1. (a, b, d)

$$\frac{dK}{dt} = \gamma t, \text{So } K = \frac{\gamma t^2}{2}$$

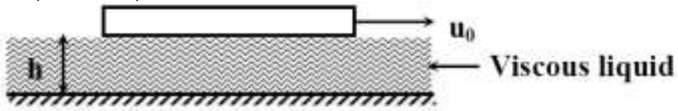
$$\frac{1}{2}mv^2 = \frac{\gamma t^2}{2} \text{ or } v = t\sqrt{\frac{\gamma}{m}}$$

$$\frac{dv}{dt} = \sqrt{\frac{\gamma}{m}}$$

$$F = \sqrt{m\gamma}$$

(i) force is constant.
(ii) speed is proportional to t
(iii) Force is constant, so it is conservative

2. (a, c, d)



$$F_v \propto \frac{1}{h}, F_v \propto A \text{ and}$$

$$\text{Shear Stress } = \frac{F}{A} \propto \eta$$

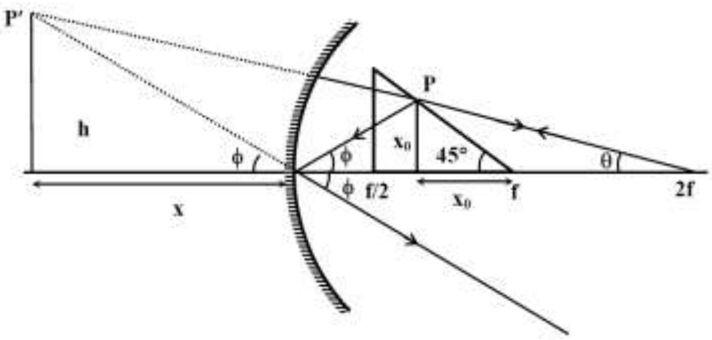
3. (a, b) Charge inside the spherical shell = $\lambda .2R\cos 30^0 = \lambda R\sqrt{3}$

So, electric flux $= \dfrac{\sqrt{3}R\lambda}{\varepsilon_0}$

4. (d)

$$\tan\phi = \frac{x_0}{f - x_0}$$

$$\frac{h}{x} = \frac{x_0}{f - x_0} \Rightarrow x = \frac{h}{x_0}(f - x_0)$$



$$\tan\theta = \frac{x_0}{f + x_0}$$

$$\frac{h}{x + 2f} = \frac{x_0}{f + x_0}$$

$$\Rightarrow \frac{h}{x_0}(f + x_0) = x + 2f$$

$$\Rightarrow \frac{h}{x_0}(f + x_0) + \frac{h}{x_0}(f - x_0) + 2f$$

$$\Rightarrow 2hx_0 = 2fx_0$$

$$\therefore h = f$$

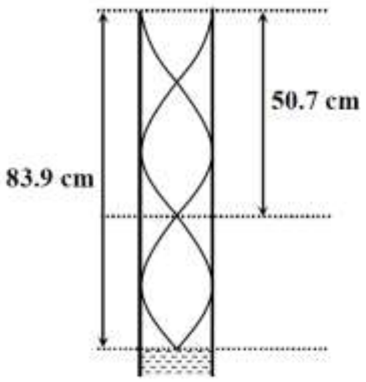
5. (a, c)

$${}^{232}_{90}Th \rightarrow {}^{212}_{82}Pb + N_\alpha \alpha + N_\beta \beta$$

No. or $\alpha = \frac{232-212}{4} = 5$

No. of β^- = 2(from conservation of charge)

6. (a, c)



$$\frac{\lambda}{2} = 83.9 - 50.7 = 33.2$$

So, $\lambda = 66.4cm$

$$v = n\lambda = \frac{500 \times 66.4}{100} = 332m/s$$

and $e = -0.9cm$

7. (6.30)

Using impulse momentum theorem

J = mv_0

$$v_0 = \frac{J}{m} = \frac{1}{0.4} = 2.5m/s$$

$$v = v_0 e^{\frac{-t}{\tau}}$$

$$\int dx = v_0 \int_0^\tau e^{\frac{-t}{\tau}} dt$$

$$\Delta x = v_0 \tau \left[1 - e^{\frac{-t}{\tau}}\right]$$

at $t = \tau$ sec

$\Delta x = 6.30m$

8. (30.00)

$$H_0 = \frac{(v_0 \sin 45^0)^2}{2g} = \frac{v_0^2}{4g}$$

$$\frac{v_0^2}{4g} = 120 \qquad \ldots(i)$$

Now, $\frac{1}{2}mv^2 = \frac{1}{2} \times \frac{1}{2} mv_0^2$

$$v = \frac{v_0}{\sqrt{2}}$$

Now, $H = \frac{(v \sin 30^0)^2}{2g} = \frac{v^2}{8g} = \frac{v_0^2}{16g} = \frac{H_0}{4} = 30m$

9. (2.00)

$$m\frac{dv}{dt} = qE$$

$$\int_0^v dv = \frac{qE_0}{m}\int_0^t \sin\omega t dt$$

$$v = \frac{qE_0}{m}\frac{(1-\cos\omega t)}{\omega} = \frac{1\times 1(1-\cos\omega t)}{10^{-3}\times 10^{3}}$$

$$v = (1-\cos\omega t)$$

$$\therefore v_{\max} = 2m/s$$

10. (5.56)

$$NiAB = C\theta$$

$$i_g = \frac{C\theta_{\max}}{NAB} = \frac{10^{-4}\times 0.2}{50\times 2\times 10^{-4}\times 0.02} = 0.1A$$

$$\therefore S = \frac{i_g R_g}{(I-i_g)} = \frac{0.1\times 50}{(1-0.1)} = \frac{50}{9} = 5.555\Omega$$

$\therefore$ shunt resistance, S = 5.56 Ω

11. (3.00)

$$\Delta l = \frac{Fl}{AY}$$

$$= \frac{4Fl}{\pi d^2 Y}$$

$$= \frac{4\times 12\times 1}{\pi\times 25\times 10^{-8}\times 2\times 10^{11}} = 0.3mm$$

10VSD = 9 MSD

$$1VSD = \frac{9}{10}MSD$$

$\therefore$ Least count, L.C. = 1 MSD - 1 VSD

$$= \left(1-\frac{9}{10}\right)MSD$$

$$= \frac{1}{10}MSD = 0.1mm$$

$\therefore$ 3rd vernier scale division coincides with a main scale division.

