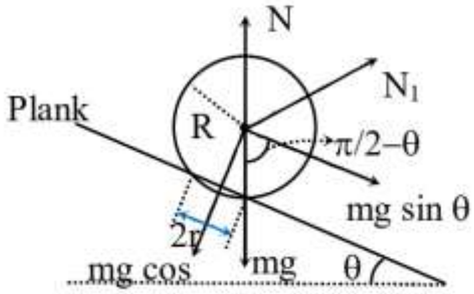
1. (a)



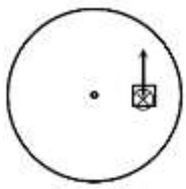
At the verge of toppling, $N_1 = 0$

$mg\sin\theta h = mg\cos\theta r$ and $\cos\theta = \frac{h}{R}$

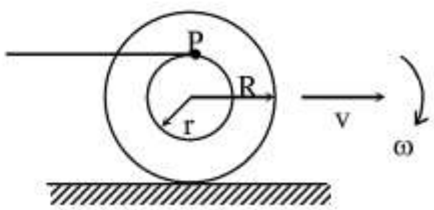
So, $mg\sin\theta h = mg\frac{h}{R}r$

$\Rightarrow \sin\theta = r/R$

2. (b) ⊗ is decreasing, so clockwise current in the square loop is induced.
So, force (along the arrow) is acting on the disc.
So, movement of disc is along the direction of motion of magnet.



3. (b)



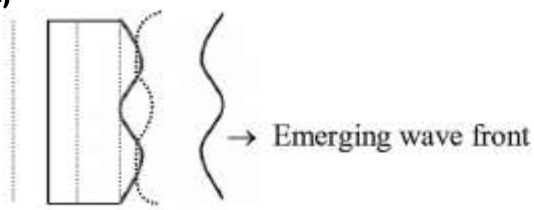
$v - \omega R = 0$

$v_p = v + \omega r = v + \frac{v}{R}r = v\left(1 + \frac{r}{R}\right)$

So, $\Delta x_p = 50\left(1 + \frac{5}{10}\right) = 75cm$

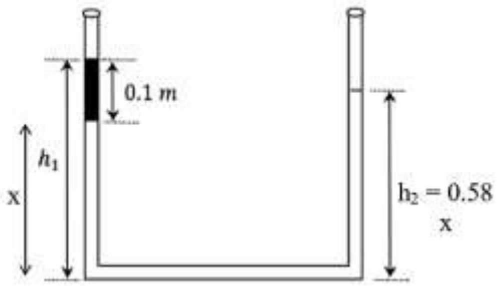
4. (b) $A.M = \int \tau dt = \int Nl\pi R^2 B dt = N\pi R^2 B_0 Q$

5. (a)



6. (b)

800 x 0.1 + 1000 x x = 1000 x (0.58 - x)
2000 x= 500
x = 0.25m



So, h_1 = 0.35 m, h_2 = 0.33m

$\frac{h_1}{h_2} = \frac{35}{33}$

7. (b, c)

$F = -\frac{dV}{dr}$

$\frac{mv^2}{R} = F$

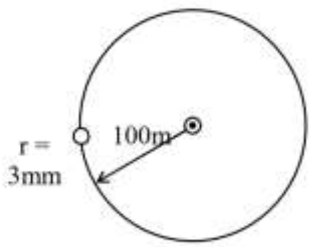
$mvr = \frac{nh}{2\pi}$

Solving above equations

$R^3 = \frac{n^2h^2}{4\pi^2 m}$

$v \propto n^{1/3}$

8. (b, c, d)



$P = A\sigma T^4$

$= 64\text{x}10^{-6}\text{x}5.67\text{x}10^{-8}\text{x}(2500)^4 \approx 140\text{W}$

Power entering in eye

$= \frac{140}{4\pi(100)^2}\pi\left(3\times10^{-3}\right)^2 = \frac{140\times9\times10^{-10}}{4} = 3.15\times10^{-8}W$

$\lambda T = b \Rightarrow \lambda = \frac{2.93\times10^{-6}}{2500} \Rightarrow \lambda = 1160$

Photons

$n = \frac{12420}{1740} = \frac{3.15\times10^{-8}}{1.6\times10^{-19}} = 2.75\times10^{11}$

9. (a, b)

r = x^{α}

v = x^{β}

a = x^p

P = x^q

F = x^r

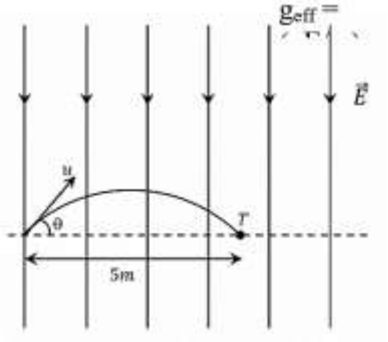
(a) ra$\rightarrow$ $\left[x^{\alpha+p}\right]$

$\left[\frac{Energy}{mass}\right] = [2\beta]$

(b) $\left[\frac{aP}{F}\right]=[at]=[v]=\beta$

$P+q-r=\beta$

10. (b, c)



Range = R= $\frac{2u\sin\theta}{g_{eff}}u\cos\theta=5m$

$\sin 2\theta=\left(\frac{qER}{mu^2}\right)=\frac{\sqrt{3}}{2}$

$\theta=30^0$

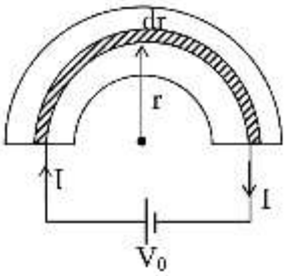
So, for same range angle of projection is either 30° or 60°

So, Option (b) is correct

$T=\frac{2u\sin\theta}{g_{eff}}=\left(\sqrt{\frac{10}{3}}\times10^{-6}\right)\sin\theta$

$T_1\left(\theta=30^0\right)=\sqrt{\frac{5}{6}}\mu s$

11. (a, c, d)



