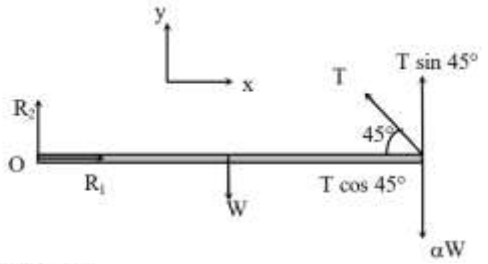
1. (a, b, d)

Free body diagram of rod



$\Sigma F_x = 0$

$R_1 = T\cos 45°$

$R_1 = \frac{T}{\sqrt{2}}$ ...(i)

$\Sigma F_y = 0$

$R_2 + T\sin 45° = W + \alpha W$

$R_2 + \frac{T}{\sqrt{2}} = W(1+\alpha)$ ...(ii)

$\Sigma \tau_0 = 0$

$W\frac{L}{2} + \alpha WL = \frac{T}{\sqrt{2}}L$

$T = \sqrt{2}W\left[\alpha + \frac{1}{2}\right]$ ...(iii)

From (ii) and (iii)

$R_2 + W\left[\alpha + \frac{1}{2}\right] = W(1+\alpha)$

$R_2 = \frac{W}{2}$

Hence, option (A) is correct.

From (i) and (iii)

$R_1 = W\left[\alpha + \frac{1}{2}\right]$

$\alpha = 0.5,\ R_1 = W$

Hence, option (B) is correct.

From equation (iii) if $\alpha = 0.5$

$T = \sqrt{2}W$

$T_{max} = 2\sqrt{2}\ W$

For rope to break

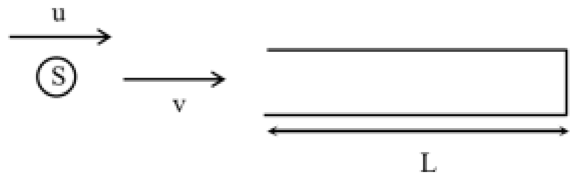
$T > 2\sqrt{2}W$

$\sqrt{2}W\left[\alpha + \frac{1}{2}\right] > 2\sqrt{2}W$

$\alpha > \frac{3}{2}$

Hence, option (D) is correct.

2. (a, d)



Natural frequency of closed pipe
$f=(2n+1)f_0$
f_0 is fundamental frequency
n = 0, 1,2,..........
frequency of source received by pipe

$$f' = f_s\left[\frac{v-0}{v-u}\right]$$

For resonance
F' = f

$$f_s\left[\frac{v}{v-u}\right] = (2n+1)f_0$$

If u = 0.8 v
$f_s = f_0$

$$f' = \frac{v}{0.2v} f_0 = 5f_0$$

for n = 2 pipe can be in resonance
Hence, option (a) is correct.
If u = 0.8v $f_s = 2f_0$

$$f' = \frac{v}{0.2v} \times 0.5 f_0 = 2.5 f_0$$

Not possible
If u = 0.5v, f_s = 1.5ft

$$f' = \frac{v}{0.5v} \times 1.5 f_0 = 3f_0$$

For n=1 $f=3f_0$
pipe can be in resonance

3. (b, c)

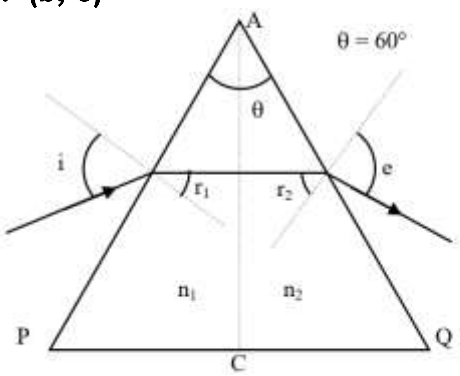


Diagram at minimum deviation for $n_1 = n_2 = n$
n=1.5
$r_1=r_2 = \theta/2 = 30^0$
for face AQ
$n\sin r_2 = \sin e$

$$1.5 \sin 30° = \frac{3}{2}\times\frac{1}{2} = \sin e$$

$$\sin e = \frac{3}{4}, \cos e = \frac{\sqrt{7}}{4}$$

When n_2 is given small variation there will be no change in path of light ray inside prism. As deviation on face AC is zero.
So, $r_2 = 30°$ C
Now for face AQ
$n_2 \sin 30° = \sin e$
for small change in n_2 change in e is given by $dn_2 \sin 30° = \cos e\, de$

or $dn_2 = \Delta n$ $\quad$ $de = \Delta e$

$\Delta n \sin 30° = \cos e\ \Delta e$

$$\Delta n \frac{1}{2} = \frac{\sqrt{7}}{4} \Delta e$$

$$\Delta n = \frac{\sqrt{7}}{2} \Delta e \ldots \text{(i)} \quad \Delta n > \Delta e$$

$$\Delta n \propto \Delta e$$

4. (b, d)

$\vec{S} = \vec{E} \times \vec{B}$

Method-1:

$\vec{S}$ is pointing vector which is defined as energy flowing per unit area in unit time.

