1. (c) Least count = $\left(1-\frac{9}{10}\right)(0.1)$ = 0.01 cm

Zero error = -0.1 + 0.06 = -0.04 cm
Final reading = 3.1 + 0.01 x 1 = 3.11 cm
So correct measurement = 3.11 + 0.04 = 3.15 cm

2. (c) $\Delta Q_1 = nC_P\Delta T > 0$

$\Delta Q_2 = nC_V\Delta T < 0$

$\Delta Q_3 = nC_P\Delta T < 0$

$\Delta Q_4 = nC_V\Delta T > 0$

3. (b) $\frac{1}{f_1} = (1.5-1)\left(\frac{1}{20}+\frac{1}{20}\right) = \frac{1}{20}$

$\frac{1}{f_2} = (1.5-1)\left(-\frac{1}{20}-\frac{1}{20}\right) = -\frac{1}{20}$

So, $\frac{1}{v}-\frac{1}{-10}=\frac{1}{20}$

So, v = - 20 cm

and $\frac{1}{v'}-\frac{1}{-30}=\frac{1}{-20}$

So, v' = -12 cm

So total magnification = $\left(\frac{-20}{-10}\right)\left(\frac{-12}{-30}\right)$ = 0.8

4. (d) Total no. of decays in 60 minutes = 1000 -1000 $\left(\frac{1}{2}\right)^3$ = 875

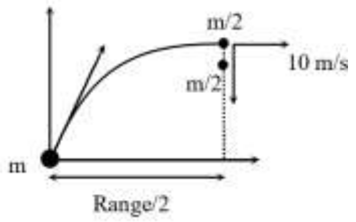
So, no. of α-decay = 875 x 0.6 = 525

5. (0.50) After splitting 1st mass takes 0.5 sec to reach ground.
Initial velocity is same for both mass at the highest point in vertical direction. Displacement and acceleration in vertical direction is also same
So, 2nd mass will also take 0.5 sec to reach ground.

6. Velocity of projectile at highest point $5m/s\hat{i}$

Since, there is no external force in horizontal direction so by conservation of momentum

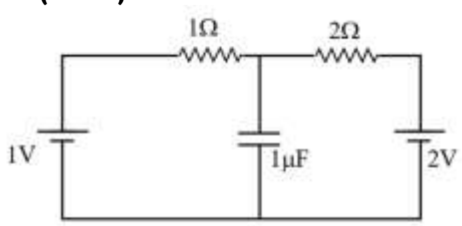
$m(5)=\frac{m}{2}(0)+\frac{m}{2}(v)$



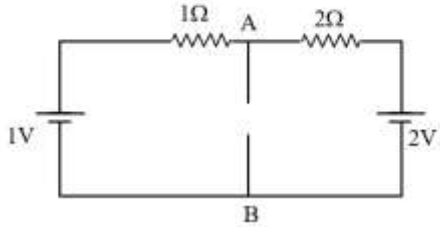
$\vec{v}=10m/s\hat{i}$

Distance covered by second mass before landing = $\frac{Range}{2}+10(t)=7.5m$

7. (1.33)



After long time we can replace the capacitor by open circuit.

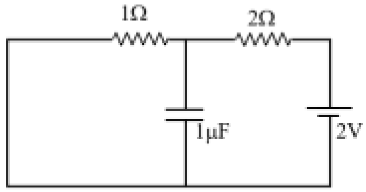


current in circuit = 1/3 ampere

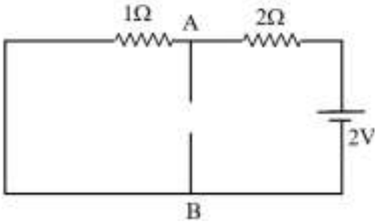
$$v_{AB} = 2 - 2\left(\frac{1}{3}\right) = \frac{4}{3} volt$$

So, $q_1 = CV = \frac{4}{3}\mu C$

8. 0.67



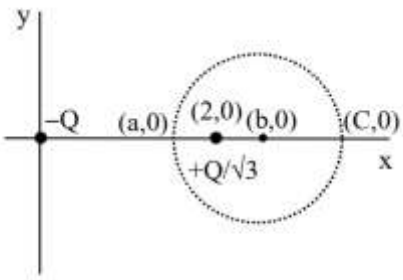
After long time i circuit = $\frac{2}{3}$



$$v_{AB} = 2 - 2\left(\frac{2}{3}\right) = \frac{2}{3} volt$$

So, $q_2 = CV = (1)\left(\frac{2}{3}\right) = \frac{2}{3}\mu C$

9. (1.73)



Lets take two points (a, 0) and (C, 0) on equipotential circle.
Net potential at (C, 0) = 0

$$\frac{K(-q)}{C} + \frac{\frac{Kq}{\sqrt{3}}}{(C-2)} = 0$$

$$\frac{1}{C} = \frac{1}{\sqrt{3}(C-2)}$$

$$\Rightarrow \sqrt{3}C - 2\sqrt{3} = C$$

$$\Rightarrow (\sqrt{3}-1)C = 2\sqrt{3}$$

$$\Rightarrow C = \frac{2\sqrt{3}}{\sqrt{3}-1}$$

Potential net at (a, 0) = 0

$$\frac{K(-q)}{a} + \frac{K\frac{q}{\sqrt{3}}}{(2-a)} = 0$$

$$\Rightarrow \frac{1}{a} = \frac{1}{\sqrt{3}(2-a)}$$

$$\Rightarrow 2\sqrt{3} - \sqrt{3}a = a$$

$$\Rightarrow a = \frac{2\sqrt{3}}{1+\sqrt{3}}$$

$$\text{So, Radius} = \frac{C-a}{2} = \frac{\frac{2\sqrt{3}}{\sqrt{3}-1} - \frac{2\sqrt{3}}{\sqrt{3}+1}}{2}$$

$$= \sqrt{3}\left(\frac{1}{\sqrt{3}-1} - \frac{1}{\sqrt{3}+1}\right) = \sqrt{3}\left(\frac{\sqrt{3}+1-\sqrt{3}+1}{3-1}\right)$$

Radius = $\sqrt{3}$

10. (3.00)

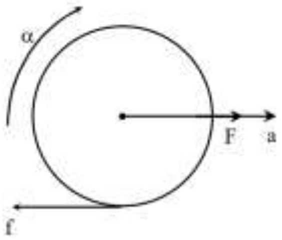
b = a + radius

$$= \frac{2\sqrt{3}}{\sqrt{3}+1} + \sqrt{3} = \frac{2\sqrt{3}+3+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{3\sqrt{3}+3}{1+\sqrt{3}} = 3$$

