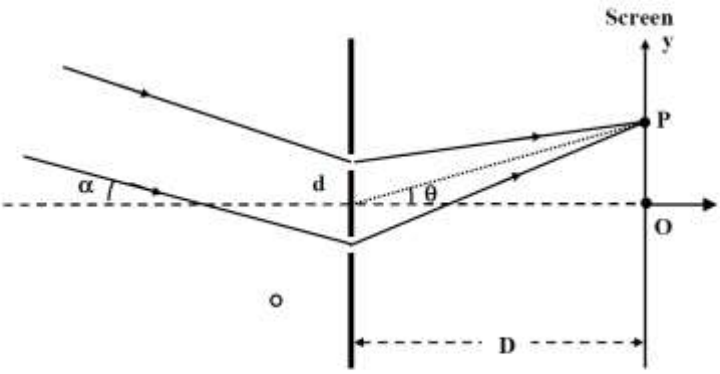
1. (c)



Given $\alpha = \frac{0.36}{\pi}$ degree = $\frac{1}{500}$ radian.

Path difference at O = d sin α = 0.3 x 10^{-3} x $\frac{1}{500}$ = 6 x 10^{-7} = λ

Path difference at P = d(sin α + sin θ)

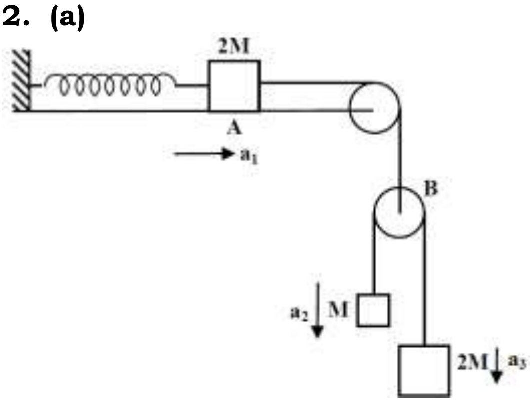
$= d\left(\frac{1}{500} + \frac{11}{1000}\right)$

$= \frac{d}{500}\left(1 + \frac{11}{2}\right)$

$= \lambda(6.5)$

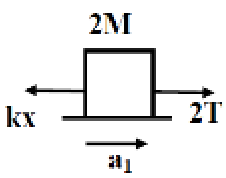
For α = 0, path difference at P = 5.5 λ

2. (a)



In the frame of pulley B, the hanging masses have accelerations : M $\rightarrow$ ($a_2 - a_1$), 2M $\rightarrow$ ($a_3 - a_1$): downward.

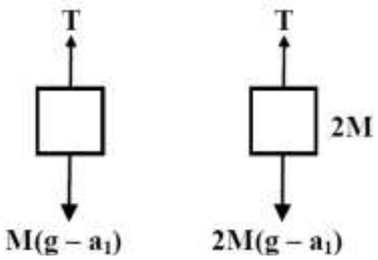
$\therefore$ ($a_2 - a_1$) = -(a_3 – a=) [constant] Assuming that the extension of the spring is x We consider the FED of A :



$2M.\frac{d^2x}{dt^2} = 2T - kx$

where $a_1 \equiv \frac{d^2x}{dt^2}$ ...(i)

and the FED of the rest of the system in the frame of pulley B :



Upward acceleration of block M w.r.t. the pulley B = Downward acceleration of block 2M w.r.t the pulley

$$\frac{T-M(g-a_1)}{M}=\frac{2M(g-a_1)-T}{2M}$$

$$\Rightarrow T=\frac{4M}{3}(g-a_1)\ldots\text{(ii)}$$

substituting in equation (i),we get

$$2M.a_1=\frac{8M}{3}(g-a_1)-kx\ \ldots\text{(iii)}$$

This is the equation of SHM

Maximum extension = 2 x Amplitude

i.e. $x_0=2\times\frac{8Mg}{3k}$

At $\frac{x_0}{4}$, acceleration is easily found from equation (iii):

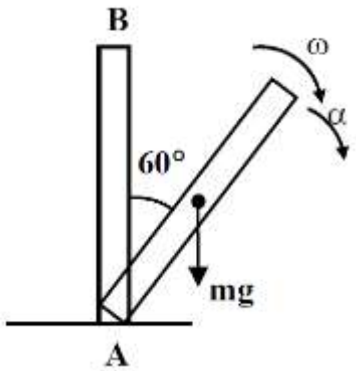
$$\frac{14M}{3}a_1=\frac{8Mg}{3}-\frac{4Mg}{3}$$

$$a_1=\frac{2g}{7}$$

At $\frac{x_0}{2}$, speed of the block (2M) = ω x amplitude

$$=\sqrt{\frac{3k}{14M}}\times\frac{8Mg}{3k}$$

3. (a, b, d)



To find ω, we apply COE: $\frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2=Mg\frac{L}{2}\left(1-\cos 60^0\right)$

$$\omega=\sqrt{\frac{3g}{2L}}$$

$$a_{CM},radial=\omega^2\frac{L}{2}=\frac{3g}{4}$$

To find α, we take torque about A

$$Mg\frac{L}{2}\sin 60^\circ=\frac{1}{3}ML^2\alpha$$

so, $\alpha = \frac{3g}{2L}\sin 60^0$

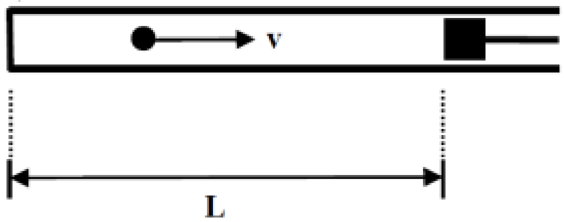
$a_{cm,\tan g} = \alpha\frac{L}{2} = \frac{3g}{4}\sin 60^0$

$\therefore Mg - N = \left(a_{cm;radial}\cos 60^0 + a_{cm,\tan g}\sin 60^0\right)$

$= M\left(\frac{3g}{8} + \frac{9g}{16}\right)$

$\therefore N = \frac{Mg}{16}$

4. (a, b)



The rate of collision of the particle with the piston is $\frac{1}{2L/v} = \frac{v}{2L}$

The speed of the particle after a collision with the piston is :
v + 2V, when it collides with it with a speed v.
if the piston moves inward by dL the speed of the particle increases by

$dv = 2V \times \frac{dL}{V} \times \frac{v}{2L}$

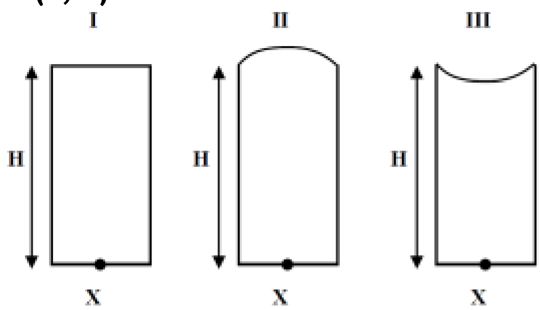
i.e. $\frac{dv}{dv} = \frac{dL}{L}$

Since kinetic energy, $K = \frac{1}{2}mv^2, \frac{dK}{K} = \frac{2(-dL)}{L}$ (after putting the proper sign)

Integrating, KL^2 = constant.

$\therefore K_f = 4K_i$

5. (a, b)



We apply, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\frac{1}{-H_1} = \frac{1.5}{-30} = \frac{1-1.5}{\infty} \ldots$(i)

$\Rightarrow H_1 = 20$cm

$\frac{1}{-H_2} - \frac{1.5}{-30} = \frac{-0.5}{-300} \ldots$(ii)

$\Rightarrow H_2 = \frac{300}{14.5}cm$

$\frac{1}{-H_3} - \frac{1.5}{-30} = \frac{-0.5}{300} \ldots.$ (iii)

$\Rightarrow H_3 = \frac{300}{15.5} cm$

6. (b, c)

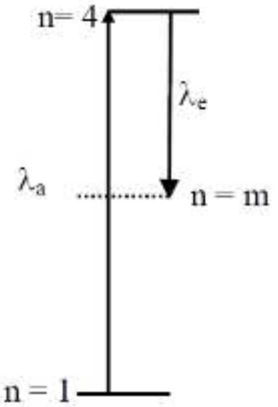
At B, $\vec{E}$ due to dipole is tangential

$\Rightarrow$ The other field must at least cancel this $\vec{E}$ due to dipole.