Area of the strip = (t.dr)

$dR=\rho\left(\frac{\pi r}{tdr}\right)$

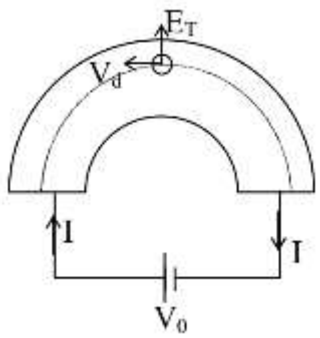
$\sum\frac{1}{dR}=\int_{R_1}^{R_2}\frac{tdr}{\pi\rho r}$

$\frac{1}{R_{eq}}=\left(\frac{t}{\pi\rho}\right)L_n\left(\frac{R_2}{R_1}\right)$

$R_{eq}=\frac{\pi\rho}{t\times\ell n\left(\frac{R_2}{R_1}\right)}$

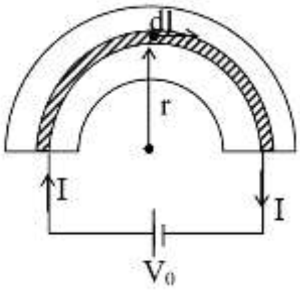
So, $I=\frac{V_0}{R_{eq}}=\left(\frac{V_0t}{\pi\rho}\right)\ell n\left(\frac{R_2}{R_1}\right)$

So, option (a) is correct.



Electrons are revolving in a circular path so there is a centripetal force on the electron, which provide a transverse electric field. The direction of transverse electric field is radially outside hence inner surface is at higher potential.
So, option (c) is correct.

Current density $J(r) = \frac{dl}{(tdr)}$



$$J(r) = \frac{V_0}{(dr)(tdr)}$$

$$J(r) = \frac{V_0}{\left(\frac{\rho\pi r}{tdr}\right)(tdr)}$$

$$J(r) = \frac{V_0}{(\rho\pi r)}$$

$I = neAV_d$

$J(r) = neV_d$

$$\Rightarrow V_d = \left(\frac{J(r)}{ne}\right) = \left(\frac{V_0}{\rho\pi rne}\right) \quad \ldots(i)$$

Centripetal force

$$eE_T = \frac{m_e v_d^2}{r} \quad \ldots(ii)$$

From (i) and (ii) $E_T \propto V_0^2$ ....(iii)

$\Delta V = \int E_T dr$

$\Rightarrow \Delta V \propto E_T$ ....(iv)

From (iii) and (iv)

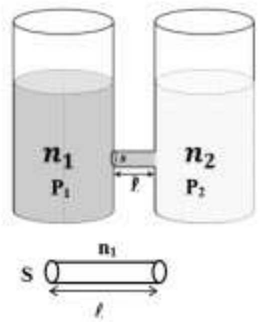
$\Delta V \propto V_0^2$

$I \propto V_0$

Hence

$\Delta V \propto I^2$

12. (a, b, c)



PV = NKT

$P_1 = n_1K_BT$

$P_2 = n_2K_BT$

$n_1, n_2 \rightarrow$ no. of molecules per unit volume

Force acting on the molecules

$F = \Delta PS$

$=(P_1-P_2)S$

$F=(n_1-n_2)K_BTS=\Delta nk_BTS$ ...(i)

$F = (n_1 - n_2)K_BTS = \Delta nk_BTS$... (i)

So Option (a) is correct

Total no. of molecules in the tube = (n_1Sl)

Force acting on each molecules

$F=-\beta v$

So total force acting on (n_1sl) molecules = - $(\beta v)(n_1sl)$ ...(ii)

From (i) and (ii)

$\Delta nK_BTS=(\beta v)(n_1Sl)$

So, $\Delta nK_BT = (\beta vn_1l)$ ...(iii)

So, Option (b) is correct.

No. of molecules going across the tube per second

$= (svn_1)$

$=(sn_1)\left(\frac{\Delta nK_\beta T}{\beta n_1 l}\right)=\left(\frac{S\Delta nK_\beta T}{\beta l}\right)$

So, Option (c) is correct.

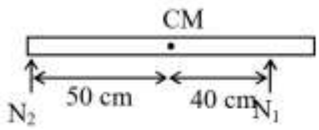
With respect to time Δn changes hence rate of molecules getting transferred through the tube changes with time.

So, Option (d) is incorrect.

13. (25.6 (25.60 to 25.60)

Initially

$40N_1 = 50N_2$



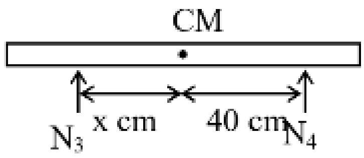
For first move

$\mu_k N_3 = \mu_s N_4$ ....(i)

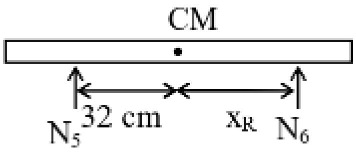
$xN_3 = 40\ N_4$ ... (ii)

x = 32cm

For second move



$\mu_k N_3 = \mu_s N_4 \ldots$(i)
$xN_3 = 40N_4 \ldots$(ii)
x=32cm
For second move



$\mu_s N_5 = \mu_k N_6 \ldots$(i)
$32(N_5) = X_R N_6 \ldots$(ii)
$X_R = 25.6cm$

14. (3.75 (3.65 to 3.85)

$$S(2R) = \left(\frac{\rho g h}{2}\right) h(2R)$$

h = 3.75 x 10^{-3} mm

15. (3.14 (0.50 to 0.50) or (3.13 to 3.15)

Potential energy of a system after a small displacement.