Center = (3, 0)

11. (b,d)



F-f=ma

fR = Iα (about center of mass)

a = Rα (pure rolling)

For hollow cylinder $a = \frac{F}{2m}, f = \frac{F}{2}$

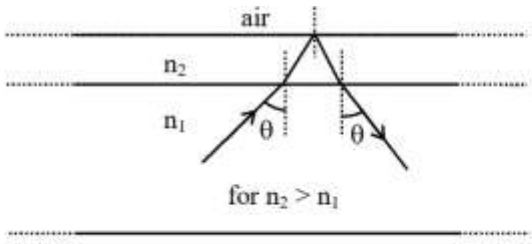
For solid cylinder, $a = \frac{2F}{3m}, f = \frac{F}{3}$

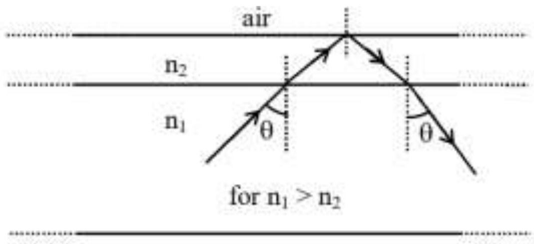
Also for solid cylinder $\frac{F}{2} \le \mu mg$

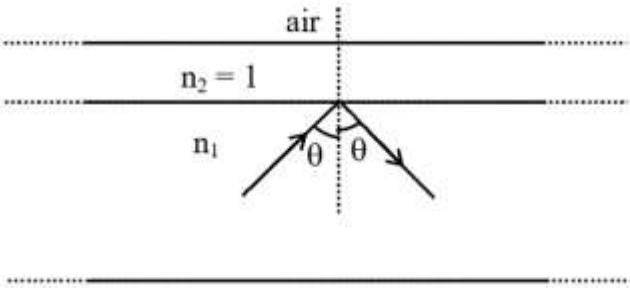
Therefore $a \le 2\mu g$

12. (b,c,d)

The ray diagram for the following conditions are







The light finally must reflected back in medium of refractive index n_1 for all values of n_2.

13. (a, b, c)

Time taken to reach from (-l, -h) to (l, -h) is given by

$2l = \frac{1}{2}at^2$

$20 = \frac{1}{2}(10)t^2$ t = 2sec

$\vec{L} = mvhk = mathk$

$\vec{L} = (0.2)(10)2(1)k = 4k$ [when particle passes through (l, -h)]

$\vec{\tau} = \frac{d\vec{L}}{dt} = mahk = (0.2)(10)(1)k = 2k$ (always)

14. (a, d)

For hydrogen atom, z = 1

$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ where R = 1.0973 x 10^7 m^{-1} = 1.1 x 10^7 m^{-1} = Rydberg constant.

For the Lyman series, n_1=1 and n_2 = 2, 3, 4,....... ∞

$\frac{1}{\lambda} = R\left(1 - \frac{1}{n_2^2}\right)$

$\lambda = \lambda_{max}$, when $n_2 = 2$

$\frac{1}{\lambda_{max}} = \frac{3R}{4} \Rightarrow \lambda_{max} = \frac{4}{3R} = 121.5nm$

$\lambda = \lambda_{min}$, when $n_2 = \infty$

$\frac{1}{\lambda_{min}} = R \Rightarrow \lambda_{min} = \frac{1}{R} = 91.1nm$

Also, $\lambda = \left(\frac{n_2^2}{n_2^2 - 1}\right)\frac{1}{R} = \left[1 + \frac{1}{(n_2^2 - 1)}\right]\lambda_0 = \left(1 + \frac{1}{m^2}\right)\lambda_0$

$\lambda = \left(1 + \frac{1}{m^2}\right)\lambda_0$

where,

$m^2 = (n_2^2 - 1)$ = an integer

$m = \sqrt{n_2^2 - 1}$ = not an integer

For the Balmer series, $n_1 = 2$ and $n_2 = 3, 4, 5, 6, \ldots\ldots\ldots\ldots, \infty$

$$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{n_2^2}\right)$$

$\lambda = \lambda_{max}$, when $n_2 = 3$

$$\frac{1}{\lambda_{max}} = \frac{5R}{36}$$

$$\lambda_{max} = \frac{36}{5R} = 656.2 \text{ nm}$$

$\lambda = \lambda_{min}$, when $n_2 = \infty$

$$\Rightarrow \lambda_{min} = \frac{4}{R} = 364.5 \text{ nm}$$

Hence, for the Balmer series,

$$\frac{\lambda_{max}}{\lambda_{min}} = \frac{36/5R}{4/R} = \frac{9}{5}$$

For the Paschen series, $n_1 = 3$ and $n_2 = 4, 5, 6, \ldots\ldots\ldots\ldots, \infty$

$$\frac{1}{\lambda} = R\left(\frac{1}{9} - \frac{1}{n_2^2}\right)$$

$\lambda = \lambda_{max}$, when $n_2 = 4$

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{9} - \frac{1}{16}\right) = \frac{7R}{144}$$

$$\Rightarrow \lambda_{max} = \frac{144}{7R} = 1874.7 \text{ nm}$$

$\lambda = \lambda_{min}$, when $n_2 = \infty$

$$\frac{1}{\lambda_{min}} = \frac{R}{9} \Rightarrow \lambda_{min} = \frac{9}{R} = 820.2 \text{ nm}$$

15. (a, c) Emf induced across the semi-circular conducting rod.

$$\varepsilon = \int_1^4 \frac{\mu_0 Ivdx}{2\pi x} = \frac{\mu_0 Iv}{2\pi}\ln(4) = \frac{\mu_0 Iv}{\pi}\ln(2)$$

Since the semi-circular conducting rod is moving with a constant speed v = 3 m/s, then

$$\varepsilon = \frac{\mu_0 Iv}{\pi}\ln(2) = \text{constant}$$

Maximum current through the resistor R is

$$i_{max} = \frac{\varepsilon}{R} = \frac{\mu_0 Iv}{\pi R}\ln(2) = \frac{4\times10^{-7}\times2\times3\times0.7}{1.4} = 1.2\times10^{-6} \text{ ampere.}$$

