1. (b)

$$\rho = \frac{1}{4\pi G r^2}\frac{d\left(gr^2\right)}{dr} \;\ldots.\;\text{(i)}$$

Because $\frac{mv^2}{r}$ = mg

so $\frac{1}{2}mv^2 = K = \frac{mgr}{2}$

so, from equation (i)

$$\rho = \frac{1}{4\pi G r^2}\frac{d}{dr}\left(\frac{2k}{m}r\right) = \frac{K}{2\pi G m r^2}$$

So, $\frac{\rho}{m} = \frac{K}{2\pi G m^2 r^2}$

2. (c)

$$V_0 = \frac{\sigma 4\pi R^2}{4\pi\varepsilon_0 R} \Rightarrow \sigma = \frac{V_0\varepsilon_0}{R}$$

so, V at $R/2 = v_0 - \frac{1}{4\pi\varepsilon_0}\frac{2}{R}\frac{V_0\varepsilon_0}{R}\alpha 4\pi R^2 = V_0\left(1-2\alpha\right)$

and V at center $= V_0 - \frac{1}{4\pi\varepsilon_0}\frac{1}{R}\frac{V_0\varepsilon_0\alpha 4\pi R^2}{R} = V_0\left(1-\alpha\right)$

3. (d)

At equilibrium, C $\frac{dT}{dt}$ = P

$$\frac{dT}{dt} = \frac{T_0\beta}{4}t^{-\frac{3}{4}}$$

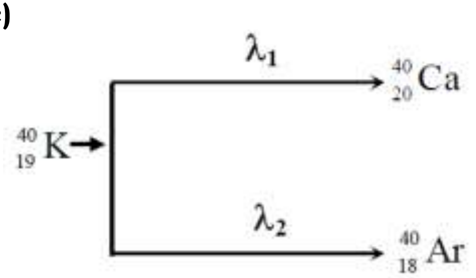
So, heat capacity $C = \frac{4P}{\beta T_0}t^{\frac{3}{4}}$

From the given equation $\frac{T(t)-T_0}{\beta T_0} = t^{\frac{1}{4}}$

so $t^{\frac{3}{4}} = \frac{\left(T(t)-T_0\right)^3}{\beta^3 T_0^3}$

So $C = \frac{4P}{\beta^4 T_0^4}\left(T(t)-T_0\right)^3$

4. (c)

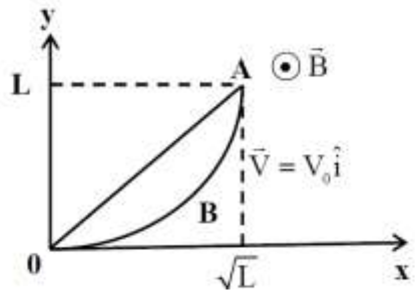


So equivalent decay constant $\lambda_1 + \lambda_2 = 5\times10^{-10}$ per year

$\frac{N}{N_0} = e^{-\lambda_{eq}t}$ and given that $\frac{N_0 - N}{N = 99}$

So t = 9.2×10^9 year

5. (a, b, d)



These is no change in flux through the loop OABO due to the movement of loop. So potential difference developed in curved wire and the straight wire OA is same.

For $\beta = 0, |\Delta\phi| = 2B_0V_0L$

For $\beta = 2, |\Delta\phi| = \int_0^L B_0\left(1+\frac{y^2}{L^2}\right)V_0 dy$

$= \frac{4}{3}B_0V_0L$

6. (b, c, d)

When $n_1 = n_2 = n$

$$\frac{1}{f} = (n-1)\left(\frac{2}{R}\right) \qquad \ldots(i)$$

When, $n_1 = n$ and $n_2 = n + \Delta n$

$$\frac{1}{f+\Delta f} = (n-1)\left(\frac{1}{R}\right) + (n+\Delta n-1)\left(\frac{1}{R}\right) \ldots(ii)$$

So from equation (i) and (ii)

$$\frac{1}{f} - \frac{1}{f+\Delta f} = -(\Delta n)\left(\frac{1}{R}\right)$$

$$\Rightarrow \frac{\Delta f}{f^2} = -(\Delta n)\left(\frac{1}{R}\right)$$

So $\frac{\Delta f}{f} = \frac{\Delta n}{2(n-1)} \approx -\frac{\Delta n}{2n}$

7. (a, c, d)

When T_1 is in contact with water

then $h = \frac{2T\cos\theta_1}{r\rho g}$ = 7.5cm<8cm.

But in option (b) height is insufficient.

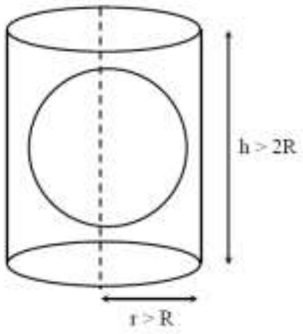
When T_2 is in contact with water

then h = $\frac{2T\cos\theta_2}{r\rho g}$ = 3.75cm<5cm

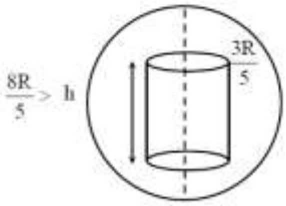
Volume of water in the meniscus depends upon the angle of contact.

8. (a, b, d)

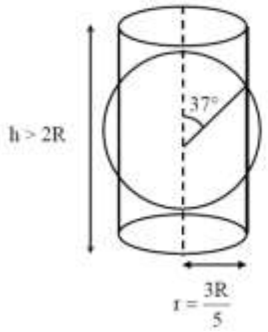
Option (B)



Option (A)



Option (D)



$$\phi = \frac{Q}{4\pi\varepsilon_0}(2)2\pi(1-\cos 37^\circ)$$

9. (a, b, d)