S.I. unit of S is Watt/m^2.

Method 2:

$$\because B = \mu_0 ni \Rightarrow \frac{1}{\mu_0} = \frac{ni}{B}$$

$$S = \frac{1}{\mu_0} EB\cos\theta = niE\cos\theta$$

S.I. unit of $S = \frac{1}{meter} \times \frac{q}{time} \times \frac{Force}{q} = \frac{Force}{meter - time}$

5. (a, c, d)

Energy released during process

$Q = \delta c^2$ ... (1)

$\because$ momentum is conserved in process.

$$\Rightarrow 0 = m_P v_P - m_Q . v_Q$$

$$\Rightarrow \frac{v_P}{v_Q} = \frac{m_Q}{m_P} \qquad \text{.... (2)}$$

$$E_P = \frac{1}{2} m_P V_P^2$$

$$E_Q = \frac{1}{2} m_Q v_Q^2$$

$$\Rightarrow \frac{E_P}{E_Q} = \frac{m_Q}{m_P} \qquad \text{... (3)}$$

Solving (1) and (3),

$$E_P = \frac{m_Q}{m_P + m_Q} \delta c^2$$

$$E_Q = \frac{m_P}{m_P + m_Q} \delta c^2$$

Momentum $P = \sqrt{2m_P E_P} = \sqrt{2m_Q E_Q}$

$$= \sqrt{2m_P \frac{m_Q}{m_P + m_Q} . \delta c^2} = c\sqrt{\frac{2m_P m_Q}{m_P + m_Q} . \delta}$$

6. (a, b) Consider a circular loop of radius r in x-y plane and having center at origin

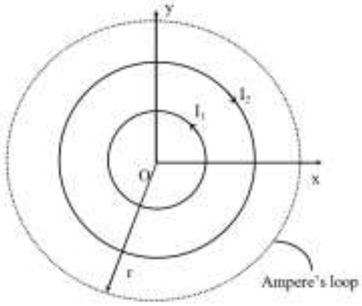
$$\oint \vec{B} . \vec{dl} = 0$$

$$B \oint dl \cos\theta = 0$$

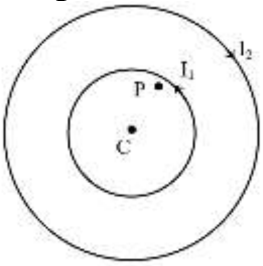
$\because B \neq 0$ for given r

$$\Rightarrow \cos\theta = 0$$

$$\theta = 90^0$$



Here dl is in xy plane $\Rightarrow$ B is normal to plane (B can't be in xy plane as its magnetic lines would have been in radial direction)
Also, for given r, B must be same in magnitude for all points on loop of radius r.



At center $B=\left(\frac{\mu_0 i_1}{2R}-\frac{\mu_0 i_2}{4R}\right)$ (inwards)

For point P, Let field of inner loop increases xi times and that of outer loop increases x_2 times
$\Rightarrow$magnetic field at P

$$B_P=\left(x_1\frac{\mu_0 i_1}{2R}-x_2\frac{\mu_0 i_2}{4R}\right)$$

For $B_P=0, i_2=\left(\frac{x_1}{x_2}\right).(2i_1)$

$\because$ B changes more rapidly as point P come closer to circumference.
$\Rightarrow x_1 > x_2$
Or $i_2 > 2i_1$ (which is given condition)
So, there are points inside inner loop where magnetic field will be zero.

7. (0.30) For equilibrium of the test tube

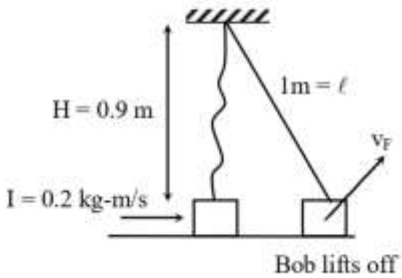
$$mg=(v_{tube}+v_{air})\rho_w g$$

So, $5=\left(\frac{5}{2.5}+v_{air}\right)$ (1)

So, v_{air} = 3cc
So, Δv = 0.3 cc

8. (10.00) _
For isothermal process $(10^5)(3.3) = (P)(3)$
So, $P=1.1 \times 10^5$Pa
So, $\Delta P = (1.1 - 1) \times 10^5 = 10 \times 10^3$ Pa

9. (0.18)



$J_i = MV_iH\sin 90^0$

= 0.2 x 0.9 = 0.18 kg-m²/s J

10. **(0.16)** As bob lifts off due to impulse of string angular momentum will be conserved about suspension point