Maximum charge on the capacitor C_0 is

$$q_{max} = C_0\varepsilon = C_0\left(\frac{\mu_0 Iv}{\pi}\ln(2)\right) = 5\times10^{-6}\times4\times10^{-7}\times2\times3\times0.7 = 8.4\times10^{-12} \text{ coulomb.}$$

16. (a, c)

$$(P_1 - P_2)ds = \rho dsd\sqrt{2}(g\sin 45 - a)$$

$$(P_1 - P_2) = \rho d\left(g - a\sqrt{2}\right)$$

$$\beta = \frac{(P_1 - P_2)}{\rho gd} = \left(1 - \frac{a\sqrt{2}}{g}\right)$$

When $a = g/\sqrt{2}, \beta = 0$

When a=g/2, $\beta = \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$

17. (4)

$$r_\alpha = \frac{\sqrt{2m_\alpha q_\alpha V}}{q_\alpha B}$$

$$r_S = \frac{\sqrt{2m_S q_S V}}{q_S B}$$

$$\frac{r_S}{r_\alpha} = \sqrt{\frac{m_S}{q_S}\frac{q_\alpha}{m_\alpha}} = \sqrt{\left(\frac{32}{1}\right)\left(\frac{2}{4}\right)}$$

$$\frac{r_S}{r_\alpha} = 4$$

18. (49) Angular momentum of disc about O is

$$L_{DO} = M\left(\frac{3a}{4}\right)\left(\frac{3a}{4}\right)\Omega + \frac{M}{2}\left(\frac{a}{4}\right)^2(4\Omega)$$

Angular momentum of rod about O is

$$L_{RO} = \frac{Ma^2}{3}\Omega$$

So, $L_0 = L_{DO} + L_{RO} = \frac{49}{48}\left(Ma^2\Omega\right)$

So, n = 49

19. (9)

$$-C\frac{dT}{dt} = \left(T^4 - T_S^4\right)$$

$$\int_{200}^{100} \frac{dT}{T^4 - T_S^4} = \int_0^{t_1} -\frac{1}{c}dt$$

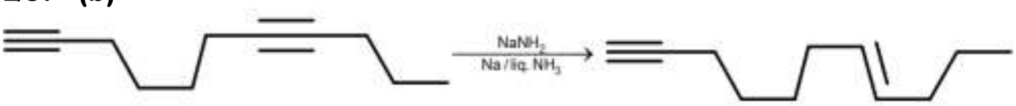
$$-\frac{1}{3}\left[\frac{1}{T^3}\right]_{200}^{100} = -\frac{1}{c}(t_1)$$

$$\Rightarrow \left[\frac{1}{(100)^3} - \frac{1}{(200)^3}\right] = \frac{3}{c}t_1$$

Similarly, $\left[\frac{1}{(50)^3} - \frac{1}{(200)^3}\right] = \frac{3}{c}t_2$

$$\frac{t_2}{t_1} = \frac{\left[\frac{1}{(50)^3} - \frac{1}{(200)^3}\right]}{\left[\frac{1}{(100)^3} - \frac{1}{(200)^3}\right]} = 9$$

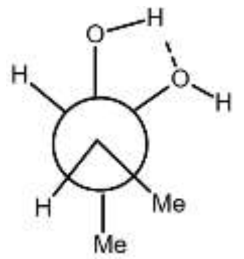
20. (b)



Na in liquid NH_3 reduces non terminal alkyne into trans alkene.

21. (b) In option (b), given configuration represents meso - butane - 2, 3-diol and due to intramolecular hydrogen bonding, the gauche form is more stable.

Option (c) and (d) does not represent meso - isomer.



22. (b)

Packing fraction (f) = $\dfrac{3\times\frac{4}{3}\pi r_+^3+1\times\frac{4}{3}\pi r_-^3}{a^3}$

$$=\frac{1\times\frac{4}{3}\pi\left[3\left(\frac{r_+}{r_-}\right)^3+1\right]}{\left(\frac{a}{r_-}\right)^3}$$

Now $2r_- = a$

Also, $\dfrac{r_+}{r_-}=0.414$

So, $f=\dfrac{1\times\frac{4}{3}\times 3.14\left[3\times(0.414)^2+1\right]}{(2)^3}$

= 0.634 ≈ 0.63

23. (a)

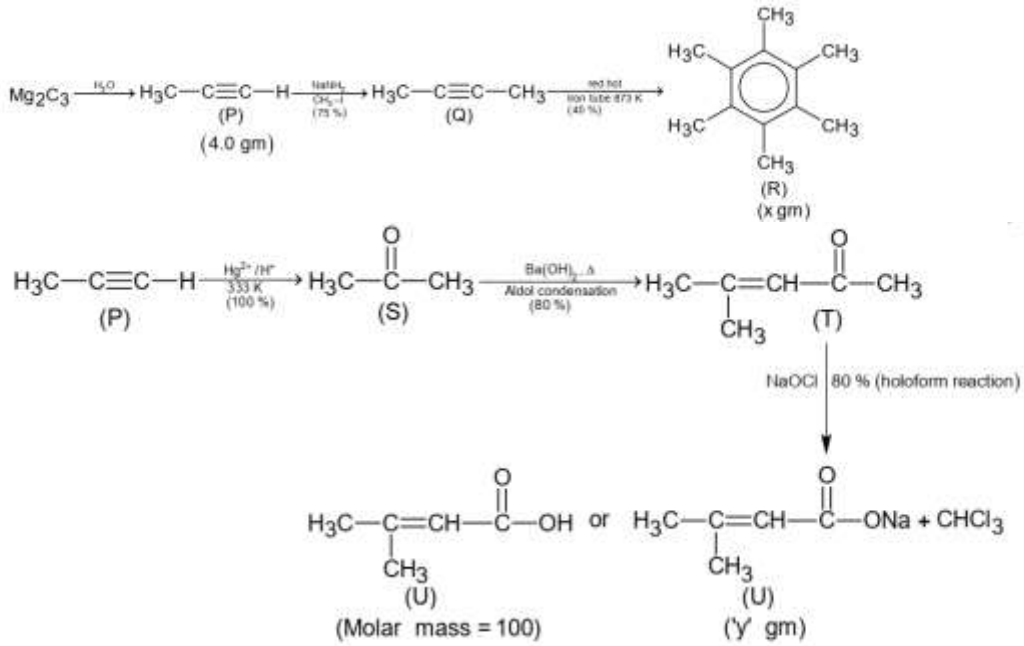
$[Cr(NH_3)_6]^{3+}$ has d^3 configuration, so as per CFT,

N = 3 and $\mu=\sqrt{3(3+2)}$ 3.87 BM

$[CuF_6]^{3-}$, has d^8 configuration and weak field ligand.