$$U = \frac{1}{2}kx^2 + \frac{k' pq}{(l+x)^2}$$

$$\frac{dU}{dx} = kx - \frac{2k' pq}{(l+x)^3}$$

$$\frac{d^2U}{dx^2} = k + \frac{6k' pq}{(l+x)^4} = k + 3k = 4k$$

$$f = \frac{1}{2\pi}\sqrt{\frac{4k}{m}}$$

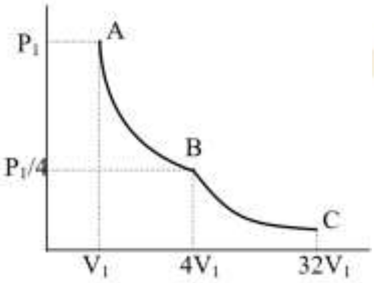
$$f = \frac{1}{\pi}\sqrt{\frac{k}{m}}$$

So, $\delta = 3.14$
and for angular frequency

$$\omega = \frac{1}{\delta}\sqrt{\frac{k}{m}}$$

$\delta = 0.5$

16. (1.77 (1.77 to 1.79)



$A \rightarrow B$

$P_1V_1 = P_B \times 4V_1$

$\Rightarrow P_B = \frac{P_1}{4}$

$W_{iso} = 1 \times R \times T \ell n \frac{4V_1}{V_1}$

$P_1V_1 \ell n4 = 2P_1V_1 \ell n2$

$B \rightarrow C$

$P_B V_B^{\gamma} = P_C V_C^{\gamma}$

$\frac{P_1}{4}(4V_1)^{\gamma} = P_C(32V_1)^{\gamma}$

$\Rightarrow P_C = \frac{1}{4}\left(\frac{1}{8}\right)^{\gamma} P_1 = \frac{1}{4}\left(\frac{1}{2^3}\right)^{5/3} P_1 = \frac{P_1}{128}$

$$W_{adic} = \frac{\frac{P_1}{128} \times 32V_1 - \frac{P_1}{4} \times 4V_1}{1 - \frac{5}{3}}$$

$$= \frac{\frac{P_1V_1}{4} - P_1V_1}{-\frac{2}{3}} = \frac{-\frac{3}{4}P_1V_1}{-\frac{2}{3}} = \frac{9}{8}P_1V_1$$

$$\frac{W_{iso}}{W_{adio}} = \frac{2P_1V_1 \ell n2}{\frac{9}{8}P_1V_1} = \frac{16}{9}\ell n2$$

Comparing with given expression of = 16/9 = 1.78

17. (0.62 (0.62 to 0.64)

$$f = f_0 \frac{320}{320-2} = \frac{320}{318} f_0$$

$$\Rightarrow \Delta f = \frac{2}{318} f_0 \qquad \ldots(1)$$

$$\because f_0 = \frac{(2n+1)V}{4L}$$

$$\frac{2}{318} = -\frac{\Delta L}{L}$$

$$100 \times \frac{\Delta L}{L} = \frac{200}{318} = 0.62983 = 0.63$$

18. (6.40 (6.40 to 6.40)

Total charge on disc

$$q_0 = \int_0^R \sigma_0\left(1 - \frac{r}{R}\right) 2\pi r dr = \sigma.2\pi\left[\int_0^R r dr - \frac{1}{R}\int_0^R r^2 dr\right]$$

$$= 2\pi\sigma\left[\frac{R^2}{2} - \frac{R^2}{3}\right]$$

$$q_0 = 2\pi\sigma\left[\frac{R^2}{6}\right] \ldots(1)$$

Change enclosed by second sphere

$$q_0 = \sigma.2\pi\left[\int_0^{R/4} r dr - \frac{1}{R}\int_0^{R/4} r^2 dr\right]$$

$$= 2\pi\sigma\left[\frac{R^2}{32} - \frac{R^2}{192}\right]$$

$$q_0 = 2\pi\sigma\left[\frac{5R^2}{192}\right]$$

19. **(b)** Since the graph is symmetrical about the peak. So, the average speed and most probable speed will coincide but rms speed will remain, $\sqrt{\frac{3RT}{M}}$

So, $u_{mp} = \sqrt{\frac{2RT}{M}} = u_{av}$

$$u_{rms} = \sqrt{\frac{3RT}{M}}$$

$$u_{mp} : u_{av} : u_{rms} = \sqrt{2} : \sqrt{2} : \sqrt{3} = 1:1:\sqrt{\frac{3}{2}}$$

=1:1:1.224

20. **(b)**

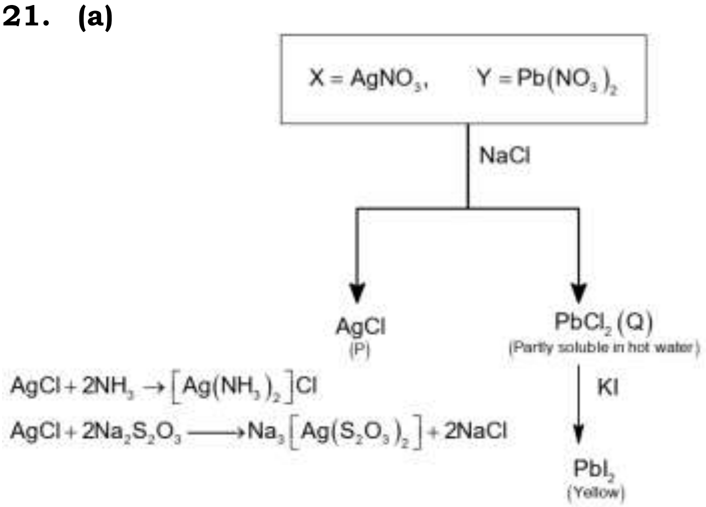
$Pb_3O_4 + H_2O \rightarrow$ No reaction