$\Rightarrow J_f = J_i$

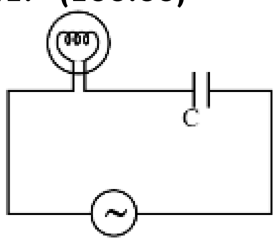
$\Rightarrow Mv_f l \sin 90^0 = 0.18$

$\Rightarrow v_f = 1.8, S$

Kinetic energy after lifting off

$\Rightarrow K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}\times 0.1\times(1.8)^2 = 0.162J$

11. **(100.00)**



P_{lamp} = 500 W

$\Rightarrow$ V_{lamp} i_{lamp} = 500

$\Rightarrow (100)i_{lamp} = 500$

$i_{lamp} = 5A$

Impedance of circuit $Z = \frac{V_{source}}{i_{source}} = \frac{200}{5} = 40\Omega$

Resistance of lamp $R = \frac{V_{lamp}}{i_{source}} = 20\Omega$

Reactance of capacitance

$X_C = \sqrt{Z^2 - R^2} = 20\sqrt{3}\Omega$

$\frac{1}{\omega C} = 20\sqrt{3}$

$C = \frac{1}{20\sqrt{3}\times 2\times\pi\times 50} = 100\mu F$

12. **(60.00)**

$\cos\phi = \frac{R}{Z} = \frac{20}{40} = \frac{1}{2}$

$\Rightarrow \phi = 60^0$

13. **(a)** $\frac{\mu_0}{2\pi}\frac{m}{r^3} = \frac{\mu_0 i_1}{2a}$

$\Rightarrow i_1 \propto \frac{m}{r^3}$

14. **(c)**

$dW = F.dx = \frac{-Km(i_1\pi a^2)}{r^4}(dr)$

$i_1 = \frac{ma}{\pi r^3}$

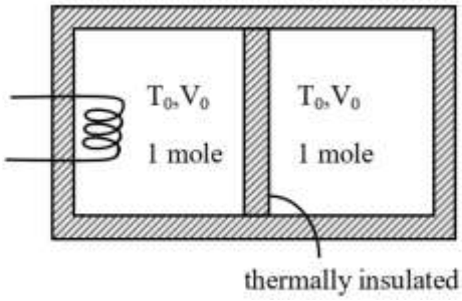
$W = \int_{\infty}^{r} \frac{km\left(\frac{ma}{\pi r^3}\times\pi a^2\right)dr}{r^3} = km^2a^2\int\frac{dr}{r^7}$

$W \alpha \frac{m^2}{r^6}$

15. (a)

16. (b)

Pressure on either side is equal



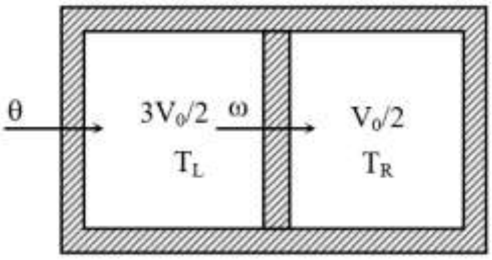
$C_v = 2R; C_P = 3R \Rightarrow \gamma = 3/2$

Left chamber

$Q = \Delta U_1 + \Delta W_1$

Right chamber

$0 = \Delta U_2 + \Delta W_2$



$\Delta W_1 + \Delta W_2 = 0$

$\Rightarrow Q = \Delta U_1 + \Delta U_2 = 2R(T_L - T_0) + 2R(T_R - T_0) \ldots$(i)

Also pressure each side of piston is equal

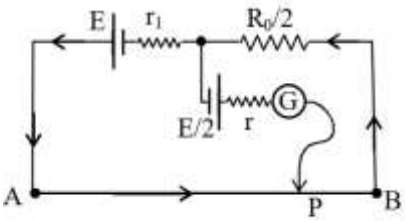
$\Rightarrow \frac{RT_L}{3V_0/2} = \frac{RT_R}{V_0/2} \qquad \ldots$(ii)

$\Rightarrow (T_L/3) = T_R$

$Q = 2R\left(3\sqrt{2}T_0 - T_0\right) = 2R\left(\sqrt{2}T_0 - T_0\right) = 8\sqrt{2}RT_0 - 4RT_0$

$\Rightarrow \frac{Q}{RT_0} = 8\sqrt{2} - 4$

17. (3)



$R_{AB} = 50\Omega$

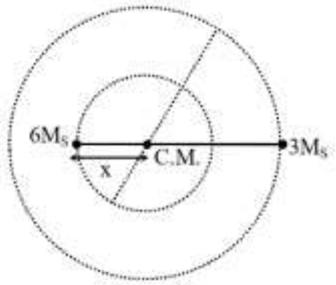
So, $R_{AP} = \frac{50}{100} \times 72 = 36\Omega$

$I = \frac{\varepsilon}{r_1 + 50 + 25} \ldots$(i)

$-36I - \frac{\varepsilon}{2} - Ir_1 + \varepsilon = 0 \ldots$(ii)

Solving equation (i) and (ii)
$R_1 = 3\,\Omega$

18. (9)