So N = 2 and $\mu=\sqrt{2(2+2)}$ 2.84 BM

24. (1.62)



Molar mass of P=40

$$\underset{(0.1 mole)}{P} \xrightarrow{75\%} \underset{\left(0.1\times\frac{3}{4}\right)}{Q}$$

$$\underset{\left(0.1\times\frac{3}{4}\right)}{3Q} \xrightarrow{40\%} \underset{\left(\frac{1}{3}\times0.1\times\frac{3}{4}\times0.4\right)}{R}$$

So, moles of R = 0.01 mole
Molar mass of (R) =162
So, x = 0.01 x 162= 1.62 g

25. (3.20 - 3.90)

Molar mass of 'U' = 122 g or 100 g

$$\underset{(0.1 mole)}{P} \xrightarrow{100\%} \underset{(0.1 mole)}{S}$$

$$\underset{(0.1 mole)}{2S} \xrightarrow[(80\%)]{Aldol\ condensation} \underset{\left(\frac{0.1}{2}\times0.8\right)}{T} \xrightarrow{80\%} \underset{\left(\frac{0.1}{2}\times0.8\times0.8\right)}{U}$$

So, mass of 'U' = $\frac{0.1}{2}$ x 0.8 x 0.8 x 100 = 3.20

Or

Mass of 'U' = $\frac{0.1}{2}$ x 0.8 x 0.8 x 122 = 3.90

26. (166.28)

X(s) ⇌ Y(s) + Z(g)

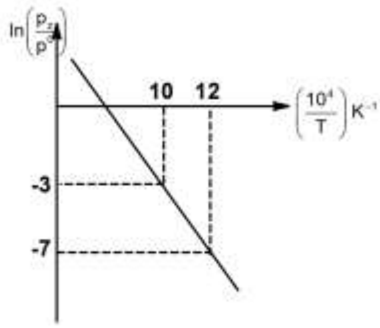
$K_p = \frac{p_z}{p^0}$ also $\Delta G° = -RT \ln k_p$

$$= -RT \ln\left(\frac{p_z}{p^0}\right)$$

Now, $\Delta G° = \Delta H° - T\Delta S°$

$$-RTkn\left(\frac{p_z}{p^0}\right) = \Delta H^0 - T\Delta S^0$$

$$\ln\left(\frac{p_z}{p^0}\right) = -\left(\frac{\Delta H^0}{R}\right)\frac{1}{T} + \frac{\Delta S^0}{R} \ldots (1)$$



$$(1) \Rightarrow \ln\left(\frac{p_z}{p^0}\right) = -\left(\frac{\Delta H^0}{10^4 R}\right)\times\frac{10^4}{T} + \frac{\Delta S^0}{T} \ldots. (2)$$

Slope of the line $= -\frac{\Delta H^0}{10^4 R} = \frac{[-7-(-3)]}{12-10} = -2$

$\therefore \Delta H^0 = 2R\times10^4$

= 2 x 8.314 x 10^{-3} x 10^4 = 1.66.28 kJmol^{-1} K^{-1}

27. (141.34)

Putting the value of ΔH^0 in equation (2), we get

$$-3 = -\left(\frac{2R\times10^4}{10^4 R}\right)\times\frac{10^4}{7} + \frac{\Delta S^0}{R}$$

$$-3 = -2R \times \frac{10^4}{T} + \frac{\Delta S^0}{R}$$
$$-3 = -2 \times \frac{10^4}{1000} + \frac{\Delta S^0}{R}$$
$$-3 = -20 + \frac{\Delta S^0}{R}$$
$$\therefore \frac{\Delta S^0}{R} = 17$$
$$\therefore \Delta S^0 = 17 \times 8.314 = 141.34 JK^{-1}mol^{-1}$$

28. (100.10 °C)

0.1 molal $AgNO_3$ (aq) solution

$AgNO_3 \rightarrow Ag^+$ (aq) + NO_3^- (aq)

i = 1 + (2-1)x1 = 2(α = 1, given)

ΔT_b = i x k_b x m

ΔT_b = 2 x 0.5 x 0.1 = 0.1

So, boiling point of solution 'A' is = 100.10°C = x

29. (2.5)

Let solution 'B' is prepared by mixing 1 L (=1000 g) of solution 'A' with 1 L (= 1000 g) of solution of $BaCl_2$.

	$BaCl_2$ +	$2AgNO_3$ $\rightarrow$	$2AgCl(s)$ +	$Ba(NO_3)_2$
Initial moles	0.1 moles	0.1 moles	-	0
Moles after reaction	0.1-0.05 = 0.05 moles	0	-	0.05 moles

So, molality or new solution = $\left(\frac{i_1 \times m_1 + i_2 \times m_2}{2}\right)$

$= \left(\frac{3 \times 0.05 + 3 \times 0.05}{2}\right)$

=0.

Elevation of boiling point of solution 'B' be $\left(\Delta T_b^1\right)$

$\left(\Delta T_b^1\right)$=0.15x$k_b$

= 0.15x$\frac{1}{2}$

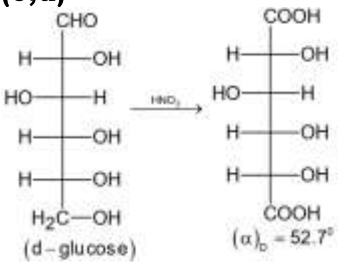
= 0.075

Now, ΔT_b^1 = 100.075 °C

So, difference of boiling point of 'A' and 'B' = 100.10 - 100.075 = 0.025 = y x 10^{-2} (given)

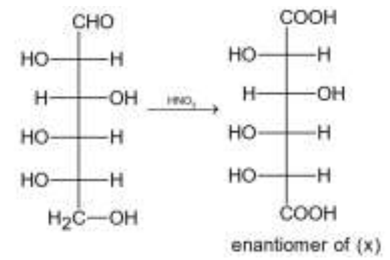
So, y=2.5

30. (c,d)

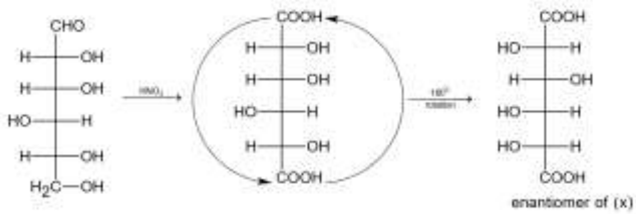


Since, we have to get the product (x) of (α)$_D$ = -52.7°, i.e. the enantiomer of above product. Which is only possible from (c) & (d).

