}{E} = \frac{4}{100} \Rightarrow \% \text{ error} = 4$$

$$4. (6) \quad I = \int \frac{2}{3} \rho 4\pi r^2 r^2 dr$$

$$I_A \propto \int (r)(r^2)(r^2) dr$$

$$I_B \propto \int (r^5)(r^2)(r^2) dr$$

$$\therefore \frac{I_B}{I_A} = \frac{6}{10}$$

5. (3) First and fourth wave interfere destructively. So, from the interference of 2nd and 3rd wave only,

$$\Rightarrow I_{net} = I_0 + I_0 + 2\sqrt{I_0}\sqrt{I_0} \cos\left(\frac{2\pi}{3} - \frac{\pi}{3}\right) = 3I_0$$

$$\Rightarrow n = 3$$

6. (2)

$$\lambda_p = \frac{1}{\tau}; \lambda_Q = \frac{1}{2\tau}$$

$$\frac{R_p}{R_Q} = \frac{(A_0 \lambda_p) e^{-\lambda_p t}}{A_0 \lambda_Q e^{-\lambda_Q t}}$$

$$\text{At } t = 2\tau; \frac{R_p}{R_Q} = \frac{2}{e}$$

7. (2) Snell's Law on 1st surface: $\frac{\sqrt{3}}{2} = n \sin r_1$

$$\sin r_1 = \frac{\sqrt{3}}{2n} \quad \dots(i)$$

$$\Rightarrow \cos r_1 = \sqrt{1 - \frac{3}{4n^2}} = \frac{\sqrt{4n^2 - 3}}{2n}$$

$$r_1 + r_2 = 60^\circ \quad \dots(ii)$$

Snell's Law on 2nd surface:

$$n \sin r_2 = \sin \theta$$

Using equation (i) and (ii)

$$n \sin(60^\circ - r_1) = \sin \theta$$

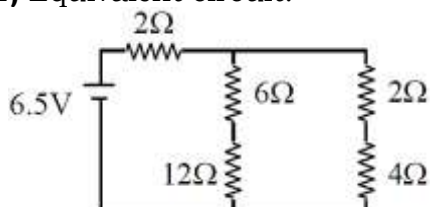
$$n \left[\frac{\sqrt{3}}{2} \cos r_1 - \frac{1}{2} \sin r_1 \right] = \sin \theta$$

$$\frac{d}{dn} \left[\frac{\sqrt{3}}{4} (\sqrt{4n^2 - 3} - 1) \right] = \cos \theta \frac{d\theta}{dn}$$

for $\theta = 60^\circ$ and $n = \sqrt{3}$

$$\Rightarrow \frac{d\theta}{dn} = 2$$

8. (1) Equivalent circuit:



$$R_{eq} = \frac{13}{2} \Omega$$

So, current supplied by cell = 1 A

9. (a) Q value of reaction = $(140 + 94) \times 8.5 - 236 \times 7.5 = 219$ Mev

So, total kinetic energy of Xe and Sr = $219 - 2 - 2 = 215$ Mev

So, by conservation of momentum, energy, mass and charge,

10. (a, d) From the given conditions, $\rho_1 < \sigma_1 < \sigma_2 < \rho_2$

From equilibrium, $\sigma_1 + \sigma_2 = \rho_1 + \rho_2$

$$V_P = \frac{2}{9} \left(\frac{\rho_1 - \sigma_2}{\eta_2} \right) g \text{ and } V_Q = \frac{2}{9} \left(\frac{\rho_2 - \sigma_1}{\eta_1} \right) g$$

$$\text{So, } \frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2} \text{ and } \vec{V}_P \cdot \vec{V}_Q < 0$$

11. (a, c)

$$Bllc \equiv VI \Rightarrow \mu_0 I^2 c \equiv VI \Rightarrow \mu_0 Ic = V$$

$$\Rightarrow \mu_0^2 I^2 c^2 = V^2$$

$$\Rightarrow \mu_0 I^2 = \epsilon_0 V^2 \Rightarrow \epsilon_0 c V = I$$

12. (d)

$$\vec{E} = \frac{\rho}{3\epsilon_0} \overrightarrow{C_1 C_2}$$

$C_1 \Rightarrow$ center of sphere and $C_2 \Rightarrow$ center of cavity.