$[M^0L^\circ T^\circ] = [ML^2T^{-1}]$

$\Rightarrow [L^2] = [T]$

Energy = $[MLT^{-2}L]$ - L^{-2}

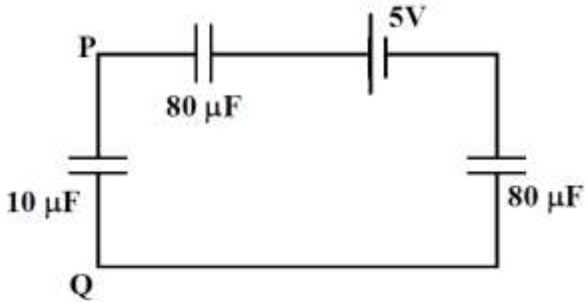
Force = $[MLT^{-2}]$ - L^{-3}

Power = $[MLT^{-2}UT^{-1}]$ - L^{-4}

linear momentum = MLT^{-1} = L^{-1}

10. (b, c)

S_1 closed for long time

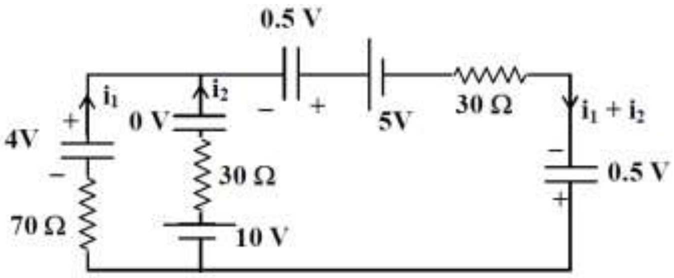


$$V_{10\mu F} = \left(\frac{40}{40+10}\right)5 = 4V$$

$V_P - V_Q = 4V$

$\Rightarrow$ t = 0, key S is closed

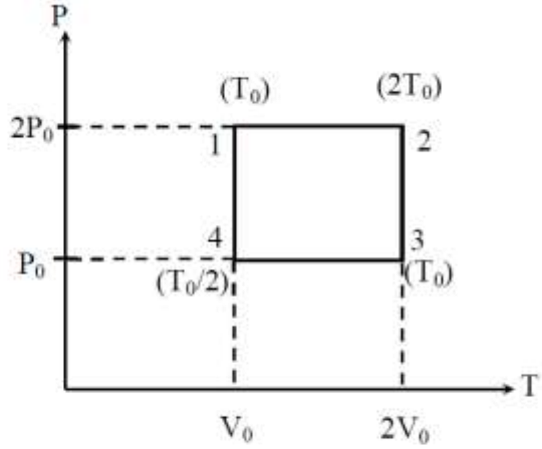
$$i = \frac{5}{70+100+30} = 25mA$$



$$-70i_1 + 30i_2 - 6 = 0$$

$$-30i_1 - 60i_2 + 6 = 0$$
$$\Rightarrow i_2 = 0.11A$$

11. **(a, b)**

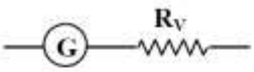


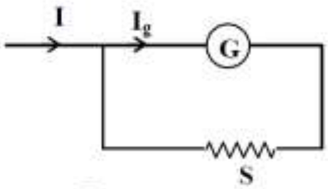
$$W_{cycle} = P_0V_0 = \frac{RT}{2}$$

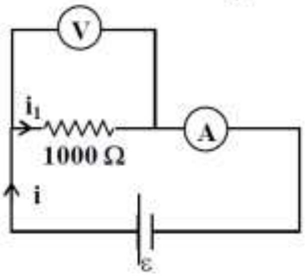
$$\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \left|\frac{nC_p(T_2 - T_1)}{nC_V(T_3 - T_2)}\right| = \left|-\frac{5}{3}\right| = \frac{5}{3}$$

$$\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \left|\frac{nC_p(T_2 - T_1)}{nC_V(T_4 - T_3)}\right| = 2$$

12. **(a, d)**







$V = I_g(G + R_v)$

$$G = R_V \square R_V = 5\times10^4\,\Omega$$

$$I_gG = (I - I_g)S$$

$$S = 20m\Omega$$

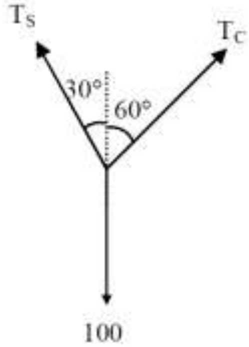
$$R_{ammeter} = \frac{GS}{G+S} = 20\times10^{-3}\,\Omega$$

$$i = \frac{\varepsilon}{\left(\frac{1000R_V}{1000 + R_V}\right)} = \frac{51\varepsilon}{5\times10^4}(R_A\ is\ small)$$

$$i_1 = i\left(\frac{1000 + R_V}{1000}\right) = \frac{\varepsilon}{1000}$$

$$R_{measure} = \frac{i_1(1000)}{i} = 980.4\Omega$$

13. (2.00)



$T_s\sin30° = T_c\sin60°$

$$\frac{\Delta l_C}{\Delta l_S} = \frac{T_C l_C}{A_C Y_C}\left(\frac{A_S Y_S}{T_S l_S}\right) = 2.00$$

14. (0.75)

$$w_{AB} = \int_0^1 \alpha y dy = -1$$

$$w_{BC} = \int_0^{0.5} 2\alpha x dy = +1$$

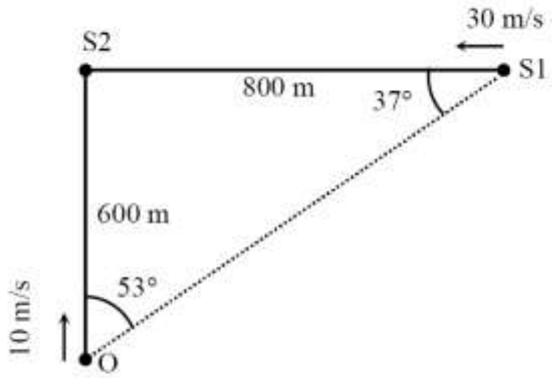
$$w_{CD} = \int_0^{0.5} \alpha y dy = +0.25$$

$$w_{DE} = \int_{0.5}^{0} 2\alpha y dy = +0.5$$

$$w_{EF} = w_{FA} = 0$$

$$W_{net} = 0.75$$

15. (8.13)



$$f_1 = 120\left(\frac{330+10\cos53}{330-30\cos37^0}\right)$$

$$f_2 = 120\left(\frac{330+10}{330}\right)$$

$$f_b = |f_1 - f_2| = 8.128Hz = 8.13Hz$$

16. (270.00)

5(s)(50) + 5L = C(30) …(i)

