1. (b)

$$eV = \left[\frac{hc}{\lambda} - \phi\right]$$

$$e(V_1 - V_2) = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = \frac{hc(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}$$

$$h = \frac{e(V_1 - V_2)\lambda_1\lambda_2}{c(\lambda_2 - \lambda_1)}$$

$$= \frac{1.6\times1\times0.3\times0.4\times10^{-12}\times10^{-19}}{3\times10^{8}\times0.1\times10^{-6}} = 6.4\times10^{-34}$$

Alternate Solution

Slope of V_0 vs $\frac{1}{\lambda} = \frac{hc}{e}$

From first two data points.

Slope = $\left(\frac{2.0-1.0}{\frac{1}{0.4}-\frac{1}{0.3}}\right)\times10^{-6}\,Vm = \frac{hc}{e}$

$$\Rightarrow 1.2\times10^{-6} = \frac{h\times3\times10^{8}}{1.6\times10^{-19}}$$

$$\Rightarrow 6.4\times10^{-34}$$

We get same value from second and third data points.

2. None On the basis of given data, it is not possible for the rod to stay at rest.

Note: If we take the normal reaction at the wall equal to the normal reaction at the floor, the answer will be D.

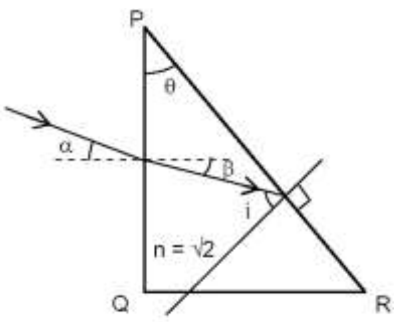
3. (b)

$(P_{heater} - P_{cooler})$x t = ms ΔT.

$\Rightarrow$ ($3\text{x}10^3$ - P) x 3 x 3600 = 120 x 4.2 x 10^3 x 20

$\Rightarrow$ P= 2067

4. (a)



$$i = \beta + \theta$$

For α = 45°; by Snell's law,

1 x sin 45° = $\sqrt{2}$ sin β

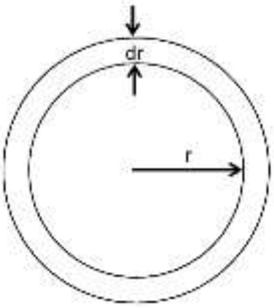
$$\Rightarrow \beta = 30^0$$

For TIR on face PR,

$$\beta + \theta = \theta_c = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^0$$

$$\Rightarrow \theta = 45^0 - \beta = 15^0$$

5. (c)



For infinite line,

$$E = \frac{\lambda}{2\pi\varepsilon r}$$

$$\Rightarrow dV = \frac{-\lambda}{2\pi\varepsilon r} dr$$

Current through an elemental shell;

$$I = \frac{|dV|}{dR} = \frac{\frac{\lambda}{2\pi\varepsilon r} dr}{\frac{1}{\sigma} \times \frac{dr}{2\pi r l}} = \frac{\lambda \sigma l}{\varepsilon}$$

This current is radially outwards so;

$$\frac{d}{dt}(\lambda l) = \frac{-\lambda \sigma l}{\varepsilon} \Rightarrow \frac{d\lambda}{\lambda} = -\left(\frac{\sigma}{\varepsilon}\right) dt$$

$$\Rightarrow \lambda = \lambda_0 e^{-(\sigma/\varepsilon)t}$$

So, $j = \frac{I}{2\pi r l} = \frac{\lambda \sigma}{2\pi\varepsilon r} = \left(\frac{\lambda_0 \sigma}{2\pi\varepsilon r}\right) e^{-(\sigma/\varepsilon)t}$

6. (a, b, d)

$$r_n = \frac{n^2}{Z} a_0$$

$$\Delta r_n = \frac{2na_0}{Z}$$

$$\frac{\Delta r_n}{r_n} = \frac{2}{n}$$

$$E_n = \frac{-13.6Z^2}{n^2}$$

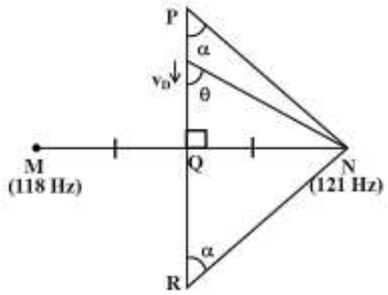
$$\Delta E_n = \frac{13.6 \times 2 \times Z^2}{n^3}$$

so, $\frac{\Delta E_n}{E_n} = -\frac{2}{n}$

$$L_n = \frac{nh}{2\pi}$$

$$\frac{\Delta L_n}{L_n} = \frac{1}{n}$$

7. (a, b, c)