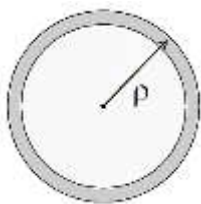
13. (a, b)

$$Y = \frac{\text{stress}}{\text{strain}}$$

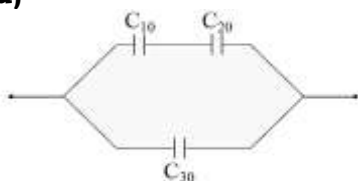
$$\Rightarrow \frac{1}{Y} = \frac{\text{strain}}{\text{stress}} \Rightarrow \frac{1}{Y_p} > \frac{1}{Y_\theta} \Rightarrow Y_p < Y_\theta$$

14. (b, c)

$$P(r) = K \left(1 - \frac{r^2}{R^2} \right)$$



15. (d)



$$C_{10} = \frac{4\epsilon_0 \frac{S}{2}}{d/2} = \frac{4\epsilon_0 S}{d}$$

$$C_{20} = \frac{2\epsilon_0 S}{d}, C_{30} = \frac{\epsilon_0 S}{d}$$

$$\frac{1}{C'_{10}} = \frac{1}{C_{10}} + \frac{1}{C_{10}} = \frac{d}{2\epsilon_0 S} \left[1 + \frac{1}{2} \right]$$

$$\Rightarrow C'_{10} = \frac{4\epsilon_0 S}{3d}$$

$$C_2 = C_{30} + C'_{10} = \frac{7\epsilon_0 S}{3d}$$

$$\frac{C_2}{C_1} = \frac{7}{3}$$

16. (b or a, b, c) P (pressure of gas) = $P_1 + \frac{kx}{A}$

$$W = \int P dV = P_1(V_2 - V_1) + \frac{kx^2}{2} = P_1(V_2 - V_1) + \frac{(P_2 - P_1)(V_2 - V_1)}{2}$$

$$\Delta U = nC_V \Delta T = \frac{3}{2}(P_2 V_2 - P_1 V_1)$$

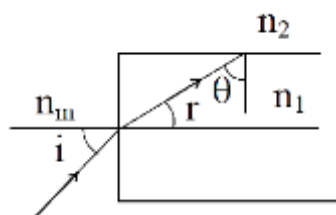
$$Q = W + \Delta U$$

$$\text{Case I: } \Delta U = 3P_1 V_1, W = \frac{5P_1 V_1}{4}, Q = \frac{17P_1 V_1}{4}, U_{\text{spring}} = \frac{P_1 V_1}{4}$$

$$\text{Case II: } \Delta U = \frac{9P_1 V_1}{2}, W = \frac{7P_1 V_1}{3}, Q = \frac{41P_1 V_1}{6}, U_{\text{spring}} = \frac{P_1 V_1}{3}$$

Note: A and C will be true after assuming pressure to the right of piston has constant value P_1 .

17. (i) (a, c)



$$\theta \geq c$$

$$\Rightarrow 90^\circ - r \geq c$$

$$\Rightarrow \sin(90^\circ - r) \geq \sin c$$

$$\Rightarrow \cos r \geq \sin c$$

Using $\frac{\sin i}{\sin r} = \frac{n_1}{n_m}$ and $\sin c = \frac{n_2}{n_1}$

We get, $\sin^2 i_m = \frac{n_1^2 - n_2^2}{n_m^2}$

Putting values, we get, correct options as A & C

(ii) (d) For total internal reflection to take place in both structures, the numerical aperture should be the least one for the combined structure & hence, correct option is D.