12. (900.00)

$$\frac{T_f}{T_i} = \left(\frac{V_i}{V_f}\right)^{\gamma-1} = \left(\frac{1}{8}\right)^{2/3}$$

$$T_f = \frac{T_i}{4} = \frac{100}{4} = 25K$$

$$\therefore \Delta U = nC_v(T_f - T_i)$$

$$= 1\times\frac{3R}{2}(25-100) = \frac{3}{2}\times 8\times(-75) = -900 Joule$$

13. (24.00)

No. of photoelectrons emitted per second

$N = \dfrac{200}{6.25\times 1.6\times 10^{-19}}$ = 2×10^{20} photoelectrons

Momentum of each electron before striking the anode

$P = \sqrt{2\times 9\times 10^{-31}\times 500\times 1.6\times 10^{-19}}$ = 1.2 x 10^{-23} kg m/s

$\therefore$ The force experienced by the anode is

F = NP = 2 x 10^{20} x 1.2 x 10^{-23} = 24 x 10^{-4} N

$\therefore$ n = 24.00

14. **(3.00)**

$$E_n = 13.6\frac{Z^2}{n^2}$$

$$E_2 - E_1 = 13.6Z^2\left(1-\frac{1}{4}\right) = 13.6Z^2 \times \frac{3}{4}$$

$$E_3 - E_2 = 13.6Z^2\left(\frac{1}{4}-\frac{1}{9}\right) = 13.6Z^2 \times \frac{5}{36}$$

Now, $13.6Z^2\left(\frac{3}{4}-\frac{5}{36}\right) = 74.8$

$\Rightarrow Z = 3$

15. **(b)**

For point charge, $E \propto \frac{1}{r^2}$

For infinite line charge, $E \propto \frac{1}{r}$

For infinite plane E = constant

For small dipole $E \propto \frac{1}{r^3}$

16. **(b)**

$v \propto \frac{1}{\sqrt{r}}$, Hence correct answer is 'B'

Also, $T \propto r^{3/2}$

$K \propto \frac{m}{r}$

17. **(c)** Process 1 is adiabatic process, hence Q = 0
Process 2 is isobaric, hence W = P. ΔV = 6 P_0V_0
Process 3 is isochoric, hence W = 0
Process 4 is isothermal

18. **(a)**
'P' is straight line with zero acceleration.
'Q' is elliptical path.
'R' is circular path
'S' is parabolic path

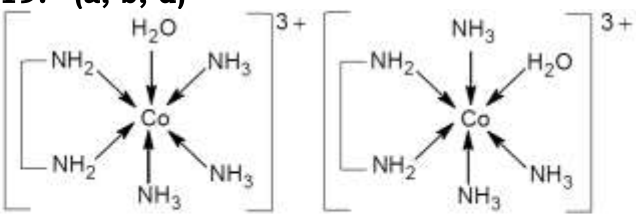
For $'S' |v| = \sqrt{\alpha^2 + \beta^2 t^2}$

$dU = -\vec{F}.d\vec{r}$

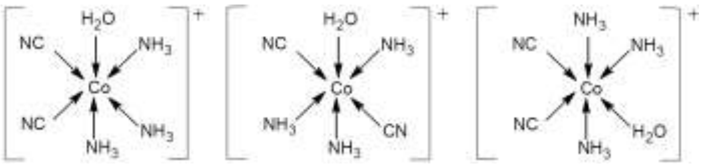
$U = -\frac{m\beta^2 t^2}{2}$

Total energy = KE + U = $\frac{1}{2}$ mα^2 = constant

19. **(a, b, d)**



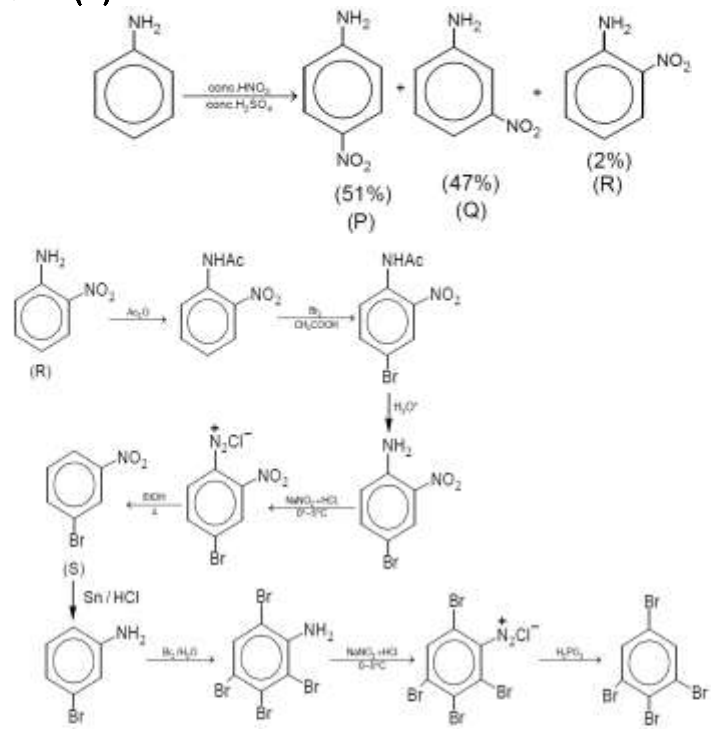
Two geometrical isomers of $[Co(en)(NH_3)_3H_2O]^{3+}$
(b) $[Co(NH_3)_3(CN)_2(H_2O)]^+$ $\Rightarrow$ 3 geometrical isomers _



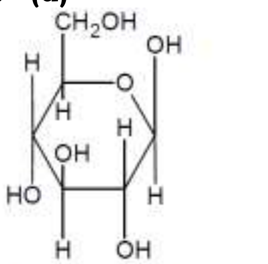
(c)
Since (en) and NH_3 are strong field ligand, so it should be diamagnetic complex
(d)
(NH_3) is a stronger ligand than
H_2O, so, $[Co(en)(NH_3)_4]^{3+}$ should absorb shorter wavelength than $[Co(en)(NH_3)_3(H_2O)]^{3+}$.

