1. (a)

$$m = \rho = \frac{4}{3}\pi R^3$$

$$0 = \rho.4\pi R^2 \frac{dR}{dt} + \frac{4}{3}\pi R^3 \frac{d\rho}{dt}$$

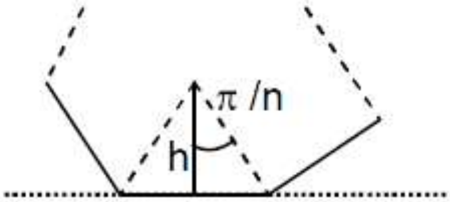
$$-\frac{1}{\rho}\frac{d\rho}{dt} = \frac{3}{R}\frac{dR}{dt}$$

$$-\frac{R}{3}\frac{1}{\rho}\frac{d\rho}{dt} = \frac{dR}{dt}$$

$$\frac{dR}{dt} \propto R$$

$$v \propto R$$

2. (b) For a regular polygon of n sides •



$$\Delta = \frac{h}{\cos\frac{\pi}{n}} - h = h\left[\frac{1}{\cos\frac{\pi}{n}} - 1\right]$$

3. (d)

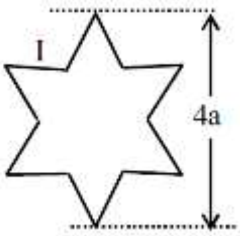
$$\frac{P^2}{2m} = \frac{hc}{\lambda} - \phi_0$$

$$\frac{h^2}{2m.\lambda_D^2} = \frac{hc}{\lambda} - \phi_0$$

$$-\frac{h^2}{m}\frac{1}{\lambda_D^3}d\lambda_D = \frac{hc}{\lambda^2}d\lambda$$

$$\frac{\Delta\lambda_D}{\Delta\lambda} \propto \frac{\lambda_D^3}{\lambda^2}$$

4. (a)



$$B = 12\left(\frac{\mu_0 I}{4\pi a}\right)(\sin 60 - \sin 30)$$

5. (a)

$$\vec{S} - \vec{P} = b\vec{R}$$

and $\vec{Q} - \vec{S} = (1-b)\vec{R}$ where $\vec{R} = \vec{Q} - \vec{P}$

From these two equations

$$\vec{S} = (1-b)\vec{P} + b\vec{Q}$$

6. (b)

$$\frac{1}{2}mv_{es}^2 - \frac{GMm}{R} - \frac{G\left(3\times10^5 M\right)\times m}{2.5\times10^4 R} = 0$$

$$\Rightarrow V_{es} = \sqrt{13} \quad v_e = 40.3km/s$$

7. (b)

$$t = \sqrt{\frac{2L}{g}} + \frac{L}{C}$$

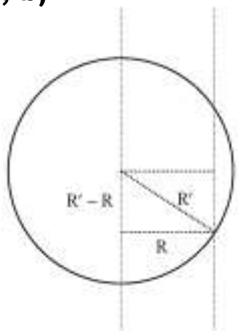
$$\frac{dt}{dL} = \sqrt{\frac{L}{g}} \times \frac{1}{2\sqrt{L}} + \frac{1}{C}$$

$$dL = \frac{dt}{\frac{1}{\sqrt{2gL}} + \frac{1}{C}}$$

$$\Rightarrow \frac{dL}{L}\times100 = \left(\frac{dt}{\frac{1}{\sqrt{2gL}} + \frac{1}{C}}\right)\frac{1}{L}\times100$$

$$= \frac{15}{16\%} \approx 1\%$$

8. (a, b)



$$R' = \frac{p}{qB}$$

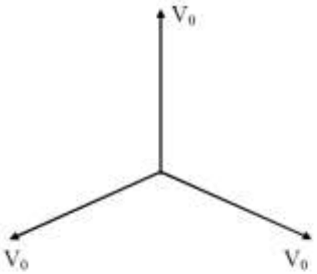
$$\left(R' - R\right)^2 + R^2 = R'$$

$$\Rightarrow R' = \frac{13}{8}R$$

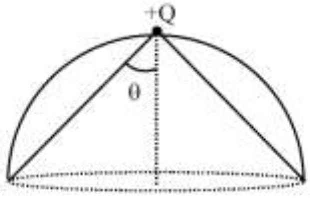
9. (a, d)

$$V_{XYO} = V_{YZO} = V_{ZXO} = \sqrt{3}V_0$$

$$V_{XX}^{rms} = V_{YZ}^{rms} = \sqrt{\frac{3}{2}}V_0$$



10. (a, d)