Both will revolve about common center of mass

$$x = \frac{3M_s}{6M_s + 3M_S} \times 9R = 3R \ldots\text{(i)}$$

$$\frac{G6M_s \times 3M_S}{(9R)^2} = 6M_S\left(\omega^2 x\right) \ldots\text{(ii)}$$

Solving equation (i) and (ii) we get

$$\omega^2 = \frac{GM_S}{81R^3}$$

$$T'^2 = \frac{4\pi^2 R^3}{GM_S} \times 81 \ldots\text{(iii)}$$

For the motion of earth around Sun

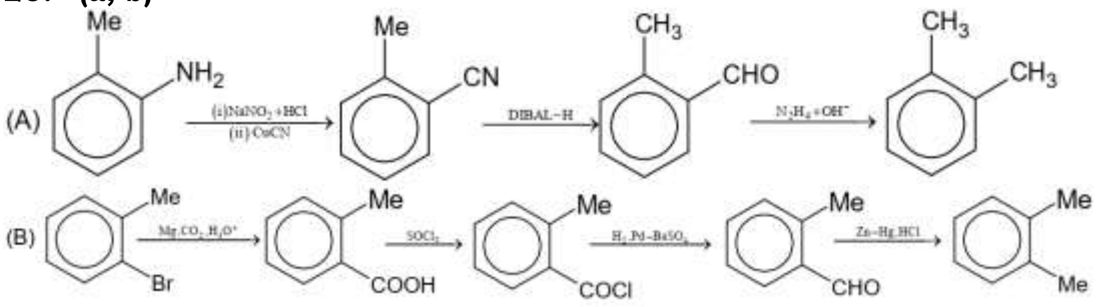
$$T^2 = \frac{4\pi^2 R^3}{GM_S} \ldots\text{(iv)}$$

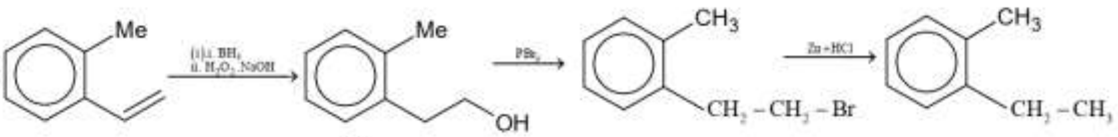
From (iii) and (iv)
$T'^2 = 81T^2$
$T' = 9T$

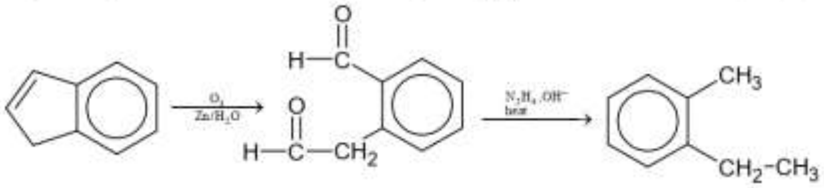
19. (6)

Let $E_R = E$
Then $E_Q = E$
$E_P = 2E$
From Einstein's equation for P, Q and R
$2E = hv - 4.0$ ...(i)
$E = hv - 4.5$ ...(ii)
$E = hv' - 5.5$ ...(iii)
Solving equation (i) and (ii) we get $E = 0.5$
So, $hv' = 6.0$ eV

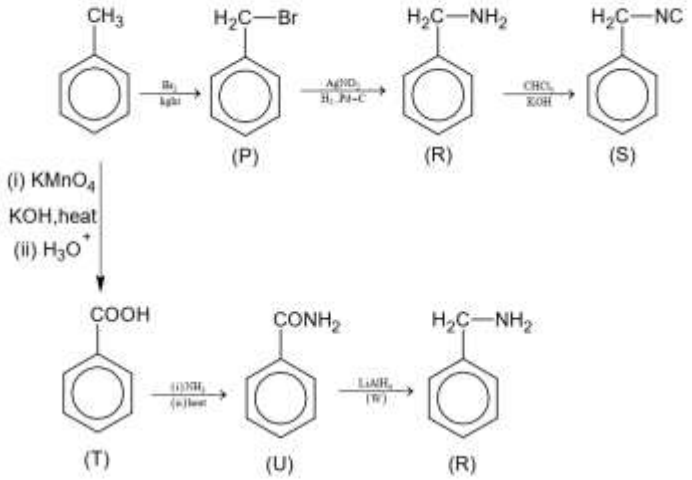
20. (a, b)



(C) 

(D) 

21. (c, d)



22. (b, c, d)

	$2X$	+	y	$\longrightarrow$	P
$t=0$	2		1		0
$t=50$	$2-2\times0.5$		$1-0.5$		
	$1\,mole$		0.5		

rate = $-\frac{1}{2}\frac{dx}{dt}=-\frac{dy}{dt}=\frac{dP}{dt}=K[X]$

$-\frac{1}{2}\frac{dx}{dt}=K[X]$

$-\frac{dx}{dt}=2K[X]=K^{1}[X]$

Half-life is t = 50 sec

$2K=\frac{0.623L}{50}\quad K=\frac{0.6932}{100}=6.332\times10^{-3}$

t =50 sec

$-\frac{dx}{dt}=2K[X]$

$-\frac{dx}{dt}=2\times6.332\times10^{-3}\times1=13.864\times10^{-3}\,mole/L/\sec$

