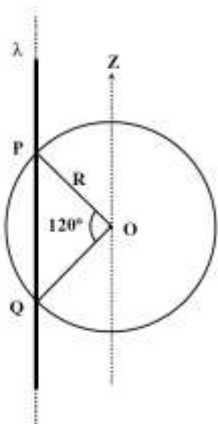


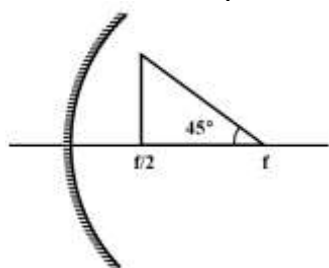
SECTION 1

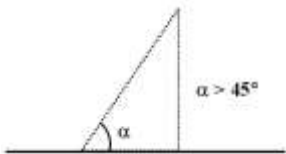
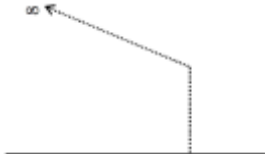
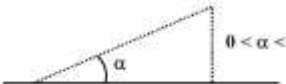
(Maximum Marks: 24)

- This section contains SIX (06) questions.
 - Each question has FOUR options for correct answer(s). ONE OR MORE THAN ONE of these four
 - option(s) is (are) correct option(s).
 - For each question, choose the correct option(s) to answer the question.
 - Answer to each question will be evaluated according to the following marking scheme: Full Marks +4 If only (all) the correct option(s) is (are) chosen.
Partial Marks +3 If all the four options are correct but ONLY three options are chosen.
Partial Marks +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks: 0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks: -2 In all other cases.
 - For Example: If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.
1. **A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x -axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true?**
 - (a) The force applied on the particle is constant
 - (b) The speed of the particle is proportional to time
 - (c) The distance of the particle from the origin increases linearly with time
 - (d) The force is conservative
 2. **Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity u_0 . Which of the following statements is (are) true?**
 - (a) The resistive force of liquid on the plate is inversely proportional to h
 - (b) The resistive force of liquid on the plate is independent of the area of the plate
 - (c) The tangential (shear) stress on the floor of the tank increases with u_0
 - (d) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid
 3. **An infinitely long thin non-conducting wire is parallel to the z -axis and carries a uniform line charge density λ . It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the center O of the spherical shell, as shown in the figure. The permittivity of free space is ϵ_0 . Which of the following statements is (are) true?**



- (a) The electric flux through the shell is $\sqrt{3}R\lambda / \epsilon_0$
 (b) The z-component of the electric field is zero at all the points on the surface of the shell
 (c) The electric flux through the shell is $\sqrt{2}R\lambda / \epsilon_0$
 (d) The electric field is normal to the surface of the shell at all points
4. A wire is bent in the shape of a right-angled triangle and is placed in front of a concave mirror of focal length f , as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire? (These figures are not to scale.)



- (a)  (b) 
- (c)  (d) 

5. In a radioactive decay chain, If ${}^{232}_{90}\text{Th}$ nucleus decays to ${}^{212}_{82}\text{Pb}$ nucleus. Let N_α and N_β be the number of α and β^- particles, respectively, emitted in this decay process. Which of the following statements is (are) true?
- (a) $N_\alpha = 5$ (b) $N_\alpha = 6$
 (c) $N_\beta = 2$ (d) $N_\beta = 4$
6. In an experiment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 Hz is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length 50.7 cm and 83.9 cm. Which of the following statements is (are) true?
- (a) The speed of sound determined from this experiment is 332 ms^{-1}
 (b) The end correction in this experiment is 0.9 cm
 (c) The wavelength of the sound wave is 66.4 cm
 (d) The resonance at 50.7 cm corresponds to the fundamental harmonic

SECTION 2
(Maximum Marks: 24)

- This section contains EIGHT (08) questions. The answer to each question is a NUMERICAL VALUE.
 - For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
 - Answer to each question will be evaluated according to the following marking scheme: Full Marks: +3 If ONLY the correct numerical value is entered as answer.
Zero Marks: 0 0 In all other cases.
7. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass $m = 0.4 \text{ kg}$ is at rest on this surface. An impulse of 1.0 Ns is applied to the block at time $t = 0$ so that it starts moving along the x -axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and $\tau = 4 \text{ s}$. The displacement of the block, in meters, at $t = \tau$ is _____. Take $e^{-1} = 0.37$.
 8. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is_____.
 9. A particle, of mass 10^{-3} kg and charge 1.0 C , is initially at rest. At time $t = 0$, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin \omega t \hat{i}$, where $E_0 = 1.0 \text{ NC}^{-1}$ and $\omega = 10^3 \text{ rad s}^{-1}$. Consider the effect of only the electrical force on the particle. Then the maximum speed, in m s^{-1} , attained by the particle at subsequent times is_____.
 10. A moving coil galvanometer has 50 turns and each turn has an area $2 \times 10^{-4} \text{ m}^2$. The magnetic field produced by the magnet inside the galvanometer is 0.02 T . The torsional constant of the suspension wire is $10^{-4} \text{ N m rad}^{-1}$. When a current flow through the galvanometer, a full-scale deflection occurs if the coil rotates by 0.2 rad . The resistance of the coil of the galvanometer is 50Ω . This galvanometer is to be converted into an ammeter capable of measuring current in the range $0 - 1.0 \text{ A}$. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance, in ohms, is _____
 11. A steel wire of diameter 0.5 mm and Young's modulus $2 \times 10^{11} \text{ Nm}^{-2}$ carries a load of mass M . The length of the wire with the load is 1.0 m . A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1.0 mm , is attached. The 10 divisions of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by 1.2 kg , the vernier scale division which coincides with a main scale division is_____. Take $g = 10 \text{ ms}^{-2}$ and is $\pi = 3.2$.
 12. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant $R = 8.0 \text{ J mol}^{-1}\text{K}^{-1}$, the decrease in its internal energy, in Joule, is_____.
 13. In a photoelectric experiment a parallel beam of monochromatic light with power of 200 W is incident on a perfectly absorbing cathode of work function 6.25 eV . The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficiency is 100% . A potential difference of 500 V is applied between the cathode and

the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$ N due to the impact of the electrons. The value of n is _____.