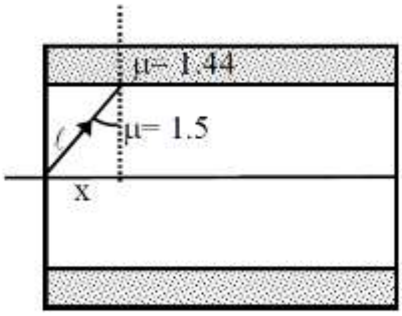
80(s)(20) = C(30) …(ii)

$\therefore$ from (i) and (ii)

$\frac{L}{S}$ = 270°C

∴ 270.00

17. (50.00)



$1.5\sin\theta_C = 1.44\sin 90^0$

$\sin\theta_C = \frac{24}{25}$

$l = \frac{x}{\sin\theta_C} = \frac{25}{4}x$

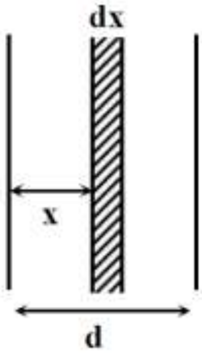
total length for light to travel

$l' = \frac{25}{4}\times 9.6 = 10m$

$\therefore$ time $= \frac{l'}{c/1.5} = 5\times10^{-8}s \Rightarrow 50\times10^{-9}s$

t=50.00

18. (1.00)



$x = \frac{d}{N}(m)$

$\frac{1}{dC} = \frac{dx}{k\left(1+\frac{m}{N}\right)\varepsilon_0 A}$

$\int\frac{1}{dC} = \int_0^d \frac{dx}{k\varepsilon_0\left(1+\frac{x}{d}\right)}$

$\Rightarrow C_{eq} = \frac{K\varepsilon_0 A}{d\ln 2}$

$\alpha = 1.00$

19. (d)

$Cr(BO_2)_3$

$Na_2B_4O_7\ .10H_2O \xrightarrow{\Delta} Na_2B_4O_7 \xrightarrow{\Delta} NaBO_2 + B_2O_3$

$Cr_2O_3+B_2O_3 \longrightarrow \underset{\text{(green colour bead)}}{Cr(BO_2)_3}$

20. (b)

21. (b)

Calamine - $ZnCO_3$

Malachite - $CuCO_3.Cu(OH)_2$

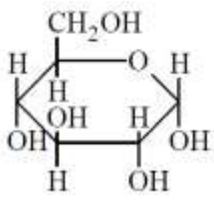
Magnetite - Fe_3O_4

Cryolite - $Na_3\,AlF_6$

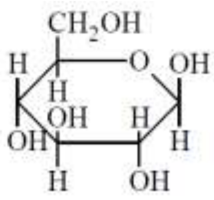
22. (c) I > II > III > IV

23. (b, c, d)

$$\xrightarrow[Hydrolysis]{H^+,H_2O} \underset{(Dextrorotatory)}{D(+)Glucose} + \underset{(leavorotatory)}{D(-)Fructose}$$



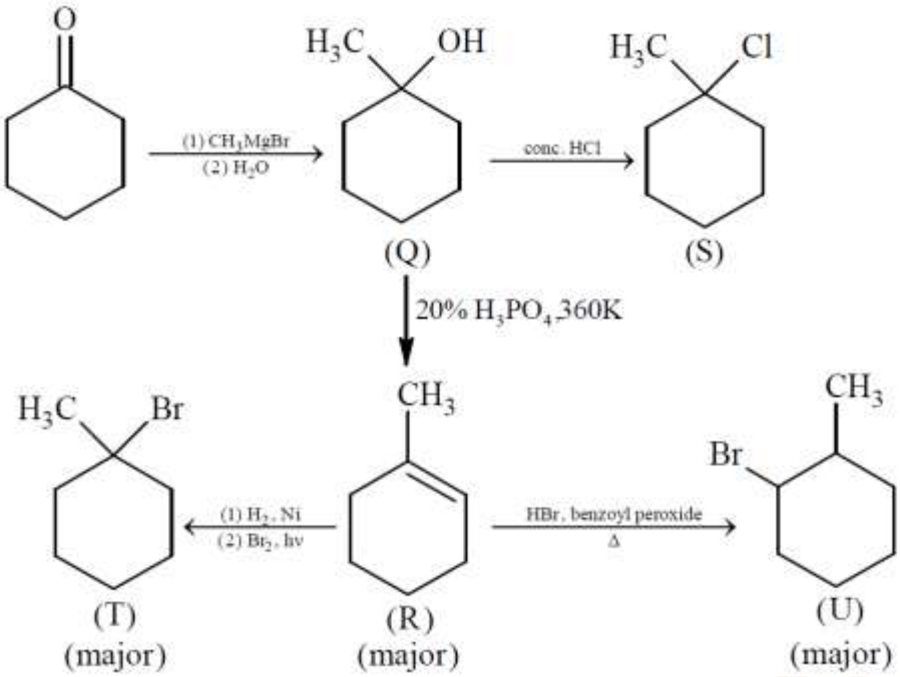
α - D - glucopyranose



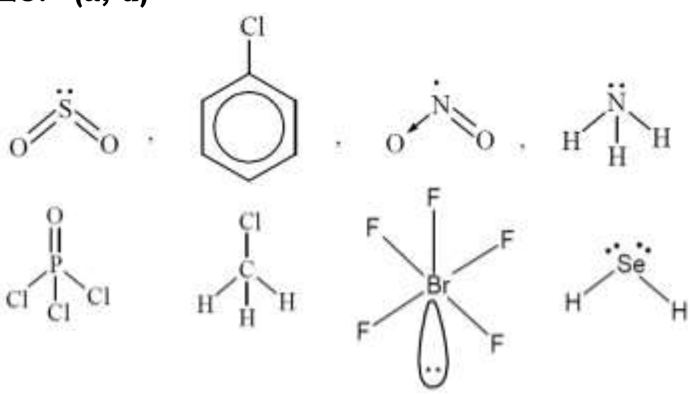
β - D - glucopyranose

α-D-glucopyranose and β-D-glucopyranose are anomers of each other.

24. (c,d)



25. (a, d)



26. (b, d) Standard enthalpy of formation of a compound is the standard enthalpy when one mole of a compound is formed from the elements in their stable state of aggregation.

