

1. (a, b, d)

Speed of transverse pulse at the point =  $\sqrt{\frac{\text{Tension in rope at the point}}{\text{Linear mass density of rope}}}$

So,  $T_{AO} = T_{OA}$

Wavelength becomes longer when speed of the pulse increases.

2. (c)

- Heat radiated by body remains unchanged even after change in room temperature.
- Energy lost by the radiation depends upon the surface area.
- From Wien's law  $\lambda_m T = \text{constant}$

3. (a, c)

$\Delta x_{cm}$  of the block & point mass system = 0

$\therefore m(x + R) + MX = 0$

where x is displacement of the block.

$$x = -\frac{mR}{M + m}$$

From conservation of momentum and mechanical energy of the combined system

$$0 = mv - MV$$

$$mgR = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

$$\therefore v = \sqrt{\frac{2gR}{1 + \frac{m}{M}}}$$

4. (a, d)

The net magnetic flux through the loops at time 't' is

$$\phi = B(2A - A)\cos\omega t = BA\cos\omega t$$

$$\text{so, } \left| \frac{d\phi}{dt} \right| = B\omega A \sin\omega t$$

$$\therefore \left| \frac{d\phi}{dt} \right| \text{ is maximum when } \phi = \omega t = \pi/2$$

The emf induced in the smaller loop,  $\varepsilon_{\text{smaller}} = -\frac{d}{dt}(BA\cos\omega t) = B\omega A \sin\omega t$

$\therefore$  Amplitude of maximum net emf induced in both the loops = Amplitude of maximum emf induced in the smaller loop alone.

5. (a, c, d)

The minimum deviation produced by a prism

$$\delta_m = 2i - A = A$$

$$\therefore i_1 = i_2 = A \text{ and } r_1 = r_2 = A/2$$

$$\therefore r_1 = i_1/2$$

Now using Snell's law

$$\sin A = \mu \sin A/2$$

$$\Rightarrow \mu = 2 \cos(A/2)$$

For this prism when the emergent ray at the second surface is tangential to the surface

$$i_2 = \pi/2$$

$$\therefore r_2 = \theta_c$$

$$\therefore r_1 = A - \theta_c$$

$$\text{so, } \sin i_1 = \mu \sin(A - \theta_c)$$

$$\text{so, } i_1 = \sin^{-1} \left[ \sin A \sqrt{4 \cos^2 \frac{A}{2} - 1} - \cos A \right]$$

For minimum deviation through isosceles prism, the ray inside the prism is parallel to the base of the prism.

6. (a, b)

$$I = \frac{v_0}{\sqrt{\left(L\omega - \frac{1}{C\omega}\right)^2 + R^2}}$$

$$f_R = \frac{1}{\sqrt{LC}} \Rightarrow \text{Resonance frequency}$$

If  $\omega \gg 10^6$  rad/s, then circuit behaves as inductive circuit.

7. (a, b, d)

8. (6)

$$\Delta U = U_f - U_i$$

$$= S \times 4\pi R^2 [K^{1/3} - 1]$$

$$\Rightarrow K^{1/3} = \frac{\Delta U}{4\pi R^2 S} + 1 = 101$$

$$\Rightarrow K \approx 10^6$$

9. 5

Final activity,

$$A_f = \frac{v}{v_{body}} \times A_0 \times e^{-\lambda t}$$

$$\Rightarrow v_{body} = \frac{v}{A_f} \times A_0 e^{\frac{\ln(2) \times t}{192}}$$

$$= 4.998 \approx 5 \text{ litres}$$

10. 5

$$\frac{v_i}{v_f} = 6.25$$

$$\Rightarrow \left(\frac{n_f}{n_i}\right)^2 = 6.25$$

$$\Rightarrow \frac{n_f}{n_i} = 2.5 = \frac{5}{2}$$

$\Rightarrow$  minimum value of nf is 5.