Also, at B, if the other field also had a component in radial direction, then this component would contribute to the net tangential component at A (since the other field is given to be uniform), which is not allowed since the given sphere is equipotential.

$\Rightarrow \vec{E}_{other} = \frac{1}{4\pi \in_0} \frac{P_0}{R^3} \left\{ \frac{\hat{i} + j}{\sqrt{2}} \right\}$

7. (b, d)



The energy level diagram is shown in the figure

It is given: $\frac{\lambda_a}{\lambda_e} = \frac{1}{5}$

or, $\frac{\left(\frac{1}{m^2} - \frac{1}{4^2}\right)}{\left(1 - \frac{1}{4^2}\right)} = \frac{\frac{1}{\lambda_e}}{\frac{1}{\lambda_a}} = \frac{1}{5}$

$\Rightarrow m = 2$

clearly $\frac{\Delta p_a}{\Delta p_e} \neq \frac{1}{2}$

The ratio of the kinetic energies is also the ratio of the corresponding total energies = $\frac{1}{4}$

$\therefore$ correct options are B, D

8. (a, b, c)

$\frac{5+1}{\gamma_m - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$

$\Rightarrow \frac{6}{\gamma_m - 1} = \frac{5}{\frac{5}{3} - 1} + \frac{1}{\frac{7}{5} - 1} = 10$

$\gamma_m = 1.6$

$P_0 V_0^{\gamma m} = P\left(\frac{V_0}{4}\right)^{\gamma m} \Rightarrow P = P_0 \times 2^{3.2} = 9.2 P_0$

$W = \frac{P_0 V_0 - (9.2 P_0) \times \frac{V_0}{4}}{0.6} = \frac{-1.3 P_0 V_0}{0.6} = \frac{13}{6} 6RT_0 = 13RT_0$

$T_0V_0^{\gamma m-1}=TV^{\gamma_m-1}\Rightarrow T=T_0\left(\frac{V_0}{V}\right)^{\gamma_m-1}=T_0(4)^{0.6}=2.3T_0$

Average kinetic energy of the gas mixture

$=5\times\frac{3}{2}RT+1\times\frac{5}{2}RT=10RT=23RT_0$

9. (135.00)

$^{226}_{88}Ra\ \underline{\alpha-decay}\ ^{22}_{86}Rn$

Total energy emitted = $(\Delta m)C^2$

= 0.005 x 931.5 MeV = E_0 (say) ... (i)

Also, E_α = 4.44 MeV ... (ii

E_{Rn} = 4.44MeV x $\frac{4}{222}$ ...(iii)

$\Rightarrow E_r = E_0 - E_a - E_{Rn} = 135\text{keV}$

10. (4.00)

Let $\frac{2u_0\sin\theta}{g}=T_0$ and $u_0\cos\theta=v_1$ (given)

Average velocity $=\dfrac{u_0\cos\theta T_0+\frac{u_0}{\alpha}\cos\theta\frac{T}{\alpha}+\frac{u_0}{\alpha^2}\cos\theta\frac{T_0}{\alpha^2}}{T_0+\frac{T_0}{\alpha}+\frac{T}{\alpha^2}+\ldots}$

$=\dfrac{u_0\cos T_0\left(1+\frac{1}{\alpha^2}+\frac{1}{\alpha^4}+....\right)}{T_0\left(1+\frac{1}{\alpha}+\frac{1}{\alpha^2}+...\right)}=v_1\frac{\alpha}{\alpha+1}=0.8V$

$\alpha=4$

11. (0.63)

$I=\frac{\varepsilon}{R}\left(1-e^{-Rt/L}\right)=\frac{Blv}{R}\left(1-e^{-Rt/L}\right)$

$=\frac{1\times0.1\times10^{-2}}{1}\left(1-e^{-1\times10^{-3}/10^{-3}}\right)=\left(1-e^{-1}\right)=0.63mA$

12. (1.00)

If v is speed of mirror just after absorption of photons

$Mv-M\frac{2h}{\lambda}=0\Rightarrow v=\frac{2Nh}{M\lambda}$

If l is the maximum compression in the spring

$\frac{1}{2}kl^2=\frac{1}{2}mv^2$

$\Rightarrow v=\sqrt{\frac{k}{m}}l$

$\Rightarrow\frac{2Nh}{M\lambda}=\Omega\times10^{-6}$

$\Rightarrow N=\Omega\times10^{-6}\times\frac{M\lambda}{2h}=8\pi\times10^{-6}\times10^{-6}\times\frac{M\lambda}{2h}$

$\frac{4\pi M\lambda}{h}\times10^{-12}=10^{24}\times10^{-12}=10^{12}$

$\Rightarrow x=1$

13. (1.50)

When angle of incidence on first face of the prism is 60° the angle of incidence on the other surface of the prism will be slightly greater than critical angle. For refraction at first surface of the prism

$\sin 60^0 = \sqrt{3}\sin r_1$

$\Rightarrow r_1 = 30^0$

For second surface r_2 = 75° - 30° = 45°

Since $r_2 \approx \theta_C$

$\Rightarrow \sin 45^0 = \frac{n}{\sqrt{3}}$

$\Rightarrow n^2 = 1.50$

14. (1.39)

$\frac{1}{v} - \frac{1}{-u} = \frac{1}{f}$

$\Rightarrow \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2} = \frac{\Delta f}{f^2}$

% error in the measurement of focal length

$= \frac{\Delta f}{f} \times 100 = \left(\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}\right) \times f \times 100$

$\left[\frac{0.5}{(60)^2} + \frac{0.5}{(30)^2}\right] \times 2 \times 100 = 1.39$

15. (i) (a)

$f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

$f_1 = \frac{1}{2L_0}\sqrt{\frac{T_0}{2\mu}} = \frac{f_0}{\sqrt{2}}$

$f_2 = \frac{1}{2L_0}\sqrt{\frac{T_0}{3\mu}} = \frac{f_0}{\sqrt{3}}$

$f_4 = \frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu}} = \frac{f_0}{2}$

(ii) (a)

$f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

For string -2

$\frac{3}{2 \times \frac{3L_0}{2}}\sqrt{\frac{T_2}{2\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

$T_2 = \frac{T_0}{2}$

For string -3

$\frac{5}{2 \times \frac{5L_0}{4}}\sqrt{\frac{T_3}{3\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

$T_3 = \frac{3T_0}{16}$

For string -4

$\frac{14}{2 \times \frac{7L_0}{4}}\sqrt{\frac{T_4}{4\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

$$T_4 = \frac{T_0}{16}$$

16. (i) (a)

$$W_{1\to2\to3} = P_0V_0 = \frac{1}{3}RT_0$$

$$\Delta U_{1\to2\to3} = nC_V\left(T_f - T_i\right) = 1\times\frac{3}{2}\left(3P_0V_0 - P_0V_0\right) = RT_0$$

$$\Delta Q_{1\to2\to3} = \Delta U + W = \frac{4}{3}RT_0$$

(ii) (d)

$$W_{1\to2\to3} = W_{1\to2} + W_{2\to3} = R\frac{T_0}{3}\ln\frac{2V_0}{V_0} + 0 = \frac{RT_0}{3}\ln 2$$

$$\Delta U_{1\to2\to3} = \Delta U_{1\to2} + \Delta U_{2\to3} = 0 + nC_v\left(T_0 - \frac{T_0}{3}\right)$$

$$= 0 + 1\times\frac{3R}{2}\times\frac{2T_0}{3} = RT_0$$

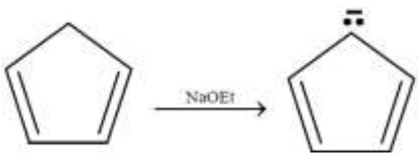
$$\Delta Q_{1\to2} = \int Tdx = \int T\left(\frac{3R}{2T}dT + \frac{R}{2}dV\right) = \frac{3R}{2}\int dT + \int_{V_0}^{2V_0}\frac{RT}{V}dV = 0 + RT\ln 2$$

$$\Delta Q_{2\to3} = \int Tdx = \frac{3R}{2}\int_{T_0/3}^{T_0} dT + \int\frac{RT}{V}dV = \frac{3R}{2}\times\frac{2T_0}{3} + 0 = RT_0$$