27. (a, b, d)

$$\underset{}{MnO_2} \xrightarrow{KOH/O_2} \underset{w}{MnO_4^{2-}} \xrightarrow[\text{Oxidation}]{\text{Electrolytic}} \underset{x}{MnO_4^-}$$

$$MnO_4^{2-} \xrightarrow{\text{acidic medium}} MnO_2 + MnO_4^-$$

y - Mn^{+6} and z = Mn^{+7}

28. (a, b, d)

$$E_{av} = \frac{3}{2}RT \qquad U_{rms} = \sqrt{\frac{3RT}{M}}$$

E_{av} does not depend on its molecular mass but depends upon absolute temperature.

29. (b, c)

$$\underset{(Q)}{SnCl_2} + Cl^- \rightarrow \underset{(X)}{SnCl_3^-}$$

$$\underset{(Q)}{SnCl_2} + Me_3N \rightarrow \underset{(Y)}{SnCl_2\left[N\left(CH_3\right)_3\right]}$$

$$\underset{(Q)}{SnCl_2} + 2CuCl_2 \rightarrow \underset{(Z)}{SnCl_4} + CuCl$$

30. (b, c, d)

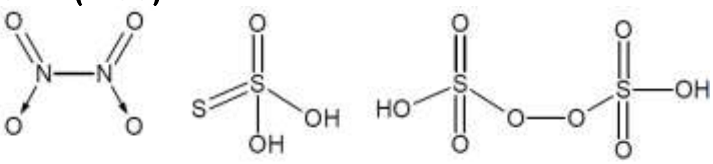
$X_1 = \alpha$

$X_2 = \beta$

$X_3 = \beta$

$X_4 = \alpha$

31. (4.00)



32. (1.02)

$$\frac{p_0 - p_s}{p_s} = i\left(\frac{n_{solute}}{n_{solvent}}\right)$$

$$\frac{650-640}{640} = 1 \times \frac{0.5 \times 78}{M \times 39}$$

$$\Rightarrow M_{solute} = 64g$$

$$\Delta T_f = K_f \times \text{molarity} = 5.12 \times \frac{0.5 \times 1000}{64 \times 39}$$

$$\Delta T_f = 1.02$$

33. (8.93)

$Fe^{2+}(aq) + S^{2-}(aq) \rightleftharpoons FeS(s)$ $K_c = 1.6 \times 10^{17}$

Initial 0.06 M 0.2 M

After mixing 0.03 M 0.1 M

? 0.07M

$$1.6 \times 10^{17} = \frac{1}{\left[Fe^{2+}\right] \times 0.07} \text{ or}$$

$$\left[Fe^{2+}\right] = \frac{10^{-17}}{1.6 \times 0.07} = \frac{10^{-15}}{11.2} = 8.928 \times 10^{-17}$$

or $8.93 \times 10^{-17} = Y \times 10^{-17}$

34. (6.75)

Rate $k[A]^x[B]^y[C]^z$

By exp. No. 1 & 2 y = 0

By exp. No. 1 & 3 z = 1

By exp. No. 1 & 4 x = 1

$Rate = k[A]^1[B]^0[C]^1$

From Exp. No. 1 $6\times10^{-5} = k(0.2)(0.1)$

$\Rightarrow k = 3\times10^{-3}$

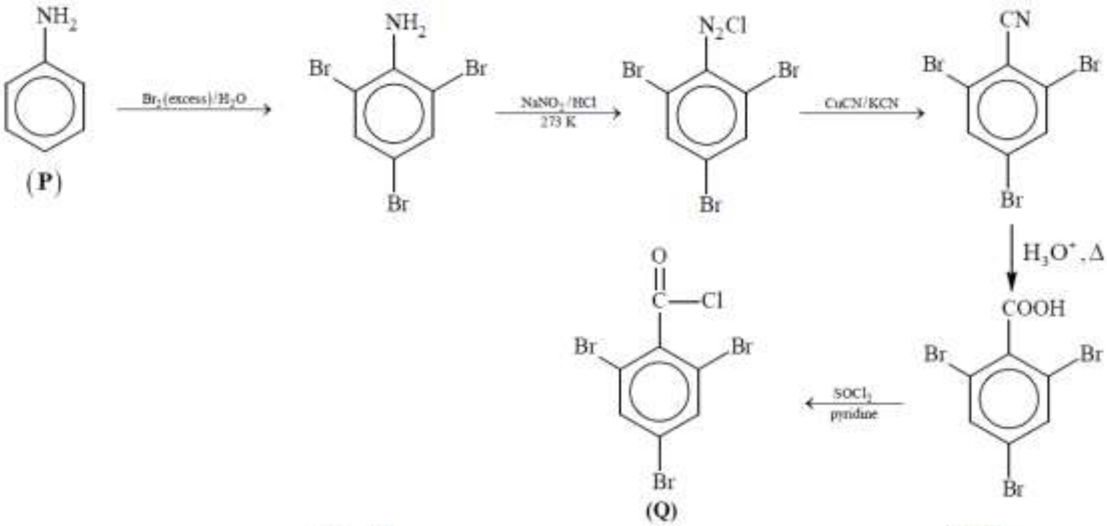
Now for [A] = 0.15 [B] = 0.25 [C] = 0.15

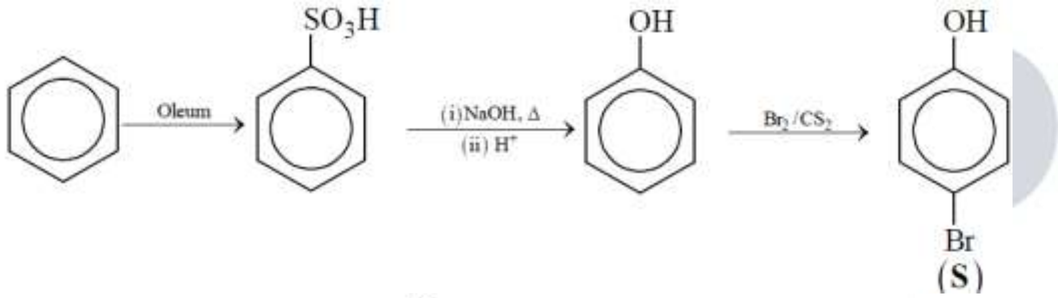
$Rate = k[A]^1[B]^0[C]^1$

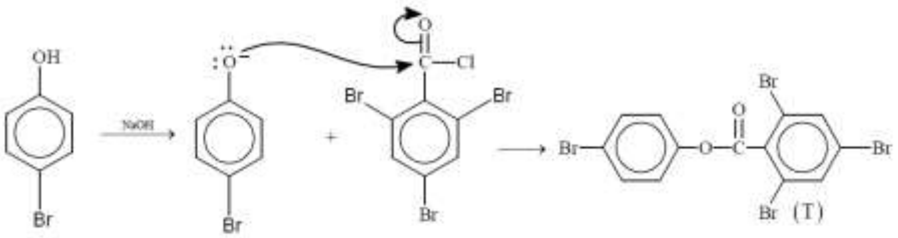
$= 3\times10^{-3}\times0.15\times1\times0.15$