11. 8

$$n \sin \theta = (n - m \Delta n) \times \sin 90^\circ$$

$$\Rightarrow m = \frac{n}{2 \Delta n}$$

$$\Rightarrow m = 8$$

12. 6

Frequency received by approaching car

$$f_1 = f_0 \left[ 1 + \frac{2}{330} \right]$$

Frequency received by source again

$$f_2 = \frac{f_1}{\left( 1 - \frac{2}{330} \right)}$$

So, beat frequency  $f_B = f_2 - f_0 = 6$  Hz.

13. (i) (a)

For constant velocity

$$q[\vec{E} + (\vec{V} \times \vec{B})] = 0$$

(ii) (c) For helical path, having its axis along z-axis, B must be along z-axis.

(iii) (d)  $\vec{E}$  and  $\vec{B}$  must be along same line (i.e. parallel or anti parallel) with initial velocity either zero or along the field ( $\vec{E}$  as well  $\vec{B}$ ).

14. (i) (b) The process must be isobaric.

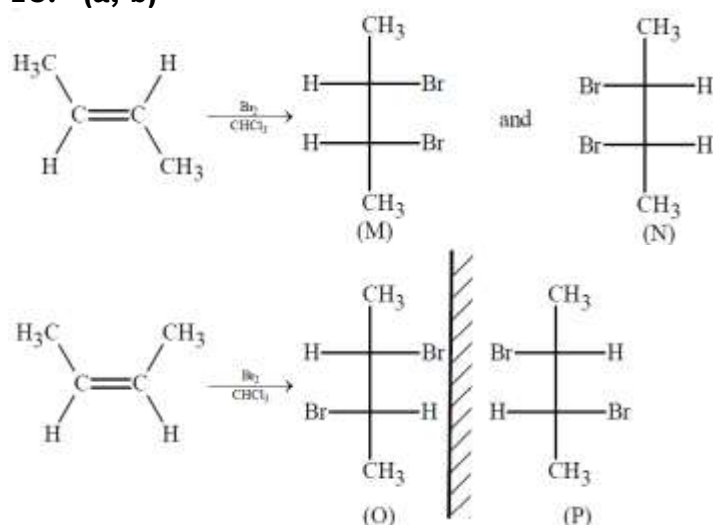
(ii) (a) The correct combination is for isochoric process.

(iii) (d) The process must be adiabatic, which is used in Laplace correction over Newton's law for sound speed in air.

15. (a, b, c)

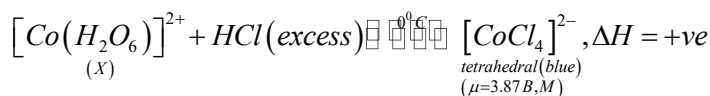
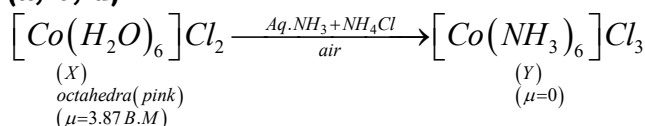


16. (a, b)



(M and O) and (N and P) have no mirror image relationship. Hence these two pairs are diastereomers. Bromination proceeds through trans-addition in both the reactions.

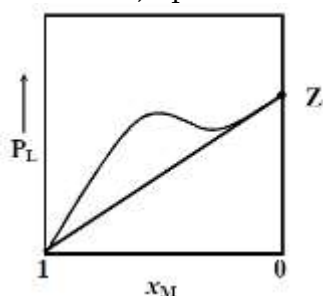
17. (a, b, d)



18. (a, c)

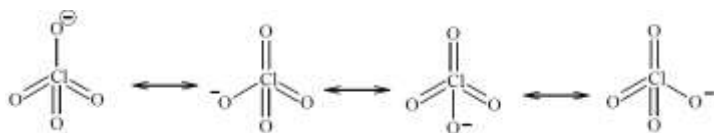
From graph it is clear that there is +ve deviation w.r.t L. Therefore, option A is correct. When  $x_L \rightarrow 1$ , then Z will have value equal to  $P_L^0$  (vapour pressure of pure L).