20. (b, d) Cu^{2+} shows the characteristic green colour in the flame test.
Ksp of CuS < Ksp of MnS
$E^0_{Cu^{2+}/Cu} > E^0_{Mn^{2+}/Mn}$

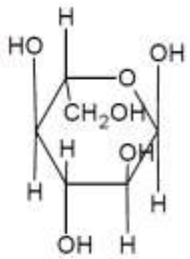
21. (d)



22. (d)



β – (D) – glucopyranose



β – (L) – glucopyranose

23. (a, d)

	A(g)	$\longrightarrow$	2B(g)	+	C(g)
at t = 0	p_0		0		0
	$(p_0 - p)$		2p		p

$P_t = P_0 - P + 2P + P = P_0 + 2P$

$$p = \frac{(p_t - p_0)}{2}$$

$$t = \frac{1}{k}\ln\frac{2p_0}{3p_0 - p_t}$$

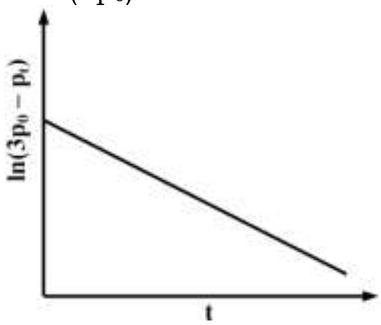
$kt = \ln 2p_0 - \ln(3p_0 - p_t)$

$\ln(3p_0 - p_t) = \ln 2p_0 - kt$

$y = mx + c$

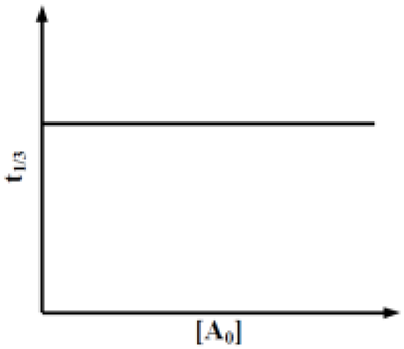
So, $m = -k$

$c = \ln(2p_0)$



$$kt_{1/3} = \ln \frac{[A_0]}{\frac{[A_0]}{3}}$$

$$t_{1/3} = \frac{\ln 3}{k}$$



24. (a, c)

$A \rightleftharpoons P$

$(P)_{eq} > 5, (A)_{eq} < 5$

$$K_{eq} = \frac{[P]}{[A]} > 1$$

$\Delta G^0 = -RT \ln K_{eq}, \Delta G^0 < 0$

$$\frac{\ln K_{T_1}}{\ln K_{T_2}} > \frac{T_2}{T_1} > 1$$

$$\Rightarrow \frac{K_{T_1}}{K_{T_2}} > 1$$

$\Rightarrow K_{T_2} < K_{T_1}$ (exothermic)

$\Delta H° < 0$, since (P) at T_2 < at T_1

$\Delta G° = \Delta H° - T\Delta S°$

$T\Delta S° = \Delta H° - \Delta G°$

$$\Delta S^0 = \frac{\Delta H^0 - \Delta G^0}{T}; (\Delta H^0) > (\Delta G^0)$$

$\Delta S^0 < 0$

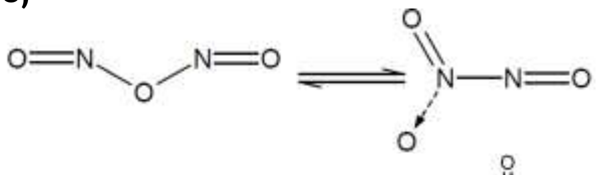
Also, $-T_1 \ln K_{T_1} < -T_2 \ln K_{T_2}$

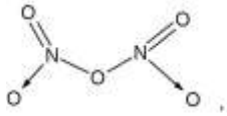
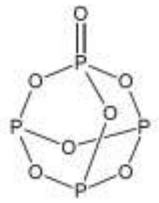
$\Delta G^0_{T_1} < \Delta G^0_{T_2}$

$\Delta H^0_{T_1} - T\Delta S^0_{T_1} < \Delta H^0_{T_2} - T\Delta S^0_{T_2}$

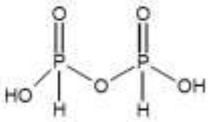
It is possible only if $\Delta S^0 < 0$

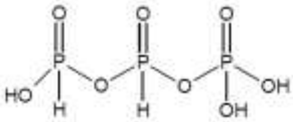
25. (6)

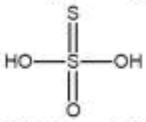


 ,  

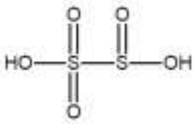
$2H_3PO_3 \rightarrow H_4P_2O_5 + H_2O$







$2H_2SO_3 \longrightarrow H_2S_2O_5 + H_2O$



26. (6.47 kg)

$2PbS + 3O_2 \rightarrow 2PbO + 2SO_2$

$2PbO + PbS \rightarrow 3Pb + SO_2$

$3PbS + 3O_2 \rightarrow 3Pb + 3SO_2$

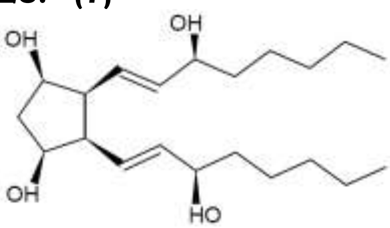
32kg 207 kg

$1kg \rightarrow \frac{207}{32} = 6.47kg$

27. (126) Number of meq of $MnCl_2$ = number of meq of $KMnO_4$ = number of meq of $H_2C_2O_4$ = 5