$KO_2 + H_2O \rightarrow KOH + H_2O_2 + O_2$

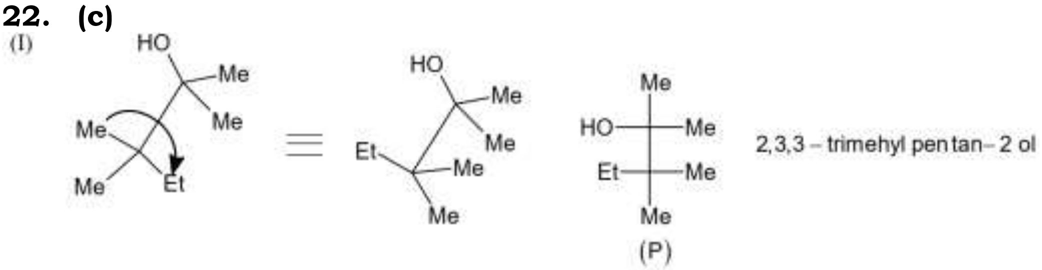
$Na_2O_2 + H_2O \rightarrow NaOH + H_2O_2$

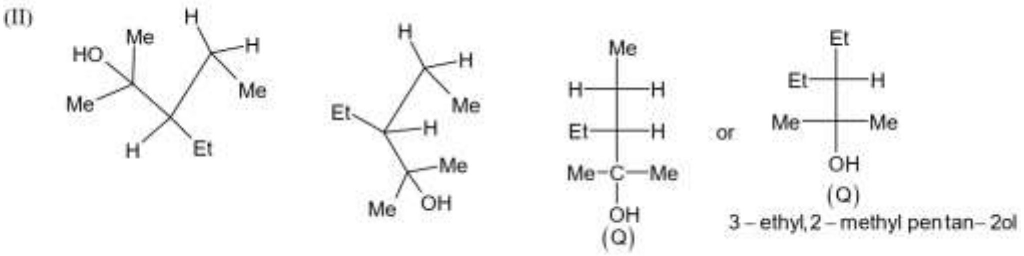
$Li_2O_2 + H_2O \rightarrow LiOH + H_2O_2$

21. **(a)**

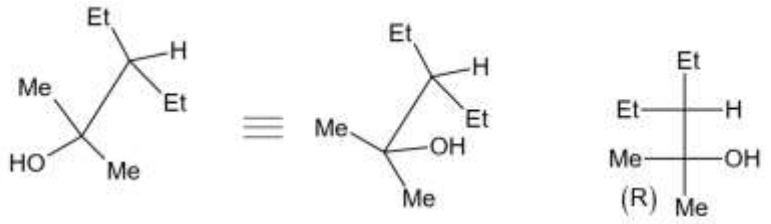


22. **(c)**



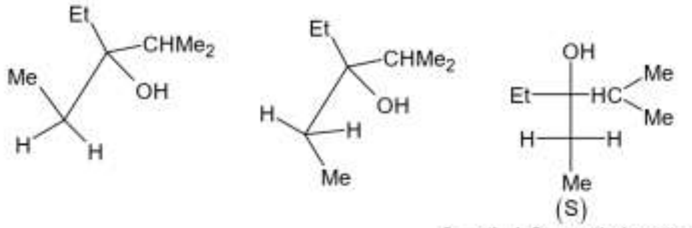


(III)



3 – ethyl, 2 – methylpentan– 2ol

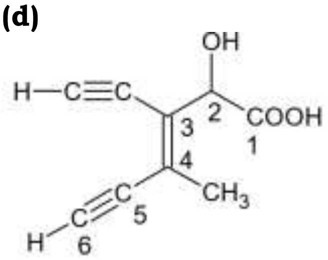
(IV)



3 – ethyl, 2 – methyl pentan – 3ol

Only (Q & R) are same.

23. (d)



3-ethynyl, 2-hydroxy, 4-methyl hex-3-en-5-ynoic acid

24. (c) Given structure: (2R, 3R)

P $\Rightarrow$ (2R,3R), so, identical. Hence P $\rightarrow$ 2

Q $\Rightarrow$ (2S,3R), so, Diastereomers. Hence Q $\rightarrow$ 1

R $\Rightarrow$ (2R,3S), so, Diastereomer s. Hence R $\rightarrow$ 1

S $\Rightarrow$ (2S,3S), so, Enantiomers. Hence S $\rightarrow$ 3

25. (a, b, c)

$W_{rev} = -\int PdV$

For one mole of a real gas,

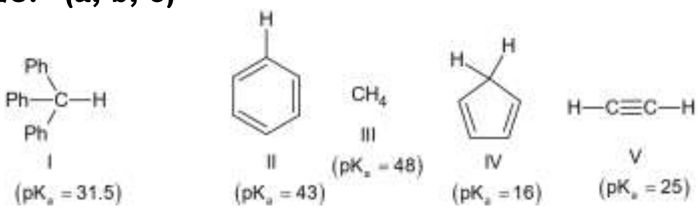
$$\left(P+\frac{a}{V^2}\right)(V-b)=RT$$

$$p=\left(\frac{RT}{(V-b)}-\frac{a}{V^2}\right)$$

So, $W_{rev} = -\int\left(\frac{RT}{(V-b)}-\frac{a}{V^2}\right)dV \quad ....(1)$

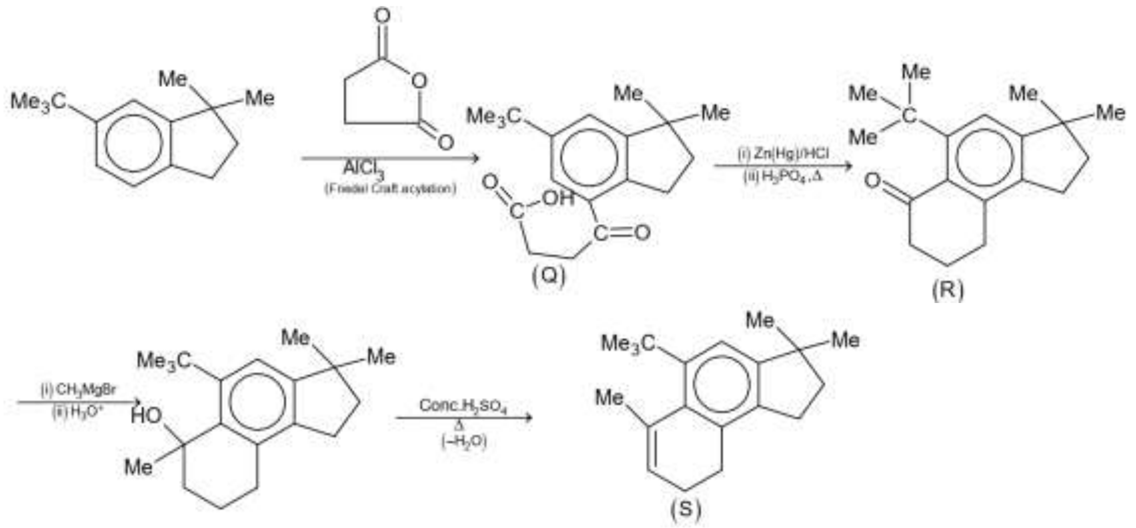
So, above Equation is valid for all reversible processes and for real gases.