Mass of the electron $m_e = 9 \times 10^{-31}$ kg and $1.0 \text{ eV} = 1.6 \times 10^{-19}$ J.

14. Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the $n = 2$ to $n = 1$ transition has energy 74.8 eV higher than the photon emitted in the $n = 3$ to $n = 2$ transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of Z is _____.

SECTION 3

(Maximum Marks: 12)

- This section contains FOUR (04) questions.
- Each question has TWO (02) matching lists: LIST - I and LIST - II.
- FOUR options are given representing matching of elements from LIST - I and LIST - II. ONLY ONE of these four options corresponds to a correct matching.
- For each question, choose the option corresponding to the correct matching.
- For each question, marks will be awarded according to the following marking scheme:
Full Marks: +3 If ONLY the option corresponding to the correct matching is chosen.
Zero Marks :0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks: -1 In all other cases.

15. The electric field E is measured at a point $P(0, 0, d)$ generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d . List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II.

LIST-I

P. E is independent of d

Q. $E \propto \frac{1}{d}$

R. $E \propto \frac{1}{d^2}$
with

S. $E \propto \frac{1}{d^3}$
density parallel

density $-\lambda$. Take

with

(a) $P \rightarrow 5$; $Q \rightarrow 3, 4$; $R \rightarrow 1$; $S \rightarrow 2$

(b) $P \rightarrow 5$; $Q \rightarrow 3$; $R \rightarrow 1, 4$; $S \rightarrow 2$

(c) $P \rightarrow 5$; $Q \rightarrow 3$; $R \rightarrow 1, 2$; $S \rightarrow 4$

(d) $P \rightarrow 4$; $Q \rightarrow 2, 3$; $R \rightarrow 1$; $S \rightarrow 5$

LIST-II

1. A point charge Q at the origin

2. A small dipole with point charges Q at $(0, 0, l)$ and $-Q$ at $(0, 0, -l)$. Take $2l \ll d$

3. An infinite line charge coincident with the x -axis, uniform linear charge density λ .

4. Two infinite wires carrying uniform linear charge density λ to the x -axis. The one along $(y=0, z=l)$ has a charge density $2l \ll d$

5. Infinite plane charge coincident with the xy -plane uniform surface charge density

16. A planet of mass M , has two natural satellites with masses m_1 and m_2 . The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force between the satellites. Define v_1 , L_1 , K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and v_2 , L_2 , K_2 and

T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II.

LIST-I

LIST-II

P. $\frac{v_1}{v_2}$

1. $\frac{1}{8}$

Q. $\frac{L_1}{L_2}$

2. 1

R. $\frac{K_1}{K_2}$

3. 2

S. $\frac{T_1}{T_2}$

4. 8

(a) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 3$

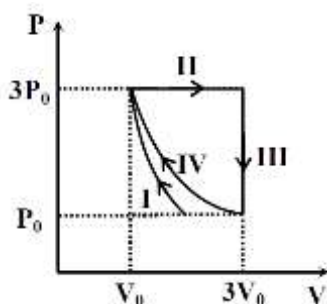
(b) $P \rightarrow 3; Q \rightarrow 2; R \rightarrow 4; S \rightarrow 1$

(c) $P \rightarrow 2; Q \rightarrow 3; R \rightarrow 1; S \rightarrow 4$

(d) $P \rightarrow 2; Q \rightarrow 3; R \rightarrow 4; S \rightarrow 1$

17. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, $3P_0$ one is isochoric, one is isothermal and one is adiabatic.

Match the processes mentioned in List-1 with the corresponding statements in List-II.



LIST-I

LIST-II

P. In process I

1. Work done by the gas is zero

Q. In process II

2. Temperature of the gas remains unchanged

R. In process III

3. No heat is exchanged between the gas and its surroundings

S. In process IV

4. Work done by the gas is $6P_0V_0$

(a) $P \rightarrow 4; Q \rightarrow 3; R \rightarrow 1; S \rightarrow 2$

(b) $P \rightarrow 1; Q \rightarrow 3; R \rightarrow 2; S \rightarrow 4$

(c) $P \rightarrow 3; Q \rightarrow 4; R \rightarrow 1; S \rightarrow 2$

(d) $P \rightarrow 3; Q \rightarrow 4; R \rightarrow 2; S \rightarrow 1$

18. In the List-I below, four different paths of a particle are given as functions of time. In these functions, a and P are positive constants of appropriate dimensions and $\alpha \neq \beta$. In each case, the force acting on the particle is either zero or conservative. In List-II, five physical quantities of the particle are mentioned: $\vec{p}$ is the linear momentum, $\vec{L}$ is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy. Match each path in List-I with those quantities in List-II, which are conserved for that path.

LIST-I

LIST-II

P. $\vec{r}(t) = \alpha t \hat{i} + \beta t \hat{j}$

1. $\vec{p}$

Q. $\vec{r}(t) = \alpha \cos \omega t \hat{i} + \beta \sin \omega t \hat{j}$

2. $\vec{L}$

R. $\vec{r}(t) = \alpha (\cos \omega t \hat{i} + \sin \omega t \hat{j})$

3. K

S. $\vec{r}(t) = \alpha t \hat{i} + \frac{\beta}{2} t^2 \hat{j}$

4. U

5. E

(a) $P \rightarrow 1,2,3,4,5$; $Q \rightarrow 2, 5$; $R \rightarrow 2,3,4,5$; $S \rightarrow 5$

(b) $P \rightarrow 1,2,3,4,5$; $Q \rightarrow 3, 5$; $R \rightarrow 2,3,4,5$; $S \rightarrow 2, 5$

(c) $P \rightarrow 2,3,4$; $Q \rightarrow 5$; $R \rightarrow 1,2,4$; $S \rightarrow 2,5$

(d) $P \rightarrow 1,2,3,5$; $Q \rightarrow 2,5$; $R \rightarrow 2,3,4,5$; $S \rightarrow 2,5$

PART H-CHEMISTRY

SECTION 1

(Maximum Marks: 24)

This section contains SIX (06) questions.