Weight of $MnCl_2$ taken = $5 \times 10^{-3} \times \frac{126}{5}$ gm = 126 mg

28. (7)



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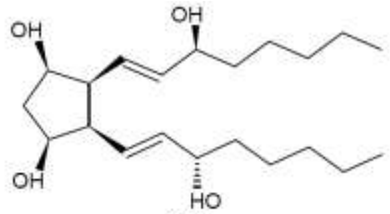

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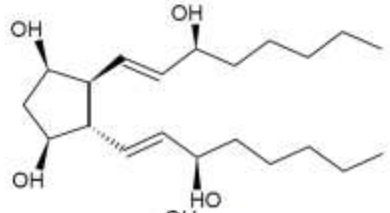

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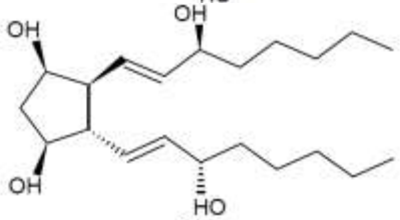

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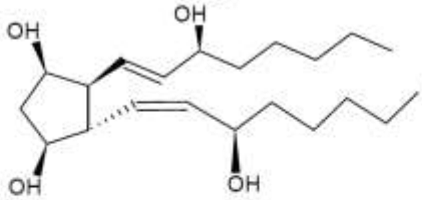

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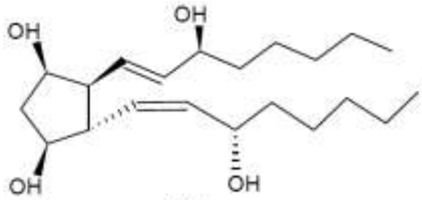

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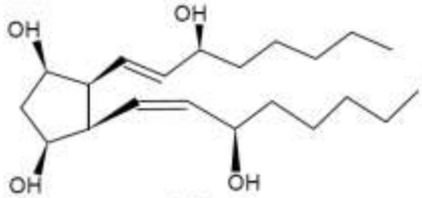

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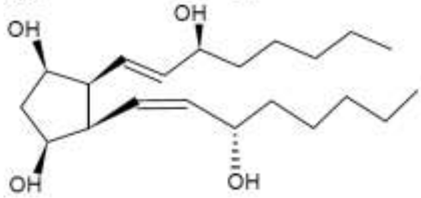
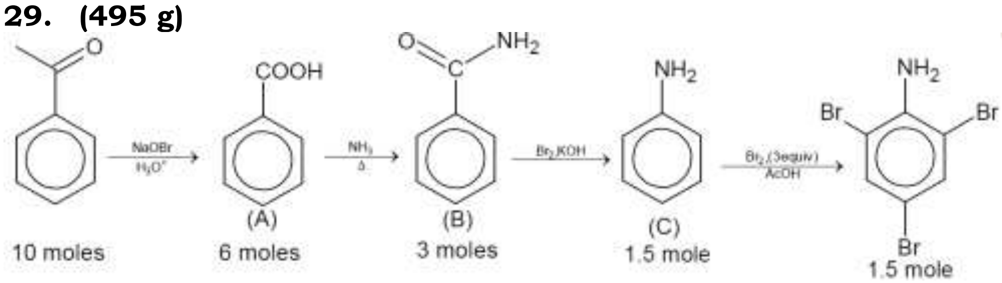


(optically active)

29. (495 g)



Amount of (d) = 1.5 x 330 = 495.0 g

30. (-14.6)

From the given data:

For $2Cu(s) + H_2O(g) \rightarrow Cu_2O(s) + H_2(g)$

$\Delta G° = 100000$

Hence $\Delta G°$ -100000 = -RT ln Kp and $K_P = \frac{P_{H_2}}{P_{H_2O}}\left(P_{H_2O(g)} = 0.01bar\right)$

On calculating; $\ln P_{H_2}$ =-14.6

31. (8500)

A(g) + B(g) ⇌ AB(g)

$(E_a)_b - (E_a)_f = 2RT$

$\frac{A_f}{A_b} = 4$

$\Delta G^0 = -RT \ln K_{eq}$

$K_f = A_f e^{-(E_a)_f/RT}$

$K_b = A_b e^{-(E_a)_b/RT}$

$K_{eq} = \frac{K_f}{K_b} = \frac{A_f}{A_b} \times e^{-\frac{(E_a)_f}{RT}} \times e^{+\frac{(E_a)_b}{RT}} = 4 \times e^{\frac{(E_a)_b - (E_a)_f}{RT}}$

$K_{eq} = 4 \times e^2$

$\Delta G^0 = -RT \times \ln\left(4 \times e^2\right)$

$\Delta G^0 = -RT\left(\ln 4 + 2\ln e\right)$

$\Delta G^0 = -RT\left(2 \times 0.7 + 2\right)$

$\Delta G^0 = -RT\left(1.40 + 2\right)$

$\Delta G^0 = -RT\left(3.40\right)$

$\Delta G^0 = -2500 \times 3.40$

$\Delta G^0 = -8500J$

32. (-11.62 J mol^{-1} K^{-1})

$A(s)|A^{n+}(aq, 2M)||B^{2n+}(aq, 1M)|B(s)$

$2A(s) \rightarrow 2A^{n+} + 2ne^-$

$B^{2n+} + 2ne^- \rightarrow B(s)$

$2A(s) + B^{2n+} \rightarrow 2A^{n+} + B(s)$

$Q = \frac{\left[A^{n+}\right]^2}{\left[B^{2n+}\right]} = \frac{2 \times 2}{1} = 4$