(c)

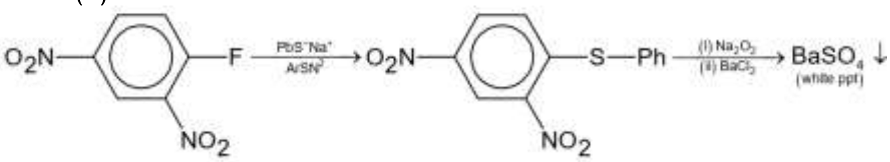


(d)

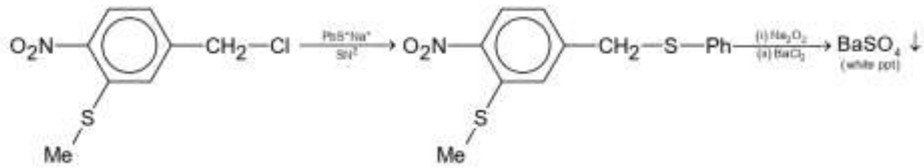


31. **(a,d)** In option (b) and (c), NO_2 group (an EWG) is not present ortho or para position wrt the leaving group, so $ArSN^2$ reaction will not be possible.

(a)



(b)



32. **(b, c)**

(a) Process of precipitating colloidal solution by using an electrolyte is called "COAGULATION" and not peptisation.

(b) Since, molar mass of sol is much higher than true solutions, so magnitude of any colligative properties is smaller than true solutions.

$\left(\Delta T_f\right)_{sol} < \left(\Delta T_f\right)_{\text{true solution}}$

So, freezing point of sols > freezing point of true solution. So, option (b) is correct.

(c) Micells are formed greater than or equal to CMC and above KRAFT temperature. So option (c) is also correct.

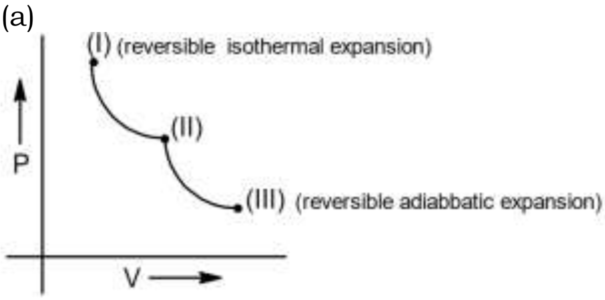
(d) Micelles are ASSOCIATED colloids and not Macromolecular colloids.

33. **(a, b, d)**

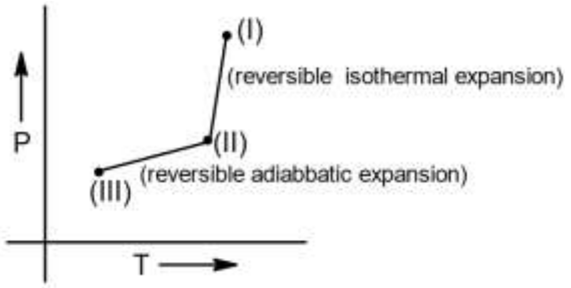
$(I)\xrightarrow[\text{expansion}]{\text{Reversible isothermal}}(II)\xrightarrow[\text{expansion}]{\text{Reversible adiabatic}}(III)$

So, the correct option is/are:

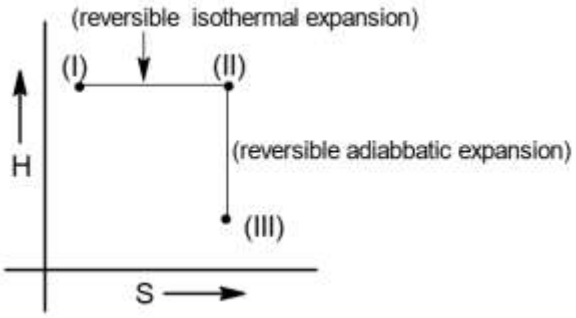
(a)



(b)



(d)



Reversible isothermal process is isoenthalpic while reversible adiabatic process is isoentropic.

34. (a, c, d)

(a) $PbS + 2PbO \xrightarrow{\text{self reduction}} 3Pb + SO_2 \uparrow$

So option (a) is correct.

(b) In the extraction Cu from copper pyrite $CuFeS_2$

SiO_2 is added to remove FeO as slag $FeSiO_3$.

So option (b) is wrong.

(c) $\underset{\text{concentrated ore}}{CuFeS_2} \xrightarrow{\text{roasting}} \underbrace{Cu_2S + FeS}_{\text{(matte)}} + SO_2$

$FeS + O_2 \rightarrow FeO + SO_2$

$Cu_2S + O_2 \rightarrow Cu_2O + SO_2$

$Cu_2S + 2Cu_2O \rightarrow 6Cu + SO_2$

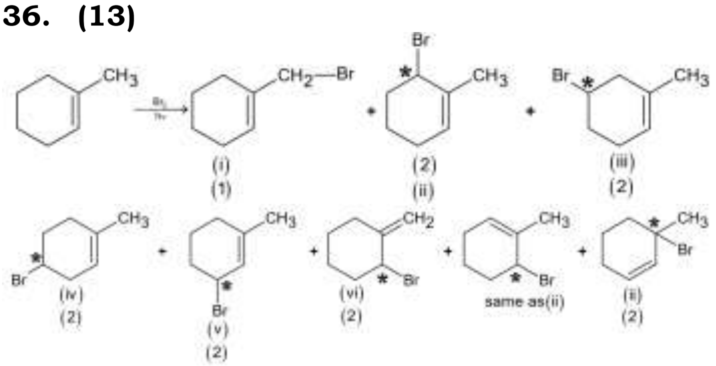
(Blister copper)

So, option (c) is correct.