$-\frac{dy}{dt}=K[X]=6.332\times10^{-3}\left(\frac{1}{2}\right)=3.46\times10^{-3}\,mole/L/\sec^{-1}$

23. (a,b,c)

(a) $Cd + Ni^{+2} \rightarrow Cd^{+2} + Ni$

$E_{cell}=0.40+(-24)-\frac{0.0591}{2}\log\frac{0.001}{0.1}$

$=0.16+\frac{0.0591}{2}\times2=6.64+0.551=0.71(+ve)$

(b) $E_{cell} = 0.40 + (-0.44) - \frac{0.591}{2}\log\frac{0.01}{0.1}$

$= -0.04 + \frac{0.591}{2} \times 2 = -0.04 + 0.06 = 0.02(+ve)$

(c) $E_{cell} = 0.24 + (-0.13) + \frac{.0591}{2} \times 2$

= 0.11 + 0.06 = 0.33(+ve)

(d) $E_{cell} = 0.24 + (-0.44) + \frac{0.0591}{2} \times 2$

= -0.20 + 0.06 = -0.14(-ve)

24. (a, b, d)

(a) $[FeCl_4]^-$

$Fe^{+3} = 3d^5$, Cl^- weak field liganol sp^3

$[Fe(CO)_4]^{-2}$

$Fe^{-2} = 3d^8 4s^2$

CO strong field liganol pairing occurs

$Fe^{-2} = 3d^{10}$ hence sp^3

(b) $[Co(CO)_4]^-$

Co = $3d^8\ 4s^2$ due to CO pairing occurs

Hence = $3d^{10}$

$[CoCl_4]^{-2}$

$Co^{+2} = 3d^7$, Cl^- weak field liganol sp^3

(d) $[Cu(Py)_4]^{+1}$

$Cu^{+1} = 3d^{10}$, sp^3

$[Cu(CN)_4]^{-3}$

$Cu^{+1} = 3d^{10}$, sp^3

25. (a, b, d)

$H_3PO_3 \xrightarrow{\Delta} H_3PO_4 + PH_3$

P-H bond is responsible for it is reducing character, H_3PO_4 does not have.

H_3PO_3 is a dibasic acid in oxyacid's of phosphors O-H bond is ionisalbe whereas P-H bond is non ionizable.

26. (0.22)

$\alpha_1 = \frac{\wedge_m^c}{\wedge_m^0} = \frac{y \times 10^2}{4 \times 10^2} = \frac{y}{4} = \alpha$

On dilution conductivity increases three times

$\alpha_2 = \frac{3y \times 10^2}{4 \times 10^2} = 3\alpha_1 = 3\alpha$

HA	$\rightleftharpoons$	H^+	+	A^-
C		O		O
$C(1-\alpha)$		$C\alpha$		$C\alpha$

$K_a = \frac{C\alpha^2}{1-\alpha}$

Since temperature is constant K_a will be constant

$\frac{C_1\alpha_1^2}{1-\alpha_1} = \frac{C_2\alpha_2^2}{1-\alpha_2}$

$$\frac{C\times\alpha^2}{1-\alpha}=\frac{\left(\frac{C}{20}\right)(3\alpha)^2}{1-3\alpha}$$

$$\frac{1}{1-\alpha}=\frac{9}{20}-\frac{1}{(1-3\alpha)}$$

$$20-60\alpha=9-9\alpha$$

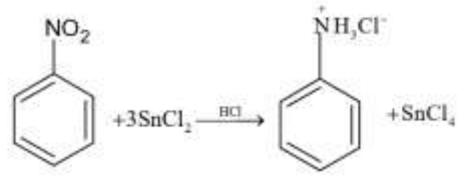
$$\alpha=\frac{11}{51}=0.2156$$

$$A=0.22$$

27. (0.88) $\alpha=\frac{y}{4};y=4\alpha$ = 4x0.22 -0.88

28. (3.57)

Sn + 2HC1 $\rightarrow$ $SnCl_2$ + H_2



Moles of Sn = $\frac{x}{119}$

Moles of nitrobenzene = $\frac{x}{119}\times\frac{1}{3}$

Moles of anilium chloride = $\frac{x}{119}\times\frac{1}{3}$

Moles of nitrobenzene = $\frac{x}{357}$

$\frac{y}{123}=\frac{x}{357}$ ...(1)

Moles of anilium chloride = $\frac{x}{357}$ ... (2)

$\frac{1.29}{129}=\frac{x}{357}$

x = 3.57

29. (1.23)

y = 1.225s 1.23

30. (1.875)

meq of Fe^{+2} = meq of $KMnO_4$

$x\times10^{-2}$x 1000x1 = 12.5x0.03x5x10 x= 1.875 mole

31. (18.75)