26. (a, b, c)



Correct order and acidity = IV > V > I > II > III (Decreasing acidity)

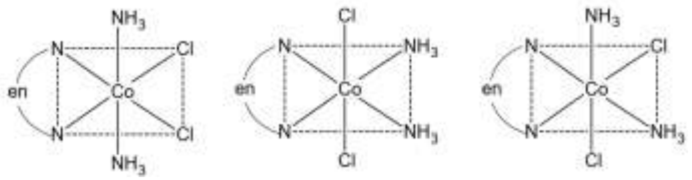
27. (b, d)



28. (a, c)

(a) $[FeCl_4]^-$: $\underset{(d^5)}{Fe^{3+}}$, weak ligand, so tetrahedral.

(b) $[Co(en)(NH_3)_2Cl_2]^+$: Three geometrical isomers.



(c) $\left[FeCl_4\right]^- : N = 5. \mu = \sqrt{5(5+2)} = \sqrt{35} BM$

$[Co(en)(NH_3)_2 Cl_2]^+$: Co^{3+} (d^6), diamagnetic

(d) $[Co(en)(NH_3)_2Cl_2]^+$:d^2sp^3. $N = 0, \mu = 0 \,\&\, \left[FeCl_4\right]^- \Rightarrow$ Tetrahedral (sp^3)

29. (a, b, d)

(a) HClO < $HClO_3$ < $HClO_4$ (increasing acidic character)
So, ClO- is the strongest conjugate base.

(b) Only Cl – O^- and ClO_3^- has lone pair of electrons on Cl-atom. But Cl – O^- is always linear ion as it is diatomic, so, no effect of lone pair on shape but in ClO_3 due the lone pair on Cl shape becomes pyramidal.

(c) $\left.\begin{array}{l} ClO^- \longrightarrow Cl^- + ClO_3^- \\ ClO_3^- \longrightarrow Cl^- + ClO_4^- \end{array}\right]$ (No same set of ions)

(d) $ClO^- + SO_3^{2-} \rightarrow Cl^- + SO_4^{2-}$

30. (a, c)

It can be assumed to be CsCl type solid (bcc)

So, $r_+ + r_- = \dfrac{\sqrt{3}a}{2} = 0.866$

Formula of solid = MX

$0.732 \le \dfrac{r_+}{r_-} \le 0.999$

31. (0.11)

Vol. of NaOH used at equivalent point = 9 mL
So, mequivalent of NaOH used up to end point = 9 x (M x 1)
5x0.1x2 = 9xM

M = $\dfrac{1}{9}$ = 0.11 9

32. (0.25)

K_1 at 1000 K = $\frac{10}{1}$ = 10

K_2 at 2000 K = $\frac{100}{1}$ = 100

ΔG_1^0 =-$RT_1\ln 10$ = -2.303RT_1

ΔG_2^0 =-2.303$RT_2\log 100$ = -2x2.303RT_2

$$\frac{\Delta G_1^0}{\Delta G_2^0} = \frac{1\times 1000}{2\times 2000} = \frac{1}{4} = 0.25$$

33. (13.32)

I. $\frac{1}{2}O_2(g) + 2H^+ + 2e^- \longrightarrow H_2O(\ell)$, $E^\circ = 1.23V$

II. $H_2(g) \longrightarrow 2H^+ + 2e^-$, $E^\circ = 0.00$ V

$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O$, $E^\circ = 1.23$ V

$$\Delta G^0 = -2\times 96500\times 1.23\times 1\times 10^{-3}\times 0.7 = -166.173J$$

W=166.173J

$$W_{adiabatic} = \frac{nR}{(\gamma-1)}(T_2 - T_1)$$

$$166.173 = \frac{1\times 8.314}{\left(\frac{5}{3}-1\right)}(T_2 - T_1)$$

$$\Delta T = \frac{166.173}{8.314}\times\frac{2}{3} = 13.32$$

34. (06.15)

$$2Al + \underset{(LR)}{3H_2SO_4} \longrightarrow Al_2(SO_4)_3 + 3H_2(g)$$

$\frac{5.4}{27}$ 0.25

$= 0.2$ mol 0.25 mol

$$V_{H_2} = \frac{0.25\times 0.0821\times 300}{1} = 6.1575 litre$$

35. (1.20)

$${}^{238}_{92}U \rightarrow {}^{206}_{82}Pb + 8\left({}^{4}_{2}He\right) + 6\left({}^{0}_{-1}\beta\right)$$