- Each question has FOUR options for correct answer(s). ONE OR MORE THAN ONE of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks: +4 If only (all) the correct option(s) is (are) chosen.
Partial Marks: +3 If all the four options are correct but ONLY three options are chosen.
Partial Marks: +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks: +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks: 0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks: -2 In all other cases.
- For Example: If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option) without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks

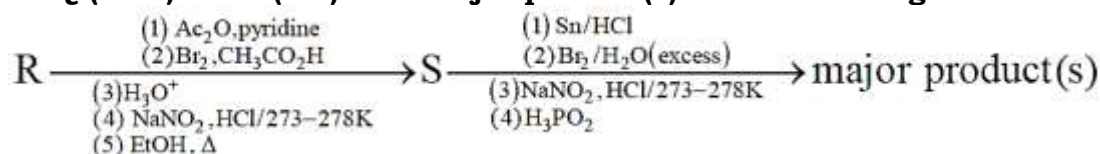
19. The correct option(s) regarding the complex $[\text{Co}(\text{en})(\text{NH}_3)_3(\text{H}_2\text{O})]^{3+}$ ($\text{en} = \text{H}_2\text{NCH}_2\text{CH}_2\text{NH}_2$) is (are)

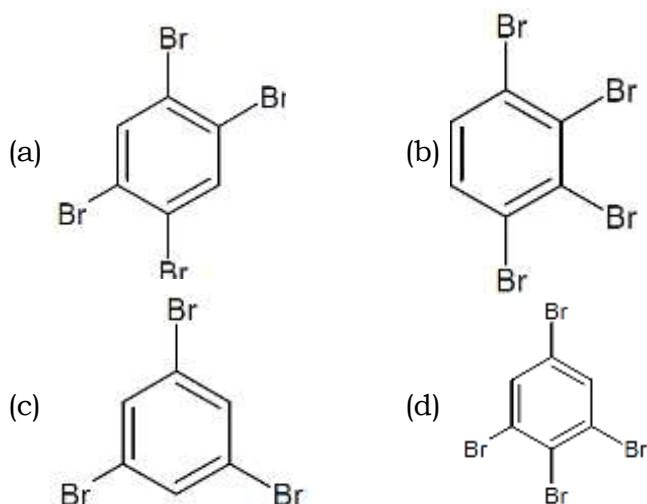
- (a) It has two geometrical isomers
- (b) It will have three geometrical isomers if bidentate 'en' is replaced by two cyanide ligands
- (c) It is paramagnetic
- (d) It absorbs light at longer wavelength as compared to $[\text{Co}(\text{en})(\text{NH}_3)_4]^{3+}$

20. The correct option(s) to distinguish nitrate salts of Mn^{2+} and Cu^{2+} taken separately is (are)

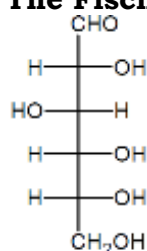
- (a) Mn^{2+} shows the characteristic green colour in the flame test
- (b) Only Cu^{2+} shows the formation of precipitate by passing H_2S in acidic medium
- (c) Only Mn^{2+} shows the formation of precipitate by passing H_2S in faintly basic medium
- (d) Cu^{2+}/Cu has higher reduction potential than Mn^{2+}/Mn (measured under similar conditions)

21. Aniline reacts with mixed acid (cone. HNO_3 and cone. H_2SO_4) at 288 K to give P (51%), Q (47%) and R(2%). The major product(s) of the following reaction sequence is (are)



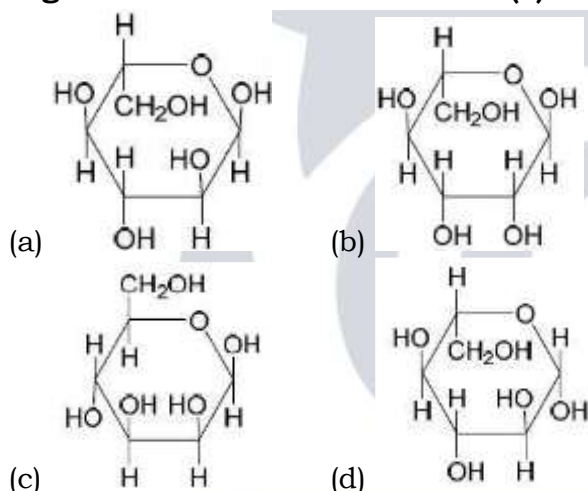


22. The Fischer presentation of D-glucose is given below CHO

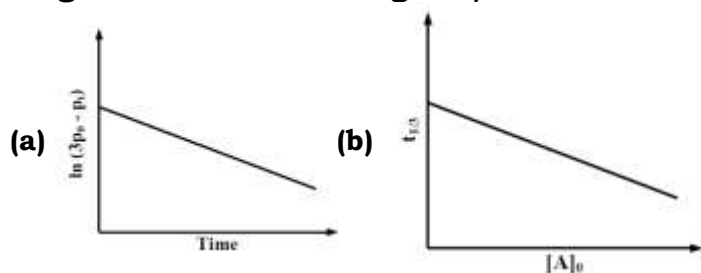


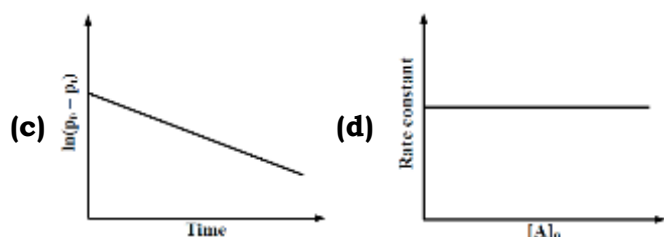
D-glucose

D-glucose The correct structure(s) of β -L-glucopyranose is(are)