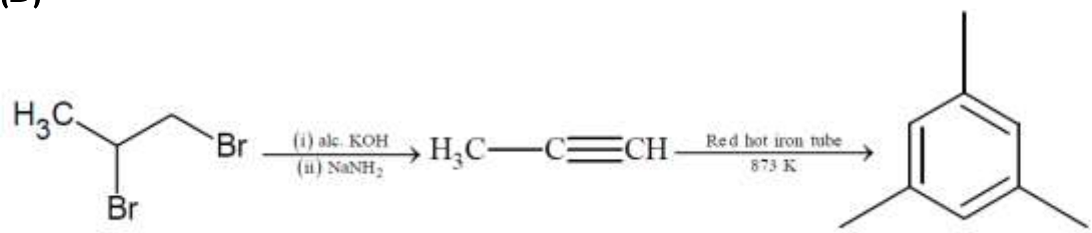
$$\Delta Q_{1\to2\to3} = RT\ln 2 + RT_0 = \frac{1}{3}RT_0\left(3+\ln 2\right)$$

17. (b, d)

(B)



(D)



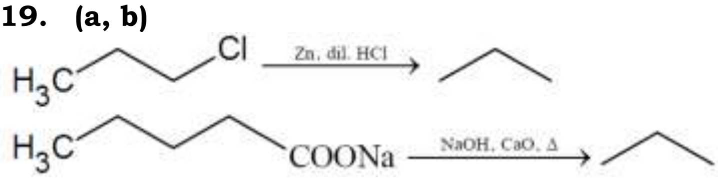
18. (b, c, d)

$$Zn + \underset{(hot\,\&\,conc.)}{H_2SO_4} \rightarrow \underset{(G)}{ZnSO_4} + \underset{(R)}{SO_2} + \underset{(X)}{H_2O}$$

$$Zn + conc.NaOH \rightarrow \underset{(T)}{Na_2ZnO_2} + \underset{(Q)}{H_2}$$

$$ZnSO_4 + H_2S + 2NH_4OH\left(aq\right) \rightarrow \underset{(Z)}{ZnS}\downarrow + \underset{(Y)}{\left(NH_4\right)_2SO_4} + \underset{(X)}{2H_2O}$$

19. (a, b)



20. (b, d)

21. (a, b)

$$-3.4=\frac{-13.6\times 2^2}{n^2}$$

n = 4

l = 2

Subshell = 4d

Angular nodes = l = 2

Radial nodes = n- l-1 = 4-2-1 = 1

22. (a, b, d)

$$Au \xrightarrow[H_2O]{CN^-,Q} R \xrightarrow{T} Au + Z$$

$$Au + 2CN^- + O_2 \rightarrow \left[Au(CN)_2\right]^-$$

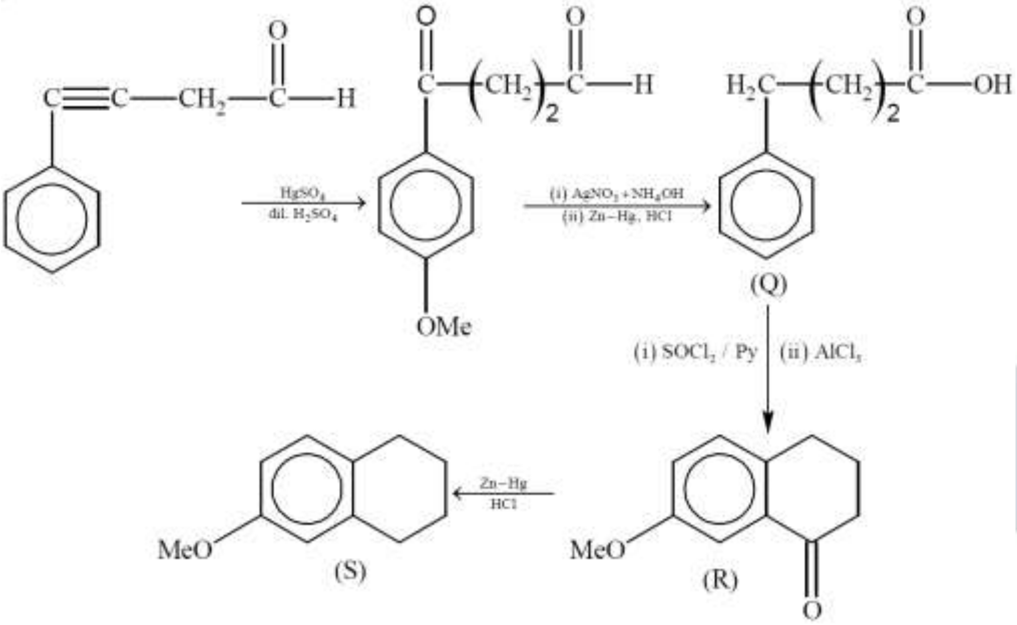
$$\left[Au(CN)_2\right]^- + Zn \rightarrow Au + \left[Zn(CN)_4\right]^{-2}$$

23. (a, c)

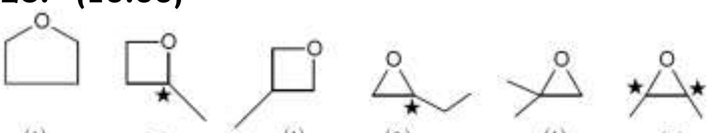
Au + HNO_3 + 3HC1 $\rightarrow$ $HAuCl_4$ + NO + H_2O

Yellow colour of aqua - regia is due to NOC1 and $C1_2$. In aqua regia HC1 and HNO_3 are in 3:1 molar ratio

24. (a, b)



25. (10.00)



(1) (2) (1) (2) (1) (3)

26. (2.98)

Let mole of urea = x

$$\frac{x}{x+\frac{900}{18}}=0.05, x=\frac{50}{19}$$

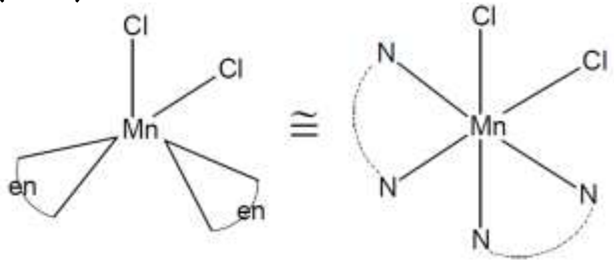
Mass of solution = $\frac{50}{19}$ x 60+ 900 = $\frac{20100}{19}$

Volume of solution = $\frac{201000}{19\times 1.2}$ ml

$$=\frac{20.1}{19\times 1.2}\text{litre}$$

$$\text{Molarity} = \frac{\frac{50}{19}}{\frac{20.1}{19\times1.2}} = \frac{50\times1.2}{20.1} = \frac{60}{20.1} \cong 2.985$$

27. (6.00)



28. (288.00)

$S_8 + 48HNO_3 \rightarrow 8H_2SO_4 + 48NO_2 + 16H_2O$

Mass of H_2O = 18 x 16 = 288 gram

29. (2.30)

Unit of K represent it is first order reaction.