Therefore, option C is also correct.



19. (a, b, d)

20. (a, c, d)



Conjugate base of  $\text{HClO}_4$  has four canonical structures.

$\text{Cl}-\text{O}^-$  (Conjugate base of  $\text{HOC1}$ ) is not resonance stabilized

$\Rightarrow$  The central atoms Cl in  $\text{HClO}_4$  and O in  $\text{HOC1}$  respectively are  $\text{sp}^3$  hybridized.

$\Rightarrow \text{HClO}_4$  is stronger acid than  $\text{H}_3\text{O}^+$ , so  $\text{ClO}_4^-$  is weaker base than  $\text{H}_2\text{O}$ .

21. (b, c) Highest occupied molecular orbital (HOMO)  $\Rightarrow \pi^*$

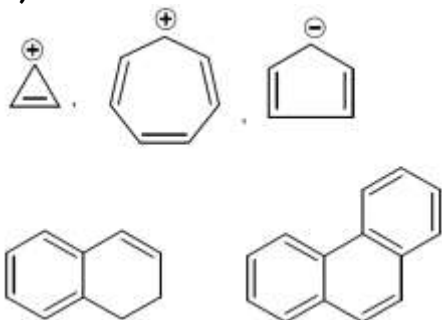
Lowest unoccupied molecular orbital (LUMO)  $\Rightarrow \sigma^*$

On descending the group gap between  $\pi^*$  and  $\sigma^*$  decreases.

22. (6)  $\text{H}_2$ ,  $\text{Li}_2$ ,  $\text{Be}_2$ ,  $\text{C}_2$ ,  $\text{N}_2$  and  $\text{F}_2$  are diamagnetic species.

\* However, because  $\text{Be}_2$  does not exist the answer may well be 5

23. (5)



24. (6)

$$\kappa = G \times \frac{l}{a}$$

$$\kappa = 5 \times 10^{-7} \times \frac{120 \text{ cm}}{1 \text{ cm}^2} = 6 \times 10^{-5} \text{ Scm}^{-1}$$

$$\wedge_m^c = \frac{\kappa \times 1000}{C} = \frac{6 \times 10^{-5} \times 1000}{0.0015}$$

$$\text{pH} = 4, [H^+] = 10^{-4} = C\alpha$$

$$\alpha = \frac{10^{-4}}{0.0015}$$

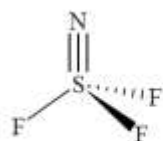
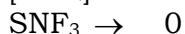
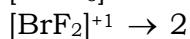
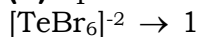
$$\alpha = \frac{\wedge_m^c}{\wedge_m^o}$$

$$\frac{10^{-4}}{0.0015} = \frac{6 \times 10^{-5} \times 1000}{0.0015 \times \wedge_m^o}$$

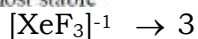
$$\wedge_m^o = 6 \times 10^2$$

$$Z = 6$$

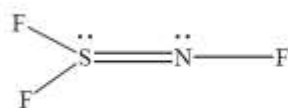
25. (6) Species      Number of lone pairs



most stable



$\therefore$  Sum is  $= 1 + 2 + 0 + 3 = 6$  lone pair



however an alternate structure also exists

26. (2)

$$d = \frac{Z \times M}{N_A \times a^3}$$

$$8 = \frac{4 \times M}{6.022 \times 10^{23} \times (400 \times 10^{-10})^3}$$

$$M = 76.8 \text{ g mol}^{-1}$$

$$76.8 \text{ g contain} = 6 \times 10^{23} \text{ atoms}$$

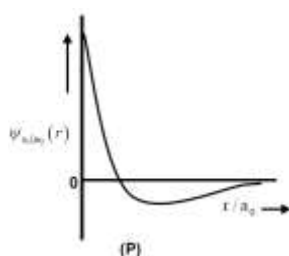
$$\therefore 256 \text{ g will contain} = 20 \times 10^{23} \text{ atoms}$$

$$= 2 \times 10^{24} \text{ atoms}$$

$$\therefore N = 2$$

27. (i) (b)

2s orbital— One radial node ( $n - 1 - 1$ )



(ii) (d)

Is orbital cannot have  $\theta$  function (angular function).  
Therefore, D is incorrect.