$$v_P = (121-118)\left(\frac{v+v_D\cos\alpha}{v}\right) = 3\left(\frac{v+v_D\cos\alpha}{v}\right)Hz$$

$$v_Q = 121-118 = 3Hz$$

$$v_R = (121-118)\left(\frac{v-v_D\cos\alpha}{v}\right) = 3\left(\frac{v-v_D\cos\alpha}{v}\right)$$

$$\therefore v_p + v_R = 2v_Q$$

Now, v(between P and Q) $= 3\left(\dfrac{v+v_D\cos\theta}{v}\right)$

$$\frac{dv}{d\theta} = -\frac{3v_D}{v}\sin\theta$$

Now, sin θ =1 at Q (maximum)

∴ rate of change in beat frequency is maximum at Q

8. (c, d)

When filament breaks up, the temperature of filament will be higher so according to wein's law $\left(\lambda_m \propto \dfrac{1}{T}, v_m \propto T\right)$ the filament emits more light at higher band of frequencies. As voltage is constant, so consumed electrical power is $P = \dfrac{V^2}{R}$

As R increases with increase in temperature so the filament consumes less electrical power towards the end of the life of the bulb.

9. (a, d)

For refraction through lens,

$\dfrac{1}{v} - \dfrac{1}{-30} = \dfrac{1}{f}$ and $-2 = \dfrac{v}{u}$

∴ v = -2u = 60 cm

∴ f=+20cm

For reflection

$$\frac{1}{10} + \frac{1}{-30} = \frac{2}{R} \Rightarrow R = 30cm$$

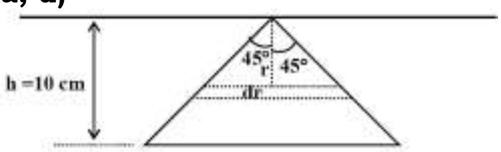
$$(n-1)\left(\frac{1}{R}\right) = \frac{1}{f} = \frac{1}{20}$$

$$\therefore n = \frac{5}{2}$$

10. (b, d)

$k_BT \approx FL, \varepsilon \approx \dfrac{Q^2}{FL^2}$ and $n \approx L^{-3}$; where F is force, Q is charge and L is length

11. (a, d)



$$\phi_{lw} = \int_0^h \frac{\mu_0 I}{2\pi r} 2r dr = \frac{\mu_0 I h}{\pi}$$

So, Mutual inductance $M_{lw} = \frac{\mu_0 h}{\pi}$

$$\therefore \varepsilon_w = \frac{\mu_0 h}{\pi}\frac{di}{dt} = \frac{\mu_0}{\pi}$$

Due to rotation there is no change in flux through the wire, so there is no extra induced emf in the wire. From Lenz's Law, current in the wire is rightward so repulsive force acts between the wire and loop.

12. (a, b, d)

$$\vec{v} = \frac{d\vec{r}(t)}{dt} = 3\alpha t^2 \hat{i} + 2\beta t j, \vec{a} = \frac{d\vec{v}}{dt} = 6\alpha t\hat{i} + 2\beta j$$

At t = 1 s, $\vec{v} = \left(10\hat{i} + 10j\right)ms^{-1}$

$$\vec{a} = 20\hat{i} + 10 jms^{-2}$$

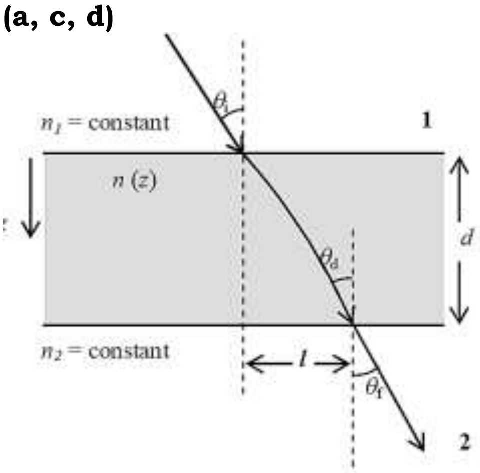
$$\vec{r} = \frac{10}{3}\hat{i} + 5jm$$

$$\vec{L}_0 = \vec{r} \times m\vec{v} = \left(-\frac{5}{3}k\right)Nms$$

$$\vec{F} = m\frac{d\vec{v}}{dt} = \left(2\hat{i} + j\right)N$$

$$\vec{\tau}_0 = \vec{r} \times \vec{F} = \vec{r} \times m\vec{a} = \left(-\frac{20}{3}k\right)Nm$$

13. (a, c, d)



From Snell's Law

$n_1 \sin\theta_i = n(d)\sin\theta_d = n_2 \sin\theta_f$

The deviation of ray in the slab will depend on n (z)

Hence, l will depend on n (z) but not on n_2.