18. (i) (a, d) $I_1 = I_2$

$$\Rightarrow n_1 A_1 v_1 = n_2 A_2 v_2$$

$$\Rightarrow d_1 w_1 v_1 = d_2 w_2 v_2$$

Now, potential difference developed across MK

$$V = Bvw$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{v_1 w_1}{v_2 w_2} = \frac{d_2}{d_1}$$

& hence correct choice is A & D

(ii) (a, c) As $I_1 = I_2$

$$n_1 w_1 d_1 v_1 = n_2 w_2 d_2 v_2$$

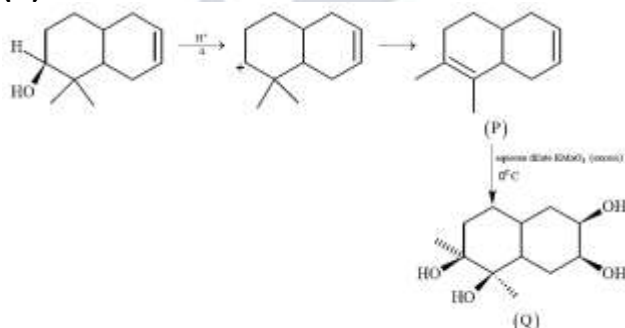
$$\text{Now, } \frac{V_2}{V_1} = \frac{B_2 v_2 w_2}{B_1 v_1 w_1} = \left(\frac{B_2 w_2}{B_1 w_1} \right) \left(\frac{n_1 w_1 d_1}{n_2 w_2 d_2} \right) = \frac{B_2 n_1}{B_1 n_2}$$

∴ Correct options are A & C

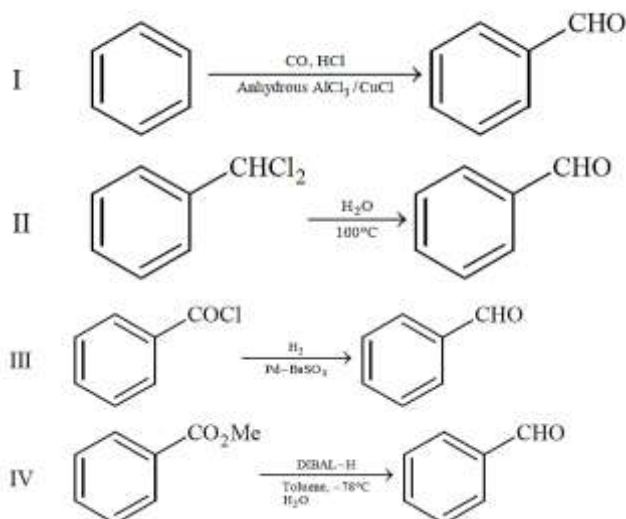
19. (8) $[Fe(C_2O_4)(H_2O)]^{2-} + MnO_4^{2-} + 8H^+ \rightarrow Mn^{2+} + Fe^{3+} + 4CO_2 + 6H_2O$

So, the ratio of rate of change of $[H^+]$ to that of rate of change of $[MnO_4^-]$ is 8.

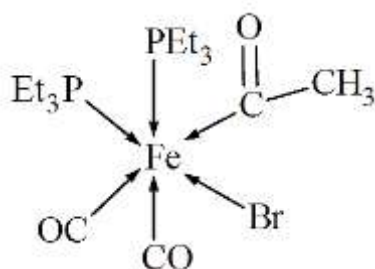
20. (4)



21. (4)



22. (3)



The number of Fe - C bonds is 3.

23. (5) $[Co(en)_2Cl_2]^+$ → will show cis - trans isomerism

$[CrCl_2(C_2O_4)_2]^{3-}$ → will show cis - trans isomerism

$[Fe(H_2O)_4(OH)_2]^+$ → will show cis - trans isomerism

$[Fe(CN)_4(NH_3)_2]^-$ → will show cis - trans isomerism

$[Co(en)_2(NH_3)Cl]^{2+}$ → will show cis - trans isomerism

$[Co(NH_3)_4(H_2O)Cl]^{2+}$ → will not show cis - trans isomerism (Although it will show geometrical isomerism)

24. (6) $B_2H_6 + 6MeOH \rightarrow 2B(OMe)_3 + 6H_2$

1 mole of B_2H_6 reacts with 6 mole of MeOH to give 2 moles of $B(OMe)_3$.