(iii) (d)

For H-atom:

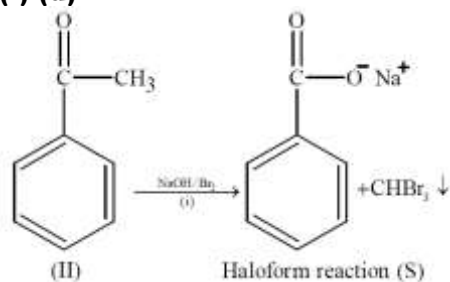
$$1s \text{ orbital} - \psi_{nlm} \propto \left(\frac{Z}{a_0}\right)^{3/2} e^{-\left(\frac{Zr}{a_0}\right)}, S$$

$$E_4 - E_2 = -\frac{13.6}{16} - \left(-\frac{13.6}{4}\right) = \frac{3 \times 13.6}{16}$$

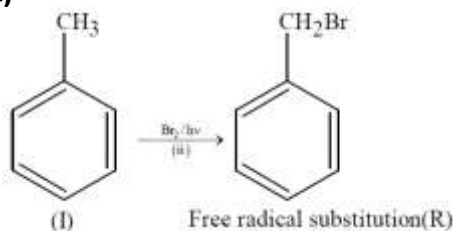
$$E_6 - E_2 = -\frac{13.6}{36} - \left(-\frac{13.6}{4}\right) = \frac{8 \times 13.6}{36}$$

$$E_4 - E_2 \text{ is } \frac{27}{32} \text{ times of } E_6 - E_2$$

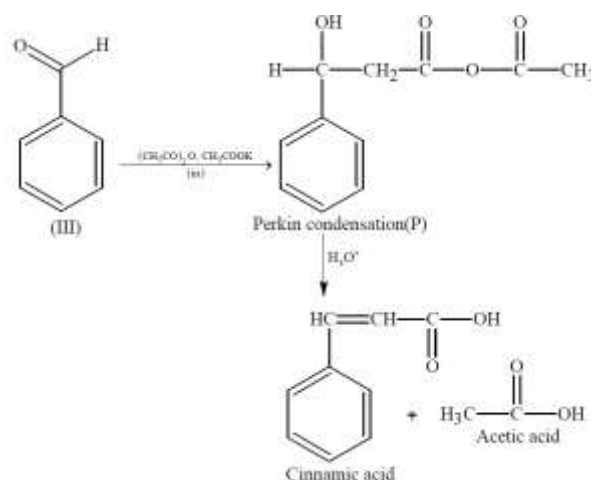
28. (i) (d)



(ii) (a)



(iii) (b)



29. (a, b)

$|A|^2$  cannot be -ve

$\Rightarrow$  A and B option cannot be square of matrix

C option is  $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}^2$

D option is  $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{vmatrix}^2$

30. (d)

$$y^2 = 16x$$

$$2x + y = p \dots (i)$$

Equation of chord having mid-point (h, k) is  $T = S_1$

$$yk - 8(x + h) = k^2 - 16h$$

$$\Rightarrow yk - 8x = k^2 - 8h \dots (ii)$$

Comparing (i) and (ii)

$$\frac{k}{1} = -\frac{8}{2} = \frac{k^2 - 8h}{p}$$

$$\Rightarrow k = -4 \text{ and } k^2 - 8h = -4p$$

$$\Rightarrow 16 - 8h = -4p$$

$$\Rightarrow 2h - p = 4$$

Clearly  $h = 3, p = 2$ .