$$2N_2O_5 \rightarrow 2N_2O_4 + O_2$$

t=0	1	0	0
t = t	1-P	P	P/2

$1\text{-}P + P + \frac{P}{2} = 1.45$

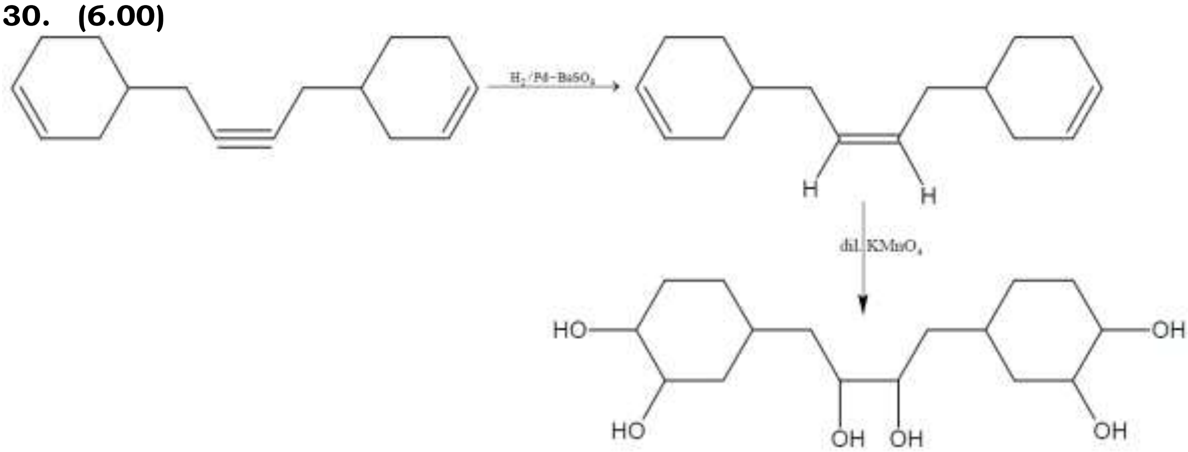
$\frac{P}{2} = 0.45, p = 0.9$

$$t = \frac{2.303}{2\times5\times10^{-4}}\log\frac{1}{1-P}$$

$$y\times10^{-3} = \frac{2.303}{2\times5\times10^{-4}}\log\frac{1}{1-0.9} = \frac{2.303}{2\times5\times10^{-4}}\log_{10}$$

Y=2.30

30. (6.00)

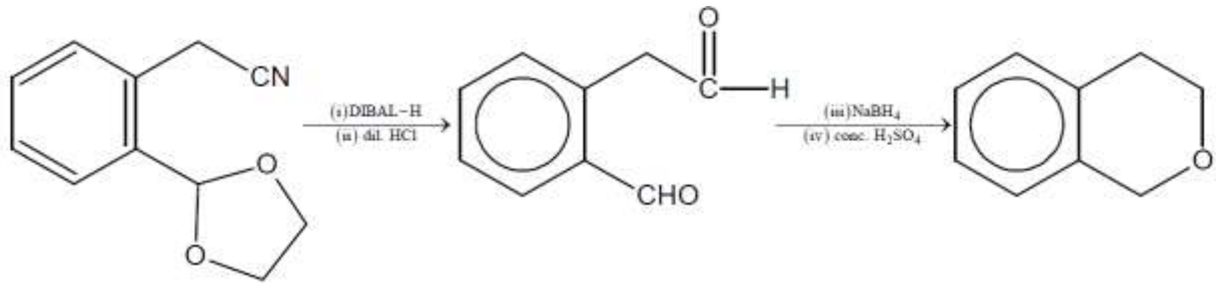


31. (a)

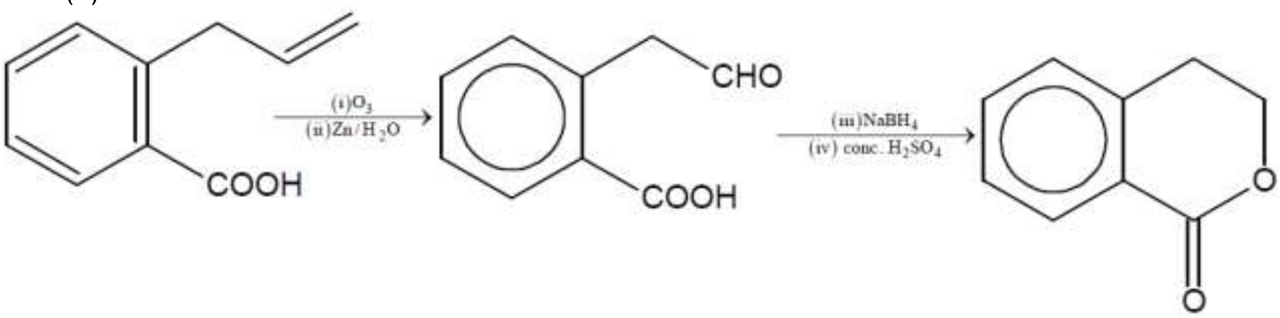
32. (a)

33. (b)

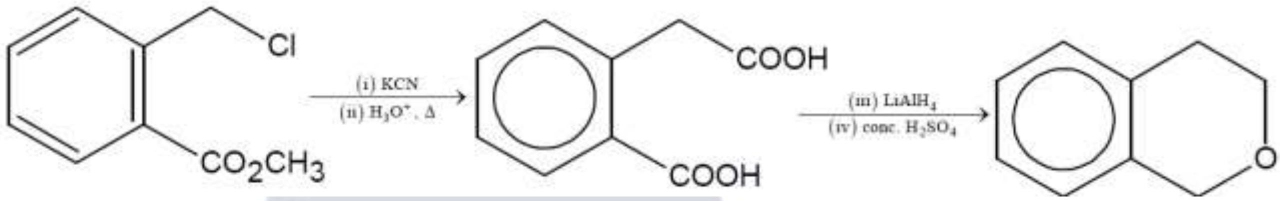
(i)



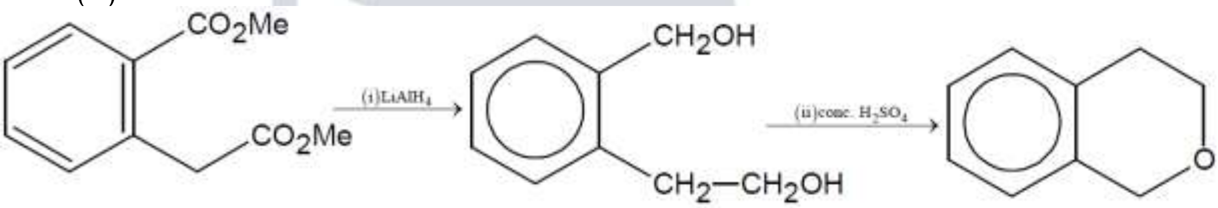
(ii)



(iii)

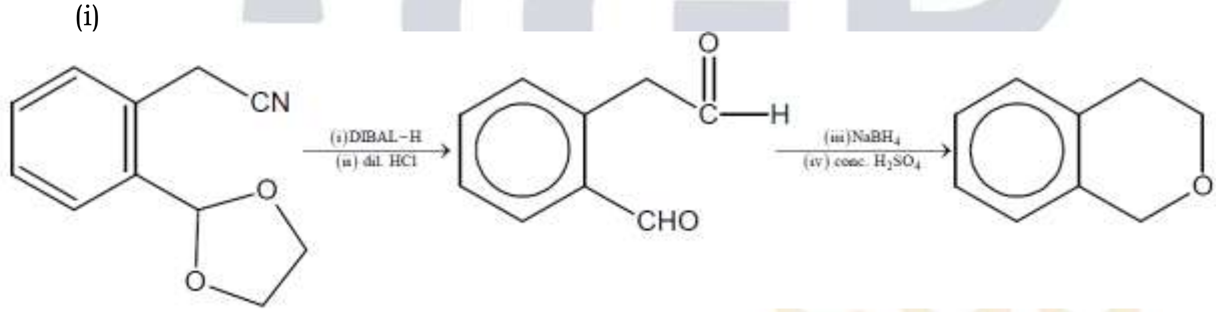


(iv)