3 mole of B_2H_6 will react with 18 mole of MeOH to give 6 moles of $B(OMe)_3$

25. (3) $HX \rightleftharpoons H^+ + X^-$

$$K_a = \frac{[H^+][X^-]}{[HX]}$$

$HY \rightleftharpoons H^+ + Y^-$

$$K_a = \frac{[H^+][Y^-]}{[HY]}$$

$\wedge_m$ for $HX = \wedge_{m_1}$

$\wedge_m$ for $HY = \wedge_{m_2}$

$$\wedge_{m_1} = \frac{1}{10} \wedge_{m_2}$$

$$K_a = C\alpha^2$$

$$K_{a_1} = C_1 \times \left(\frac{\wedge_{m_1}}{\wedge_{m_1}^0} \right)^2$$

$$K_{a_2} = C_2 \times \left(\frac{\wedge_{m_2}}{\wedge_{m_2}^0} \right)^2$$

$$\frac{K_{a_1}}{K_{a_2}} = \frac{C_1}{C_2} \times \left(\frac{\wedge_{m_1}}{\wedge_{m_2}} \right)^2 = \frac{0.01}{0.1} \times \left(\frac{1}{10} \right)^2 = 0.001$$

$$pK_{a_1} - pK_{a_2} = 3$$

26. (9) In conversion of ${}_{92}^{238}U$ to ${}_{82}^{206}Pb$, 8 α - particles and 6 β particles are ejected.

The number of gaseous moles initially = 1 mol

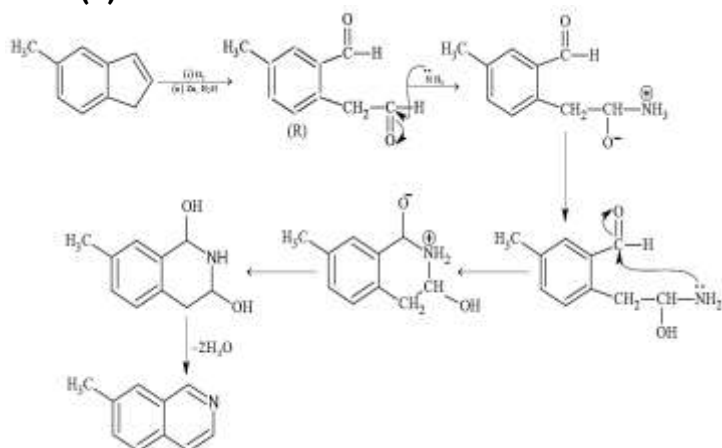
The number of gaseous moles finally = 1 + 8 mol; (1 mol from air and 8 mol of $2He_4$)

So the ratio = 9/1 = 9

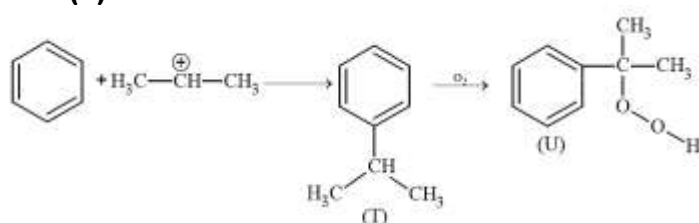
27. (c) At large inter-ionic distances (because $a \rightarrow 0$) the P.E. would remain constant.

However, when $r \rightarrow 0$; repulsion would suddenly increase.

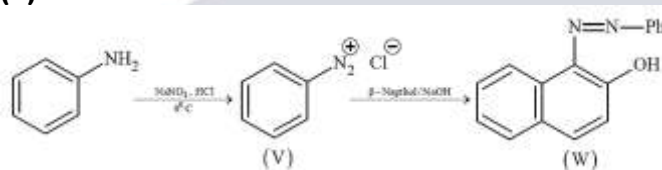
28. (a)



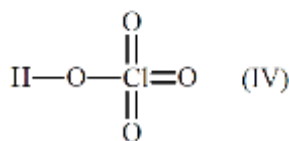
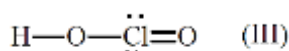
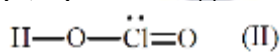
29. (b)



30. (a)