14. (9)

Power radiated $P = e\sigma AT^4$

At 487°C; $P_1 = e\sigma A(760)^4 \ldots$(i)

Given, $\log_2 \frac{P_1}{P_0} = 1 \Rightarrow P_0 = \frac{P_1}{2}$

At 2767°C; $P_2 = e\sigma A(3040)^4$

$$\therefore \text{Reading } \log_2\left(\frac{P_2}{P_0}\right) = \log_2\left[\frac{e\sigma A(3040)^4 \times 2}{e\sigma A(760)^4}\right] = \log_2\left(4^4 \times 2\right) = 9$$

15. (9)

Q value = $\left[12.014u - \left(12u + 4.041\frac{MeV}{c^2}\right)\right]c^2$

e = (0.014ux931.5) MeV-(4.041)MeV =9MeV

Hence, β particle will have a maximum KE of 9 MeV

16. (6)

Note: Unit of $\frac{hc}{e}$ in original paper is incorrect

Energy available $E = \frac{hc}{\lambda} = \frac{1.237\times10^{-6}\,eVm}{970\times10^{-10}\,m} = 12.75eV$ (4^{th} energy level of hydrogen atom)

17. (3)

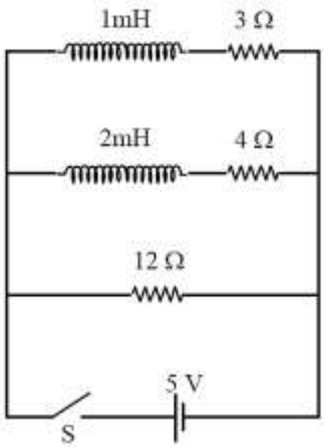
Terminal velocity $v_T = \frac{2}{9}\frac{r^2}{\eta}(\rho-\sigma)g$, where ρ is the density of the solid sphere and σ is the density of the liquid

$$\therefore \frac{v_P}{v_Q} = \frac{(8-0.8)\times\left(\frac{1}{2}\right)^2\times2}{(8-1.6)\times\left(\frac{1}{4}\right)^2\times3}$$

18. (8)

At t = 0, current will flow only in 12 Ω resistance

$I_{\min} = \frac{5}{12}$

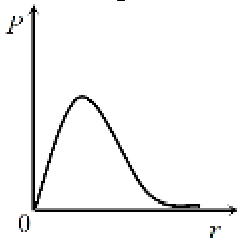


At $t\to\infty$ both L_1 and L_2 behave as conducting wires

$\therefore R_{eff} = \frac{3}{2}$

$I_{\max} = \frac{10}{3}$

19. (d) The probability distribution curve for Is electron of hydrogen atom.



20. (c)

Isothermal process, $\Delta U = 0$

dq = -dW = P_{ext} ($V_2 - V_1$) = 3 L - atm = 3 x 101.3 Joule

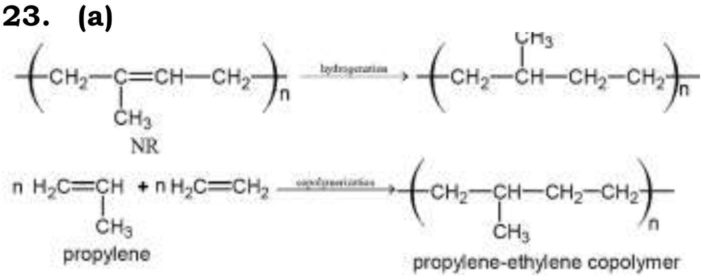
$\Delta S_{surrounding} = -\frac{3\times101.3}{300}$ JouleK^{-1} = -1.013 Joule K^{-1}

$\therefore \Delta\ S_{surr}$ = -1.013 Joule K^{-1}

21. **(b)** Increasing order of atomic radius of group 13 elements Ga < Al < In < Tl. Due to poor shielding of d-electrons in Ga, its radius decreases below Al.

22. **(b)** Number of paramagnetic compounds are 3.
Following compounds are paramagnetic.
$[NiCl_4]^{2-}$,$Na_3[CoF_6]$,CsO_2

23. **(a)**



24. **(b, c, d)**

25. **(b, d)**

$\frac{n}{p} < 1$

Positron, emission

${}_aX^b \rightarrow {}^0_{+1}e + {}_{a-1}Y^b$

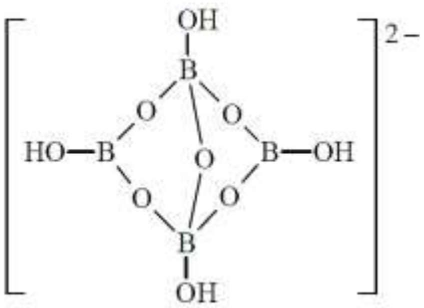
$\Rightarrow$K - electron capture

${}_aX^b + {}_{-1}e^0 \rightarrow {}_{a-1}Y^b$

Both process cause an increase in n/p ratio towards 1 thus stabilizing the nucleus.