Total moles of uranium = = $\frac{68\times 10^{-6}}{238}$

= 0.286 x10^{-6} mol

So, after 3 half-lives fraction left = 0.125

Fraction of uranium reacted = 0.875

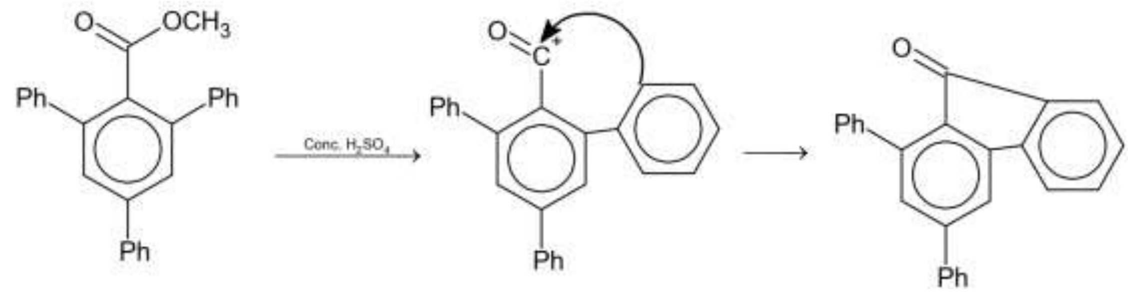
So, number of moles of a - particles = = $\frac{8\times 68\times 10^{-6}}{238}$ x 0.875 mol

So, number or α - particle = $\frac{8\times 68\times 10^{-6}\times 0.875\times 6.023\times 10^{23}}{238}$

= 12.046 x10^{17}

= 1.20x10^{18}

36. (18)



Degree of unsaturation = 18

37. (d)

Given a + b = -20, ab = -2020, c + d = 20, cd = 2020-09-28

$a^2c - ac^2 + a^2d - ad^2 + b^2c - b^2c - be^2 + b^2d - bd^2$

$= (a^2 + b^2)(c + d) - (a + b)(c^2 + d^2)$

$= ((a + b)^2 - 2ab)(c + d) - (a + b)((c + d)^2 - 2cd)$

= (400 - 2 x 2020)(20) - (-20)(400 - 2 x 2020)

= 20x800=16000

38. (c) f(x) is odd, continuous function

$$f(x)=\begin{cases} x(x-\sin x), & x\ge 0 \\ -x(x-\sin x), & x<0 \end{cases}$$

for $x \ge 0$, $f'(x) = 2x - \sin x - x\cos x = x(1 - \cos x) + (x - \sin x) \ge 0$

for $x < 0$, $f(x) = -2x + \sin x + x\cos x = x(\cos x - 1) - (x - \sin x) > 0$ as $x < 0$

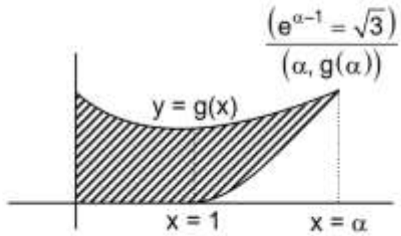
$\Rightarrow$ So f(x) strictly increases in $(-\infty,\infty) \Rightarrow f(x)$ is one-one

$x\to\infty\ f(x)\to\infty$

$x\to-\infty\ f(x)\to-\infty$.

So f(x) is onto

39. (a)



$$f(x)=\begin{cases} 0 & , x\le 1 \\ e^{x-1}-e^{1-x} & , x>1 \end{cases}$$

$$g(x)=\frac{1}{2}(e^{x-1}+e^{1-x})$$

$f(x) = g(x)$

$$e^{x-1}-e^{1-x}=\frac{1}{2}(e^{x-1}+e^{1-x})$$

$e^{x-1} = 3e^{1-x}$

$e^{x-1} = \sqrt{3}$

$e^{\alpha-1} = \sqrt{3}$. Let $x = \alpha$ be P.O.I

Required area = $\int_0^1 \frac{1}{2}\left(e^{x-1}+e^{1-x}\right)dx+\int_1^{\alpha}\left(\frac{1}{2}\left(e^{x-1}+e^{1-x}\right)-\left(e^{x-1}-e^{1-x}\right)\right)dx$

$=\frac{1}{2}\left|e^{x-1}-e^{1-x}\right|_0^1+\left|-\frac{1}{2}e^{x-1}-\frac{3}{2}e^{1-x}\right|_1^{\alpha}$

$=\frac{1}{2}\left\{\left(e^0-e^0\right)-\left(e^{-1}-e^1\right)\right\}-\frac{1}{2}\left\{e^{\alpha-1}+3e^{1-\alpha}-\left(e^0+3e^0\right)\right\}$

$=\frac{1}{2}\left(e-\frac{1}{e}\right)-\frac{1}{2}\left(\sqrt{3}+\frac{3}{\sqrt{3}}-4\right)$

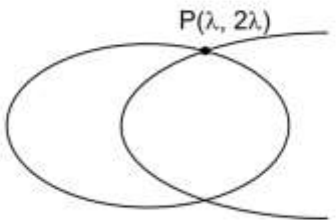
$=-\left(\sqrt{3}-2\right)+\frac{1}{2}\left(e-\frac{1}{e}\right)$

$=\left(2-\sqrt{3}\right)+\frac{1}{2}\left(e-\frac{1}{e}\right)$

40. (a)

Given a, b, $\lambda \in R^+$

As P lies on ellipse $\frac{\lambda^2}{a^2}+\frac{y\lambda^2}{b^2}=1$



So equation of tangent at P to ellipse ss, $m=\frac{-\lambda}{a^2}\times\frac{b^2}{2\lambda}$

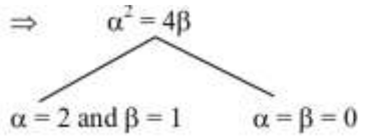
(Slope of tangent for parabola at P) (Slope of tangent at P for ellipse) = -1

$\Rightarrow -\frac{b^2}{2a^2}=-1, b^2=2a^2$

$e=\sqrt{1-\frac{1}{2}}=\frac{1}{\sqrt{2}}$

41. (b) For $C_1 P(h)=\frac{2}{3}$, for $C_2 P(h)=\frac{1}{3}$, given roots of the equation $x^2-\alpha x+\beta=0$ equal real roots

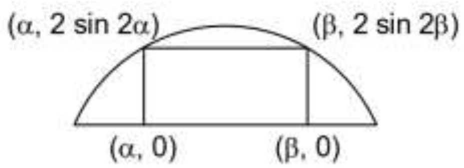
$\Rightarrow$



So required probability = $=\left(\frac{2}{3}\right)^2 . {}^2C_1 . \frac{1}{3} . \frac{2}{3}+\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2=\frac{16}{81}+\frac{4}{81}=\frac{20}{81}$