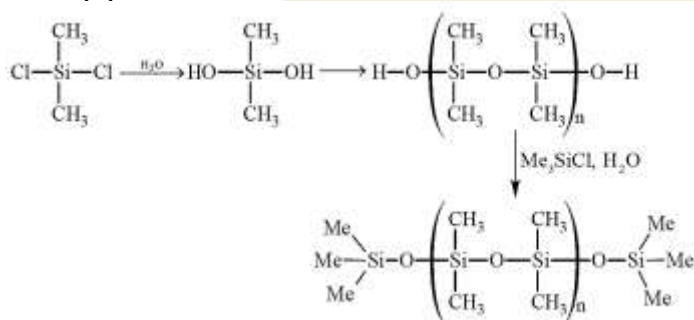


31. (b, c) H-O-Cl (I)



32. (c, d) Cu^{2+} , Pb^{2+} , Hg^{2+} , Bi^{3+} give ppt with H_2S in presence of dilute HCl .

33. (b)



34. (b, c, d) Adsorption of O_2 on metal surface is exothermic.

* During electron transfer from metal to O_2 electron occupies π_{2p}^* orbital of O_2 .

* Due to electron transfer to O_2 the bond order of O_2 decreases hence bond length increases.

35. (i) (a) $2 \text{HCl} + \text{NaOH} \rightarrow \text{NaCl} + \text{H}_2\text{O}$

$n = 100 \times 1 = 100 \text{ m mole} = 0.1 \text{ mole}$

Energy evolved due to neutralization of HCl and NaOH = $0.1 \times 57 = 5.7 \text{ kJ} = 5700 \text{ Joule}$

Energy used to increase temperature of solution = $200 \times 4.2 \times 5.7 = 4788 \text{ Joule}$

Energy used to increase temperature of calorimeter = $5700 - 4788 = 912 \text{ Joule}$

ms. $\Delta t = 912$

m.s $\times 5.7 = 912$

ms = $160 \text{ Joule}/^{\circ}\text{C}$ [Calorimeter constant]

Energy evolved by neutralization of CH_3COOH and NaOH

= $200 \times 4.2 \times 5.6 \times 160 \times 5.6 = 5600 \text{ Joule}$

So energy used in dissociation of 0.1 mole $\text{CH}_3\text{COOH} = 5700 - 5600 = 100 \text{ Joule}$

Enthalpy of dissociation = 1 kJ/mole

(ii) (b) $\text{CH}_3\text{COOH} = \frac{1 \times 100}{200} = \frac{1}{2}$

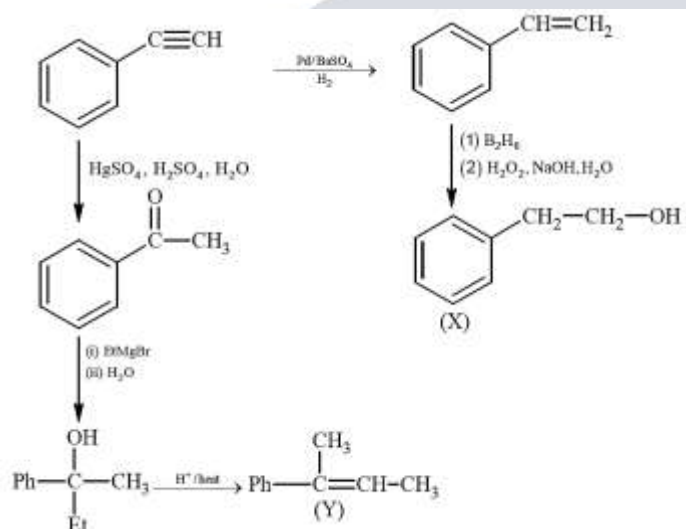
$$\text{CH}_3\text{CONa} = \frac{1 \times 100}{200} = \frac{1}{2}$$

$$\text{pH} = \text{pK}_a + \log \frac{[\text{salt}]}{[\text{acid}]}$$

$$\text{pH} = 5 - \log 2 + \log \frac{1/2}{1/2}$$

$$\text{pH} = 4.7$$

36. (i) (c) $\text{C}_8\text{H}_6 \rightarrow$ double bond equivalent = $8 + 1 - \frac{6}{2} = 6$