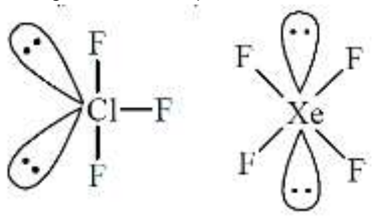
26. **(a, c, d)**

The structure of anion of borax is



27. **(b, c)**

C1F$_3$ and XeF$_4$ contain two lone pair of electrons on the central atom.

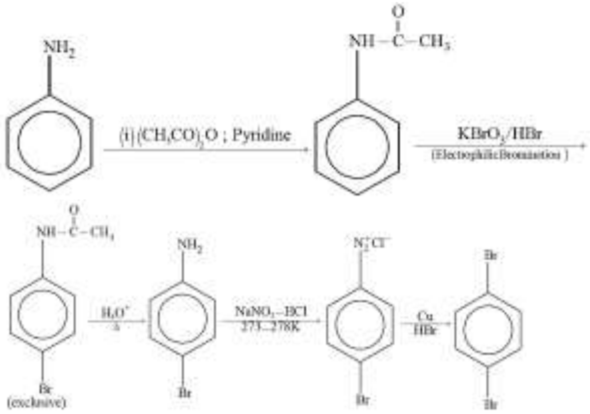


28. **(a)** The reagent that can selectively precipitate S^{2-} and SO_4^{2-} in aqueous solution is $CuCl_2$.

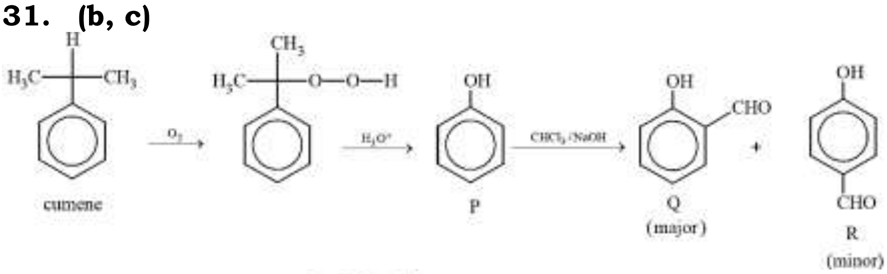
$S^{2-} + CuCl_2 \rightarrow CuS\downarrow + 2Cl^-$

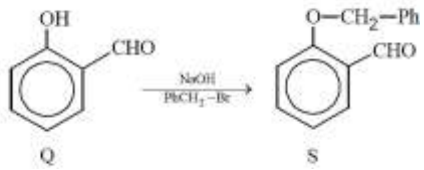
29. **(a, b)** Beside aldehyde α-hydroxy ketones can also show Tollen's test due to rearrangement in aldehyde via ene diol intermediate, (however this needs a terminal α-carbon)

30. **(b)**



31. (b, c)





Q (not R) is steam volatile due to intramolecular hydrogen bonding.
Q gives violet colouration due to phenolic functional group.
S gives yellow precipitate due to aldehydic group with 2, 4 -DNP.
S does not have free phenolic group to respond to $FeCl_3$ test.

32. (9)

$$m = \frac{X_A \times 1000}{X_B \times M_A}$$

$$m = \frac{1000}{9M_A} \quad \ldots \text{(i)}$$

$$M = \frac{n_B \times 1000 \times d}{n_A \times M_A + n_B \times M_B} = \frac{X_B \times 1000 \times d}{X_A \times M_A + X_B \times M_B}$$

$$= \frac{200}{0.9M_A + 0.1M_B}$$

$$\Rightarrow \frac{2000}{9M_A + M_B} \ldots\text{(ii)}$$

As m=M

$$\frac{1000}{9M_A} = \frac{2000}{9M_A + M_B}$$

$$9M_A + M_B = 18M_A$$

$$\therefore 9M_A = M_B$$

$$\therefore \frac{M_B}{M_A} = 9$$

33. (4)

Mean free path $(l) \propto \dfrac{T}{P}$

Mean speed (C_{av}) $\propto \sqrt{T}$

Diffusion coefficient (d) is proportional to both mean free path (l) and mean speed.

$\therefore D \propto \dfrac{T^{3/2}}{P}$

At temperature T_1 and P_1

$D_1 = \dfrac{KT_1^{3/2}}{P_1}$ ...(1)

If T_1 is increased 4 times and P_1 is increased 2 times.