Solid angle subtended by flat surface at position of charge +Q

$= 2\pi(1-\cos\theta)$

$= 2\pi(1-\cos 45^0)$

$= 2\pi\left(1-\frac{1}{\sqrt{2}}\right)$

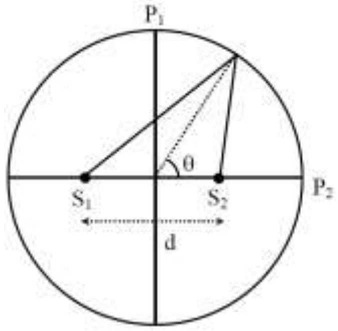
Flux entering through curved surface = flux leaving through flat surface

$\Rightarrow$ Flux entering through curved surface $= -\frac{Q}{\varepsilon_0}\times\frac{1}{4\pi}\times 2\pi\left(1-\frac{1}{\sqrt{2}}\right)$

Also all points of circumference are at equal distance from the charge

11. **(b, c)**

Path difference at point P



$\Delta x = d\cos\theta \ldots$(i)

At $P_1, \theta = 90^0 \Rightarrow \Delta x = 0 \rightarrow$ maxima

At $P_2, \theta = 0^0 \Rightarrow \Delta x = d = n\lambda$

$\Rightarrow n = \frac{1.8\times10^{-3}}{600\times10^{-9}} = 3000 \rightarrow$ maxima

For maxima at P, d $\cos\theta = n\lambda$

or $d[-\sin\theta]d\theta = (dn)\lambda$

Angular fringe width, $\left|\frac{d\theta}{dn}\right| = \frac{\lambda}{d\sin\theta}$

as $\theta\uparrow d\theta\uparrow$

12. **(a, b, c)**

Let current through R, L_1 and L_2 is i, i_1 and i_2 respectively

$L_1 i_1 = L_2 i_2$ …(i)

$i_1 + i_2 = i = \frac{V}{R}$ … (ii) After long time

Solving, $i_1 = \frac{L_2}{L_1+L_2}.\frac{V}{R}, i_2 = \frac{L_1}{L_1+L_2}.\frac{V}{R}$

13. **(a, c, d)**

• There is no horizontal force on rod during its motion

$\Rightarrow$ C.M. will fall vertically downwards

• Net torque about point B(on the ground) = mgL sin θ

• Displacement mid-point $=\frac{L}{2}-\frac{L}{2}\cos\theta$

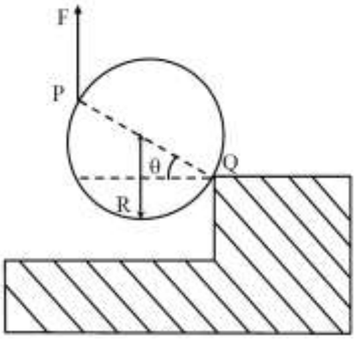
14. (a)

• As the force acts as shown and considering its F' direction to be constant, then

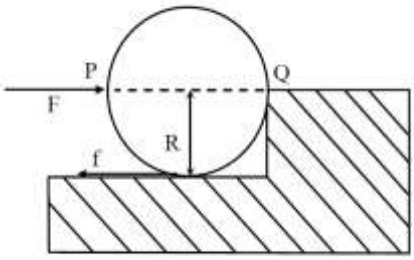
$\tau_{net}=F(2R\cos\theta)-mgR\cos\theta=R\cos\theta(2F-mg)$

Which definitely infers that as θ increases,

τ_{net} decreases.



As soon as force is applied as shown, it leads to frictional force between bottom point and horizontal surface, which will produce non-zero torque about point Q.