Moles of Fe^{+2} = $x\times10^{-2}$ = 1.875 x $10\times^{2}$

wt. of Fe^{+2}=1.875x$10\times^{2}$x56

Hence percentage of Fe^{+2}

$$=\frac{1.875\times10^{-2}\times56}{5.6}\times100\%=18.75\%$$

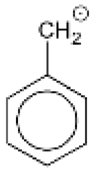
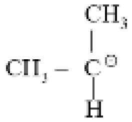
32. (a) As S character increases bond dissociation energy increases

H-C = CH H-CH = CH_2

sp sp^2

H-CH$(CH_3)_2$ H-CH_2-Ph

sp^3 sp^3

And  is more stable then 

33. **(d)** $CH_4 + Cl_2 \xrightarrow{light} CH_3Cl + HCl$ this reaction is obtained from given reaction.

$CH_3\text{-}H \rightarrow CH_3^{\bullet} + H^{\bullet}$ ...(1)

$Cl\text{-}Cl \rightarrow Cl^{\bullet} + Cl^{\bullet}$ ...(2)

$CH_3\text{-}Cl \rightarrow CH_3^{\bullet} + Cl^{\bullet}$ ...(3)

$H\text{-}Cl \rightarrow H^{\bullet} + Cl^{\bullet}$ ...(4)

(1) + (2)-(3)-(4)

Hence ΔH-105+ 58-85-103--25 KCal/mole

34. **(c)** $FeSO_4 + K_4[Fe(CN)_6] \rightarrow K_2Fe[Fe(CN)_6]$

While ppt.

35. **(d)** $FeSO_4 + NaNO_3 + H_2SO_4 \rightarrow [Fe(H_2O)_5 NO]^{+2} + SO_4^{-2}$

36. **(10)** 1 st process is adiabatic since entropy is constant.

$W_1 = \Delta U$

ΔU = 450R - 2250R = -1800R

W_1=-1800R ...(1)

In 2nd process internal energy is constant it means it is a isothermal process.

$W_2 = -2.303nRT\log\frac{V_3}{V_2}$ ....(2)

$= -nRT\ln\frac{V_3}{V_2}$ ....(3)

Given, n = 1 mole, here temperature is unknown

$U = nC_vT$ for process II

$450R = 1 \text{x} \frac{5}{2}RT$

$T = \frac{450 \times 2}{5}$ = 180K

Equation (1) = equation (2)

$W_1 = W_2$

$-1800R = -1\text{x}R\text{x}180\ln\frac{V_3}{V_2}$

$\ln\frac{V_3}{V_2} = \frac{1800}{180} = 10$

37. **(30)**

$\lambda = \frac{h}{m(\Delta V)}$

$330 \times 10^{-9} = \frac{6.6 \times 10^{-34}}{\left(\frac{4 \times 10^{-3}}{6 \times 10^{23}}\right) \times \Delta V}$

$\Delta V = \frac{6.6 \times 6 \times 10^{23} \times 10^{-34}}{4 \times 10^{-3} \times 330 \times 10^{-9}}$

= 0.30m/s

= 0.30x100 cm/s

= 30 cm/s

38. **(6)** $2ClO_2 + 2O_3 \rightarrow Cl_2O_6 + 2O_2$

Average oxidization state of Cl atom is +6

39. (a, b, d)

$n_1 = 10 \times 10 \times 10 = 10^3$

$n_2 = {}^8C_2 + 2\ {}^8C_1 = 44$

$n_3 = {}^{10}C_4 = 210$

$n_4 = {}^{10}P_4 = 5040$

40. (a, b)

(a) $\cos P = \dfrac{q^2 + r^2 - p^2}{2qr}, q^2 + r^2 \geq 2qr$, by $A.M. \geq G.M$

$\Rightarrow \cos P \geq 1 - \dfrac{p^2}{2qr}$

(b) By triangle inequality q + p > r

by projection formula

r cosP + p cosR + q cosR + rcosQ > p cosQ + q cosP

$\Rightarrow \cos R > \dfrac{(q-r)}{(p+q)}\cos P + \dfrac{(p-r)}{(p+q)}\cos Q$

So inequality is true (equality does not hold)

(c) By AM. $\geq$ GM.