42. (c)

$0 \le y \le 2\sin 2x, 0 < x < \frac{\pi}{2}$



for rectangle $2\sin 2\alpha = 2\sin 2\beta \Rightarrow \alpha+\beta=\frac{\pi}{2}$

Perimeter = $2(\beta-\alpha)+4\sin 2\beta = 2\left(\beta-\left(\frac{\pi}{2}-\beta\right)\right)+4\sin 2\beta$

$=4\beta-\pi+4\sin 2\beta$

$\frac{dp}{d\beta}=4+8\cos 2\beta=0, \cos 2\beta=-\frac{1}{2}, \beta=\frac{\pi}{3}$

$\frac{d^2 p}{d\beta^2}=-16\sin 2\beta<0$ so P is max

So, area of rectangle = $(\beta-\alpha)2\sin 2\beta=\left(\frac{\pi}{3}-\frac{\pi}{6}\right).2\sin\left(\frac{2\pi}{3}\right)$

$=\frac{\pi}{6}\times 2.\frac{\sqrt{3}}{2}=\frac{\pi}{2\sqrt{3}}$

43. (a c)

f:R$\rightarrow$R

f(x) = $x^3 - x^2 + (x - 1)\sin x$

g:R$\rightarrow$R

(a) If g is continuous at x = 1 then fg is differentiable

Let h(x) = f(x>g(x)

$$\text{RHD} = h'(1^+) = \lim_{h\to 0}\frac{f(1+h)g(1+h)-f(1)g(1)}{h}$$

$$= \lim_{h\to 0}\frac{f(1+h)g(1+h)-0}{h}$$

$$= \lim_{h\to 0} g(1+h)\cdot \lim_{h\to 0}\frac{f(1+h)-f(0)}{h}$$

$= g(1)\, f'(1)$

LHD at x = 1

$$= \lim_{h\to 0}\frac{f(1-h)g(1-h)-f(1)g(1)}{-h}$$

$$= \lim_{h\to 0} g(1-h)\cdot \lim_{h\to 0}\frac{f(1-h)-f(1)}{-h} = g(1)\, f'(1)$$

So, h(x) is differentiable at x = 1

(b) Given h(x) = f(x) . g(x) is differentiable

$$\lim_{h\to 0}\frac{f(1+h)g(1+h)-f(1)g(1)}{h}=\lim_{h\to 0}\frac{f(1-h)g(1-h)-f(1)g(1)}{-h}$$

$$\lim_{h\to 0} g(1+h)\cdot f'(1)=\lim_{h\to 0} g(1-h)\cdot f'(1)$$

f'(1) $\neq$ 0 and g(1) is not define

So, cannot comment over continuity and differentiability

(c) Given g(x) is differentiable So, h(x) = f(x) g(x)

h'(x) = f (x) g(x) + g'(x) f(x), as g(x) is differentiable = f(1)g(1) + 0 will exist

(d) Same as for B

$\lim_{h\to 0} g(1+h)=\lim_{h\to 0} g(1-h)$

cannot say about differentiability of the g(x)

44. (b c, d)

$M^{-1} = \text{adj (adj M)}$

$= |M|\, M$

$\Rightarrow |M^{-1}| = ||M|\, M|$

$\frac{1}{|M|} = |M|^3 |M| \Rightarrow |M| = 1 \quad \text{..... (1)}$

$\frac{\text{adj } M}{|M|} = |M| M$

$\text{adj } M = |M|^2 M \Rightarrow \text{adj } M = M \text{ as } |M| = 1$

$\Rightarrow M\,(\text{adj } M) = M^2$

$\Rightarrow M^2 = I$

$\text{adj } M = M \text{ as } |M| = 1$

$(\text{adj } M)^2 = I \text{ (as adj } M = M)$

$M \text{ adj } M = |(M)|\, I$

replace M by adj M

$(\text{adj } M) \text{ adj (adj } M) = |\text{adj } M|\, I = |M|^2 I$

Multiplying by M

$M(\text{adj } M) \text{ adj (adj } M) = |M|^2 M$

$|M| \text{ adj(adj } M) = |M|^2 M$

$\text{adj (adj } M) = |M|\, M$

45. (b, c)

$|z^2 + z + 1| = 1$

$$\left|\left(Z + \frac{1}{2}\right)^2 + \frac{3}{4}\right| = 1$$

Using Δ inequality

$$\left|z + \frac{1}{2}\right|^2 + \frac{3}{4} \geq \left|z^2 + z + 1\right| = 1$$

$$\left|z + \frac{1}{2}\right|^2 \geq \frac{1}{4}$$

$$\left|z + \frac{1}{2}\right|^2 \geq \frac{1}{2} \ldots(1)$$

$$\left|\left|z^2 + z\right| - 1\right| \leq 1$$

$$0 \leq \left|z^2 + z\right| \leq 2 \Rightarrow \left|z^2 + z\right| \leq 2$$

$$|z|^2 - |z| \leq 2 \Rightarrow |z|^2 - |z| - 2 \leq 0$$

$$(|z| - 2)(|z| + 1) \leq 0$$

$$|z| \leq 2$$

46. (b, c)

$$\tan\frac{X}{2} + \tan\frac{Z}{2} = \frac{2y}{2x + y + z}$$

$$\Rightarrow \frac{\Delta}{s(s-x)} + \frac{\Delta}{s(s-z)} = \frac{2y}{2x} \Rightarrow \Delta = (s-x)(s-z)$$

$\Rightarrow s(s - y) = (s - x)(s - z) \Rightarrow z^2 + x^2 = y^2$

$\Rightarrow$ Triangle is right angled, $y = \frac{\pi}{2}$

$\Rightarrow Y = X + Z$

$$\tan\frac{X}{2} = \frac{\Delta}{s(s-x)} = \frac{\frac{1}{2}xz}{\left(\frac{x+y+z}{2}\right)\left(\frac{y+z-x}{2}\right)} = \frac{x}{y+z}$$