$D_2 = \dfrac{K(4T_1)^{3/2}}{2P_1}$ ...(2)

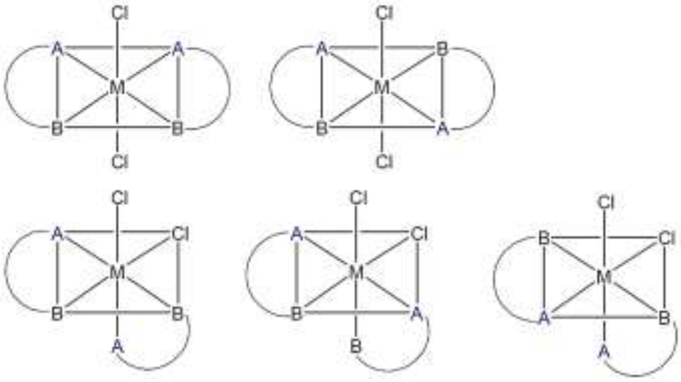
$\dfrac{D_2}{D_1} = \dfrac{(4)^{3/2}}{2} = \dfrac{2^3}{2} = 4$

34. **(6)**

$8MnO_4^- + 3S_2O_3^{2-} + H_2O \xrightarrow{\text{faintly alkaline}} 8MnO_2 + 6SO_4^{2-} + 2OH^-$

$\therefore$ 8 moles MnO_4^- produce 6 moles SO_4^{2-}.

35. **(5)** Number of geometrical isomers = 5.

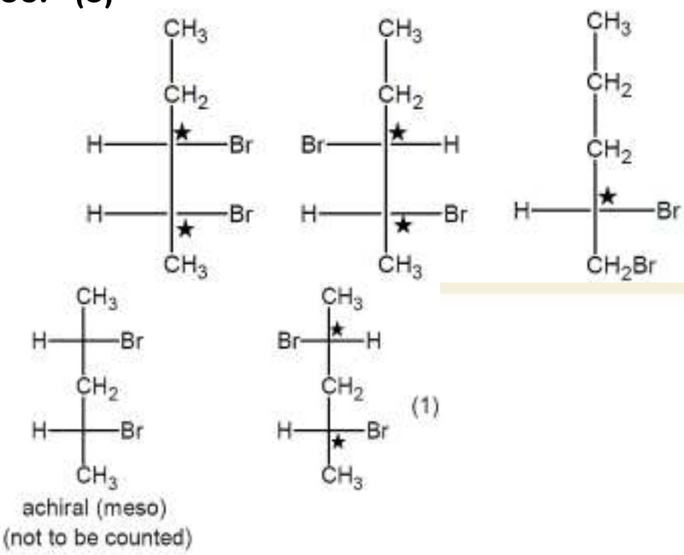


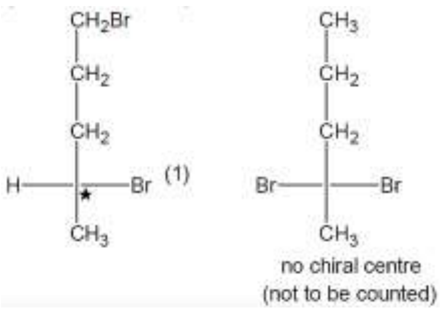
$AB = H_2NCH_2CH_2O^-$

A site is N

B site is O

36. **(5)**





37. (c)

$\left(\alpha_1,\beta_1\right)=\sec\theta\pm\tan\theta$

Since $\alpha_1>\beta_1$

$\alpha_1=\sec\theta-\tan\theta$

$\beta_1=\sec\theta+\tan\theta$

$\left(\alpha_2,\beta_2\right)=-\tan\theta\pm\sec\theta$

Since $\alpha_2>\beta_2$

$\alpha_2=-\tan\theta+\sec\theta$

$\beta_2=-\tan\theta-\sec\theta$

38. (a)

If a boy is selected then number of ways = ${}^4C_1.{}^6C_3$

If a boy is not selected then number of ways = ${}^6C_4$

Captain can be selected in ${}^4C_1$ ways

Required number of ways = ${}^4C_1.{}^6C_3$-${}^4C_1$ + ${}^6C_4.{}^4C_1$ = 380

39. (c)

$\left(\sqrt{3}\sin x+\cos x\right)=2\cos 2x$

$\cos\left(x-\frac{\pi}{3}\right)=\cos 2x$

$x-\frac{\pi}{3}=2n\pi\pm 2x$

$\Rightarrow x=-\frac{\pi}{3},-\frac{5\pi}{9},\frac{\pi}{9},\frac{7\pi}{9}$

$\Rightarrow\sum x_i=0$

40. (c)