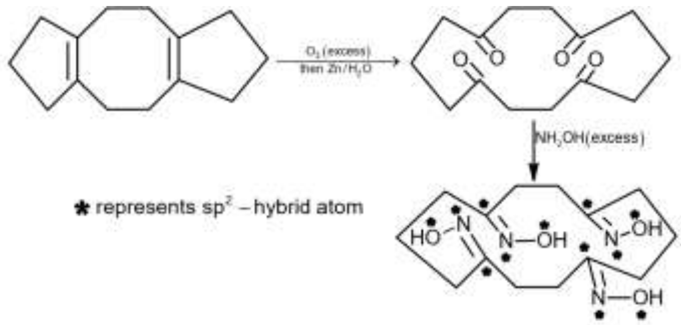
(d) $Zn + 2Na[Au(CN)_2] \rightarrow Na_2[Zn(CN)_4] + 2Au \downarrow$

35. (a,b,c) $Pb(OH)_2$, $Zn(OH)_2$ and $Bi(OH)_3$ are white precipitates but $Hg(OH)_2$ (unstable) is not. $PbCl_2$ is white ppt.

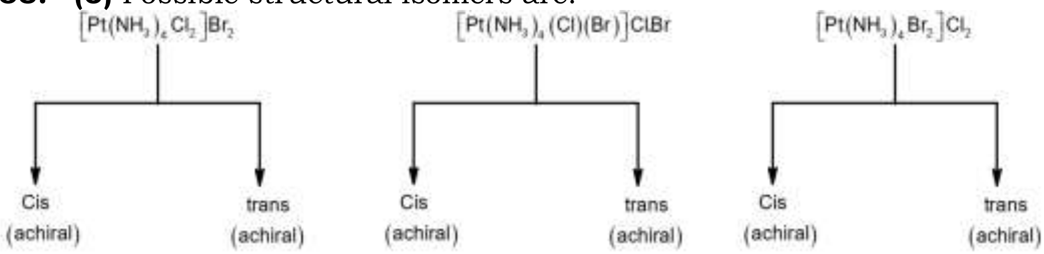
36. (13)



37. (12)



38. (6) Possible structural isomers are:



39. (b)

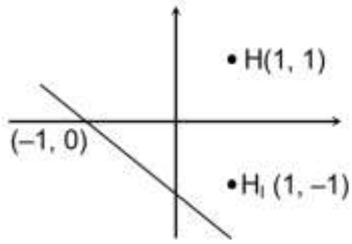
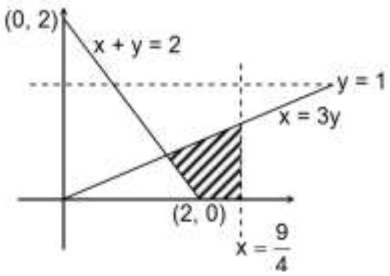


Image of orthocenter about any side of the triangle lies on its circumcircle.
We can observe that ,
$x^2 + y^2 + x + 3y = 0$ is satisfied by (-1, 0) and (1, -1) both

40. (a)



Area of shaded region (0,2)

$$=\frac{1}{2}\left(\frac{2}{3}+\frac{3}{4}\right)\frac{1}{4}+\frac{1}{2}\times\frac{2}{3}\times\frac{1}{2}$$

$$=\frac{11}{32}$$

41. (a)

	S_1	$F_2 = F_1 \cup S_1$	S_2	$G_1 \cup S_2 = G_2$	S_3
(i)	{1,2}	{1,2,3,4)	{1,x}	{1,2,3,4,5}	{1,2}
(ii)	{2,3}	{1,2,3,4}	{1,x}	{1,2,3,4,5}	{2,3}
			{x, y} (where x and y	or {2, 3, 4,5}	
(iii)	{1,3}	{1,3,4}	{1,x} are other than 1)	{1,2,3,4,5}	{1,3}

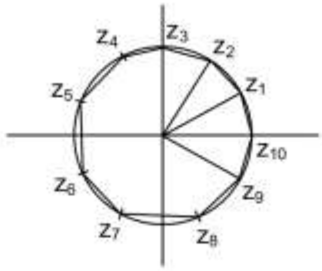
(i) $P_1 = \frac{{}^2C_2}{{}^3C_2} \cdot \frac{{}^3C_1}{{}^4C_2} \cdot \frac{{}^2C_1}{{}^5C_2} = \frac{1}{60}$

(ii) $P_2 = \frac{1}{3}\left(\frac{{}^3C_1}{{}^4C_2} \times \frac{{}^2C_2}{{}^5C_2} + \frac{{}^3C_2}{{}^4C_2} \times \frac{{}^2C_2}{{}^4C_2}\right) = \frac{2}{45}$

(iii) $P_3 = \frac{1}{{}^3C_1} \times \frac{{}^2C_1}{{}^3C_2} \times \frac{{}^2C_2}{{}^5C_2} = \frac{1}{45}$

Conditional probability $= \frac{P_1}{P_1 + P_2 + P_3} = \frac{1}{5}$

42. (c)



$\because z_1 = e^{i\theta_1}$

So, $z_2 = e^{i(\theta_1+\theta_2)}$

$z_3 = e^{i(\theta_1+\theta_2+\theta_3)}$

$\vdots$

$z_{10} = e^{i(\theta_1+\theta_2+\ldots+\theta_{10})} = e^{i(2\pi)}$

Sum of all the chord length < Circumference

So, $\sum|z_2 - z_1| \le 2\pi$

Also, $2|z_2 - z_1| \ge |z_2^2 - z_1^2|$

Hence, $2(|z_2 - z_1| + \ldots + |z_{10} - z_1|) \le 2(2\pi) = 4\pi$

So for, we have $P \le 2\pi$ and $Q \le 4\pi$

43. (76.25)

P_1 = 1 - (Probability that 3 chosen numbers are less than 81)

$= 1 - \left(\frac{80}{100}\right)^3 = 1 - \frac{64}{125} = \frac{61}{125}$

So, $\frac{625}{4}$

$P_1 = \frac{625}{4} \times \frac{61}{125} = 76.25$

44. (24.5) P_2 = 1 - (Probability that 3 chosen numbers are greater than 40)

$= 1 - \left(\frac{60}{100}\right)^3 = 1 - \left(\frac{3}{5}\right)^3 = \frac{98}{125}$

So, $\frac{125}{4}$

$P_2 = \frac{125}{4} \times \frac{98}{125} = 24.5$

45. $x + 2y + 3z = \alpha$ .....(1)

$4x + 5y + 6z = \beta$ .....(2)

$7x + 8y + 9z = \gamma - 1$ .....(3)