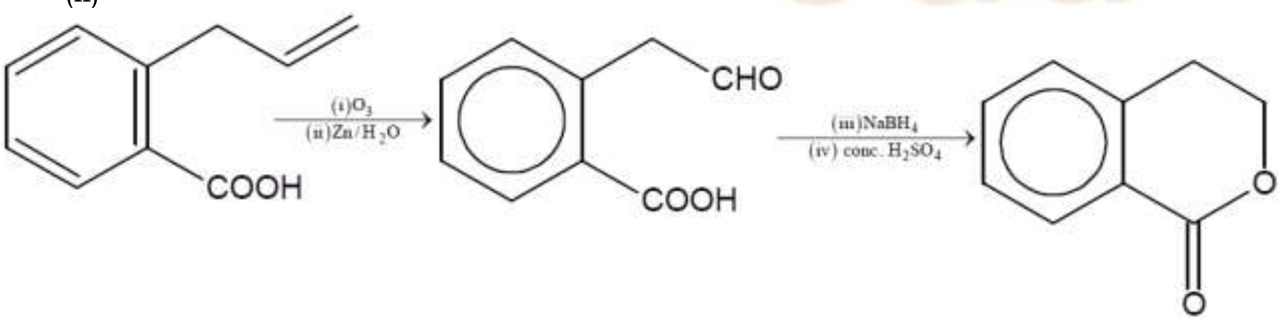


34. (b)

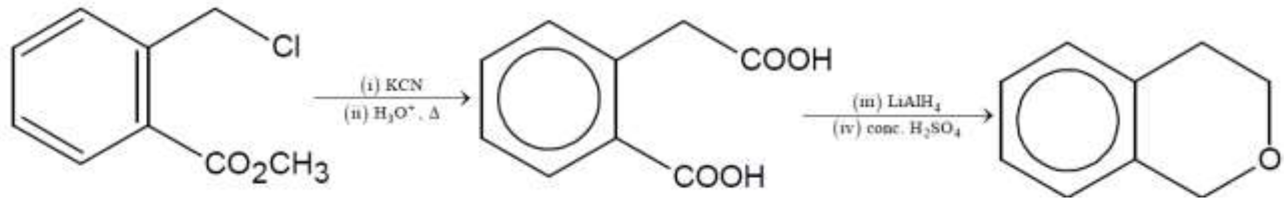
(i)



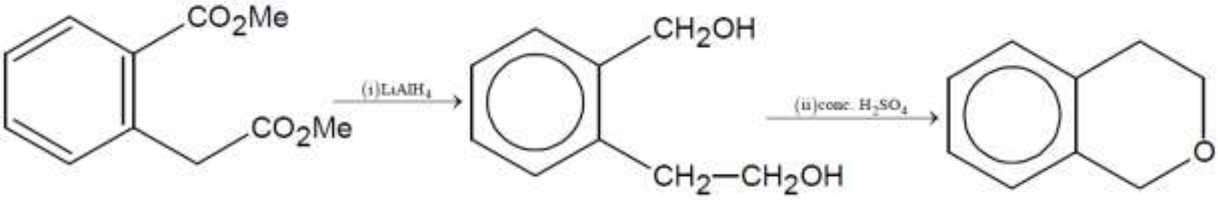
(ii)



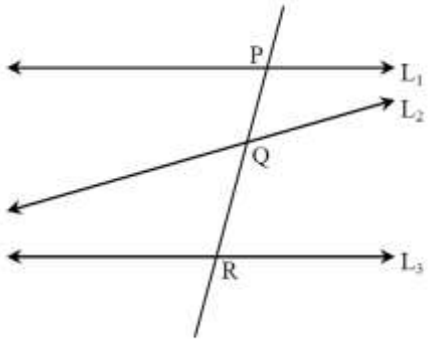
(iii)



(iv)



35. (a, b)



P. V. of point P, $\vec{p} = \lambda\hat{i}$

P. V. of point $q = \mu j + k$

P. V. of point R, $\vec{r} = \hat{i} + j + rk$

PQR are collinear. Hence $x\left(\overrightarrow{PQ}\right) = y\left(\overrightarrow{PR}\right)$

$\Rightarrow \frac{x}{y} = \frac{1-\lambda}{-\lambda} = \frac{1}{\mu} = r$

$\Rightarrow \vec{q} = \frac{1}{r} j + k$ or $\vec{q} = \frac{\lambda}{\lambda - 1} j + k$, where $r \neq 0, \lambda \neq 0, \frac{\lambda}{\lambda-1} \neq 1$

$\Rightarrow \mu \neq 0,1$

Hence, $\vec{q} \neq k$ or $j + k$

36. (a, c)

R = PQP^{-1}

det (R) = det (PQP^{-1})

|R| = $|PQP^{-1}|$

= $|P|.|Q|.|P^{-1}|$

$= |P|.|Q|.\frac{1}{|P|}$

$= |Q|$

$\det(Q) = \begin{vmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{vmatrix} = 4\begin{vmatrix} 2 & x \\ x & 6 \end{vmatrix}$

$= 4\left(12 - x^2\right) = 48 - 4x^2$

$\det(R) = \det(Q) = 48 - 4x^2$

(A) For x=0

$$PQ=\begin{bmatrix}1&1&1\\0&2&2\\0&0&8\end{bmatrix}\begin{bmatrix}2&0&0\\0&4&0\\0&0&6\end{bmatrix}$$

$$=\begin{bmatrix}2&4&6\\0&8&12\\0&0&18\end{bmatrix}$$

$$P^{-1}=\frac{1}{|P|}(AdjP)'$$

$$=\frac{1}{6}\begin{bmatrix}6&-3&0\\0&3&-2\\0&0&2\end{bmatrix}$$

$$R=PQP^{-1}$$

$$=\begin{bmatrix}2&4&6\\0&8&12\\0&0&18\end{bmatrix}\frac{1}{6}\begin{bmatrix}6&-3&0\\0&3&-2\\0&0&2\end{bmatrix}$$

$$=\frac{1}{6}\begin{bmatrix}12&6&4\\0&24&8\\0&0&36\end{bmatrix}=\begin{bmatrix}2&1&\frac{2}{3}\\0&4&\frac{4}{3}\\0&0&6\end{bmatrix}$$

Given, $R\begin{bmatrix}1\\a\\b\end{bmatrix}=6\begin{bmatrix}1\\a\\b\end{bmatrix}$

$$\Rightarrow(R-6I)\begin{bmatrix}1\\a\\b\end{bmatrix}=0$$

$$\Rightarrow\begin{bmatrix}-4&1&\frac{2}{3}\\0&-2&\frac{4}{3}\\0&0&0\end{bmatrix}\begin{bmatrix}1\\a\\b\end{bmatrix}=0$$

$$\Rightarrow -4+a+\frac{2}{3}b=0\ \ldots(1)$$

$$-2a+\frac{4}{3}b=0\ldots(2)$$

From (1) and (2)

2a = 4 ⇒ a = 2 and b = 3

So, a + b = 5.

(B) For x = 1

det (R) = 48 - 4x^2 = 48 - 4 = 40

det (R) ≠ 0

$$R\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$\Rightarrow \alpha = \beta = \gamma = 0$

Hence, $\alpha\hat{i} + \beta j + rk$ cannot a unit vector.

(c) $\det\begin{vmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{vmatrix} + 8$

$= 4\begin{vmatrix} 2 & x \\ x & 6 \end{vmatrix} + 8$

= $4(10-x^2) + 8$

= $40-4x^2 + 8 = 48-4x^2$.

(D) PQ = QP

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix} = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

If we equate a_{12} from both

x + 4 + x = 2 + 2x

$\Rightarrow$4 = 2

$\Rightarrow x \in \phi$, no value exists.