(i) (d)

37. (9)

$$\vec{s} = 4\vec{p} + 3\vec{q} + 5\vec{r}$$

$$\vec{s} = x(-\vec{p} + \vec{q} + \vec{r}) + y(\vec{p} - \vec{q} + \vec{r}) + z(-\vec{p} - \vec{q} + \vec{r})$$

$$\vec{s} = (-x + y - z)\vec{p} + (x - y - z)\vec{q} + (x + y + z)\vec{r}$$

$$\Rightarrow -x + y - z = 4$$

$$\Rightarrow x - y - z = 3$$

$$\Rightarrow x + y + z = 5$$

$$\text{On solving we get } x = 4, y = \frac{9}{2}, z = -\frac{7}{2}$$

$$\Rightarrow 2x + y + z = 9$$

38. (4) For any integer k, let $\alpha_k = \cos\left(\frac{k\pi}{7}\right) + i \sin\left(\frac{k\pi}{7}\right)$, where $i = \sqrt{-1}$. The value of the expression

$$\frac{\sum_{k=1}^{12} |\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^3 |\alpha_{4k-1} - \alpha_{4k-2}|} \text{ is}$$

39. (9) Let seventh term be 'a' and common difference be 'd'

$$\text{Given } \frac{S_7}{S_1} = \frac{6}{11} \Rightarrow a = 15d$$

$$\text{Hence, } 130 < 15d < 140$$

$$\Rightarrow d = 9$$

40. (8) x^9 can be formed in 8 ways

i.e. x^9 , x^{1+8} , x^{2+7} , x^{3+6} , x^{4+5} , x^{1+2+6} , x^{1+3+5} , x^{2+3+4} and coefficient in each case is 1

$$\Rightarrow \text{Coefficient of } x^9 = 1 + 1 + 1 + \dots (8 \text{ times}) + 1 = 8$$

41. (4) The equation of P_1 is $y^2 - 8x = 0$ and P_2 is $y^2 + 16x = 0$

Tangent to $y^2 - 8x = 0$ passes through $(-4, 0)$

$$\Rightarrow 0 = m_1(-4) + \frac{2}{m_1} \Rightarrow \frac{1}{m_1^2} = 2$$

Also, tangent to $y^2 + 16x = 0$ passes through $(2, 0)$

$$\Rightarrow 0 = m_2 \times 2 - \frac{4}{m_2} \Rightarrow m_2^2 = 2$$

$$\Rightarrow \frac{1}{m_1^2} + m_2^2 = 4$$

42. (2) $\lim_{\alpha \rightarrow 0} \frac{e^{\cos(\alpha^n)} - e}{\alpha^m} = -\frac{e}{2}$

$$\lim_{\alpha \rightarrow 0} \frac{e \left(e^{(\cos(\alpha^n) - 1)} - 1 \right) (\cos \alpha^n - 1)}{(\cos(\alpha^n) - 1) \alpha^m \alpha^{2n}} \alpha^{2n} = -\frac{e}{2} \text{ if and only if } 2n - m = 0$$

43. (9) $\alpha = \int_0^1 e^{(9x+3\tan^{-1}x)} \left(\frac{12+9x^2}{1+x^2} \right) dx$

Put $9x + 3 \tan^{-1} x = t$

$$\Rightarrow \left(9 + \frac{3}{1+x^2} \right) dx = dt$$

$$\Rightarrow \alpha = \int_0^{9+\frac{3\pi}{4}} e^t dt = e^{9+\frac{3\pi}{4}} - 1$$

$$\Rightarrow \left(\log_e |1 + \alpha| - \frac{3\pi}{4} \right) = 9$$

44. (7) $G(1) = \int_{-1}^1 t |f(f(t))| dt = 0$

$f(-x) = -f(x)$

$$\text{Given } f(1) = \frac{1}{2}$$

$$\lim_{x \rightarrow 1} \frac{F(x)}{G(x)} = \lim_{x \rightarrow 1} \frac{\frac{F(x) - F(1)}{x - 1}}{\frac{G(x) - G(1)}{x - 1}} = \frac{f(1)}{|f(f(1))|} = \frac{1}{14}$$