$= 3 \times 0.0225 \times 10^{-3} = 6.75 \times 10^{-5}$ mol L^{-1} sec^{-1}

35. (4.00)

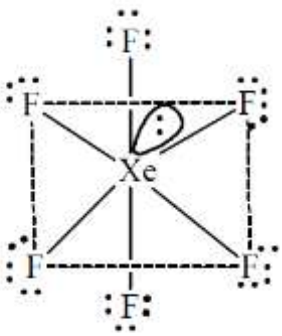






36. (19.00)

$XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2$



37. (d)

$$M=\alpha\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\beta\begin{bmatrix}\cos^4\theta & 1+\sin^2\theta\\ -1-\cos^2\theta & \sin^4\theta\end{bmatrix}$$

on comparing we have

$$\sin^4\theta=\alpha+\frac{\cos^4\theta}{|M|}$$

$$-1-\sin^2\theta=\alpha+\frac{\beta\left(1+\sin^2\theta\right)}{|M|}$$

$$\alpha=\sin^4\theta+\cos^4\theta$$

Now $\alpha=\left(\sin^2\theta+\cos^2\theta\right)^2-2\sin^2\theta\cos^2\theta$

$$=1-\frac{\sin^2 2\theta}{2}$$

$$\Rightarrow \alpha^*=\frac{1}{2}$$

We have, $|M|=\sin^4\theta\cos^4\theta+\left(1+\sin^2\theta\right)\left(1+\cos^2\theta\right)$

$$=2+\sin^2\theta\cos^2\theta+\sin^4\theta\cos^4\theta$$

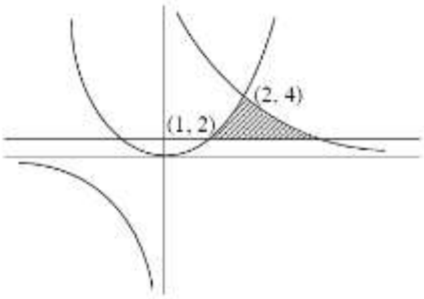
$$=\left(\sin^2\theta\cos^2\theta+1/2\right)^2+7/4$$

$$\Rightarrow \beta=-|M|=-\frac{7}{4}-\left(\sin^4\theta\cos^2\theta+\frac{1}{2}\right)^2$$

$$\Rightarrow \beta^*=-\frac{7}{4}-\left(\frac{1}{4}+\frac{1}{2}\right)^2=-\frac{7}{4}-\frac{9}{16}=-\frac{37}{16}$$

$$\alpha^*+\beta^*=-\frac{29}{16}$$

38. (c)

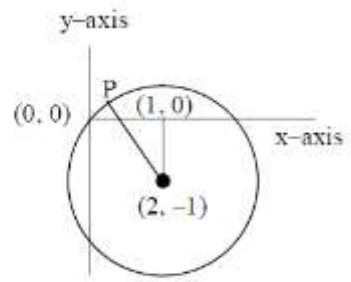


Required area = $\int_1^4\left(\frac{8}{7}-\sqrt{y}\right)dy$

$$=\left(8\ln y-\frac{2}{3}y^{3/2}\right)_1^4$$

$$=16\ln 2-\frac{14}{3}$$

39. (c)

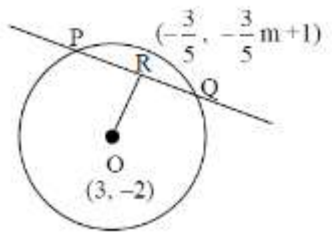


Clearly location of required point z_0 is at P with abscissa < 1 & ordinate > 0

Now $\arg\left[\dfrac{4-z_0-\overline{z_0}}{z_0-\overline{z_0}+zi}\right] = Arg\left(\dfrac{x-2}{y+1}\right)i = Arg\, ki \;\&\; k<0$

$\Rightarrow$Required argument = $-\pi/2$

40. (c)



$PQ \perp OR \Rightarrow$ Stope OR $= -\dfrac{1}{m} = \dfrac{-\frac{3}{5}m+1+2}{-\frac{3}{5}-3}$

$\Rightarrow$ $m^2 - 5m + 6 = 0$

$\Rightarrow$ m = 2, 3

41. (b, c, d)

Clearly, we have $\alpha+\beta=1 \& \alpha\beta=-1$

$\alpha,\beta=\dfrac{1\pm\sqrt{5}}{2}$

As $b_n = a_{n-1}+a_{n+1}$

$=\dfrac{\alpha^{n-1}-\beta^{n-1}}{\alpha-\beta}+\dfrac{\alpha^{n+1}-\beta^{n+1}}{\alpha-\beta}$

$=\dfrac{\alpha^{n-1}\left(1+\alpha^2\right)-\beta^{n-1}\left(1+\beta^2\right)}{\alpha-\beta}$

$=\dfrac{\alpha^{n-1}\left(\frac{5+\sqrt{5}}{2}\right)-\beta^{n-1}\left(\frac{5-\sqrt{5}}{2}\right)}{\alpha-\beta}$

$=\dfrac{\sqrt{5}\left(\alpha^n+\beta^n\right)}{\alpha-\beta}=\alpha^n+\beta^n$: As $\alpha-\beta=\sqrt{5}$

(d) $\displaystyle\sum_{n=1}^{\infty}\frac{a_n}{10^n}=\sum_{n=1}^{\infty}\frac{\alpha^n-\beta^n}{(\alpha-\beta)10^n}$