$\Delta G^0 = \Delta H^0 - T\Delta S^0$

$\Delta H^0 = 2\Delta G^0$

$E^0_{cell} = \frac{RT}{2nF} \ln 4$

$\Delta G^0 = -2n \times F \times \frac{RT}{2nF} \ln 4$

$\Delta G^0 = -RT \ln 4$

$\Delta S^0 = \frac{\Delta H^0 - \Delta G^0}{T} = \frac{\Delta G^0}{T} = -\frac{RT\ln 4}{T}$

$\Delta S^0 = -8.314 \times 1.4$

$= -11.62 Jmol^{-1}K^{-1}$

33. (c)

dsp^2: $[Ni(CN)_4]^{2-}$

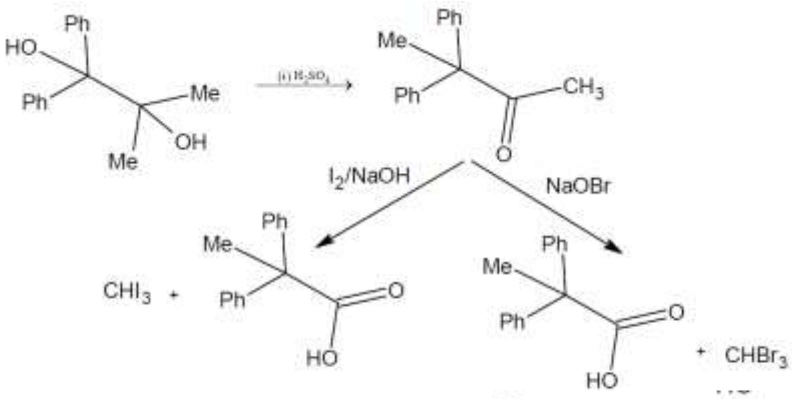
sp^3: $[FeCl_4]^{2-}$; $[Ni(CO)_4]$

sp^3d^2: $[FeF_6]^{4-}$

d^2sp^3: $[Ti(H_2O)_3Cl_3]$, $[Cr(NH_3)_6]^{3+}$

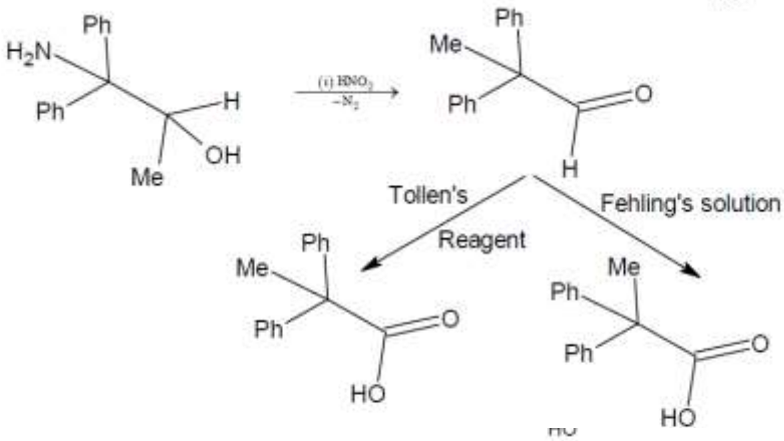
34. (d)

(P)



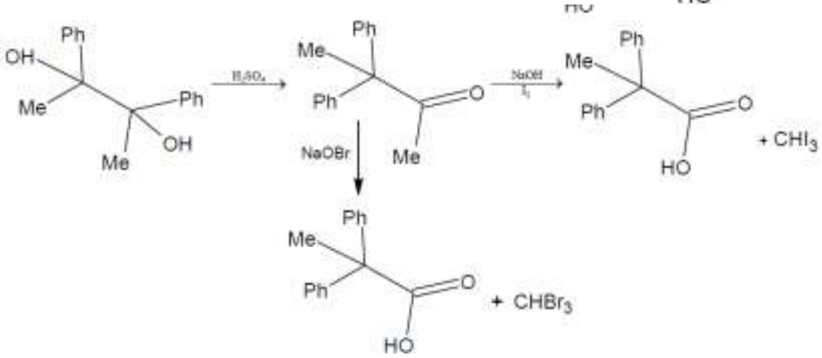
HO
Ph
Me
OH
(i) H_2SO_4
CH_3
O
I_2/NaOH
NaOBr
CHI_3 +
+ $CHBr_3$

(Q)



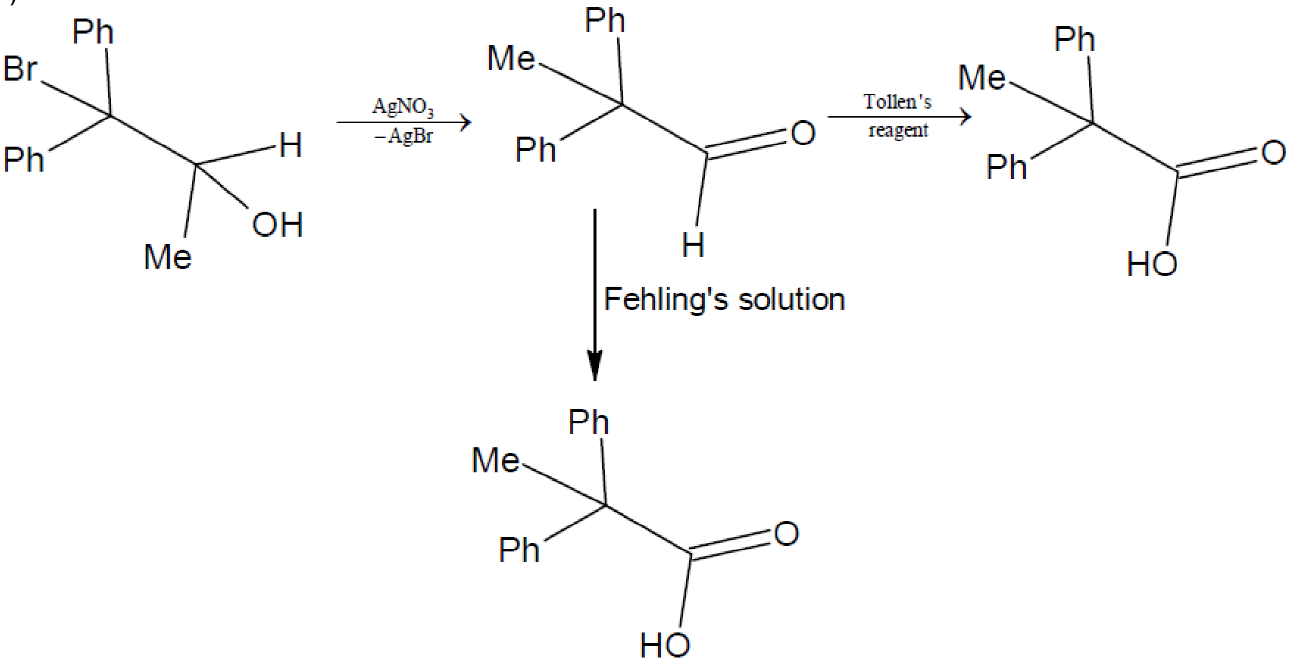
H_2N
Ph
H
OH
Me
(i) HNO_2
$-N_2$
O
Tollen's Reagent
Fehling's solution
HO