15. (i) (a)

$W_b=CV_0 x V_0$

$\Delta U=\frac{1}{2}CV_0^2=E_C$

Heat dissipated = W_b -E_c or E_c = E_D = $\frac{1}{2}$ CV_0^2

(ii) (c)

If capacitor is charged from V_i to V_f; heat dissipated is

$H=W_{battery}-\Delta U$

$=C(V_f-V_i)V_f-\left[\frac{1}{2}C(V_f^2-V_i^2)\right]$

$=\frac{1}{2}C(V_f-V_i)^2$

Total heat dissipated

$=\frac{1}{2}C\left\{\left(\frac{V_0}{3}-0\right)^2+\left(\frac{2V_0}{3}-\frac{V_0}{3}\right)^2+\left(V_0-\frac{2V_0}{3}\right)^2\right\}$

$=\frac{1}{2}CV_0^2$

16. (i) (c)

(ii) (a)

17. (c)

$\Delta T_f=K_f\times m$

$=2\times\frac{34.5\times 2}{46}$

$= 2 \times 1.5$

= 3 Freezing point of ethanol + water mixture = 273- 3 = 270

18. (b)

$Zn - 2e \longrightarrow Zn^{2+}$

$Cu^{2+} + 2e \longrightarrow Cu$

$Zn + Cu^{2+} \longrightarrow Zn^{2+} + Cu$

$$E_{cell} = E^0_{cell} - \frac{2.303RT}{nF}\log\frac{[Zn^{2+}]}{[Cu^{2+}]}$$

$$E_{cell} = 1.1 - \frac{2.303RT}{2F}\log 10$$

$$= 1.1 - \frac{2.303RT}{2F}$$

$$\Delta G = -nFE_{cell}$$

$$= -2F\left(1.1 - \frac{2.303RT}{2F}\right)$$

= 2.303RT-2.2F

19. (a)

$\Delta G = PdV$

$$\left[\Delta_f G^0_{(diamond)} - \Delta_f G^0_{(graphite)}\right] = PdV$$

$2.9 \times 10^3\ Jmol^{-1} = P \times 2 \times 10^{-6}\ m^3\ mol^{-1}$

$P = 1.45 \times 10^9 Pa$

$P = 1.45 \times 10^9 \times 10^{-5}$ bar

$P = 1.45 \times 10^4$ bar

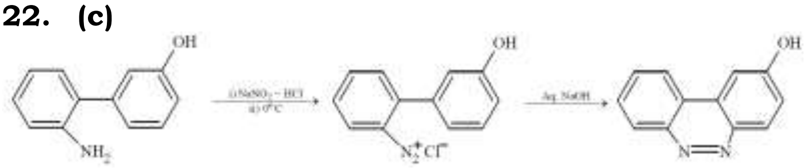
P = 14500 bar

20. (c) $Zn + 2NaOH \rightarrow Na_2ZnO_2 + H_2\uparrow$

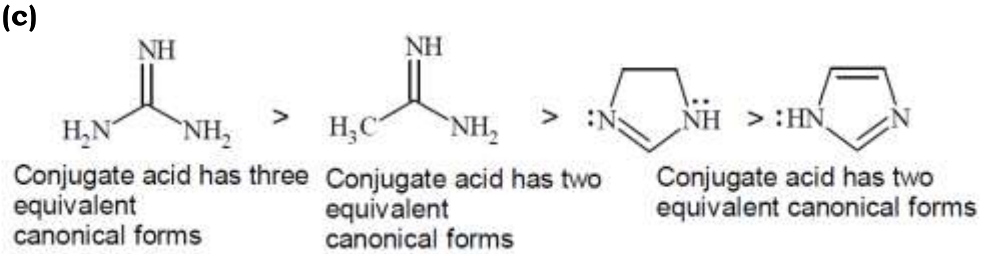
21. (c)

Species	Oxidation state of P
H_3PO_4	+5
$H_4P_2O_6$	+4
H_3PO_3	+3
H_3PO_2	+1

22. (c)



23. (c)



24. (a, b) In adsorption process both AH & AS is - ve. Higher the critical temperature of a gas higher the extent of adsorption. Cloud is not an emulsion. Brownian motion depends on the size of the particles.