Equation (1) + (3) - (2) = 0. Equation (2) provides

$\alpha + \gamma - 1 - 2\beta = 0$

$$|M|=\begin{vmatrix}\alpha & 2 & \gamma\\ \beta & 1 & 0\\ -1 & 0 & 1\end{vmatrix}(C_1\to C_1+C_3)=\begin{vmatrix}\alpha+\gamma & 2 & \gamma\\ \beta & 1 & 0\\ 0 & 0 & 1\end{vmatrix}(R_1\to R_1-2R_2)$$

$$\begin{vmatrix}\alpha+\gamma-2\beta & 0 & \gamma\\ \beta & 1 & 0\\ 0 & 0 & 1\end{vmatrix}=\begin{vmatrix}1 & 0 & \gamma\\ \beta & 1 & 0\\ 0 & 0 & 1\end{vmatrix}=1$$

46. x + 2y+3z = α .....(1)

4x + 5y+6z = β .....(2)

7x + 8y+9z = γ -1 .....(3)

Equation (1) + (3) - (2) = 0. Equation (2) provides

$\alpha+\gamma-1-2\beta=0$

(1.5)

Plane Pisx-2y + z-1=0

$$D=\left(\frac{|-2-1|}{\sqrt{1+4+1}}\right)^2=1.5$$

47. (9)

Locus $C=\left|\frac{\left(x\sqrt{2}+y-1\right)\left(x\sqrt{2}-y+1\right)}{\sqrt{3}}\right|=\lambda^2$

$2x^2 - (y-1)^2 = \pm3\lambda^2$ for intersection with y = 2x + 1

$2x^2 - (2x)^2 = \pm3\lambda^2$

$2x^2 = -3\lambda^2$ (taking -ve sign)

$$x=\pm\sqrt{\frac{3}{2}}\lambda$$

Distance between R and $S=2\left|\sqrt{\frac{3}{2}}\lambda\right|\sec\theta$ (tan θ is slope of line)

$$=\sqrt{6}|\lambda|\sqrt{5}$$

So, $\sqrt{30}|\lambda|=\sqrt{270}\,(\lambda=\pm3)$

$\lambda^2=9$

48. (77.14)

Equation of perpendicular bisector y = $-\frac{1}{2}$x +1

For point of intersection $2x^2-\frac{1}{4}x^2=\pm3\lambda^2$

$x=\pm\sqrt{\frac{12}{7}}\lambda$ (taking +ve sign)

Distance=$\left|2.\sqrt{\frac{12}{7}}.3.\sec\theta\right|=2.\sqrt{\frac{12}{7}}.3.\sqrt{\frac{5}{2}}=3.\sqrt{\frac{60}{7}}$

$$D=\frac{9\times60}{7}=77.14$$

49. (a, b, d)

Let A = [C_1 C_2 C_3], $B=\begin{bmatrix}R_1\\ R_2\\ R_3\end{bmatrix}$

AP = [C_1 C_3 C_2] and $PB = \begin{bmatrix} R_1 \\ R_3 \\ R_2 \end{bmatrix}$

and $P^2 = I$

$$P(EP) = P\begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 3 \\ 8 & 18 & 13 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{bmatrix} = F$$

$$|E| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{vmatrix} (R_3 \to R_3 - 3R_2 - 2R_1) = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$\Rightarrow |F| = 0$

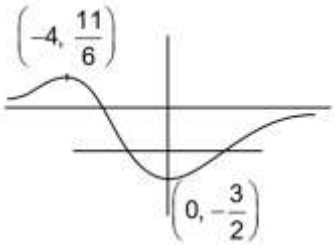
$|EQ + PFQ^{-1}| = |EQ + PPEPQ^{-1}| = |EQ + EPQ^{-1}| = |E||Q + PQ^{-1}| = 0$

$|EQ| = |E|\,|Q| = 0,\ |PFQ^{-1}| = |P||F||Q^{-1}| = 0$

(D) $P^{-1}EP + F = PEP + F = 2F$ (as $P^{-1} = P$)

$E + P^{-1}FP = E + P^{-1}PEPP = 2E$ (trace (E) = trace (F))

50. (a, b)



$$f'(x) = \frac{5x(x+4)}{\left(x^2+2x+4\right)^2}$$

$f(0) = -\frac{3}{2}$ (point of local minima)

f (-4) = $\frac{11}{6}$ (point of local maxima)

51. (a, b, c)

$P(E) = \frac{1}{8},\ P(F) = \frac{1}{6},\ P(G) = \frac{1}{4},\ P(E \cap F \cap G) = \frac{1}{10}$

(A) $P(E \cap F \cap G^C) = P(E \cap F) - P(E \cap F \cap G)$

$\le P(E) - P(E \cap F \cap G)$

$\le \frac{1}{8} - \frac{1}{10}$

$\le \frac{5-4}{40}$

$\le \frac{1}{40}$

(B) $P(E^C \cap F \cap G) = P(F \cap G) - P(E \cap F \cap G)$

$\le P(F) - P(E \cap F \cap G)$

$\le \frac{1}{6} - \frac{1}{10}$

$\le \frac{10-6}{60}$

$\le \frac{4}{60}$

$\le \frac{1}{15}$

(C) $P(E \cup F \cup G) \le P(E) + P(E) + P(G)$

$\le \frac{1}{8} + \frac{1}{6} + \frac{1}{4}$

$\le \frac{13}{24}$

(D) $P(E^C \cap F^C \cap G^C) = 1 - P(E \cup F \cup G)$

$\ge 1 - \frac{13}{24}$

$\ge \frac{11}{24} > \frac{10}{24} > \frac{5}{12}$

52. (a, b, c)

G(I - EF) = (I - EF)G = I

$\Rightarrow$ G - GEF = G - EFG = I .....(1)

(a) |FE| = |I - FE| |FGE| = |FGE - FE FGE|

= |FGE-F(G-I)E|=|FGE-FGE+FE|=|FE|

(b) (I - FE)(I + FGE) = I + FGE - FE - FEFGH

= I + FGE - FE - F(G - I)E = I + FGE - FE - FGE + FE = I

(c) From (I) it is true

(d) (I - FE)(I - FGE) = I - FGE - FE + FEFGE

= I - FGE - FE + F(G - I)E = I - FGE - FE + FGE - FE

= I - 2FE

53. (a,b)

$S_n = \sum_{k=1}^{n} \cot^{-1}\left\{\frac{1+k(k+1)x^2}{x}\right\}$; $\sum_{k=1}^{n} \cot^{-1}\left\{\frac{k(k+1)x^2+1}{(k+1)x-kx}\right\}$