31. (a, d)

$$\operatorname{Im}\left(\frac{a(x+iy)+b}{x+iy+1}\right) = y$$

$$\Rightarrow (x+1)^2 + y^2 = 1$$

$$\Rightarrow x = -1 \pm \sqrt{1-y^2}$$

32. (a, d)

$$P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)} = \frac{1}{2}$$

$$P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)} = \frac{2}{5}$$

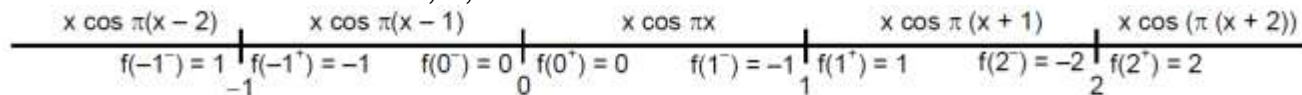
$$\Rightarrow P(X \cap Y) = \frac{2}{15}$$

$$P(Y) = \frac{4}{15} \text{ also } P(X \cup Y) = \frac{7}{15}$$

$$P\left(\frac{X'}{Y}\right) = 1 - P\left(\frac{X}{Y}\right) = 1 - \frac{1}{2} = \frac{1}{2}$$

33. (a, c, d)

Discontinuous at  $x = -1, 1, 2$



34. (b, c, d)

Tangent to  $\frac{x^2}{a^2} - \frac{y^2}{4^2} = 1$

$$y = mx \pm \sqrt{a^2 m^2 - 16}$$

Comparing with  $y = 2x + 1$

$$m = 2$$

$$\Rightarrow 4a^2 - 16 = 1$$

$$\Rightarrow 4a^2 - 16 = 1$$

$$a^2 = \frac{17}{4}$$

$$a = \frac{\sqrt{17}}{2}$$

Only 2a, 4, 1 are sides of a right-angled triangle

35. (b, d)

(a) no solution for  $e^x = \int_0^x f(t) \sin t dt, x \in (0, 1)$

As LHS is  $> 1$  and RHS is less than 1

(b) Let  $h(x) = x^9 - f(x)$

$$h(0) = -f(0) < 0$$

$$h(1) = 1 - f(1) > 0$$

by IVT (As  $h(x)$  is cont. in  $[0, 1]$ )

$h(x) = 0$  will be for some  $x \in (0, 1)$

(c) both are +ve function so no solution

(d)  $h(x) = x - \int_0^{\frac{\pi}{2}-x} f(t) \cos t dt$

$$h(0) = -ve$$

$$h(1) = +ve$$

so as  $h(x)$  is cont. in  $[0, 1]$

so  $h(x) = 0$  for some  $x \in (0, 1)$

36. (6)

Let sides be  $a - d, a, a + d, (d > 0)$

$$\Rightarrow a^2 + (a-d)^2 = (a+d)^2$$

$$\Rightarrow a = 4d$$

$$\Rightarrow \text{sides are } 3d, 4d, 5d$$

As area is 24

$$\Rightarrow \frac{1}{2} \times 3d \times 4d = 24$$

$$\Rightarrow d = 2$$

$$\Rightarrow \text{sides are } 6, 8, 10$$

$$\Rightarrow \text{smallest side is } 6.$$

37. (2)

$$x^2 + y^2 + 2x + 4y - p = 0$$

$$(x+1)^2 + (y+2)^2 = p+5$$

$$\Rightarrow p+5 = 4 \text{ (when it touches the x-axis)}$$

$$p = -1$$

$$\text{and } p+5 = 5 \text{ (when circle passes through the origin)}$$

$$p = 0$$

$$\Rightarrow \text{Two possible values of } p$$

38. (1)

$$\text{Here, } D = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{vmatrix} = 0$$