(R)



OH
Ph
Me
H_2SO_4
NaOH
I_2
NaOBr
O
HO
+ CHI_3
+ $CHBr_3$

(S)



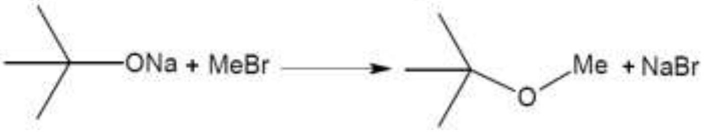
Br
Ph
H
OH
Me
$AgNO_3$
$-AgBr$
O
Tollen's reagent
Fehling's solution
HO

35. (b)

(P) 

(Q) 

(R) 

(S) 

36. (d)

$$\frac{K_w}{K_a} = \frac{[CH_3COO^-][OH^-]}{[CH_3COOH]}$$

$$[OH^-] = \left(\frac{K_w}{K_a} \times C\right)^{1/2}$$

(P) is a buffer, so $[H^+]$ does not change on dilution, as [salt] = [acid].

(Q) contains only CH_3COONa

So $CH_3COO^- + H_2O \rightleftharpoons CH_3COOH + OH^-$

$[OH^-] = \sqrt{k_h \times C} \Rightarrow [H^+]$ decreases by $\sqrt{2}$ times

(R) is also salt hydrolysis

So, $NH_4^+ + H_2O \rightleftharpoons NH_4OH + H^+$

$$[H^+] = \sqrt{\frac{K_w}{K_b} \times C}$$

So, C is made $\frac{1}{2}$ so, $[H^+]$ becomes $\frac{1}{\sqrt{2}}$

(S) it is a solubility equilibria

So dilution does not effect $[H^+]$ or $[OH^-]$

37. (d)

$$f_n(z) = \sum_{j=1}^{n} \tan^{-1}(x+j) - \tan^{-1}(x+j-1)$$

$$= \tan^{-1}(x+n) - \tan^{-1} x$$

$$= \tan^{-1}\left(\frac{n}{1+x(x+n)}\right)$$

(a) & (b) zero is not in the domain

(c) $\lim_{x\to\infty} \tan(f_n(x)) = 0$

(d) $\lim_{x\to\infty} \sec^2(f_n(x)) = 1$

38. (b, d)

Locus of M is circle center (0, 1) and radius $\sqrt{40}$, excluding points P and Q

$\Rightarrow x^2 + y^2 - 2y = 39$ excluding points P and Q

$\therefore$ (-2, 7) is excluded from locus and 0, $\left(0, \frac{3}{2}\right)$ does not lie in E_1

Locus of point in E_2 is circle with diameter (0, 1) and (1, 1), excluding midpoints of chords passing through P or Q

$\Rightarrow x^2 - x + (y-1)^2 = 0$

$\therefore \left(\frac{1}{2},1\right)$ does not lie in E_2 and $\left(\frac{4}{5},\frac{7}{5}\right)$ doesn't lie in E_2 since latter is midpoint of chord through P

39. (a, d)

$\Delta = \begin{vmatrix} -1 & 2 & 5 \\ 2 & -4 & 3 \\ 1 & 2 & -2 \end{vmatrix} = 0$ given equation represent family of lines

$\Rightarrow$ 2x - 4y + 3z - b_2 + λ (-x + 2y + 5z – b_1) = 0 is same as x - 2y + 2z - b_3 = 0 for same λ

$\frac{2-\lambda}{1} = \frac{2\lambda-4}{-2} = \frac{5\lambda+3}{2} = \frac{-(b_1\lambda+b_2)}{-b_3}$

$\lambda = \frac{1}{7}$ and $13b_3 = b_1 + 7b_2$

$A \to \begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 2 & 6 \end{vmatrix} \neq 0$ as planes are non-parallel is must represent family of planes for solution to exist

$(5\mu + 1)x + (2\mu + 1)y + (6\mu + 3)z = b_1 + \mu b_2$

Must be 2x + b + 3z + b_3 = 0 which gives

μ = 1, $b_1 + b_2 + 3b_3 = 0$ which is not true always

C $\to$ x - 2y + 5z = $-b_1$, x - 2y + 5z = $\frac{b_2}{2}$, x - 2y + 5z = b_3 as these are parallel plane for specific case when $-b_1 = \frac{b_2}{2} = b_3$

$D \to \begin{vmatrix} 1 & 2 & 5 \\ 2 & 0 & 3 \\ 1 & 4 & -5 \end{vmatrix} \neq 0$

40. (a, c)