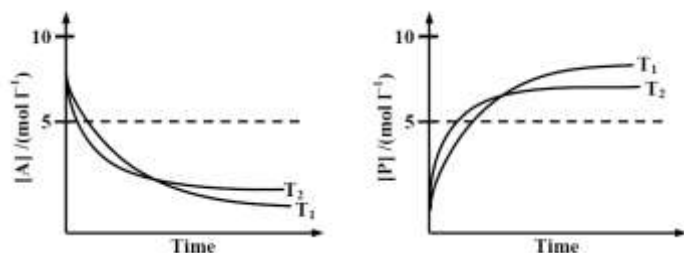


23. For a first order reaction $A(g) \rightarrow 2B(g) + C(g)$ at constant volume and 300 K, the total pressure at the beginning ($t = 0$) and at time t are P_0 and P_t , respectively. Initially, only A is present with concentration $[A]_0$, and $t_{1/3}$ is the time required for the partial pressure of A to reach $1/3^{\text{rd}}$ of its initial value. The correct option(s) is (are) (Assume that all these gases behave as ideal gases)





24. For a reaction, $A \rightleftharpoons P$, the plots of $[A]$ and $[P]$ with time at temperatures T_1 and T_2 are given below.



If $T_2 > T_1$, the correct statement(s) is (are) (Assume ΔH^θ and ΔS^θ are independent of temperature and ratio of $\ln K$ at T_1 to $\ln K$ at T_2 is greater than T_2/T_1 . Here H, S, G and K are enthalpy, entropy, Gibbs energy and equilibrium constant, respectively.)

- (a) $\Delta H^\theta < 0$, $\Delta S^\theta < 0$ (b) $\Delta G^\theta < 0$, $\Delta H^\theta > 0$
 (c) $\Delta G^\theta < 0$, $\Delta S^\theta < 0$ (d) $\Delta G^\theta < 0$, $\Delta S^\theta > 0$

SECTION 2

(Maximum Marks: 24)

- This section contains **EIGHT** (08) questions. The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
 Full Marks: +3 If ONLY the correct numerical value is entered as answer.
 Zero Marks: 0 In all other cases

25. The total number of compounds having at least one bridging oxo group among the molecules given below is ____.



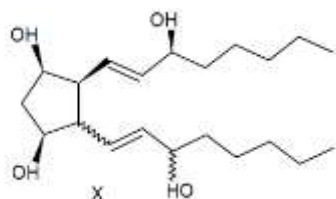
26. Galena (an ore) is partially oxidized by passing air through it at high temperature. After some time, the passage of air is stopped, but the heating is continued in a closed furnace such that the contents undergo self-reduction. The weight (in kg) of Pb produced per kg of O_2 consumed is ____.

(Atomic weights in g mol⁻¹: O = 16, S = 32, Pb = 207)

27. To measure the quantity of $MnCl_2$ dissolved in an aqueous solution, it was completely converted to $KMnO_4$ using the reaction, $MnCl_2 + K_2S_2O_8 + H_2O \rightarrow KMnO_4 + H_2SO_4 + HCl$ (equation not balanced). Few drops of concentrated HCl were added to this solution and gently warmed. Further, oxalic acid (225 mg) was added in portions till the colour of the permanganate ion disappeared. The quantity of $MnCl_2$ (in mg) present in the initial solution is ____.

(Atomic weights in g mol⁻¹: Mn = 55, Cl = 35.5)

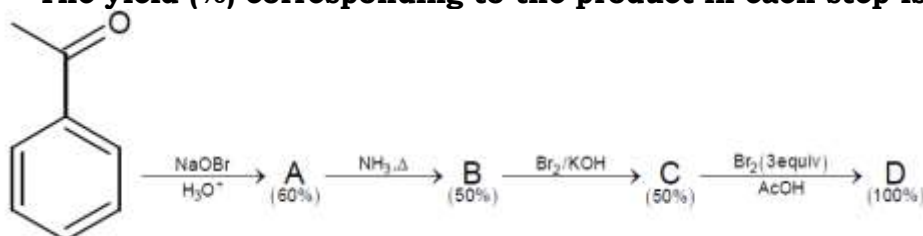
28. For the given compound X, the total number of optically active stereoisomers is ____