$$\Rightarrow \frac{1/2}{|f(1/2)|} = \frac{1}{14}$$

$$\Rightarrow f\left(\frac{1}{2}\right) = 7$$

45. (d) $\frac{192}{3} \int_{1/2}^x t^3 dt \leq f(x) \leq \frac{192}{2} \int_{1/2}^x t^3 dt$

$$16x^4 - 1 \leq f(x) \leq 24x^4 - \frac{3}{2}$$

$$\int_{1/2}^1 (16x^4 - 1) dx \leq \int_{1/2}^1 f(x) dx \leq \int_{1/2}^1 \left(24x^4 - \frac{3}{2}\right) dx$$

$$1 < \frac{26}{10} \leq \int_{1/2}^1 f(x) dx \leq \frac{39}{10} < 12$$

46. (a, d) Here, $0 < (x_1 - x_2)^2 < 1$

$$\Rightarrow 0 < (x_1 + x_2)^2 - 4x_1x_2 < 1$$

$$\Rightarrow 0 < \frac{1}{\alpha^2} - 4 < 1$$

$$\Rightarrow \alpha \in \left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$$

47. (b, c) $\frac{\pi}{2} < \alpha < \pi, \pi < \beta < \frac{3\pi}{2} \Rightarrow \frac{3\pi}{2} < \alpha + \beta < \frac{5\pi}{2}$

$$\Rightarrow \sin \beta < 0; \cos \alpha < 0$$

$$\Rightarrow \cos(\alpha + \beta) > 0$$

48. (a, b) For the given line, point of contact for $E_1: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\left(\frac{a^2}{3}, \frac{b^2}{3}\right)$

and for $E_2: \frac{x^2}{B^2} + \frac{y^2}{A^2} = 1$ is $\left(\frac{B^2}{3}, \frac{A^2}{3}\right)$

Point of contact of $x + y = 3$ and circle is $(1, 2)$

Also, general point on $x + y = 3$ can be taken as $\left(1 \mp \frac{r}{\sqrt{2}}, 2 \pm \frac{r}{\sqrt{2}}\right)$ where, $r = \frac{2\sqrt{2}}{3}$

So, required points are $\left(\frac{1}{8}, \frac{8}{3}\right)$ and $\left(\frac{5}{3}, \frac{4}{3}\right)$

Comparing with points of contact of ellipse,

$$a^2 = 5, B^2 = 8$$

$$b^2 = 4, A^2 = 1$$

$$\therefore e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{10}} \text{ and } e_1^2 + e_2^2 = \frac{43}{40}$$

49. (a, b, d) Tangent at P, $xx_1 - yy_1 = 1$ intersects x axis at $M\left(\frac{1}{x_1}, 0\right)$

$$\text{Slope of normal} = -\frac{y_1}{x_1} = \frac{y_1 - 0}{x_1 - x_2}$$

$$\Rightarrow x_2 = 2x_1 \Rightarrow N \equiv (2x_1, 0)$$

$$\text{For centroid } l = \frac{3x_1 + \frac{1}{x_1}}{3}, m = \frac{y_1}{3}$$

$$\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}$$

$$\frac{dm}{dy_1} = \frac{1}{3}, \frac{dm}{dx_1} = \frac{1}{3} \frac{dy_1}{dx_1} = \frac{x_1}{3\sqrt{x_1^2 - 1}}$$

50. (a, c) Let $\int_0^\pi e^t (\sin^6 at + \cos^4 at) dt = A$

$$I = \int_\pi^{2\pi} e^t (\sin^6 at + \cos^4 at) dt$$

Put $t = \pi + x$

$dt = dx$

for $a = 2$ as well as $a = 4$

$$I = e^\pi \int_0^\pi e^x (\sin^6 ax + \cos^4 ax) dx$$

$$I = e^\pi A$$

Similarly, $\int_{2\pi}^{3\pi} e^t (\sin^6 at + \cos^4 at) dt = e^{2\pi} A$

$$\text{So, } L = \frac{A + e^\pi A + e^{2\pi} A + e^{3\pi} A}{A} = \frac{e^{4\pi} - 1}{e^\pi - 1}$$

For both $a = 2, 4$

51. (b, c) Let $H(x) = f(x) - 3g(x)$

$H(-1) = H(0) = H(2) = 3$.