37. (a, b, c)

$$f(x) = \frac{\sum_{k=0}^{n} 2\sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n} 2\sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

$$= \frac{\sum_{k=0}^{n}\cos\frac{\pi}{n+2} - \sum_{k=0}^{n}\cos\left(\frac{2k+3}{n+2}\pi\right)}{\sum_{k=0}^{n}\left(1-\cos\left(\frac{2k+2}{n+2}\pi\right)\right)}$$

$$= \frac{(n+1)\cos\left(\frac{\pi}{n+2}\right) - \frac{\cos\left(\frac{n+3}{n+2}\pi\right).\sin\left(\frac{n+1}{n+2}\pi\right)}{\sin\left(\frac{\pi}{n+2}\right)}}{(n+1) - \frac{\cos\pi.\sin\left(\frac{n+1}{n+2}\pi\right)}{\sin\left(\frac{\pi}{n+2}\right)}}$$

$$= \frac{(n+1)\cos\left(\frac{\pi}{n+2}\right) - \frac{\cos\left(\pi+\frac{\pi}{n+2}\right).\sin\left(\pi-\frac{\pi}{n+2}\right)}{\sin\left(\frac{\pi}{n+2}\right)}}{(n+1) - \frac{\cos\pi.\sin\left(\pi-\frac{\pi}{n+2}\right)}{\sin\left(\frac{\pi}{n+2}\right)}}$$

$$= \frac{(n+1)\cos\left(\frac{\pi}{n+2}\right) + \cos\left(\frac{\pi}{n+2}\right)}{n+1+1}$$

$$=\frac{(n+2)\cos\left(\frac{\pi}{n+2}\right)}{n+2}=\cos\left(\frac{\pi}{n+2}\right)$$

(A) $f(4)=\cos\left(\frac{\pi}{4+2}\right)=\cos\left(\frac{\pi}{4+2}\right)=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$

(B) $\alpha=\tan\left(\cos^{-1}\left(\cos\left(\frac{\pi}{8}\right)\right)\right)=\tan\frac{\pi}{8}$

$$\tan\frac{\pi}{4}=\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}\Rightarrow 1=\frac{2\alpha}{1-\alpha^2}$$

$$\Rightarrow \alpha^2+2\alpha-1=0$$

(C) $\sin\left(7\cos^{-1}\left(\cos\frac{\pi}{7}\right)\right)$

$$=\sin\left(7\times\frac{\pi}{7}\right)=\sin\pi=0$$

(D) $\lim\limits_{n\to\infty} f(n)=\lim\limits_{n\to\infty}\cos\left(\frac{\pi}{n+2}\right)=\cos(0)=1$

38. (a, d)

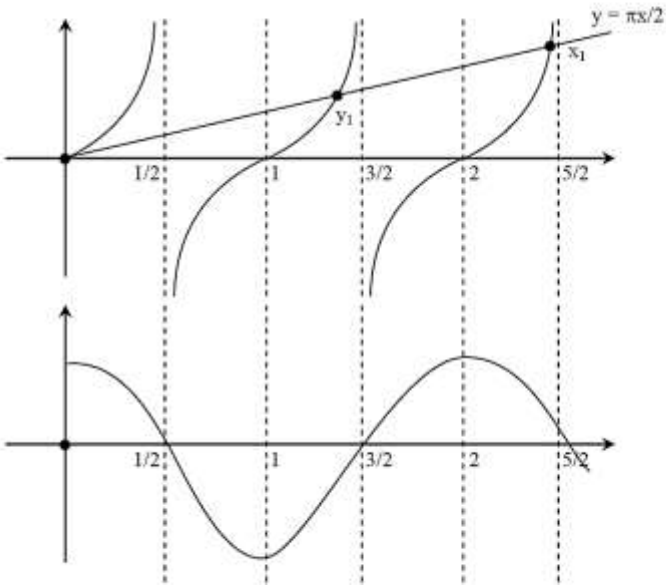
$$\lim_{n\to\infty}\frac{\sqrt[3]{1}+\sqrt[3]{2}+\ldots+\sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(xa+1)^2}+\frac{1}{(xa+2)^2}+\ldots+\frac{1}{(xa+n)^2}\right)}=54$$

$$=\lim_{n\to\infty}\frac{\sum\limits_{r=1}^{n} r^{1/3}}{n^{7/3}\left(\sum\limits_{r=1}^{n}\frac{1}{(na+r)^2}\right)}=\lim_{n\to\infty}\frac{\sum\limits_{r=1}^{n}\left(\frac{r}{n}\right)^{1/3}}{\left(\sum\limits_{r=1}^{n}\frac{1}{\left(a+\frac{r}{n}\right)^2}\right)}$$

$$=\frac{\int_0^1 x^{1/3}dx}{\int_0^1\frac{dx}{(a+x)^2}}=\frac{\frac{3}{4}x^{4/3}\Big|_0^1}{-\left(\frac{1}{a+x}\right)\Big|_0^1}=\frac{\frac{3}{4}}{-\left(\frac{1}{a+1}-\frac{1}{a}\right)}=54$$

$$\Rightarrow\frac{3}{4\left(\frac{1}{a(a+1)}\right)}=54\Rightarrow a=8\text{ or } a=-9$$

39. (b, c, d)



$f(x)=\frac{\sin \pi x}{x^2}$

$\Rightarrow f'(x)=\frac{\pi x^2 \cos \pi x - 2x \sin \pi x}{x^4}$

$=\frac{2\cos \pi x\left(\frac{\pi x}{2}-\tan \pi x\right)}{x^3}$

$f'(x)=0 \Rightarrow \cos \pi x = 0 \text{ or } \frac{\pi x}{2}=\tan \pi x$

$\Rightarrow \pi x=(2n+1)\frac{\pi}{2} \text{ or } \frac{\pi x}{2}=\tan \pi x$

$x=\frac{(2n+1)}{2}, n\in I$

from graph, we can see that $\forall\ x=\frac{2n+1}{2}$

$\Rightarrow$ f'(x) doesn't change sign so these points are neither local maxima nor local minimum.

Similarly, $\forall x : \frac{\pi x}{2}=\tan \pi x$

Where $y_n \in \left(2n-1, 2n-\frac{1}{2}\right) \forall$ n = 1, 2, 3, ...

and $x_n \in \left(2n-1, 2n+\frac{1}{2}\right) \forall$ n = 1, 2, 3, ...

$x_{n+1} - y_{n+1} > 1$ and $y_{n+1} - x_n > 1 \Rightarrow x_{n+1} - x_n > 2$.

40. (a, b, c)

$F(x)=\int_0^x f(t)\,dt$

F'(x) = f (x) = (x - 1) (x - 2) (x - 5)

$\Rightarrow$ x = 1, 5 is point of local minima for x > 0

x = 2 is point of local maxima for x > 0

$F(2)=\int_0^2 f(t)\,dt<0 \Rightarrow F(x)<0 \forall x\in(0,5)$

41. (b, d, d)

$$P_1 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, P_5 \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, P_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$X = \sum P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_k^T$$

Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix}$

$A = A^T$

$X^T = \left(P_1AP_1^T + P_2AP_2^T + \ldots + P_6AP_6^T\right)^T$

=X

So X is symmetric matrix

Let $Q = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$XQ = P_1AP_1^TQ + P_2AP_2^TQ + \ldots + P_6AP_6^TQ$

$= P_1AQ + P_2AQ + \ldots. + P_6AQ$

$= \left(P_1 + P_2 + \ldots + P_6\right)AQ, AQ = \begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix}$

$= \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 30 \\ 30 \\ 30 \end{bmatrix} = 30Q$

$XQ = 30Q \Rightarrow \left(X - 30I\right)Q = 0$

So, $|X - 30I| = 0$, has non-trivial solution.

So, not invertible.

Where trace (P_k A P'_k) = 3

$\Rightarrow$Trace X = 3x6= 18.

42. (c, d)

(a) f(x) = x|x|

$= \lim\limits_{h \to 0} \frac{h|h|}{h^2}$

$\lim\limits_{h \to 0} \frac{|h|}{h}$ does not exist.

(b) f(x) = sinx

$\lim\limits_{h \to 0} \frac{\sin h - 0}{h^2}$ does not exist.