$S_n = \sum_{k=1}^{n} \cot^{-1}(kx) - \cot^{-1}(k+1)x$

$t_1 = \cot^{-1}(x) - \cot^{-1}(2x)$

$t_2 = \cot^{-1}(2x) - \cot^{-1}(3x)$

$t_3 = \cot^{-1}(3x) - \cot^{-1}(4x)$

$\vdots$

$t_n = \cot^{-1}(nx) - \cot^{-1}((n+1)x)$

$S_n = \cot^{-1}(x) - \cot^{-1}((n+1)x)$

$\Rightarrow S_n = \cot^{-1}\left(\frac{(n+1)x^2+1}{nx}\right)$

$S_{10} = \cot^{-1}\left(\frac{11x^2+1}{10x}\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{11x^2+1}{10x}\right)$

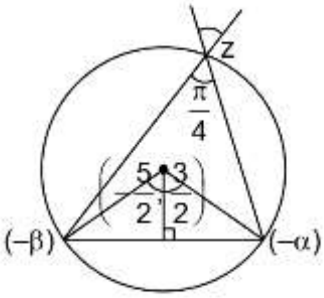
(B) $\lim_{n\to\infty} \cot(S_n(x)) = \lim_{n\to\infty} \cot\left(\cot^{-1}\left(\frac{(n+1)x^2+1}{nx}\right)\right) = \lim_{n\to\infty} \frac{nx^2+x^2+1}{nx} = x$

(C) $S_3(x) = \cot^{-1}\left(\frac{4x^2+1}{3x}\right) = \frac{\pi}{4} \Rightarrow \frac{1+4x^2}{3x} = 1$

$\Rightarrow 4x^2 - 3x + 1 = 0$ have imaginary roots

(D) $\tan(S_n(x)) = \tan\left(\cot^{-1}\left(\frac{1+(n+1)x^2}{nx}\right)\right) = \frac{nx}{1+(n+1)x^2} = \frac{1}{\frac{1}{nx} + \frac{(n+1)x}{n}}$

54. (b,d)



w = c + id, arg(w) $\in(-\pi,\pi]$

$\arg\left(\frac{z+\alpha}{z+\beta}\right)=\frac{\pi}{4}; z_0=-\frac{5}{2}+\frac{3}{2}i$

$z_0+\alpha=(z_0+\beta)i; z_0+\alpha=z_0 i+\beta i; z_0(1-i)=\beta i-\alpha$ (by Rotation)

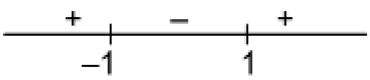
$-\frac{5}{2}+\frac{3}{2}i+\alpha=\left(-\frac{5}{2}+\frac{3}{2}i\right)i+\beta i$

$-\frac{5}{2}+\frac{3}{2}i+\alpha=-\frac{5}{2}i-\frac{3}{2}+\beta i$

$-\frac{5}{2}+\frac{3}{2}+\frac{3}{2}i+\frac{5}{2}i+\alpha=\beta i; -1+4i=\beta i-\alpha$ (As α, β are real number)

$\Rightarrow -\alpha=-1, \alpha=1; \beta=4$

55. (4)



$3x^2 - 4|x^2 - 1| + x - 1 = 0$

Case-I: $|x| \geq 1$

$3x^2 - 4x^2 + 4 + x - 1 = 0$

$-x^2 + x + 3 = 0$

$x^2 - x - 3 = 0$

$x=\frac{1\pm\sqrt{1+12}}{2}=\frac{1\pm\sqrt{13}}{2}=\frac{1+\sqrt{13}}{2},\frac{1-\sqrt{13}}{2}$

$|x|<1$

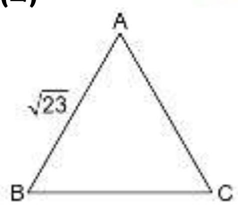
$3x^2 + 4x^2 - 4 + x - 1 = 0$

$7x^2 + x - 5 = 0$

$x=\frac{-1\pm\sqrt{1+20\times 7}}{14}=\frac{-1\pm\sqrt{141}}{14}$

So, number of real roots = 4

56. (2)



AB = $\sqrt{23}$ = C

BC = 3 = a

CA = 4 = b

$\frac{\cot A+\cot C}{\cot B}; \frac{\frac{\cos A}{\sin A}+\frac{\cos C}{\sin C}}{\frac{\cos B}{\sin B}}$

$$= \frac{\cos A \sin C + \cos C \sin A}{\sin A . \sin C . \frac{\cos B}{\sin B}} = \frac{\sin(A+C) . \sin B}{\sin A . \sin C . \cos B} = \frac{\sin B . \sin B}{\sin A . \sin C . \cos B}$$

$$= \frac{b^2}{ac . \frac{(a^2+c^2-b^2)}{2ac}} = \frac{2b^2}{a^2+c^2-b^2} = \frac{2\times 16}{9+23-16} = \frac{2\times 16}{32-16} = \frac{2\times 16}{16} = 2$$

57. (7)

$$[\vec{u}\ \vec{v}\ \vec{w}]^2 = \begin{vmatrix} \vec{u}\cdot\vec{u} & \vec{u}\cdot\vec{v} & \vec{u}\cdot\vec{w} \\ \vec{v}\cdot\vec{u} & \vec{v}\cdot\vec{v} & \vec{v}\cdot\vec{w} \\ \vec{w}\cdot\vec{u} & \vec{w}\cdot\vec{v} & \vec{w}\cdot\vec{w} \end{vmatrix} = \begin{vmatrix} 1 & \vec{u}\cdot\vec{v} & 1 \\ \vec{v}\cdot\vec{u} & 1 & 1 \\ 1 & 1 & 4 \end{vmatrix} = 2$$

$$= 1(3) - \vec{u}\cdot\vec{v}(4\vec{u}\cdot\vec{v}-1) + 1(\vec{u}\cdot\vec{v}-1) = 2$$

$$= 3 - 4(\vec{u}\cdot\vec{v})^2 + \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{v} - 1 = 2 \ = -4(\vec{u}\cdot\vec{v})^2 + 2\vec{u}\cdot\vec{v} = 0$$

$$(\vec{u}\cdot\vec{v})(2-4(\vec{u}\cdot\vec{v})) = 0\ ;\ \vec{u}\cdot\vec{v} = 0,\ \vec{u}\cdot\vec{v} = \frac{1}{2}$$

$$|3\vec{u}+5\vec{v}|^2 = 9|\vec{u}|^2 + 30\vec{u}\cdot\vec{v} + 25|\vec{v}|^2 = 9 + 15 + 25 = 49$$

$$\therefore\ |3\vec{u}+5\vec{v}| = 7$$