$=\dfrac{\dfrac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}}-\dfrac{\frac{\beta}{10}}{1-\frac{\beta}{10}}}{(\alpha-\beta)}$

$$=\frac{\alpha(10-\beta)-\beta(10-\alpha)}{(10-\alpha)(10-\beta)(\alpha-\beta)}=\frac{10}{89}$$

(a) $\sum_{n=1}^{\infty}\frac{b_n}{10^n}=\sum_{n=1}^{\infty}\frac{\alpha^n+\beta^n}{10^n}=\frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}}+\frac{\frac{\beta}{10}}{1-\frac{\beta}{10}}$

$$=\frac{\alpha(10-\beta)+\beta(10-\alpha)}{(10-\alpha)(10-\beta)}$$

$$=\frac{10(\alpha+\beta)-2\alpha\beta}{100-(\alpha+\beta)10+\alpha\beta}=\frac{12}{89}$$

(c) $a_1+a_2+a_3+\ldots+a_n=\sum_{r=1}^{n}a_r=\sum_{r=1}^{n}\frac{\alpha^r-\beta^r}{\alpha-\beta}$

$$=\frac{\frac{\alpha(1-\alpha^n)}{1-\alpha}-\frac{\beta(1-\beta^n)}{1-\beta}}{\alpha-\beta}$$

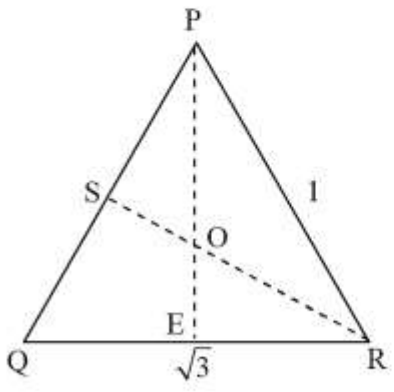
$$=\frac{(\alpha-\alpha\beta)(1-\alpha^n)-\beta(1-\alpha)(1-\beta^n)}{(\alpha-\beta)(1-\alpha)(1-\beta)}$$

$$=\frac{(\alpha-\beta)-\alpha^n(1+\alpha)+\beta^n(1+\beta)}{-(\alpha-\beta)}$$

$=\frac{(\alpha-\beta)-\alpha^{n+2}+\beta^{n+2}}{-(\alpha-\beta)}$; As $1+x=x^2$

$$=-1+\frac{\alpha^{n+2}-\beta^{n+2}}{\alpha-\beta}=-1+a_{n+2}$$

42. (a, b, c)



Circum radius = 1

By sine rule P

$\frac{\sin P}{P}=\frac{1}{2R}\Rightarrow \sin P=\frac{\sqrt{3}}{2}\Rightarrow P=60^0$ or 120^0

$\frac{\sin\theta}{2}=\frac{1}{2R}\Rightarrow \sin\theta=\frac{1}{2}\Rightarrow \theta=30^0,150^0$

so only possible combination, P = 120° & Q = 30° $\Rightarrow$ $\angle R=30^0$

(c) Length of RS $\Rightarrow$ $(PR)^2+(QR)^2=2(RS^2+PS^2)$ $\Rightarrow RS=\frac{\sqrt{7}}{2}$

(a) One $\Delta PQR=\frac{1}{2}PQ.PR\sin 120^0=\frac{\sqrt{3}}{4}\Rightarrow PE=\frac{1}{2}$

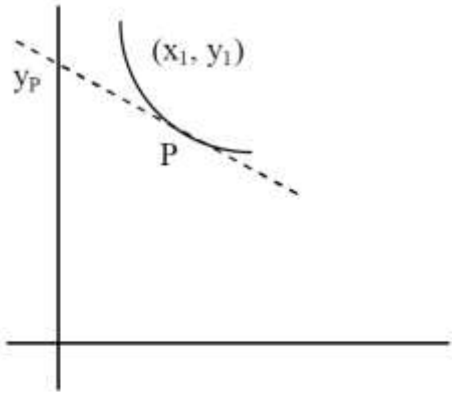
$\Rightarrow OE = 1/3\, PE = 1/6$

(b) $r = \frac{\Delta}{s} = \frac{2\Delta}{p+q+r} = \frac{2.\frac{\sqrt{3}}{4}}{\sqrt{3}+2} = \frac{\sqrt{3}}{2\left(2+\sqrt{3}\right)} = \frac{\sqrt{3}}{2}\left(2-\sqrt{3}\right)$

(d) are of $\Delta OSE = \frac{1}{2} SE.OE \sin\angle SEO$

$\frac{1}{2}.\frac{1}{2}.\frac{1}{6}\sin 60^0 = \frac{\sqrt{3}}{48} unit^2$

43. (a, b)



Equation tangent

$$y - y_1 = \frac{dy_1}{dx_1}(x - x_1)$$

$$y_p\left(0, y_1 - x_1\frac{dy_1}{dx_1}\right)$$

$$Py_p = \sqrt{x_1^2 + \left(-x_1\frac{dy_1}{dx_1}\right)^2} = 1$$

$$\Rightarrow \frac{dy}{dx} = \pm\frac{\sqrt{1-x^2}}{x} \Rightarrow dy = \pm\int\frac{\sqrt{1-x^2}}{x}dx, 1 - x^2 t^2, -xdx = tdt$$

$$dy = \pm\int\frac{-t^2}{1-t^2}dt$$

$$y = \pm\left(\int 1dt + \int\frac{1}{1-t^2}dt\right)$$

$$y = \pm\left(t - \frac{1}{2}\ln\left|\frac{1+t}{1-t}\right|\right) + c$$

$$y = \pm\left(\sqrt{1-x^2} - \frac{1}{2}\left|\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\right|\right) + c$$

y is passing through (1, 0) so c = 0

$$y = \pm\left(\sqrt{1-x^2} - \frac{1}{2}\ln\left(\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\right)\right)$$

$$y = \pm\left(\sqrt{1-x^2} - \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)\right)$$

As curve y = y(x) lies in the first quadrant so option A and B will only satisfy.
so AB are correct.