(c) f(x) = |x|

$\lim\limits_{h \to 0} \frac{|h| - 0}{\sqrt{|h|}} = 0$

(d) f(x) = $x^{2/3}$

$$\lim_{h\to 0}\frac{h^{2/3}}{\sqrt{|h|}}=0$$

43. (0.50)

$$I=\int_0^{\pi/2}\frac{3\sqrt{\cos\theta}d\theta}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5}$$

$$=\int_0^{\pi/2}\frac{3\sqrt{\sin\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5}d\theta$$

$$\Rightarrow 2I=3\int_0^{\pi/2}\frac{d\theta}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^4}=\int_0^{\pi/2}\frac{3d\theta}{\cos^2\theta\left(1+\sqrt{\tan\theta}\right)^4}$$

$$\Rightarrow I=\frac{3}{2}\int_0^{\pi/2}\frac{\sec^2\theta d\theta}{\left(1+\sqrt{\tan\theta}\right)^4}$$

Let $1+\sqrt{\tan\theta}=t\Rightarrow\frac{1}{2\sqrt{\tan\theta}}\sec^2\theta d\theta=dt$

$$I=\frac{3}{2}\int_1^{\infty}\frac{2(t-1)}{t^4}dt=\frac{3}{2}\left|\frac{2}{-2t^2}+\frac{2}{3t^3}\right|_1^{\infty}$$

$$=\frac{3}{2}\left(\frac{2}{2}-\frac{2}{3}\right)=\frac{1}{2}=0.5$$

44. (18.00)

$$\frac{\vec{c}.\left(\vec{a}+\vec{b}\right)}{\left|\vec{a}+\vec{b}\right|}=3\sqrt{2}$$

$$\Rightarrow\frac{\alpha\vec{a}.\vec{a}+\beta\vec{b}.\vec{b}+(\alpha+\beta)\vec{a}.\vec{b}}{3\sqrt{2}}=3\sqrt{2}$$

$$\Rightarrow 6\alpha+6\beta+(\alpha+\beta)3=18$$

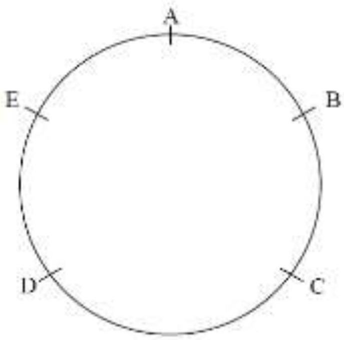
$$\Rightarrow\alpha+\beta=2$$

$$\left(\vec{c}-\left(\vec{a}\times\vec{b}\right)\right).\vec{c}=\alpha^2\left|\vec{a}\right|^2+\beta^2\left|\vec{b}\right|^2+2\alpha\beta\vec{a}.\vec{b}-\left(\vec{a}\times\vec{b}\right).\vec{c}$$

$=6\alpha^2+6\beta^2+6\alpha\beta$ (as $\vec{c}$ is linearly dependent on $\vec{a}$ & $\vec{b}$)

$$=6\left[(\alpha+\beta)^2-\alpha\beta\right],\max\alpha\beta=1$$

45. (30.00)



Maximum number of hats of same colour are 2 and minimum colour required are 3. So, the selection of hats will have to be of the type AABBC

${}^3C_1$ ways to select hat having single colour. Then distribute that single hat in ${}^5C_1$ ways. ${}^2C_1$ ways to distribute hat to adjacent person, after that alternate coloured hat will be given so 3x5x2 =30

46. (6.20)

$$\begin{vmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^nC_k k^2 \\ \sum_{k=0}^{n} {}^nC_k k & \sum_{k=0}^{n} {}^nC_k 3^k \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \frac{n(n+1)}{2} & n(n+1)2^{n-2} \\ n2^{n-1} & 4^n \end{vmatrix} = 0$$

$$\Rightarrow n(n+1)2^{n-1} \begin{vmatrix} \frac{1}{2} & 2^{n-2} \\ n & 2^{n+1} \end{vmatrix} = 0$$

$$\Rightarrow n = 0, -1, \Rightarrow n = 4$$

$$\sum_{k=0}^{4} \frac{{}^4C_k}{k+1} = \sum_{k=0}^{4} \frac{1}{5} {}^5C_{k+1} = \frac{1}{5}\left(2^5 - 1\right) = \frac{31}{5} = 6.2$$

47. (422.00)

Let $|A| = x$, $|B| = y$, $|A \cap B| = P$, then $\frac{P}{6} = \frac{x}{6} \cdot \frac{y}{6} \Rightarrow 6P = xy$

If p = 1 $\Rightarrow$ (i) x = 6, y = 1 $\Rightarrow$ ${}^6C_6 - {}^6C_1 = 6$

or $\Rightarrow$ (ii) x = 3, y = 2 $\Rightarrow {}^6C_3 . {}^3C_1 . {}^3C_1 = 180$

If P = 2 $\Rightarrow$ (i) x = 6, y = 2 $\Rightarrow$ ${}^6C_6 - {}^6C_2 = 15$

or (ii) x = 4, y = 3 $\Rightarrow$ ${}^6C_4 . {}^4C_2 . {}^2C_1 = 180$

If p = 3 $\Rightarrow$ x = 6, y = 3 $\Rightarrow$ ${}^6C_6 . {}^6C_3 = 20$

If p = 4 $\Rightarrow$ x = 6, y = 4 $\Rightarrow$ ${}^6C_6 . {}^6C_4 = 15$

If p = 5 $\Rightarrow$ x = 6, y = 5 $\Rightarrow$ ${}^6C_6 . {}^6C_5 = 6$

48. (0.00)

$$\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10} \sec\left(\frac{7\pi}{12} + \frac{k\pi}{2}\right) \sec\left(\frac{7\pi}{12} + \frac{(k+1)\pi}{2}\right)\right)$$

$$= \sec^{-1}\left(-\frac{1}{4}\sum_{k=0}^{10} \sec\left(\frac{7\pi}{12} + \frac{k\pi}{2}\right) \operatorname{cosec}\left(\frac{7\pi}{12} + \frac{k\pi}{2}\right)\right)$$

$$= \sec^{-1}\left(-\frac{1}{2}\sum_{k=0}^{10} \frac{1}{\sin\left(\frac{7\pi}{6}\right).(-1)^k}\right)$$

$$= \sec^{-1}\left(-\frac{1}{2}\frac{1}{\sin\left(\frac{7\pi}{6}\right)}\right) = \sec^{-1}(1) = 0$$

49. (c)

f(x) = 0 $\Rightarrow$ sin(π cosx) = 0

$$\Rightarrow \pi \cos x = n_1\pi, n_1 \in I$$

$$\Rightarrow \cos x = -1, \cos x = 0, \cos x = 1$$

$$\Rightarrow x = \frac{n_2\pi}{2}, n_2 \in I$$

$$\Rightarrow X = \left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi....\right\}$$

$$f'(x) = 0 \Rightarrow -\cos(\pi\cos x)\pi\sin x = 0$$

$$\Rightarrow \sin = 0 \text{ or } \cos(\pi\cos x) = 0$$

$\Rightarrow x = n_3\pi$ or $\pi\cos x = (2x_4+1)\pi/2$

$\Rightarrow \cos x = -\frac{1}{2}, \frac{1}{2}$

$\Rightarrow x = n_5\pi \neq \pi/3$

$Y = \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{6\pi}{3} ....\right\}$

$g(x) = 0 \Rightarrow \cos(2\pi\sin x) = 0$

$\Rightarrow 2\pi\sin x = (2n_6+1)\frac{\pi}{2}$

$\Rightarrow \sin x = \frac{2n_6+1}{4} = -\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{3}{4}$

$\Rightarrow Z = \left\{x \middle| \sin x = \pm\frac{1}{4}, \pm\frac{3}{4}, x > 0\right\}$