25. (b, d) (b) With increase in temperature, the value of K for endothermic reaction increases because unfavourable change in entropy of the surroundings decreases

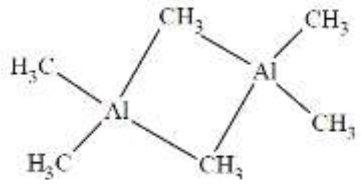
(d) With increase in temperature, the value of K for exothermic reaction decreases because favourable change in entropy of the surrounding decreases

26. (a, b) (a) The activation energy of the reaction is unaffected by the value of the steric factor
(b) Experimentally determined value of frequency factor is higher than that predicted by Arrhenius equation

27. (a, b, c, d)

Benzylic and 3° halides both follow S_N1 mechanism.
Benzylic and 1° halides both follow S_N2 mechanism.
Benzylic 2° halides can undergo inversion of configuration.
The order of reactivity would be IV > I > III if both S_N1 and S_N2 are considered suitably for substrates.

28. (a, b, d) Both $A1(CH_3)_3$ and BH_3 has 3c - 2e bonds in the dimeric structure.

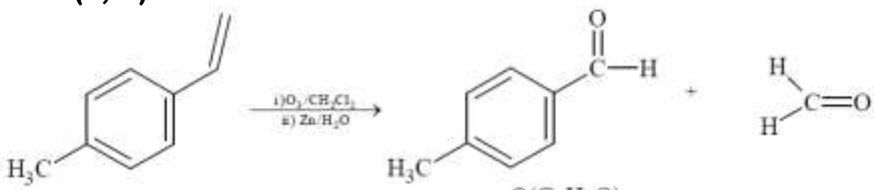


$BC1_3$ is stronger Lewis acid than Aids.

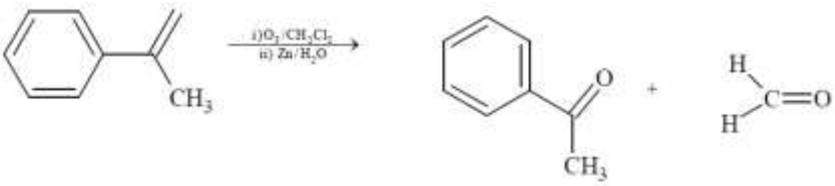
29. (a, d)

Amphoteric oxides are
ZnO, $A1_2O_3$, PbO, PbO_2, BeO, SnO, SnO_2, Cr_2O_3
NO is a neutral oxide.
CrO is a basic oxide.
B_2O_3 is an acidic oxide

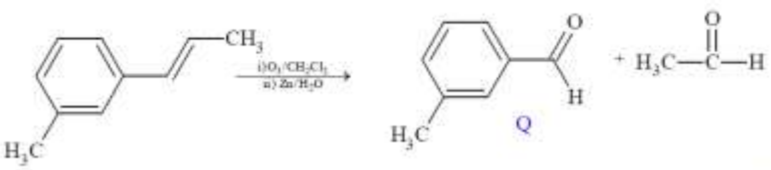
30. (a, b)



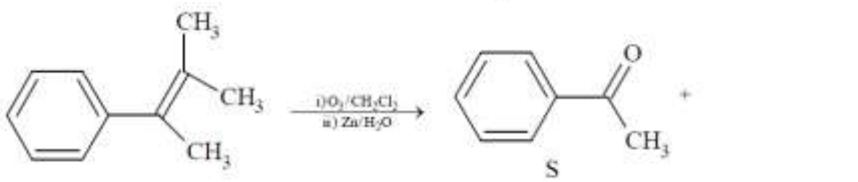
$Q(C_8H_8O)$
Undergoes Cannizzaro but not haloform



Undergoes Haloform but not Cannizzaro



Undergoes Cannizzaro but not haloform



Undergoes Haloform but not Cannizzaro

For (c) and (d) options no. of carbons are not matching

31. (i) (c)