E_1: Computer is produced by plant T_1

E_2: Computer is produced by plant T_2

A : Computer is defective

Now, $P(A/E_1)$ = $10P(A/E_2)$

$\Rightarrow\frac{P\left(A\cap E_1\right)}{P\left(A\cap E_1\right)}=\frac{5}{2}$

Let $P\left(E_2\cap\overline{A}\right)=\frac{x}{100}$

$\Rightarrow P\left(E_2\cap A\right)=\frac{80-x}{100}$

$P\left(E_1\cap A\right)=\frac{x-73}{100}$

$\Rightarrow x=78$

$\Rightarrow P\left(E_2/\overline{A}\right)=\frac{P\left(E_2\cap\overline{A}\right)}{P\left(\overline{A}\right)}=\frac{78}{93}$

41. (c)

$4\alpha x^2+\frac{1}{x}\geq 1, x>0$

Let $f(x)=4\alpha x^2+\frac{1}{x}\Rightarrow f'(x)=8\alpha x-\frac{1}{x^2}$

$f'(x)=0$ at $x=\frac{1}{2\alpha^{1/3}}$

$3\alpha^{1/3}=1\Rightarrow \alpha=\frac{1}{27}$

Alternate

$\frac{4\alpha x^2+\frac{1}{2x}+\frac{1}{2x}}{3}\geq(\alpha)^{1/3}$

$\Rightarrow 4\alpha x^2+\frac{1}{x}\geq 3(\alpha)^{1/3}$

$\Rightarrow 3\alpha^{1/3}=1$

$\alpha=\frac{1}{27}$

42. (b, c, d)

Points O, P, Q, R, S are (0, 0, 0), (3, 0, 0), (3, 3, 0) (0, 3, 0), $\left(\frac{3}{2},\frac{3}{2},3\right)$ respectively.

$\Rightarrow$ Angle between OQ and OS is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Equation of plane containing the points O, Q and S is x - y = 0

$\Rightarrow$ Perpendicular distance from P(3, 0, 0) to the plane

$x-y=0$ is $\left|\frac{3-0}{\sqrt{2}}\right|=\frac{3}{\sqrt{2}}$

Perpendicular distance from O(0, 0, 0) to the line RS:

$\frac{x}{1}=\frac{y-3}{-1}=\frac{z}{2}$ is $\sqrt{\frac{15}{2}}$

43. (a)

$f'(x)+\frac{1}{x}f(x)=2\forall x\in(0,\infty)$

$\Rightarrow f(x)=x+\frac{c}{x}, c\neq 0$ as $f(1)\neq 1$

(a) $\lim\limits_{x\to 0^+} f'\left(\frac{1}{x}\right)=\lim\limits_{x\to 0^+}\left(2-1-cx^2\right)=1$

(b) $\lim\limits_{x\to 0^+}\left(xf\left(\frac{1}{x}\right)\right)=\lim\limits_{x\to 0^+}\left(1+cx^2\right)=1$

(c) $\lim\limits_{x\to 0^+} x^2 f'(x)=\lim\limits_{x\to 0^+}\left(x^2-c\right)=-c\neq 0$

(d) for $c\neq 0$ f(x) is unbounded function for x $\in$ (0, 2)

44. (b, c)

$P\left(\frac{Q}{k}\right)=I$

$P^{-1}=\left(\frac{Q}{k}\right)$

$$\left(P^{-1}\right)_{23}=\frac{q_{23}}{k}=-\frac{1}{8}$$

$$-\frac{(3\alpha+4)}{20+12\alpha}=-\frac{1}{8}\alpha=-1$$

$$\Rightarrow \det(P)=20+12\alpha=8$$

$$(\det P)\left(\det\left(\frac{Q}{k}\right)\right)=1$$

$$\frac{8\det(Q)}{k^3}=1\Rightarrow |Q|=\frac{k^3}{8}$$

$$\Rightarrow \frac{k^3}{8}=\frac{k^2}{2}\Rightarrow k=4\Rightarrow \det(Q)=8$$

$$\det(P.adjQ)=\det P.\det adjQ$$

$$=\det P(\det Q)^2=8\times 8^2=2^9$$

$$\det Q.adjP=\det Q(\det P)^2=8\times 8^2=2^9$$

Alternate

$$|P|.|Q|=k^3\Rightarrow |P|=2k$$

$$\Rightarrow 6\alpha+10=k\ .....(1)$$

Also PQ = kI

|P|Q = k adj(P)

2kQ = k adj(P)