$g'(x) = 0 \Rightarrow -2\pi\cos x\sin(2\pi\sin x) = 0$

$\cos x = 0$ or $\sin(2\pi\sin x) = 0$

$\Rightarrow x = (2n_7+1)\frac{\pi}{2}$ or $2\pi\sin x = n_8\pi$

$\Rightarrow \sin x = -1, -\frac{1}{2}, 0, \frac{1}{2}, 1$

$W = \left\{x \middle| \sin x = \pm 1, \pm\frac{1}{2}, 0, x > 0\right\}$

$\Rightarrow I - (P), (Q)$

$II - (Q), (T)$

$III - (R)$

$IV - (P), (R), (S)$

50. **(a)**

f(x) = 0 $\Rightarrow$ sin(π cosx) = 0

$\Rightarrow \pi\cos x = n_1\pi, n_1 \in I$

$\Rightarrow \cos x = -1, \cos x = 0, \cos x = 1$

$\Rightarrow x = \frac{n_2\pi}{2}, n_2 \in I$

$\Rightarrow X = \left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi ....\right\}$

$f'(x) = 0 \Rightarrow -\cos(\pi\cos x)\pi\sin x = 0$

$\Rightarrow \sin = 0$ or $\cos(\pi\cos x) = 0$

$\Rightarrow x = n_3\pi$ or $\pi\cos x = (2x_4+1)\pi/2$

$\Rightarrow \cos x = -\frac{1}{2}, \frac{1}{2}$

$\Rightarrow x = n_5\pi \neq \pi/3$

$Y = \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{6\pi}{3} ....\right\}$

$g(x) = 0 \Rightarrow \cos(2\pi\sin x) = 0$

$\Rightarrow 2\pi\sin x = (2n_6+1)\frac{\pi}{2}$

$$\Rightarrow \sin x = \frac{2n_6+1}{4} = -\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{3}{4}$$

$$\Rightarrow Z = \left\{ x \middle| \sin x = \pm\frac{1}{4}, \pm\frac{3}{4}, x > 0 \right\}$$

$$g'(x) = 0 \Rightarrow -2\pi \cos x \sin(2\pi \sin x) = 0$$

$$\cos x = 0 \text{ or } \sin(2\pi \sin x) = 0$$

$$\Rightarrow x = (2n_7+1)\frac{\pi}{2} \text{ or } 2\pi \sin x = n_8 \pi$$

$$\Rightarrow \sin x = -1, -\frac{1}{2}, 0, \frac{1}{2}, 1$$

$$W = \left\{ x \middle| \sin x = \pm 1, \pm\frac{1}{2}, 0, x > 0 \right\}$$

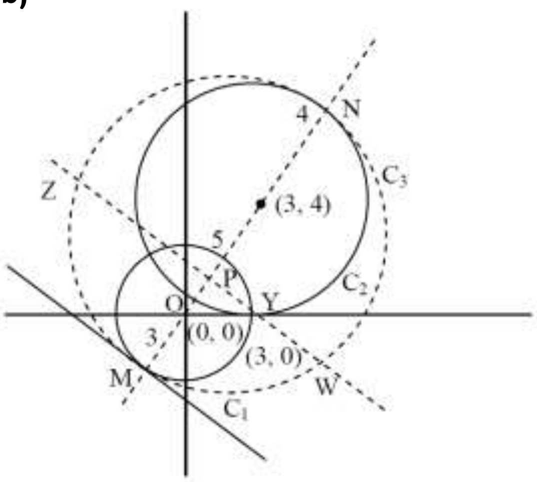
$$\Rightarrow I - (P), (Q)$$

$$II - (Q), (T)$$

$$III - (R)$$

$$IV - (P), (R), (S)$$

51. (b)



Centre of C_3 will lie on line MN : 3x = 4y at a distance of 3 units from origin in first quadrant

$$\Rightarrow (h,k) = \left(\frac{9}{2}, \frac{12}{5}\right) \ \& \ r = 6$$

Equation of ZW: $C_1 - C_2 = 0$

$\Rightarrow$ 6x + 8y-18 = 0 or 3x + 4y = 9

perpendicular distance from centre of C_3 to

$$ZW = -\frac{\left|\frac{27}{5} + \frac{48}{5} - 9\right|}{5} = \frac{6}{5}$$

Length of $ZW = 2\sqrt{36 - \frac{36}{25}} = \frac{24}{5}\sqrt{6}$

Similarly, length of $XY = 2\sqrt{9 - \frac{81}{25}} = \frac{24}{5}$

$$\frac{Area\ of\ \Delta MZN}{Area\ of\ \Delta ZMW} = \frac{ZP.MN}{MP.ZW} = \frac{\frac{ZW}{2}.MN}{MP.ZW}$$

$$=\frac{\frac{1}{2}.12}{\frac{24}{5}}=\frac{5}{4}$$

Common tangent of C_1C_3 : $C_1 - C_3 = 0$ (as they touch each other)

$$\Rightarrow \frac{18}{5}x+\frac{24}{5}y+18=0$$

or 3x + 4y + 5 = 0

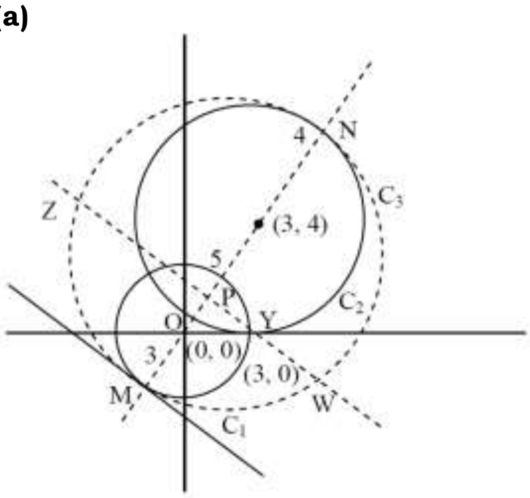
Tangent to $x^2=8\alpha y$ is $x=Py+\frac{2\alpha}{P}$

comparing $\Rightarrow \frac{3}{1}=\frac{4}{-P}=\frac{15}{-\frac{2\alpha}{P}}$

$$\Rightarrow P=-\frac{4}{3}\Rightarrow \frac{-15P}{2\alpha}=3\Rightarrow \alpha=\frac{10}{3}$$

$\Rightarrow$ I-(P)
II-(Q)
III-(R)
IV-(U)

52. (a)



Centre of C_3 will lie on line MN : 3x = 4y at a distance of 3 units from origin in first quadrant

$$\Rightarrow (h,k)=\left(\frac{9}{2},\frac{12}{5}\right) \;\&\; r=6$$

Equation of ZW: $C_1 - C_2 = 0$

$\Rightarrow$ 6x + 8y-18 = 0 or 3x + 4y = 9

perpendicular distance from centre of C_3 to

$$ZW=-\frac{\left|\frac{27}{5}+\frac{48}{5}-9\right|}{5}=\frac{6}{5}$$

Length of $ZW=2\sqrt{36-\frac{36}{25}}=\frac{24}{5}\sqrt{6}$

Similarly, length of $XY=2\sqrt{9-\frac{81}{25}}=\frac{24}{5}$

$$\frac{Area\,of\,\Delta MZN}{Area\,of\,\Delta ZMW}=\frac{ZP.MN}{MP.ZW}=\frac{\frac{ZW}{2}.MN}{MP.ZW}$$

$$=\frac{\frac{1}{2}.12}{\frac{24}{5}}=\frac{5}{4}$$

Common tangent of C_1C_3 : $C_1 - C_3 = 0$ (as they touch each other)

$$\Rightarrow \frac{18}{5}x+\frac{24}{5}y+18=0$$

or 3x + 4y + 5 = 0

Tangent to $x^2=8\alpha y$ is $x=Py+\frac{2\alpha}{P}$

comparing $\Rightarrow \frac{3}{1}=\frac{4}{-P}=\frac{15}{-\frac{2\alpha}{P}}$

$$\Rightarrow P=-\frac{4}{3}\Rightarrow \frac{-15P}{2\alpha}=3\Rightarrow \alpha=\frac{10}{3}$$

$\Rightarrow$ I-(P)
II-(Q)
III-(R)
IV-(U)