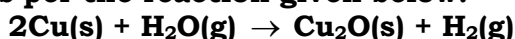
— This type of bond indicates that the configuration at the specific carbon and geometry of the double bond is fixed

~~~~~ This type of bond indicates that the configuration at the specific carbon and the geometry of the double bond is NOT fixed

29. In the following reaction sequence, the amount of D (in g) formed from 10 moles of acetophenone is \_\_\_\_\_. (Atomic weights in  $\text{g mol}^{-1}$ : H = 1, C = 12, N = 14, O = 16, Br = 80. The yield (%) corresponding to the product in each step is given in the parenthesis)



30. The surface of copper gets tarnished by the formation of copper oxide.  $\text{N}_2$  gas was passed to prevent the oxide formation during heating of copper at 1250 K. However, the  $\text{N}_2$  gas contains 1 mole % of water vapour as impurity. The water vapour oxidizes copper as per the reaction given below:



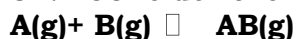
$p_{\text{H}_2}$  is the minimum partial pressure of  $\text{H}_2$  (in bar) needed to prevent the oxidation at 1250 K. The value of  $\ln(p_{\text{H}_2})$  is\_\_\_\_\_.

(Given: total pressure = 1 bar, R (universal gas constant) =  $8 \text{ J K}^{-1} \text{ mol}^{-1}$ ,  $\ln(10) = 2.3$ .  $\text{Cu(s)}$  and  $\text{Cu}_2\text{O(s)}$  are mutually immiscible.)

At 1250 K:  $2\text{Cu(s)} + \frac{1}{2} \text{O}_2\text{(g)} \rightarrow \text{Cu}_2\text{O(s)}$ ;  $\Delta G^\circ = -78,000 \text{ J mol}^{-1}$

$\text{H}_2\text{(g)} + \frac{1}{2} \text{O}_2\text{(g)} \rightarrow \text{H}_2\text{O(g)}$ ;  $\Delta G^\circ = -1,78,000 \text{ J mol}^{-1}$ ; G is the Gibbs energy)

31. Consider the following reversible reaction,



The activation energy of the backward reaction exceeds that of the forward reaction by  $2RT$  (in  $\text{J mol}^{-1}$ ). If the pre-exponential factor of the forward reaction is 4 times that of the reverse reaction, the absolute value of  $\Delta G^\circ$  (in  $\text{J mol}^{-1}$ ) for the reaction at 300 K is\_\_\_\_\_.

(Given;  $\ln(2) = 0.7$ ,  $RT = 2500 \text{ J mol}^{-1}$  at 300 K and G is the Gibbs energy)

32. Consider an electrochemical cell:  $\text{A(s)} \mid \text{A}^{n+}(\text{aq}, 2 \text{ M}) \parallel \text{B}^{2n+}(\text{aq}, 1 \text{ M}) \mid \text{B(s)}$ . The value of  $\Delta H^\circ$  for the cell reaction is twice that of  $\Delta G^\circ$  at 300 K. If the emf of the cell is zero, the  $\Delta S^\circ$  (in  $\text{J K}^{-1} \text{ mol}^{-1}$ ) of the cell reaction per mole of B formed at 300 K is\_\_\_\_\_.

(Given:  $\ln(2) = 0.7$ , R (universal gas constant) =  $8.3 \text{ J K}^{-1} \text{ mol}^{-1}$ . H, S and G are enthalpy, entropy and Gibbs energy, respectively.)

### SECTION 3

(Maximum Marks: 12)

- This section contains FOUR (04) questions.

- Each question has TWO (02) matching lists: LIST - I and LIST - II.
- FOUR options are given representing matching of elements from LIST - I and LIST - II. ONLY ONE of these four options corresponds to a correct matching.
- For each question, choose the option corresponding to the correct matching.
- For each question, marks will be awarded according to the following marking scheme:  
Full Marks: +3 If ONLY the option corresponding to the correct matching is chosen.  
Zero Marks: 0 If none of the options is chosen (i.e. the question is unanswered).  
Negative Marks: -1 In all other cases

33. Match each set of hybrid orbitals from LIST-I with complex(es) given in LIST-II.

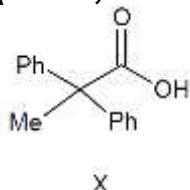
| List - I     | List - II              |
|--------------|------------------------|
| P. $dsp^2$   | 1. $[FeF_6]^{4-}$      |
| Q. $sp^3$    | 2. $[Ti(H_2O)_3Cl_3]$  |
| R. $sp^3d^2$ | 3. $[Cr(NH_3)_6]^{3+}$ |
| S. $d^2sp^3$ | 4. $[FeCl_4]^{2-}$     |
|              | 5. $Ni(CO)_4$          |
|              | 6. $[Ni(CN)_4]^{2-}$   |

The correct option is

- (a)  $P \rightarrow 5$ ;  $Q \rightarrow 4,6$ ;  $R \rightarrow 2,3$ ;  $S \rightarrow 1$   
 (b)  $P \rightarrow 5,6$ ;  $Q \rightarrow 4$ ;  $R \rightarrow 3$ ;  $S \rightarrow 1,2$   
 (c)  $P \rightarrow 6$ ;  $Q \rightarrow 4,5$ ;  $R \rightarrow 1$ ;  $S \rightarrow 2,3$   
 (d)  $P \rightarrow 4,6$ ;  $Q \rightarrow 5,6$ ;  $R \rightarrow 1,2$ ;  $S \rightarrow 3$

34. The desired product X can be prepared by reacting the major product of the reactions in LIST-I with one or more appropriate reagents in LIST-II

(Given, order of migratory aptitude: aryl > alkyl > hydrogen)



| LIST - I              | LIST - II           |
|-----------------------|---------------------|
| <b>P.</b> + $H_2SO_4$ | 1. $I_2, NaOH$      |
| <b>Q.</b> + $HNO_2$   | 2. $[Ag(NH_3)_2]OH$ |
| <b>R.</b> + $H_2SO_4$ | 3. Fehling solution |
| <b>S.</b> + $AgNO_3$  | 4. $HCHO, NaOH$     |
|                       | 5. $NaOBr$          |

The correct option is

- (a)  $P \rightarrow 1$ ;  $Q \rightarrow 2,3$ ;  $R \rightarrow 1,4$ ;  $S \rightarrow 2,4$

- (b)  $P \rightarrow 1,5$ ;  $Q \rightarrow 3,4$ ;  $R \rightarrow 4,5$ ;  $S \rightarrow 3$   
(c)  $P \rightarrow 1,5$ ;  $Q \rightarrow 3,4$ ;  $R \rightarrow 5$ ;  $S \rightarrow 2,4$   
(d)  $P \rightarrow 1,5$ ;  $Q \rightarrow 2,3$ ;  $R \rightarrow 1,5$ ;  $S \rightarrow 2,3$

35. LIST-I contains reactions and LIST-II contains major products.

| List I | List II |
|--------|---------|
| P.     | 1.      |
| Q.     | 2.      |
| R.     | 3.      |
| S.     | 4.      |
|        | 5.      |

Match each reaction in LIST-I with one or more products in LIST-II and choose the correct option.

- (a)  $P \rightarrow 1,5$ ;  $Q \rightarrow 2$ ;  $R \rightarrow 3$ ;  $S \rightarrow 4$   