$y = mx + \frac{1}{m}$

$y = mx \pm \frac{1}{\sqrt{2}}\sqrt{1+m^2} \Rightarrow \frac{2}{m^2} = 1 + m^2$

$\Rightarrow (m^2+2)(m^2-1) = 0$

$\Rightarrow m = \pm 1$

$\Rightarrow y = x + 1 \;\&\; y = -x - 1$

$e = \sqrt{1 - \frac{1/2}{1}} = \frac{1}{\sqrt{2}}$

L.R. $= \frac{2b^2}{a} = \frac{2\times 1/2}{1} = 1$

Ellipse = $\frac{x^2}{1} + \frac{y^2}{1/2} = 1 \Rightarrow y = -\frac{1}{\sqrt{2}}\sqrt{1-x^2}$

Area of region $2\int_{1/\sqrt{2}}^{1} \frac{1}{\sqrt{2}}\sqrt{1-x^2}\,dx$

$= \sqrt{2}\left(-\frac{1}{2\sqrt{2}}\frac{1}{\sqrt{2}} + \frac{\pi}{4} - \frac{\pi}{8}\right)$

$$= \sqrt{2}\left(\frac{\pi}{8}-\frac{1}{4}\right)=\frac{(\pi-2)}{4\sqrt{2}}$$

41. (a, c, d)

$sz+t\bar{z}+r=0 \Rightarrow \bar{s}\bar{z}+\bar{t}z+\bar{r}=0$

Eliminating $\bar{z}$, we get $z=\dfrac{\bar{r}t-r\bar{s}}{|s|^2-|t|^2}$

(a) L has one element exactly $\Rightarrow \begin{vmatrix} s & t \\ \bar{t} & \bar{s}\end{vmatrix} \neq 0 \Rightarrow |s|^2 \neq |t|^2$

(b) s = t = i and r = i $\Rightarrow$ L has no element

(c) Adding both equations $(s+\bar{t})z+(\bar{s}+t)\bar{z}+(r+\bar{r})=0$ which is a line if $s+\bar{t}\neq 0$

If $s+\bar{t}=0 \Rightarrow t=-\bar{s}, sz-\bar{s}\bar{z}+r=0$

Which either represents no points or a straight line

In either case, number of points of intersection of given circle and L cannot be more than 2

(d) If a and P both satisfy the given equation then any number of form $\lambda\alpha+(1-\lambda)\beta$ satisfies the given equation where $\lambda \in R$

42. (b, c, d)

Using L Hospitals rule

$$\lim_{t\to x}\frac{f(x)\cos t-f'(t)\sin x}{1}=\sin^2 x$$

$$\Rightarrow f(x)\cos x-f'(x)\sin x=\sin^2 x$$

$$\Rightarrow \frac{f(x)}{\sin x}=-x+c$$

$$\Rightarrow f(x)=-x\sin x+c\sin x$$

$$f\left(\frac{\pi}{6}\right)=-\frac{\pi}{12}=-\frac{\pi}{6}.\frac{1}{2}+c.\frac{1}{2}$$

$$\Rightarrow c=0$$

$$\Rightarrow f(x)=-x\sin x$$

43. (2)

$$I_2=\int_0^{1/2}\frac{1+\sqrt{3}}{\sqrt{1-x^2}\,(1-x)}dx$$

Let x = sin θ

$$I_2=\int_0^{\pi/6}\frac{1+\sqrt{3}}{\left(\sin\frac{\theta}{2}-\cos\frac{\theta}{2}\right)^2}d\theta=\int_0^{\pi/6}\frac{(1+\sqrt{3})\sec^2\frac{\theta}{2}}{\left(\tan\frac{\theta}{2}-1\right)}d\theta$$

Let $\tan\dfrac{\theta}{2}=y$

$$\Rightarrow y=\int_0^{2-\sqrt{3}}\frac{2(1+\sqrt{3})dy}{(y-1)^2}=2(1+\sqrt{3})\left|\frac{-1}{y-1}\right|_0^{2-\sqrt{3}}=2(1+\sqrt{3})\left|\frac{-1}{1-\sqrt{3}}-1\right|=2(1+\sqrt{3})\left(\frac{\sqrt{3}-1}{2}\right)=2$$

44. (4) If 0 is used maximum can only be 4

If 0 not used then by using only {-1, 1} can only form matrix with maximum determinant value 4

45. (119)

$$\alpha={}^7C_5.5!=2520$$

$$\beta=5^7-{}^5C_1 4^7+{}^5C_2 3^7-{}^5C_3 2^7+{}^5C_4=16800$$

$$\frac{\beta-\alpha}{5!}=119$$

46. (0.4)

$$\int_0^y \frac{dy}{25y^2-4}=\int_0^x dx \Rightarrow \frac{1}{25}\int_0^y \frac{dy}{y^2-\left(\frac{2}{5}\right)^2}=x$$

$$\Rightarrow -\frac{1}{25}\cdot\frac{1}{2\cdot\frac{2}{5}}\ln\left|\frac{\frac{2}{5}+y}{\frac{2}{5}-y}\right|=x$$

$$\Rightarrow y=-\frac{2}{5}\left(\frac{1-e^{-20x}}{1+e^{-20x}}\right)$$

$$\Rightarrow \lim_{x\to-\infty} y=0.4$$

47. (2)

$$f'(x)=\lim_{h\to 0}\frac{f(x)f'(h)+f'(x)f(h)-f(x)}{h}$$

$$f(0)=2f'(0)\text{is } f'(0)=1/2$$

for limit to exist $\frac{f(x)}{2}-f(x)+f'(x)=0$

$$\Rightarrow f'(x)=\frac{f(x)}{2}$$

$$\Rightarrow \ln(f(x))=x/2+C$$

$$f(0)=1\Rightarrow C=0$$

$$\Rightarrow f(x)=e^{x/2}$$

$$\ln(f(x))=\ln(e^2)=2$$

48. (8)