$$2KClO_3 \xrightarrow{MnO_2} 2KCl + \underset{W}{3O_2}$$

$$\underset{X}{P_4 + 5O_2 \rightarrow P_4O_{10}}$$

$$P_4O_{10} + 4NHO_3 \rightarrow 4HPO_3 + 2N_2O_5$$

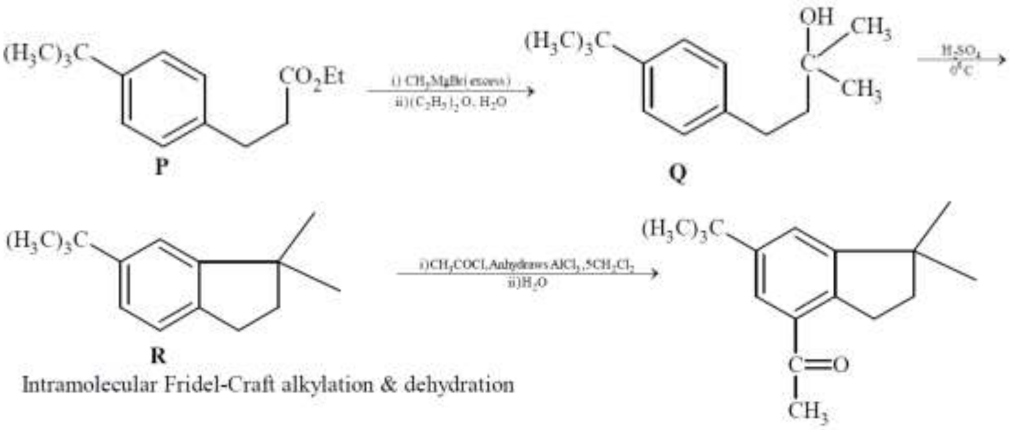
(ii) (b)

$$2KClO_3 \xrightarrow{MnO_2} 2KCl + \underset{W}{3O_2}$$

$$\underset{X}{P_4 + 5O_2 \rightarrow P_4O_{10}}$$

$$P_4O_{10} + 4NHO_3 \rightarrow 4HPO_3 + 2N_2O_5$$

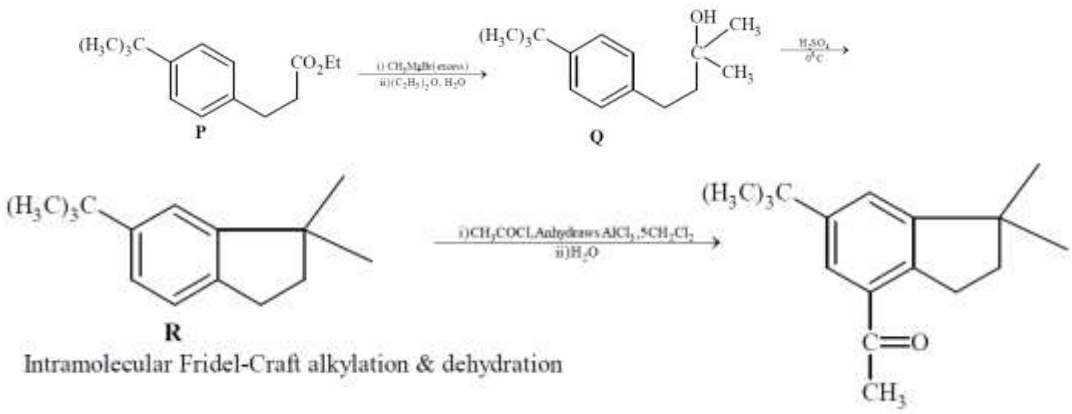
32. (i) (a)



Intramolecular Fridel-Craft alkylation & dehydration

Fridel-Craft alkylation

(ii) (c)



Intramolecular Fridel-Craft alkylation & dehydration

Fridel-Craft alkylation

33. (c)

Let direction ratios of normal be $\vec{n}$

$$\Rightarrow \vec{n} = \begin{vmatrix} \hat{i} & j & k \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = -14\hat{i} - 2j - 15k$$

$\Rightarrow$ equation of plane is

-14(x - 1) - 2(y - 1) - 15(z - 1) = 0

$\Rightarrow$ 14x + 2y+ 15z = 31

34. (d)

$$\overrightarrow{OP}.\overrightarrow{OQ} + \overrightarrow{OR}.\overrightarrow{OS} = \overrightarrow{OR}.\overrightarrow{OP} + \overrightarrow{OQ}.\overrightarrow{OS}$$

$$\Rightarrow \overrightarrow{OP}.\left(\overrightarrow{OQ} - \overrightarrow{OR}\right) = \overrightarrow{OS}.\left(\overrightarrow{OQ} - \overrightarrow{OR}\right)$$

$$\Rightarrow \left(\overrightarrow{OP} - \overrightarrow{OS}\right).\left(\overrightarrow{RQ}\right) = 0$$