(b)  $P \rightarrow 1,4$ ;  $Q \rightarrow 2$ ;  $R \rightarrow 4$ ;  $S \rightarrow 3$   
(c)  $P \rightarrow 1,4$ ;  $Q \rightarrow 1,2$ ;  $R \rightarrow 3,4$ ;  $S \rightarrow 4$   
(d)  $P \rightarrow 4,5$ ;  $Q \rightarrow 4$ ;  $R \rightarrow 4$ ;  $S \rightarrow 3,4$

36. Dilution processes of different aqueous solutions, with water, are given in LIST-I. The effects of dilution of the solutions on  $[H^+]$  are given in LIST-II.

(Note: Degree of dissociation ( $\alpha$ ) of weak acid and weak base is  $\ll 1$ ; degree of hydrolysis of salt  $\ll 1$ ;  $[f]$  represents the concentration of  $H^+$  ions)

| List I                                                                                                                                                       | List II                                                                                         |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|
| P. (10 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 60 mL                                                                                       | 1. the value of $[H^+]$ does not change on dilution                                             |
| Q. (20 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 80 mL                                                                                       | 2. the value of $[f]$ changes to half of its initial value on dilution                          |
| R. (20 mL of 0.1 M HCl + 20 mL of 0.1 M ammonia solution) diluted to 80 mL                                                                                   | 3. the value of $[H^+]$ changes to two times of its initial value of dilution.                  |
| S. 10 mL saturated solution of $Ni(OH)_2$ in equilibrium with excess solid $Ni(OH)_2$ is diluted to 20 mL (solid $Ni(OH)_2$ is still present after dilution) | 4. the value of $[H^+]$ changes to $\frac{1}{\sqrt{2}}$ times of its initial value on dilution. |
|                                                                                                                                                              | 5. the value of $[H^+]$ changes of $\sqrt{2}$ times of its initial value on dilution            |

**Match each process given in LIST-I with one or more effect(s) in LIST-II. The correct option is**

- (a)  $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 3; S \rightarrow 1$   
 (b)  $P \rightarrow 4; Q \rightarrow 3; R \rightarrow 2; S \rightarrow 3$   
 (c)  $P \rightarrow 1; Q \rightarrow 4; R \rightarrow 5; S \rightarrow 3$   
 (d)  $P \rightarrow 1; Q \rightarrow 5; R \rightarrow 4; S \rightarrow 1$

### PART III: MATHEMATICS

#### SECTION 1

**(Maximum Marks: 24)**

- This section contains SIX (06) questions.
- Each question has FOUR options for correct answer(s). ONE OR MORE THAN ONE of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:  
 Full Marks: +4 If only (all) the correct option(s) is (are) chosen.  
 Partial Marks: + 3 If all the four options are correct but ONLY three options are chosen.  
 Partial Marks: +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.  
 Partial Marks: +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.  
 Zero Marks: 0 If none of the options is chosen (i.e. the question is unanswered).  
 Negative Marks: - 2 In all other cases.
- For Example: If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.

**37. For any positive integer  $n$ , define  $f_n : (0, \infty) \rightarrow R$  as**

$$f_n(x) = \sum_{j=1}^n \tan^{-1} \left( \frac{1}{1+(x+j)(x+j-1)} \right) \text{ for all } x \in (0, \infty)$$

**(Here, the inverse trigonometric function  $\tan^{-1}x$  assumes values in  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ )**

**Then, which of the following statement(s) is (are) TRUE ?**

- (a)  $\sum_{j=1}^5 \tan^2(f_j(0)) = 55$   
 (b)  $\sum_{j=1}^{10} (1 + f'_j(0)) \sec^2(f_j(0)) = 10$   
 (c) For any fixed positive integer  $n$ ,  $\lim_{x \rightarrow \infty} \tan(f_n(x)) = \frac{1}{n}$   
 (d) For any fixed positive integer  $n$ ,  $\lim_{x \rightarrow \infty} \sec^2(f_n(x)) = 1$

**38. Let  $T$  be the line passing through the points  $P(-2, 7)$  and  $Q(2, -5)$ . Let  $F_1$  be the set of all pairs of circles  $(S_1, S_2)$  such that  $T$  is tangent to  $S_1$  at  $P$  and tangent to  $S_2$  at  $Q$ , and also such that  $S_1$  and  $S_2$  touch each other at a point, say,  $M$ . Let  $E_2$  be the set representing the locus of  $M$  as the pair  $(S_1, S_2)$  varies in  $F_1$ . Let the set of all straight line segments joining a pair of distinct points of  $E_1$  and passing through the point  $R(1, 1)$  be  $F_2$ . Let  $E_2$  be the set of the mid-points of the line segments in the set  $F_2$ . Then, which of the following statements) is (are) TRUE?**

- (a) The point  $(-2, 7)$  lies in  $E_1$

(b) The point  $\left(\frac{4}{5}, \frac{7}{5}\right)$  does NOT lie in  $E_2$

(c) The point  $\left(\frac{1}{2}, 1\right)$  lies in  $E_2$

(d) The point  $\left(0, \frac{3}{2}\right)$  does NOT lie in  $E_1$

39. Let  $S$  be the set of all column matrices  $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$  such that  $b_1, b_2, b_3 \in \mathbb{R}$  and the system of

equations (in real variables)

$$-x + 2y + 5z = b_1$$

$$2x - 4y + 3z = b_2$$

$$x - 2y + 2z = b_3$$

has at least one solution. Then, which of the following system(s) (in real variables) has (have)

at least one solution for each  $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$  ?

(a)  $x + 2y + 3z = b_1$ ,  $4y + 5z = b_2$  and  $x + 2y + 6z = b_3$

(b)  $x + y + 3z = b_1$ ,  $5x + 2y + 6z = b_2$  and  $-2x - y - 3z = b_3$