Applying Rolle's Theorem in the interval $[-1, 0]$

$H'(x) = f'(x) - 3g'(x) = 0$ for at least one $c \in (-1, 0)$.

As $H''(x)$ never vanishes in the interval

$\Rightarrow$ Exactly one $c \in (-1, 0)$ for which $H'(x) = 0$

Similarly, apply Rolle's Theorem in the interval $[0, 2]$.

$\Rightarrow H'(x) = 0$ has exactly one solution in $(0, 2)$

52. (a, b) $f(x) = (7 \tan^6 x - 3 \tan^2 x)(\tan^2 x + 1)$

$$\int_0^{\pi/4} f(x) dx = \int_0^{\pi/4} (7 \tan^6 x - 3 \tan^2 x) \sec^2 x dx$$

$$\Rightarrow \int_0^{\pi/4} f(x) dx = 0$$

$$\int_0^{\pi/4} xf(x) dx = \left[x \int f(x) dx \right]_0^{\pi/4} - \int_0^{\pi/4} \left[\int f(x) dx \right] dx$$

$$\int_0^{\pi/4} xf(x) dx = \frac{1}{12}$$

53. (i) (a, b, c) (a) $f'(x) = F(x) + xF'(x)$

$f(1) = F(1) + F'(1)$

$f'(1) = F'(1) < 0$

$f(1) < 0$

(b) $f(2) = 2F(2)$

$F(x)$ is decreasing and $F(1) = 0$

Hence $F(2) < 0$

$\Rightarrow f(2) < 0$

(c) $f'(x) = F(x) + xF'(x)$

$F(x) < 0 \quad \forall x \in (1, 3)$

$F'(x) < 0 \quad \forall x \in (1, 3)$

Hence $f'(x) < 0 \quad \forall x \in (1, 3)$

(ii) (c, d) $\int_1^3 f(x) dx = \int_1^3 xF(x) dx$

$$= \left[\frac{x^2}{2} F(x) \right]_1^3 - \frac{1}{2} \int_1^3 x^2 F'(x) dx$$

$$= \frac{9}{2} F(3) - \frac{1}{2} F(1) + 6 = -12$$

$$40 = \left[x^3 F'(x) \right]_1^3 - 3 \int_1^3 x^2 F'(x) dx$$

$$40 = 27F'(3) - F'(1) + 36 \dots (i)$$

$$f(x) = F(x) + xF'(x)$$

$$f(3) = F(3) + 3F'(3)$$

$$f(1) = F(1) + F'(1)$$

$$9f(3) - f(1) + 32 = 0.$$

54. (i) (a, b) $P(\text{Red Ball}) = P(I) \cdot P(R | I) + P(II) \cdot P(R | II)$

$$P(II | R) = \frac{1}{3} = \frac{P(II) \cdot P(R | II)}{P(I) \cdot P(R | I) + P(II) \cdot P(R | II)}$$

$$= \frac{1}{3} = \frac{\frac{n_3}{n_3 + n_4}}{\frac{n_1}{n_1 + n_2} + \frac{n_3}{n_3 + n_4}}$$

Of the given options, A and B satisfy above condition

(ii) (c, d) $P(\text{Red after Transfer}) = P(\text{Red Transfer}) \cdot P(\text{Red Transfer in II Case}) + P(\text{Black Transfer}) \cdot P(\text{Red Transfer in II Case})$

$$P(R) = \frac{n_1}{n_1 + n_2} \cdot \frac{(n_1 - 1)}{(n_1 + n_2 - 1)} + \frac{n_2}{n_1 + n_2} \cdot \frac{n_1}{n_1 + n_2 - 1} = \frac{1}{3}$$

Of the given options, option C and D satisfy above condition.