44. (a, b, d)

M adj M = |M| I $\Rightarrow$ a = 2, b = 1

$$\Rightarrow M=\begin{bmatrix}0&1&2\\1&2&3\\3&1&1\end{bmatrix}\Rightarrow |M|=-2$$

(a) (adj M)$^{-1}$ + (adj M^{-1}) = (|M|M^{-1})$^{-1}$+ |M^{-1}| M = $\frac{M}{|M|}+\frac{M}{|M|}=\frac{2M}{|M|}$ = - M

(b) $M\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}\Rightarrow\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}=-\frac{1}{2}\begin{bmatrix}-1&1&-1\\8&-6&2\\-5&3&-1\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}1\\-1\\1\end{bmatrix}$

$\alpha=1,\beta=-1,\gamma=1\Rightarrow\alpha-\beta+\gamma=3$

(c) |adjM^2| -|M^2|^2 -|M|^4=16

(d) a = 2,b=1, a + b = 3

45. (a, d)

Area of R_n = 2a cosθ x 2b sinθ

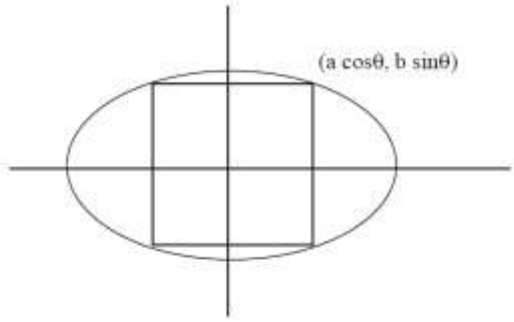
=2ab sin 2θ

which will be maximum when θ = 45°

$\therefore R_n|_{\max}=2ab$

	a	b
E_1	3	2
E_2	$\frac{3}{\sqrt{2}}$	$\frac{2}{\sqrt{2}}$
E_3	$\frac{3}{(\sqrt{2})^2}$	$\frac{2}{(\sqrt{2})^2}$
$\vdots$		
E_n	$\frac{3}{(\sqrt{2})^{n-1}}$	$\frac{2}{(\sqrt{2})^{n-1}}$

(a) Area of R_1 + Area of R_2... Area of R_n



< Area of R_1 + Area of R_2.... ∞

$<2\left(6+\frac{6}{2}+\frac{6}{4}+...\right)$

$<12\left(1+\frac{1}{2}+...\right)$

$<12\times\frac{1}{1-\frac{1}{2}}<24$

(b) $b^2 = a^2(1-e^2)$

for equation

$$\left(\frac{2}{\left(\sqrt{2}\right)^8}\right)^2=\left(\frac{3}{\left(\sqrt{2}\right)^8}\right)^2\left(1-e_9^2\right)$$

$$e_9^2=1-\frac{4}{9}=\frac{5}{9}$$

$$e_9=\frac{\sqrt{5}}{3}$$

Distance between center and focus = ae

$$=\frac{3}{\left(\sqrt{2}\right)^8}\times\frac{\sqrt{5}}{3}=\frac{\sqrt{5}}{16}$$

(c) $e_{18} = e_{19}$

(d) Latus rectum (length) $=\dfrac{2b^2}{a}=\dfrac{2\times\left(\dfrac{2}{\left(\sqrt{2}\right)^8}\right)^2}{\dfrac{3}{\left(\sqrt{2}\right)^8}}=\dfrac{1}{6}$

46. (a, b, c)

Range will contain set

$$f(x)=\begin{cases} x^5+5x^4+10x^3+10x^2+3x+1 & x<0\to(-\infty,1) \\ x^2-x+1 & 0\le x<1\to\left[\frac{3}{4},1\right] \\ \frac{2}{3}x^3-4x^2+7x-\frac{8}{3} & 1\le x<3\to\left[\frac{1}{3},1\right) \\ (x-2)\ln(x-2)-x+\frac{10}{3} & x\ge3\to\left[\frac{1}{3},\infty\right) \end{cases}$$

$$f'(x)=\begin{cases} 5\left(x^4+4x^3+6x^2+4x+1\right)-2 & x<0 \\ 2x-1 & 0\le x<1 \\ 2x^2-8x+7 & 1\le x<3 \\ \ln(x-2) & x\ge3 \end{cases}$$

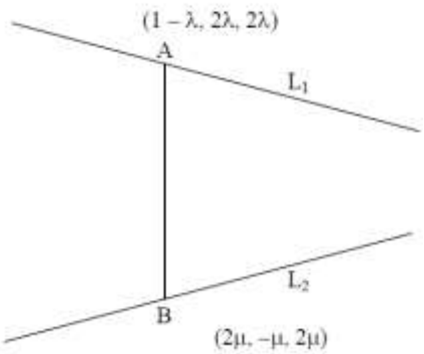
(a) f'(1) > f'(1^+) & f'(1) > f (1^-) so f (x) has local max. at x = 1

(b) L.H.D. = 2 are R.H.D. = - 2, f is not differentiable at x = 1

(c) f is containing ($-\infty$, ∞), so f is onto

(d) f (x) = 5 (x + 1)^4 - 2 is changing sign in ($-\infty$, 0), so if is not increasing

47. (a, b, c)



Equation of L_1 $\vec{r}=\hat{i}+\lambda\left(-\hat{i}+2j+2k\right)$

Equation of L_2 $\vec{r} = \mu\left(2\hat{i} - j + 2k\right)$

L_1 & L_2 are skew lines

The direction ratios of line AB which is perpendicular to L_1 and L_2 will be

$$\begin{vmatrix} \hat{i} & j & k \\ -1 & 2 & 2 \\ 2 & -1 & 2 \end{vmatrix} = 6\hat{i} + 6j - 3k$$

Hence direction ratios of AB will be (2, 2, -1)

direction ratios of AB proportional to (2, 2, -1)

$1-\lambda-2\mu = 2k$ ... (i)

$2\lambda + \mu = 2k$ ... (ii)

$2\lambda-2\mu = -k$ ... (iii)

solving (i) (ii) & (iii)

we get $\lambda = 1/9$

$\mu = 2/9$

$$A\left(\frac{8}{9},\frac{2}{9},\frac{2}{9}\right), B\left(\frac{4}{9},-\frac{2}{9},\frac{4}{9}\right)$$