(c)  $-x + 2y - 5z = b_1$ ,  $2x - 4y + 10z = b_2$  and  $x - 2y + 5z = b_3$

(d)  $x + 2y + 5z = b_1$ ,  $2x + 3z = b_2$  and  $x + 4y - 5z = b_3$

40. Consider two straight lines, each of which is tangent to both the circle  $x^2 + y^2 = \frac{1}{2}$  and the parabola  $y^2 = 4x$ . Let these lines intersect at the point  $Q$ . Consider the ellipse whose center is at the origin  $O(0, 0)$  and whose semi-major axis is  $OQ$ . If the length of the minor axis of this ellipse is  $\sqrt{2}$ , then which of the following statement(s) is (are) TRUE?

(a) For the ellipse, the eccentricity is  $\frac{1}{\sqrt{2}}$  and the length of the latus rectum is 1

(b) For the ellipse, the eccentricity is  $\frac{1}{2}$  and the length of the latus rectum is  $\frac{1}{2}$

(c) The area of the region bounded by the ellipse between the lines  $x = \frac{1}{\sqrt{2}}$  and  $x = 1$  is  $\frac{1}{4\sqrt{2}}(\pi - 2)$

(d) The area of the region bounded by the ellipse between the lines  $x = \frac{1}{\sqrt{2}}$  and  $x = 1$  is  $\frac{1}{16}(\pi - 2)$

41. Let  $s, t, r$  be non-zero complex numbers and  $L$  be the set of solutions  $z = x + iy$  ( $x, y \in \mathbb{R}$ ,  $i = \sqrt{-1}$ ) of the equation  $sz + t\bar{z} + r = 0$ , where  $\bar{z} = x - iy$ . Then, which of the following statement(s) is (are) TRUE?

(a) If  $L$  has exactly one element, then  $|s| \neq |t|$

(b) If  $|s| = |t|$  then  $L$  has infinitely many elements

(c) The number of elements in  $L \cap \{z : |z - 1 + i| = 5\}$  is at most 2

(d) If  $L$  has more than one element, then  $L$  has infinitely many elements

42. Let  $f: (0, \pi) \rightarrow \mathbb{R}$  be a twice differentiable function such that

$$\lim_{t \rightarrow x} \frac{f(x)\sin t - f(t)\sin x}{t - x} = \sin^2 x \text{ for all } x \in (0, \pi)$$

If  $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$ , then which of the following statement(s) is (are) TRUE?

- (a)  $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$
- (b)  $f(x) < \frac{x^4}{6} - x^2$  for all  $x \in (0, \pi)$
- (c) There exists  $\alpha \in (0, \pi)$  such that  $f'(\alpha) = 0$
- (d)  $f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

## SECTION 2

(Maximum Marks: 24)

- This section contains **EIGHT (08)** questions. The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:  
Full Marks: +3 If ONLY the correct numerical value is entered as answer.  
Zero Marks: 0 In all other cases.

43. The value of the integral  $\int_0^{1/2} \frac{1+\sqrt{3}}{\left((x+1)^2(1-x)^6\right)^{1/4}} dx$  is \_\_\_\_\_.

44. Let  $P$  be a matrix of order  $3 \times 3$  such that all the entries in  $P$  are from the set  $\{-1, 0, 1\}$ . Then, the maximum possible value of the determinant of  $P$  is \_\_\_\_\_.

45. Let  $X$  be a set with exactly 5 elements and  $Y$  be a set with exactly 7 elements. If  $\alpha$  is the number of one-one functions from  $X$  to  $Y$  and  $\beta$  is the number of onto functions from  $Y$  to  $X$ , then the value of  $\frac{1}{5!}(\beta - \alpha)$  is \_\_\_\_\_.

46. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function with  $f(0) = 0$ . If  $y = f(x)$  satisfies the differential equation

$$\frac{dy}{dx} = (2+5y)(5y-2), \text{ then the value of } \lim_{x \rightarrow \infty} f(x)$$

47. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function with  $f(0) = 1$  and satisfying the equation  $f(x+y) = f(x)f'(y) + f'(x)f(y)$  for all  $x, y \in \mathbb{R}$ . Then, the value of  $\log_e(f(4))$  is \_\_\_\_\_.

48. Let  $P$  be a point in the first octant, whose image  $Q$  in the plane  $x + y = 3$  (that is, the line segment  $PQ$  is perpendicular to the plane  $x + y = 3$  and the mid-point of  $PQ$  lies in the plane  $x + y = 3$ ) lies on the  $z$ -axis. Let the distance of  $P$  from the  $x$ -axis be 5. If  $R$  is the image of  $P$  in the  $xy$ -plane, then the length of  $PR$  is \_\_\_\_\_.

49. Consider the cube in the first octant with sides  $OP$ ,  $OQ$  and  $OR$  of length 1, along the  $x$ -axis,  $y$ -axis and  $z$ -axis, respectively, where  $O(0, 0, 0)$  is the origin. Let  $S\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$  be the center of the cube and  $T$  be the vertex of the cube opposite to the origin  $O$  such that  $S$  lies on the diagonal  $\vec{p} = \vec{SP}$ ,  $\vec{q} = \vec{SQ}$ ,  $\vec{r} = \vec{SR}$  and  $\vec{t} = \vec{ST}$ , then the value of  $\left|(\vec{p} \times \vec{q}) \times (\vec{t} \times \vec{t})\right|$  is \_\_\_\_\_.

50. Let

$$X = \binom{10}{C_1}^2 + 2\binom{10}{C_2}^2 + 3\binom{10}{C_3}^2 + \dots + 10\binom{10}{C_{10}}^2,$$

where  $\binom{10}{C_r}$ ,  $r \in \{1, 2, \dots, 10\}$  denote binomial coefficients. Then the value of  $\frac{1}{1430}X$  is \_\_\_\_.

### SECTION 3

(Maximum Marks: 12)

- This section contains FOUR (04) questions.
- Each question has TWO (02) matching lists: LIST - I and LIST - II.
- FOUR options are given representing matching of elements from LIST - I and LIST - II. ONLY ONE of these four options corresponds to a correct matching.
- For each question, choose the option corresponding to the correct matching.
- For each question, marks will be awarded according to the following marking scheme:  
Full Marks: +3 If ONLY the option corresponding to the correct matching is chosen.  
Zero Marks: 0 If none of the options is chosen (i.e. the question is unanswered).  
Negative Marks: -1 In all other cases.

51. Let  $E_1 = \left\{ x \in \mathbb{R} : x \neq 1 \text{ and } \frac{x}{x-1} > 0 \right\}$

and  $E_2 = \left\{ x \in E_1 : \sin^{-1} \left( \log_e \left( \frac{x}{x-1} \right) \right) \text{ is a real number} \right\}$