$$\text{which gives } \alpha = -1 \text{ or } +1$$

$$\text{For } a = 1, \text{ the equations become}$$

$$x+y+z=1$$

$$x+y+z = -1$$

$$\text{and } x+y+z = 1$$

$$\text{which give no solution}$$

$$\text{For } a = -1, \text{ the equations become}$$

$$x-y+z = 1$$

$$-x+y-z = -1$$

$$x-y+z = 1$$

$$\text{which are all same and hence infinitely many solutions}$$

$$\text{Hence, } a = -1$$

$$\Rightarrow 1+\alpha+\alpha^2 = 1$$

39. (5)

$$x=10!$$

$$y = {}^{10}C_1 \times {}^9C_8 \times \frac{10!}{2!}$$

$$\Rightarrow \frac{y}{9x} = \frac{{}^{10}C_1 \times {}^9C_8}{9 \times 2!} = \frac{10 \times 9}{9 \times 2} = 5$$

$$\Rightarrow 5$$

40. (2)

$$g(x) = \int_x^{\pi/2} [f'(t) \operatorname{cosec} t - \cot(t) \operatorname{cosec}(t) f(t)] dt$$

$$\Rightarrow g(x) = f(t) \operatorname{cosec}(t) \Big|_x^{\pi/2} = 3 - \frac{f(x)}{\sin x}$$

$$\Rightarrow \lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} \left( 3 - \frac{f(x)}{\sin x} \right)$$

$$\Rightarrow \lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} \left( 3 - \frac{f(x)}{\sin x} \right)$$

$$= 3 - f'(0) = 2$$

41. (i) (d)

$$\text{Tangent at } \left( \sqrt{3}, \frac{1}{2} \right) \text{ is } \sqrt{3}x + 2y = 4$$

$$\text{Since slope of tangent at } \left( \sqrt{3}, \frac{1}{2} \right) \text{ is -ve hence possible curve are (I), (II) only}$$

$$\Rightarrow \text{equation of curve is } \frac{x^2}{4} + y^2 = 1, \text{ i.e., } x^2 + 4y^2 = 4$$

$$\text{Hence equation of tangent is } y = mx + \sqrt{a^2m^2 + 1}$$

(ii) (a)

Tangent at (8, 16) is  $y = x + 8$

Slope of tangent is +ve hence possible curve will be  $y^2 = 4ax$

$\Rightarrow$  equation of tangent is  $my = m^2x + a$  and point of contact is

$$\left( \frac{a}{m^2}, \frac{2a}{m} \right)$$

(ii) (d)

For  $a = \sqrt{2}$  and point (-1, 1) on the curve is  $x^2 + y^2 = a^2$  equation of tangent is  $y = mx + a$

$$\sqrt{m^2 + 1} \text{ and point of contact is } \left( \frac{-ma}{\sqrt{m^2 + 1}}, \frac{a}{\sqrt{m^2 + 1}} \right)$$

42. (i) (d)

$$f(x) = x + \ln x - x \ln x$$

$$f'(x) = \frac{1}{x} - \ln x$$

$$f''(x) = -\frac{1}{x^2} - \frac{1}{x} = -\frac{(1+x)}{x^2}$$

$$f'(1) \cdot f'(e) = \left( \frac{1}{e} - 1 \right) < 0 \Rightarrow f'(x) = 0 \forall \text{ some } x \in (1, e)$$

$$\lim_{x \rightarrow \infty} \left( \frac{1}{x} - \ln x \right) = -\infty$$

$f(x)$  is decreasing for  $x \in (0, \infty)$

(ii) (d)

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} 1 - (x-1)(\ln x - 1) = -\infty$$

$$f'(e) = \frac{1}{e} - 1 < 0, f''(x) < 0$$

$\Rightarrow f(x)$  is decreasing in  $(e, e^2)$

(ii) (d)

$f(x)$  is decreasing and  $f(1) = 1$

$\Rightarrow f(x) = 0$  has no root in  $(0, 1)$