Comparing q_{23} we get

$$-\frac{k}{4}=-3\alpha-4\ \ .....(2)$$

Solving (1) and (2) we get α = -1 and k = 4

45. (a, c, d)

$$\frac{s-x}{4}=\frac{s-y}{3}=\frac{s-z}{2}=\frac{s}{9}$$

$$\Rightarrow \Delta=\frac{2\sqrt{6}}{27}s^2 \text{ and inradius } r=\frac{2\sqrt{6}}{27}s$$

$$\Rightarrow s=9, x=5, y=6, z=7$$

$$R=\frac{35}{24}\sqrt{6}$$

$$r=4R\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2}$$

$$\Rightarrow \sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2}=\frac{4}{35}$$

$$\sin^2\left(\frac{X+Y}{2}\right)=\frac{1}{2}(1-\cos(X+Y))$$

$$=\frac{1}{2}(1+\cos Z)=\frac{1}{2}\left(1+\frac{1}{5}\right)=\frac{3}{5}$$

46. (a, d)

$$\left[(x+2)^2+y(x+2)\right]\frac{dy}{dx}-y^2=0$$

$$\frac{dy}{y}=\frac{yd(x+2)-(x+2)dy}{(x+2)^2}$$

$\Rightarrow \ln y = -\dfrac{y}{x+2} + c, c = 1 + \ln 3$ since y(1) = 3

$\Rightarrow \ln\left(\dfrac{y}{3}\right) + \dfrac{y}{x+2} = 1$

For y = x + 2, $\ln\left(\dfrac{x+2}{3}\right) = 0$

$\Rightarrow$ x = 1 only

For y = $(x + 2)^2$

$\ln\left(\dfrac{(x+2)^2}{3}\right) + (x+2) = 1$

$\dfrac{(x+2)^2}{3} > \dfrac{4}{3} > 1 \ \forall x > 0$

$\Rightarrow \ln\left(\dfrac{(x+2)^2}{3}\right) + (x+2) > 2 \ \forall x > 0$

47. (b, c)

g'(f(x))-f (x) = 1

$\Rightarrow g'(2) = \dfrac{1}{3}$

h(x) = f(f(x))

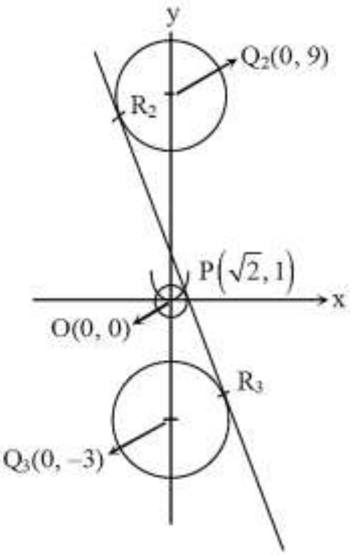
$\Rightarrow$h(0)=16

h(x) = f(f(x))

$\Rightarrow$h'(x) = f(f(x))f(x)

$\Rightarrow$h'(l) = f(f(1))-f (1) = 111 x 6 = 666

48. (a, b, c)



Equation of tangent at $P(\sqrt{2},1)$ is $\sqrt{2}x + y - 3 = 0$

If center of C_2 at (0, α) and radius equal to $2\sqrt{3}$

$2\sqrt{3} = \left|\dfrac{\alpha - 3}{\sqrt{3}}\right| \Rightarrow \alpha = -3, 9$

(a) Q_2Q_3 = 12

(b) R_2R_3 = length of transverse common tangent

$= \sqrt{(Q_2Q_3)^2 - (r_1 + r_2)^2} = \sqrt{(12)^2 - (2\sqrt{3} + 2\sqrt{3})^2} = 4\sqrt{6}$

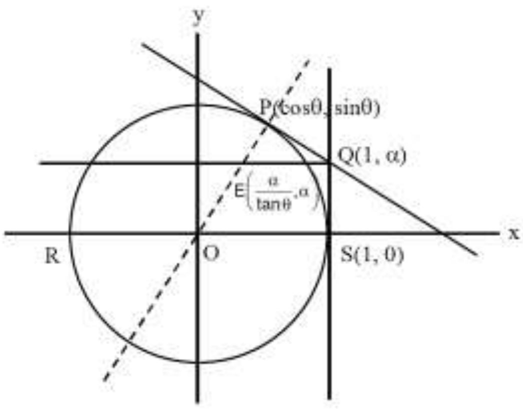
(c) Area of ΔOR_2R_3

$= \frac{1}{2}$ x R_2R_3 x perpendicular distance of O from line

$= \frac{1}{2} \times 4\sqrt{6} \times \sqrt{3} = 6\sqrt{2}$

(d) Area of ΔPQ_2Q_3 = $\frac{1}{2}$x12x $\sqrt{2}$ = $6\sqrt{2}$

49. (a, c)