$q + r \geq 2\sqrt{qr}$

$\dfrac{q+r}{p} \geq \dfrac{2\sqrt{qr}}{p}$

$\dfrac{\sin P}{p} = \dfrac{\sin Q}{q} = \dfrac{\sin R}{r}$ by sine rule

$\dfrac{q+r}{p} \geq \dfrac{2\sqrt{\sin Q \sin R}}{\sin P}$

(d) If $\cos Q > \dfrac{p}{r} \Rightarrow r\cos Q > p \ldots$ (i)

$\cos R > \dfrac{p}{q} \Rightarrow q\cos R > p \ldots$ (ii)

Add (i) and (ii)

r cosQ + q cosR > 2P $\Rightarrow$ p > 2p

$\Rightarrow$ p < 0, false

41. (a, b, c)

(a) Let g(x) = f(x) - 3 cos3x

$$\int_0^{\pi/3} g(x)dx = \int_0^{\pi/3} \left(f(x) - 3\cos 3x\right)dx = \int_0^{\pi/3} f(x)dx - \int_0^{\pi/3} 3\cos 3x dx = 0$$

$\Rightarrow$ g(x) = 0 has at least one solution in [0, π /3]

(b) Let h(x) = f(x) - 3 sin3x + 6/ π

$$\int_0^{\pi/3} h(x) = \int_0^{\pi/3} \left(f(x) - 3\sin 3x + \frac{6}{\pi}\right)dx = 0$$

$\Rightarrow$ h(x) = 0 has atleast one solution in [0, π /3]

(c) $\displaystyle \lim_{x\to 0} \frac{\int_0^x f(t)dt}{x} = \lim_{x\to 0} \frac{f(x)}{1} = f(0) = 1$

$$\Rightarrow \lim_{x\to 0} \frac{x\int_0^x f(t)dt}{x^2\left(\frac{1-e^{x^2}}{x^2}\right)} = \lim_{x\to 0} \frac{-\int_0^x f(t)}{x} \cdot \frac{x^2}{\left(e^{x^2} - 1\right)} = -1$$

(d) $\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\int_0^x f(t)}{x}=1$

42. (a, c)

$$\frac{dy}{dx}+\alpha y = xe^{\beta x}$$

$d\left(e^{\alpha x}\cdot y\right)xe^{\alpha x}\cdot e^{\beta x}$

$d\left(e^{\alpha x}\cdot y\right)=xe^{(\alpha+\beta)x}$ ... (i)

Case–I : $\alpha+\beta\neq 0$

$d\left(e^{\alpha x}\cdot y\right)=xe^{(\alpha+\beta)x}$

$e^{\alpha x}\cdot y=\frac{xe^{(\alpha+\beta)x}}{(\alpha+\beta)}-\frac{e^{(\alpha+\beta)x}}{(\alpha+\beta)^2}+C$

$y=\frac{xe^{\beta x}}{(\alpha+\beta)}-\frac{e^{\beta x}}{(\alpha+\beta)^2}+Ce^{-\alpha x}$

$\alpha=1,\beta=1\Rightarrow y=\frac{xe^x}{2}-\frac{e^x}{4}+Ce^{-x}$

as $y(1)=1\Rightarrow C=e\left(1-\frac{e}{4}\right)$

$y(x)=\frac{xe^x}{2}-\frac{e^x}{4}+\left(e-\frac{e^2}{4}\right)e^{-x}$

Case–II : $\alpha+\beta=0$

$\Rightarrow\quad \frac{dy}{dx}-\beta y=xe^{\beta x}$

$d\left(e^{-\beta x}y\right)=x$

$e^{-\beta x}\cdot y=\frac{x^2}{2}+C$

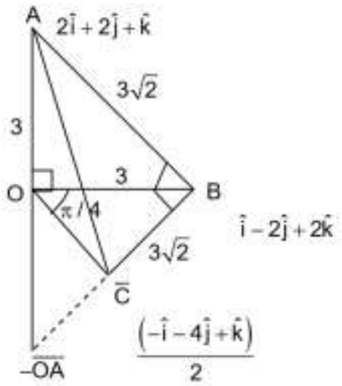
$y=\frac{e^{\beta x}x^2}{2}+Ce^{\beta x}$

$y(1)=1$

$\Rightarrow\quad C=\left(1-\frac{e}{2}\right)\frac{1}{e}\Rightarrow y=e^{\beta x}\cdot\frac{x^2}{2}+\left(1-\frac{e}{2}\right)\frac{1}{e}\cdot e^{\beta x}$

Take $\beta=-1\Rightarrow y=\frac{x^2}{2}e^{-x}+\left(1-\frac{e}{2}\right)e^{-x}$

43. (a, b, c)



$\overline{OA}\cdot\overline{OB}=0 \Rightarrow \overline{OA}\perp\overline{OB}$

$\overline{OC}=\frac{1}{2}\left((1-2\lambda)\hat{i}+(-2-2\lambda)\hat{j}+(2-\lambda)\hat{k}\right)$