Let Q $\equiv$ (0, 0, z_1) then P is image of Q in x + y = 3

$$\Rightarrow \frac{x_p-0}{1}=\frac{y_p-0}{1}=\frac{-2(-3)}{2}=3$$

$$\Rightarrow P\equiv(3,3,z_1)$$

$\Rightarrow$ $(3)^2 + z_1^2 = 25 \Rightarrow z_1 = 4$

Length of PR = 2 z_1 = 8

49. (0.5)

$$\vec{p}=\frac{\hat{i}}{2}-\frac{j}{2}-\frac{k}{2}$$

$$\vec{q}=-\frac{\hat{i}}{2}+\frac{j}{2}-\frac{k}{2}$$

$$\vec{r}=-\frac{\hat{i}}{2}-\frac{j}{2}+\frac{k}{2}$$

$$\vec{t}=\frac{\hat{i}}{2}+\frac{j}{2}+\frac{k}{2}$$

$$=\left|(\vec{p}\times\vec{q})(\vec{r}\times\vec{t})\right|=\left|[prt]\vec{q}-[qrt]\vec{p}\right|$$

$$\left|-\frac{4}{8}\vec{q}-\frac{4}{8}\vec{p}\right|$$

$$=\frac{4}{8}\left|\vec{p}\times\vec{q}\right|=0.5$$

50. (646)

$$\sum r\left(C_r\right)^2=n\,{}^{2n-1}C_{n-1}$$

$$=\frac{10\,{}^{19}C_9}{1430}=646$$

51. (a) $E_1:\left\{x \mid x\in(-\infty,0)\cup(1,\infty)\right\}$

$$E_2:\left\{x \mid x\in\left(-\infty,-\frac{1}{e-1}\right]\cup\left[\frac{e}{e-1},\infty\right)\right\}$$

$$f(x)=\ln\left(\frac{x}{x-1}\right)\Rightarrow R_f=R-\{0\}$$

$$R_g=[-1,1]-\{0\}$$

$$\Rightarrow P\to 4, Q\to 2, R\to 1, S\to 1$$

52. (c) $\alpha_1={}^6C_3-{}^5C_2$ = 20- 10 = 200

α_2 = ${}^6C_1.\ {}^5C_1 + {}^6C_2\ {}^5C_2 + {}^6C_3\ {}^5C_3 + {}^6C_4\ {}^5C_4 + {}^6C_5\ {}^5C_5$

= 30+150 + 200 + 75 + 6

= 461

α_3 = ${}^5C_2\ {}^6C_3 + {}^5C_3\ {}^6C_2 + {}^5C_4\ {}^6Ci + {}^5C_5$

= 200+150 + 30+1 = 381

α_4 = $1.[{}^4C_1 - {}^5C_2 + {}^4\ C_2{}^5C_1 + {}^4C_3] + 1.[{}^4C_2{}^5C_1 + {}^4C_3] + [{}^4C_4 + {}^4C_3 - {}^5C_1 + {}^4C_2 + {}^5C_2]$

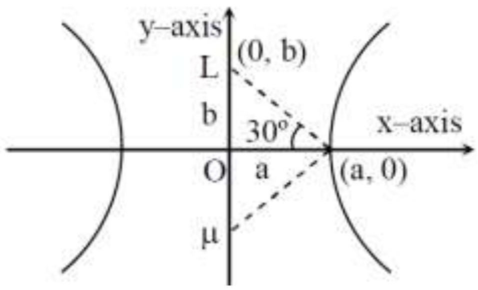
with G_1 with M_1 neither M_1 nor G_1

= (40 + 30 + 4) + (34) + (1 + 20 + 60)

= 74 + 34+81 = 189

P $\to$ 4, Q $\to$ 6, R $\to$ 5, S $\to$ 2.

53. (b)



$$ab=4\sqrt{3}$$

$$\frac{b}{a}=\frac{1}{\sqrt{3}}$$

$$\Rightarrow b=2, a=2\sqrt{3}$$

$$e=\sqrt{1+\frac{4}{4.3}}=\frac{2}{\sqrt{3}}$$

$$2ae=2.2\sqrt{3.\frac{2}{\sqrt{3}}}=8$$

L.R. $=\frac{2b^2}{a}=\frac{2.4}{2\sqrt{3}}=\frac{4}{\sqrt{3}}$

P $\to$ 4; Q $\to$ 3; R $\to$ 1; S $\to$ 2

54. (d)

(i) $RHL=\lim\limits_{h\to 0^+}\sin\left(\sqrt{1-e^{-h^2}}\right)=0=LHL=f_1(0)$

$f_1(x)$ is continuous at x = 0

$$RHD = \lim_{h\to 0}\frac{\sin\left(\sqrt{1-e^{-h^2}}\right)}{h} = \lim_{h\to 0}\frac{\sin\sqrt{1-e^{-h^2}}}{\sqrt{1-e^{-h^2}}}.\lim_{h\to 0}\frac{\sqrt{1-e^{-h^2}}}{h} = 1$$

$$LHD = \lim_{h\to 0}\frac{\sin\left(\sqrt{1-e^{-h^2}}\right)}{-h} = -1 \neq RHD$$

(ii) $LHL = \lim_{h\to 0}\frac{-\sin(-h)}{\tan^{-1}(-h)} = \lim_{h\to 0}\frac{\sinh}{-\tan^{-1}h} = -1$

$RHL = \lim_{h\to 0}\frac{\sinh}{\tan^{-1}h} = 1 \Rightarrow$ Not continuous

(iii) $LHL = \lim_{h\to 0}\left[\sin\left(\ln(-h+2)\right)\right] = \lim_{h\to\infty}\left[\sin(\ln 2)\right] = 0$

$RHL = \lim_{h\to 0}\left[\sin\left(\ln(2+h)\right)\right] = 0 = f_3(0)$ continuous at x = 0

$$LHD = \lim_{h\to 0}\frac{\left[\sin\left(\ln(-h+2)\right)\right]-0}{-h} = 0$$

$RHD = \lim_{h\to 0}\frac{\left[\sin\left(\ln(h+2)\right)\right]-0}{h} = 0$ differentiable at x = 0

$f_3'(x)$ is 0 in neighbourhood of x = 0

so $f_3'(x)$ is continuous at x = 0

(iv) f4(x) is continuous at x = 0

$f_4'(x)$ is also differentiable at x = 0