$(\overrightarrow{SP}).(\overrightarrow{RQ}) = 0$

$\Rightarrow \overrightarrow{SP} \perp \overrightarrow{RQ}$

Similarly, $\overrightarrow{SP} \perp \overrightarrow{QP}$ and $\overrightarrow{SQ} \perp \overrightarrow{PR}$

35. (a)

$$dy = \left(\frac{1}{8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)\left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)} \right) dx$$

Let $4+\sqrt{9+\sqrt{x}} = t$

$\Rightarrow \frac{1}{2\sqrt{9+\sqrt{x}}} \cdot \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow dy = \frac{dt}{2\sqrt{t}}$

$\Rightarrow 2dy = \frac{1}{\sqrt{t}} dt$

$2y = 2\sqrt{t} + c$

$\Rightarrow 2y = 2\sqrt{4+\sqrt{9+\sqrt{x}}} + c$

$y(0) = \sqrt{3} \Rightarrow c = 0$

$y = \sqrt{4+\sqrt{9+\sqrt{x}}}$

$y(256) = 3$

36. (d)

Let h(x) = f(x) - x

$h\left(\frac{1}{2}\right) = 0 = h(1)$

$\Rightarrow h'(\alpha) = 0$ for some $\alpha \in (0,1)$ by rolle's theorem

$f'(\alpha) = 1$

as f'(x) > 0 $\Rightarrow$ f'(x) is increasing

$\therefore$ f'(1) > f (a)

37. (b)

Let matrix M = $[k_{ij}]$

Then sum of diagonal entries = $\sum k_{ij}^2$

$\Rightarrow \sum k_{ij}^2 = 5$

where k_{ij} are from {0, 1,2}

$\Rightarrow$Total number of matrices = ${}^9C_5 + {}^9C_1 . {}^8Ci = 198$

38. (d) There are only 4 even numbers in S

Any subset of 5 elements of S will have at least 1 odd number.

$\Rightarrow N_1 + N_2 + N_3 + N_4 + N_5 = {}^9C_5 = 126$

39. (b) Total number of solutions = ${}^{10+3-1}C_{3-1} = 66$

Favourable number of solutions = ${}^{11}C_1 + {}^9C_1 + {}^7C_1 + {}^5C_1 + {}^3C_1 + {}^1C_1 = 36$

$P(req) = \frac{36}{66} = \frac{6}{11}$

40. Bonus

$g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t)\,dt$

g'(x) = 2 (cos 2x) $\sin^{-1}$ (sin 2x) - (cos x) $\sin^{-1}$ (sin x)

$$g'\left(\frac{\pi}{2}\right)=2(-1)(0)-\cos\left(\frac{\pi}{2}\right)(1)=0$$

$g'\left(-\frac{\pi}{2}\right)=0$ no option is matching.

41. (a, c)

$$\cos\beta(2+\cos\alpha)=1+2\cos\alpha$$

$$\frac{\cos\beta}{1}=\frac{1+2\cos\alpha}{2+\cos\alpha}$$

$$\frac{\cos\beta+1}{\cos\beta-1}=\frac{3(\cos\alpha+1)}{\cos\alpha-1}$$

$$\frac{2\cos^2\frac{\beta}{2}}{-\sin^2\frac{\beta}{2}}=\frac{3\times2\cos^2\frac{\alpha}{2}}{-2\sin^2\frac{\alpha}{2}}$$

$$\tan^2\frac{\alpha}{2}=3\tan^2\frac{\beta}{2}$$

$$\Rightarrow\tan\frac{\alpha}{2}=\pm\sqrt{3}\tan\frac{\beta}{2}$$

42. (a, c)