$$E \equiv \left[\frac{\alpha}{\tan\theta}, \alpha\right]$$

$$\cos\theta + \alpha\sin\theta = 1$$

$$\alpha = \tan\frac{\theta}{2}$$

$\therefore$ locus is $y^2 = 1 - 2x$

50. (2)

$$x^3\begin{vmatrix}1 & 1 & 1\\2 & 4 & 1\\3 & 9 & 1\end{vmatrix} + x^6\begin{vmatrix}1 & 1 & 1\\2 & 4 & 8\\3 & 9 & 27\end{vmatrix} = 10$$

$$x^3\begin{vmatrix}1 & 0 & 0\\2 & 2 & -1\\3 & 6 & -2\end{vmatrix} + x^6\begin{vmatrix}1 & 0 & 0\\2 & 2 & 6\\3 & 6 & 24\end{vmatrix}$$

$$x^3(-4+6) + x^6(48-36) = 10$$

$$2x^3 + 12x^6 = 10$$

$$6x^6 + x^3 - 5 = 0$$

$$6x^6 + 6x^3 - 5x^3 - 5 = 0$$

$$(6x^3 - 5)(x^3 + 1) = 0$$

$$x^3 = \frac{5}{6}, x^3 = -1 \Rightarrow x = \left(\frac{5}{6}\right)^{1/3}, x = -1$$

51. (5)

Here (3n + 1) $^{51}C_3$ = $\left(\text{Coefficient of } x^2 \text{in } (1+x)^2 \frac{\left[(1+x)^{48} - 1\right]}{1+x-1}\right) + {}^{50}C_2 m^2$

(3n + 1) $^{51}C_3$ = (Coefficient of x^3 in $(1 + x)^{50}$ - $(1 + x)^2$) + $^{50}C_2m^2$

$$\Rightarrow (3n+1)\,{}^{51}C_3 = {}^{50}C_3 + {}^{50}C_2 m^2 \Rightarrow n = \frac{m^2-1}{51}$$

Least positive integer m for which n is an integer is m = 16 for which n = 5

52. (1)

$$\int_0^x \frac{t^2}{1+t^4}dt = 2x-1$$

Let $f(x)=\int_0^x \frac{t^2}{1+t^4}dt-2x+1$

$$\Rightarrow f'(x)=\frac{x^2}{1+x^4}-2<0 \forall x\in[0,1]$$

Now, f(0) = 1 and f(1) = $\int_0^1 \frac{t^2}{1+t^4}dt-1$

As $0\le\frac{t^2}{1+t^4}<\frac{1}{2}\ \forall t\in[0,1]$

$$\Rightarrow \int_0^1 \frac{t^2}{1+t^4}dt<\frac{1}{2}\Rightarrow f(1)<0$$

$\Rightarrow$ f(x) = 0 has exactly one root in [0, 1].

53. (7)

$$\lim_{x\to 0}\frac{x^2\sin\beta x}{\alpha x-\sin x}\times\frac{\beta x}{\beta x}=1$$

$$\lim_{x\to 0}\frac{x^3\beta}{\alpha x-\left[x-\frac{x^3}{\lfloor 3}+\frac{x^5}{\lfloor 5}...\right]}=1$$

$$\lim_{x\to 0}\frac{x^3\beta}{(\alpha-1)x+\frac{x^3}{\lfloor 3}-\frac{x^5}{\lfloor 5}+...}=1$$

$$\alpha-1=0,\beta\times 3!=1,\beta=\frac{1}{6}$$

$$\alpha=1$$

$$6(\alpha+\beta)=6\left(1+\frac{1}{6}\right)=7$$

54. (1)

$$z=\omega$$

$$P=\begin{bmatrix}(-\omega)^r & \omega^{2s}\\ (\omega)^{2s} & \omega^r\end{bmatrix}$$

$$P^2=\begin{bmatrix}(-\omega)^r & (\omega)^{2s}\\ (\omega)^{2s} & \omega^r\end{bmatrix}\begin{bmatrix}(-\omega)^r & \omega^{2s}\\ \omega^{2s} & \omega^r\end{bmatrix}=\begin{bmatrix}(-\omega)^{2r}+\omega^{4s} & (-\omega)^r(\omega)^{2s}+\omega^{2s}\omega^r\\ (-\omega)^r\omega^{2s}+\omega^r\omega^{2s} & \omega^{4s}+\omega^{2r}\end{bmatrix}$$

$$P^2=-I$$

$$\omega^{2r}+\omega^{4s}=-1 \text{ and } \omega^{2s}(-\omega)^r+\omega^{2s}\omega^r=0$$

$$\omega^{2r}+\omega^s=1$$

$$1+\omega^s+\omega^{2r}=0$$

$$r=s=1$$