Equation of line L_3 (A, B) passing through A

$$= \left(\frac{8}{9}\hat{i} + \frac{2}{9}j + \frac{2}{3}k\right) + t\left(2\hat{i} + 2j - k\right), t \in R$$

option (a) correct

Equation of line L_3 passing through B

$$= \left(\frac{4}{9}\hat{i} - \frac{2}{9}j + \frac{4}{9}k\right) + t\left(2\hat{i} + 2j - k\right)$$

Option (c) is correct, option (b) also satisfy

48. (a, b)

$P(B_1) = \frac{3}{10}, P(B_2) = \frac{3}{10}, P(B_3) = \frac{4}{10}$

(a) $P(G) = P(B_1) \times P\left(\frac{G}{B_1}\right) + P(B_2) \times P\left(\frac{G}{B_2}\right) + P(B_3) \times P\left(\frac{G}{B_3}\right)$

$$= \frac{3}{10} \times \frac{5}{10} + \frac{3}{10} \times \frac{5}{8} + \frac{4}{10} \times \frac{3}{8}$$

$$= \frac{60+75+60}{400} = \frac{195}{400} = \frac{39}{80}$$

(b) $P\left(\frac{G}{B_3}\right) = \frac{3}{8}$

(c) $P\left(\frac{B_3}{G}\right) = \dfrac{P(B_3) \times P\left(\frac{G}{B_3}\right)}{P(G)} = \dfrac{\frac{4}{10} \times \frac{3}{8}}{\frac{39}{80}} = \frac{4}{13}$

49. (3.00)

$\left|a + b\omega + c\omega^2\right|^2$

$= (a + b\omega + c\omega^2)(a + b\overline{\omega} + c\overline{\omega}^2)$

$= a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

Let $a > b > c \Rightarrow |a - b| \geq 1, |b - c| \geq 1, |a - c| \geq 2$

$\geq \frac{1}{2}$ (1 + 1 + 4) ≥ 3

50. (4.00)

$$I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4}\frac{dx}{\left(1+e^{\sin x}\right)\left(2-\cos 2x\right)} \ldots\text{(i)}$$

x=-t

$$I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4}\frac{e^{\sin t}}{\left(1+e^{\sin t}\right)\left(2-\cos 2t\right)}dt \ldots\text{(ii)}$$

add (i) and (ii)

$$I = \frac{1}{\pi}\int_{-\pi/4}^{\pi/4}\frac{1}{2-x\cos 2t}dt = \frac{1}{\pi}\int_{-\pi/4}^{\pi/4}\frac{\sec^2 t}{1+3\tan^2 t}dt = \frac{1}{\pi}.\frac{2\pi}{3\sqrt{3}}$$

$$3\sqrt{3}I = 2 \Rightarrow 27I^2 = 4$$

51. (0.50)

$n(E_2)$ = arrangement of 7, 1 and 2 or = $\frac{9!}{7!2!}$ = 36

$n(E_1 \cap E_2)$ = both zero should be in a row or a column

= $= \begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 0 & 0\end{bmatrix}$ (number of ways of arranging of (1, 0, 0) = 3 and arrangement of row = 3

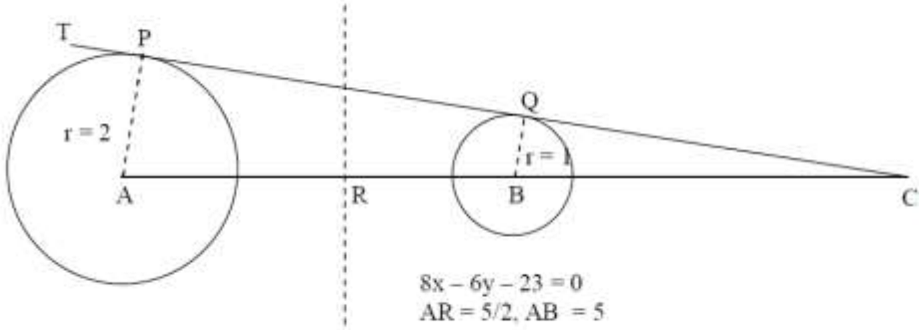
total = 9

in same way for (1, 0, 0) for columns number of ways will be = 9

total ways =18

$$P\left(\frac{E_1}{E_2}\right) = \frac{P\left(E_1 \cap E_2\right)}{p\left(E_2\right)} = \frac{18}{36} = \frac{1}{2} = 0.50$$

52. (10.00)



now Δ APC and BQC are similarly

$$\frac{BC}{AC} = \frac{1}{2} \Rightarrow 2\left(AC - AB\right) = AC$$

$$AC = 2AB = 10$$

AR=5/2, AB =5

53. (157.00)

AP (1, 3) $\equiv$ {1, 4, 7, 10,....} = {n /n = 3k + 1, k $\in$ W}

AP(2,5) $\equiv$ {2,7, 12,....}= {n/n = 5k + 2,k$\in$W}

AP(3,7) $\equiv${3, 10, 17,...} = {n/n = 7k + 3,k$\in$W}

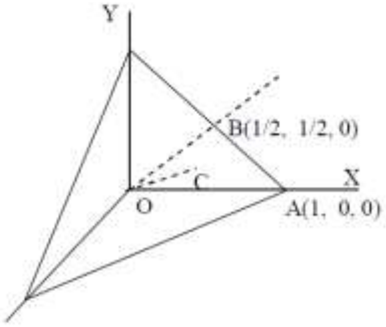
Let common term is M

M $\equiv$ 1 (mod 3), M $\equiv$ 2(mod 5), M $\equiv$ 3(mod 7)

$\Rightarrow$ M$\equiv$52(mod 105)

so, a = 52, d = 105 and a + d = 157

54. (0.75)



O is origin point C will be foot of perpendicular from O to plane

so $C\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

so, $\overrightarrow{AB} = -\frac{1}{2}\hat{i} + \frac{1}{2}j$

$\overrightarrow{AC} = -\frac{2}{3}\hat{i} + \frac{1}{3}j + \frac{1}{3}k$

$\Delta = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{1}{2}\left|\frac{\hat{i}}{6} + \frac{j}{6} + \frac{k}{6}\right| = \frac{\sqrt{3}}{12}$

$(6\Delta)^2 = \frac{3}{4} = 0.75$