Here, the inverse trigonometric function  $\sin^{-1} x$  assumes values in  $\left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$

Let  $f: E_1 \rightarrow \mathbb{R}$  be the function defined by  $f(x) = \log_e \left( \frac{x}{x-1} \right)$

and  $g: E_2 \rightarrow \mathbb{R}$  be the function defined by  $g(x) = \sin^{-1} \left( \log_e \left( \frac{x}{x-1} \right) \right)$

| List I                      | List II                                                                              |
|-----------------------------|--------------------------------------------------------------------------------------|
| P. The range of f is        | 1. $\left( -\infty, \frac{1}{1-e} \right] \cup \left[ \frac{e}{e-1}, \infty \right)$ |
| Q. The range of g contains  | 2. (0, 1)                                                                            |
| R. The domain of f contains | 3. $\left[ -\frac{1}{2}, \frac{1}{2} \right]$                                        |
| S. The domain of g is       | 4. $(-\infty, 0) \cup (0, \infty)$                                                   |
|                             | 5. $\left( -\infty, \frac{e}{e-1} \right]$                                           |
|                             | 6. $(-\infty, 0) \cup \left( \frac{1}{2}, \frac{e}{e-1} \right]$                     |

The correct option is :

- (a)  $P \rightarrow 4$ ;  $Q \rightarrow 2$ ;  $R \rightarrow 1$ ;  $S \rightarrow 1$   
(b)  $P \rightarrow 3$ ;  $Q \rightarrow 3$ ;  $R \rightarrow 6$ ;  $S \rightarrow 5$   
(c)  $P \rightarrow 4$ ;  $Q \rightarrow 2$ ;  $R \rightarrow 1$ ;  $S \rightarrow 6$   
(d)  $P \rightarrow 4$ ;  $Q \rightarrow 3$ ;  $R \rightarrow 6$ ;  $S \rightarrow 5$
52. In a high school, a committee has to be formed from a group of 6 boys  $M_1, M_2, M_3, M_4, M_5, M_6$  and 5 girls  $G_1, G_2, G_3, G_4, G_5$ .  
(i) Let  $\alpha_1$  be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

- (ii) Let  $\alpha_2$  be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.
- (iii) Let  $\alpha_3$  be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.
- (iv) Let  $\alpha_4$  be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls and such that both  $M_1$  and  $G_1$  are NOT in the committee together.

| List I                        | List II |
|-------------------------------|---------|
| P. The value of $\alpha_1$ is | 1. 136  |
| Q. The value of $\alpha_2$ is | 2. 189  |
| R. The value of $\alpha_3$ is | 3. 192  |
| S. The value of $\alpha_4$ is | 4. 200  |
|                               | 5. 381  |
|                               | 6. 461  |

The correct option is:

- (a)  $P \rightarrow 4; Q \rightarrow 6; R \rightarrow 2; S \rightarrow 1$
- (b)  $P \rightarrow 1; Q \rightarrow 4; R \rightarrow 2; S \rightarrow 3$
- (c)  $P \rightarrow 4; Q \rightarrow 6; R \rightarrow 5; S \rightarrow 2$
- (d)  $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 3; S \rightarrow 1$

53. Let  $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  where  $a > b > 0$ , be a hyperbola in the xy-plane whose conjugate axis LM subtends an angle of  $60^\circ$  at one of its vertices N. Let the area of the triangle LMN be  $4\sqrt{3}$ .

| List I                                      | List II                 |
|---------------------------------------------|-------------------------|
| P. The length of the conjugate axis of H is | 1. 8                    |
| Q. The eccentricity of H is                 | 2. $\frac{4}{\sqrt{3}}$ |
| R. The distance between the foci of H is    | 3. $\frac{2}{\sqrt{3}}$ |
| S. The length of the latus rectum of H is   | 4. 4                    |

The correct option is :

- (a)  $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 3$
- (b)  $P \rightarrow 4; Q \rightarrow 3; R \rightarrow 1; S \rightarrow 2$
- (c)  $P \rightarrow 4; Q \rightarrow 1; R \rightarrow 3; S \rightarrow 2$
- (d)  $P \rightarrow 3; Q \rightarrow 4; R \rightarrow 2; S \rightarrow 1$

54. Let  $f_1: \mathbb{R} \rightarrow \mathbb{R}, f_2: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow \mathbb{R}, f_3: (-1, e^{\pi/2} - 2) \rightarrow \mathbb{R}$  and  $f_4: \mathbb{R} \rightarrow \mathbb{R}$  be functions defined by

(i)  $f_1(x) = \sin\left(\sqrt{1 - e^{-x^2}}\right),$

(ii)  $f_2(x) = \begin{cases} \frac{|\sin x|}{\tan^{-1} x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases},$  where the inverse trigonometric function  $\tan^{-1}x$  assumes values in  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right),$

(iii)  $f_3(x) = [\sin(\log_e(x + 2))],$  where, for  $t \in \mathbb{R}, [t]$  denotes the greatest integer less than or equal to  $t,$

(iv)  $f_4(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

| List I                   | List II                                                              |
|--------------------------|----------------------------------------------------------------------|
| P. The function $f_1$ is | 1. NOT continuous at $x=0$                                           |
| Q. The function $f_2$ is | 2. continuous at $x=0$ and NOT differentiable at $x=0$               |
| R. The function $f_3$ is | 4. differentiable at $x=0$ and its derivative is continuous at $x=0$ |

The correct option is:

- (a)  $P \rightarrow 2; Q \rightarrow 3; R \rightarrow 1; S \rightarrow 4$
- (b)  $P \rightarrow 4; Q \rightarrow 1; R \rightarrow 2; S \rightarrow 3$
- (c)  $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 3$
- (d)  $P \rightarrow 2; Q \rightarrow 1; R \rightarrow 4; S \rightarrow 3$