F'(x) - 2f(x) > 0

$$\Rightarrow\frac{d}{dx}\left(f(x).e^{-2x}\right)>0$$

$\Rightarrow f(x).e^{-2x}$ is increasing function

$$\Rightarrow f(x).e^{-2x}>f(0)=1\forall x>0$$

$$f(x)>e^{2x}\forall x>0$$

$$\Rightarrow f'(x)>2f(x)>2e^{2x}>0\forall x>0$$

43. (a, d)

$$\lim_{x\to1^-}f(x)=\lim_{h\to0}\frac{h^2}{h}\cos\frac{1}{h}=0$$

$\lim_{x\to1^+}f(x)=\lim_{h\to0}\frac{-h(2+h)}{h}\cos\frac{1}{h}$ does not exist

44. (b, c)

$$f(x)=\begin{vmatrix}\cos2x & \cos2x & \sin2x\\ -\cos x & \cos x & -\sin x\\ \sin x & \sin x & \cos x\end{vmatrix}=\cos4x+\cos2x$$

Now f'(x) = -2sin2x - 4sin4x = 0

⇒ f (x) = 2sin2x (1 + 4 cos 2x) = 0

Then sin 2x = 0 or cos 2x $=-\frac{1}{4}$

For sin 2x = 0; x = 0, $\pi/2$ - $\pi/2$

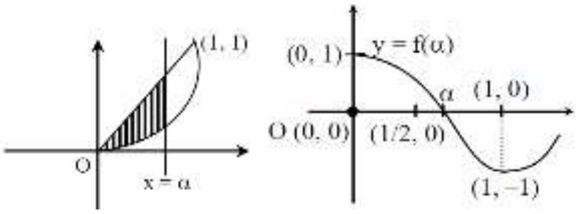
For cos 2x = - 1/4 there are four solutions.

f (x) = 0 has more than three solutions.

Again f'(x) = -(4 cos 2x + 16 cos 4x)

⇒ f'(0) < 0

45. (b, c)



$$\frac{1}{2}\left(\int_0^1 (x-x^3)dx\right)=\int_0^\alpha (x-x^3)dx$$

$$\frac{1}{8}=\left|\frac{x^2}{2}-\frac{x^4}{4}\right|_0^\alpha$$

$$\frac{1}{2}=2\alpha^2-\alpha^4$$

$$2\alpha^4-4\alpha^2+1=0$$

$$f(\alpha)=2\alpha^4-4\alpha^2+1$$

46. (b, d)

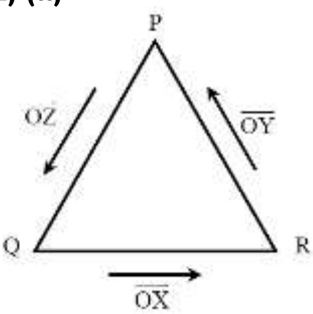
$$\sum_{k=1}^{98}\int_k^{k+1}\frac{1}{x+1}dx<\sum_{k=1}^{98}\int_k^{k+1}\frac{k+1}{x(x+1)}dx<\sum_{k=1}^{98}\int_k^{k+1}\frac{dx}{x}$$

$$\sum_{k=1}^{98}(\ln(k+2)-\ln(k+1))<I<\sum_{k=1}^{98}(\ln(k+1)-\ln k)$$

$$\ln 50<I<\ln 99$$

$$\Rightarrow \frac{49}{50}<I<\ln 99$$

47. (i) (a)



$$\left|\overrightarrow{OX}\times\overrightarrow{OY}\right|=\left|\overrightarrow{OX}\right|\left|\overrightarrow{OY}\right|\sin(\pi-R)$$

$$=\sin R=\sin(P+Q)$$

(ii)(b)

$$-(\cos P+\cos Q+\cos R)=\overrightarrow{OX}.\overrightarrow{OY}+\overrightarrow{OY}.\overrightarrow{OZ}+\overrightarrow{OZ}.\overrightarrow{OX}$$

$$=\frac{\left(\overrightarrow{OX}+\overrightarrow{OY}+\overrightarrow{OZ}\right)^2-\left(\left|\overrightarrow{OX}\right|^2+\left|\overrightarrow{OY}\right|^2+\left|\overrightarrow{OZ}\right|^2\right)}{2}$$

$$\geq -\frac{3}{2}$$

48. (i) (b)

As $a_{n+1}-a_n-a_{n-1}=0$

So $a_{12}=a_{11}+a_{10}$

(ii) (d)

$a_4 = 2a_0 + 3a_1$

$a_4 = (q-p)(3\beta) + 5p + 2q = 28$

$\Rightarrow p = q = 4.$