47. (a, b)

DCs of $L_1 : \frac{1}{\sqrt{11}}, \frac{-1}{\sqrt{11}}, \frac{3}{\sqrt{11}}$

DCs of $L_2 : \frac{-3}{\sqrt{11}}, \frac{-1}{\sqrt{11}}, \frac{1}{\sqrt{11}}$

Since $\frac{1}{\sqrt{11}}\left(\frac{-3}{\sqrt{11}}\right)+\frac{-1}{\sqrt{11}}\left(\frac{-1}{\sqrt{11}}\right)+\frac{3}{\sqrt{11}}\frac{1}{\sqrt{11}}=\frac{1}{11}>0$

So DCs of acute angle bisectors are $\frac{-2}{\sqrt{11}},\frac{-2}{\sqrt{11}},\frac{4}{\sqrt{11}}$

D.rs can be written as 1, 1, -2 $\Rightarrow l=m=1$

L_1 and L_2 intersect at (1, 0, 1) which also lies on bisector L. Point on L having y-coordinate equal to 1 is

(2, 1,-1) $\Rightarrow \alpha$ = 2, γ = -1

48. (a, b, d)

(a) $\int_0^1 x\cos x dx \ge \int_0^1 x\left(1-\frac{x^2}{2!}\right)dx$

$\ge \frac{3}{8}$

(b) $\int x\sin x dx \ge \int_0^1 x\left(x-\frac{x^3}{3!}\right)dx$

$\ge \frac{3}{10}$

(c) $\int_0^1 x^2\cos x dx \le \int_0^1 x^2 dx$

$\le \frac{1}{3}$

(d) $\int_0^1 x^2\sin x dx \ge \int_0^1 x^2\left(x-\frac{x^3}{3!}\right)dx$

$\ge \frac{2}{9}$

49. (8)

$\frac{3^{y_1}+3^{y_2}+3^{y_3}}{3}\ge 3^{\frac{y_1+y_2+y_3}{2}}$ (Using $AM\ge GM$)

$\Rightarrow \log_3\left(3^{y_1}+3^{y_2}+3^{y_3}\right)\ge \log_3 3^4$

≥ 4

$\Rightarrow m=4$

Similarly, $\frac{x_1+x_2+x_3}{3}\ge\left(x_1x_2x_3\right)^{\frac{1}{3}}$

$\log_3 3\ge\frac{1}{3}\left(\log_3 x_1+\log_3 x_2+\log_3 x_3\right)$

$3\ge \log_3 x_1+\log_3 x_2+\log_3 x_3$

$\Rightarrow M=3$

50. (1)

$2(a_1 + a_2 +.....+ a_n) = b_1 + b_2 +.....+ b_n$

$\Rightarrow 2\left(\frac{n}{2}\left(2c+(n-1)2\right)\right)=c\left(\frac{2^n-1}{2-1}\right)$

$c=\frac{2n(n-1)}{2^n-1-2n}$

$2^{x-2}>x^2-x$ if $x\ge 8$

$2^x-1>2x$ if $x>3$

So, $c<\frac{2^{n-1}}{2^n-1}\forall n\ge 8$

as c ≥ 1 values of c are not possible for n ≥ 8
possible values of n = 3, 4, 5, 6, 7; only satisfy when n = 3 and c = 12

51. (1)

$$(3-\sin 2\pi x)\sin\left(\pi x-\frac{\pi}{4}\right)-\sin\left(3\pi x+\frac{\pi}{4}\right)$$

$$=(3-\sin 2\pi x)\sin\left(\pi x-\frac{\pi}{4}\right)+\sin\left(3\pi x-\frac{3\pi}{4}\right)$$

$$=\left(3-\sin 2\pi x+3-4\sin^2\left(\pi x-\frac{\pi}{4}\right)\right)\sin\left(\pi x-\frac{\pi}{4}\right)$$

$$\Rightarrow \sin\left(\pi x-\frac{\pi}{4}\right)\ge 0$$

$$\Rightarrow \pi \ge \pi x-\frac{\pi}{4}\ge 0\left(\because x\in[0,2]\right)$$

$$\Rightarrow \frac{5}{4}\ge x\ge\frac{1}{4}\Rightarrow \beta-\alpha=1$$

52. (108)

Equation reduces to $\left(4\vec{a}+3\vec{b}\right).\vec{c}=7\vec{a}.\vec{b}$

But $\vec{a}+\vec{b}+\vec{c}=0$

So, $\left(4\vec{a}+3\vec{b}\right).\left(\vec{a}+\vec{b}\right)=-7\vec{a}.\vec{b}$

$\vec{a}.\vec{b}=-6$

$\left|\vec{a}\times\vec{b}\right|=108$

53. (5) $f = (x^2 - 1)^2(a_0 + a_1x + a_2x^2 + a_3x^3)$
$= (x + 1)^2(x - 1)^2 (a_0 + a_1x + a_2x^2 + a_3x^3)$
For min value of $m_{f'}+m_{f''}$, a_0 + aix + $a_2x^2 + a_3x^3$ must have only 1 real root which is 1 or -1, then $m_{f'}$ = 3 & $m_{f''}$ = 2 ⇒ Answer = 5

54. (1)

$$\lim_{x\to0^+}\frac{(1-x)^{1/x}-\frac{1}{e}}{x^a}=\lim_{x\to0^+}\frac{e^{\frac{\ln(1-x)}{x}}-\frac{1}{e}}{x^a}$$

$$=\lim_{x\to0^+}\frac{1}{e}\frac{\left(e^{\frac{-x-\frac{x^2}{2}}{x}+1}-1\right)}{x^a}=\lim_{x\to0}\frac{1}{e}\left(\frac{e^{-\frac{x}{2}}-1}{x^a}\right)$$

For value of limit to be a non-zero real number a = 1