$\left|\overline{OB}\times\overline{OC}\right|=\frac{9|\lambda|}{2}=\frac{9}{2}$

$\Rightarrow \quad \lambda=\pm 1$, as $\lambda>0, \lambda=1$

$\overline{OC}=\frac{\overline{OB}-\overline{OA}}{2}$

(A) Projection $\overline{OC}$ on $\overline{OA}$

$\overline{OC}\cdot\widehat{OA}=-\frac{3}{2}$

(B) Area of $\Delta OAB = 9/2$

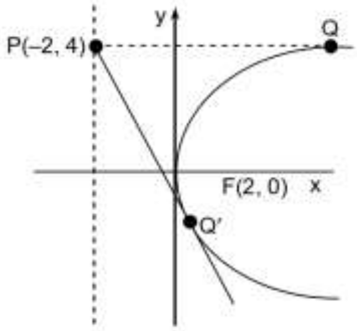
(C) Area of $\Delta ABC = 9/2$

(D) $\overline{OA}+\overline{OC}=\frac{3\hat{i}+3\hat{x}}{2}, \overline{OA}-\overline{OC}=\frac{5\hat{i}+8\hat{j}+\hat{k}}{2}$

$\left(\overline{OA}+\overline{OC}\right)\left(\overline{OA}-\overline{OC}\right)=\left|\overline{OA}+\overline{OC}\right|\left|\overline{OA}-\overline{OC}\right|\cos\theta$

$\cos\theta=\frac{1}{\sqrt{5}}$

44. (a, b, d)



E = $y^2-8x = 0$

a=2

Let $Q\left(2t_1^2, 4t_1\right), Q'\left(2t_2^2, 4t_2\right)$

$t_1t_2=-1, t_1+t_2=2$

$t_1=1+\sqrt{2}, t_2=1-\sqrt{2}$

(a) (slope of PF) (slope of FQ) = -1

$\Rightarrow \angle PFQ=\frac{\pi}{2}$

(b) (slope of PQ')(slope of PQ) = -1

$\Rightarrow \angle QPQ'=\frac{\pi}{2}$

(c) PF = $4\sqrt{2}$

(d) slope of Q'F = slope of FQ

$\Rightarrow$ Q, F, Q' are collinear

45. (1.5)

46. (2) Solution (7 and 8)

Let the equation of circle $(x - r)^2 + y^2 = r^2$

Equation of parabola $y^2 = 4 - x$ Solving them

$(x-r)^2 + 4-x = r^2$

$\Rightarrow x^2-x(2r+ 1) + 4 = 0$

Since circle and parabola meet tangentially hence

$(2r+1)^2-16 = 0$
$\Rightarrow 2r + 1 = 4$
$\Rightarrow r = 3/2$ and $x^2 - 4x + 4 = 0$
$(x-2)^2 = 0$
$\Rightarrow \alpha = 2, r = 3/2=1.5$

47. (57)

$$f_1(x)=\int_0^x \prod_{j=1}^{21}(t-j)^j dt$$

$$\Rightarrow f'_1(x)=\prod_{j=1}^{21}(x-j)^j$$

Therefore $m_1 = 6$, $n_1 = 5$
$2m_1 + 3n_1 + m_1n_1$
$= 2(6) + 3(5) + 30$
$= 12+15 + 30=57$

48. (6) $f_2'(x) = (98)(50)(x - 1)^{49} - (600)(49)(x - 1)^{48}$
$= (49)(100)(x-1)^{48}(x-1-6)$
$m_2= 1, n_2 = 0$
$6m_2 + 4n_2 + 8m_2n_2$
$6(1) + 4(0) + 8(0) = 6$

49. (2)

$$g_1:\left[\frac{\pi}{8},\frac{3\pi}{8}\right]\to R, i=1,2, f:\left[\frac{\pi}{2},\frac{3\pi}{r}\right]\to R$$

$$g_1=1, g_2=|4x-\pi|, f(x)=\sin^2 x$$

$$S_i=\int_{\pi/8}^{3\pi/8} f(x).g_i(x)dx$$

$$S_1=\int_{\pi/8}^{3\pi/8}\sin^2 x dx=\int_{\pi/8}^{3\pi/8}\sin^2\left(\frac{\pi}{2}-x\right)dx\Rightarrow 2S_1=\int_{\pi/8}^{3\pi/8}1dx$$

$$\Rightarrow S_1=\frac{1}{2}\left(\frac{3\pi}{8}-\frac{\pi}{8}\right)=\frac{\pi}{8}\Rightarrow\frac{16S_1}{\pi}=2$$

50. (1.5)

$$S_2=\int_{\pi/8}^{3\pi/8} sin^2x|4x-\pi|dx=\int_{\pi/8}^{3\pi/8}\sin^2\left(\frac{\pi}{2}-x\right)\left|4\left(\frac{\pi}{2}-x\right)-\pi\right|dx$$

$$=\int_{\pi/8}^{3\pi/8}\cos^2 x|4x-\pi|dx$$

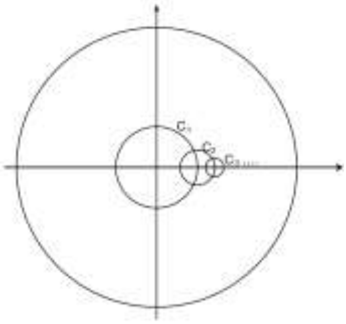
$$\Rightarrow 2S_2=\int_{\pi/8}^{3\pi/8}|4x-\pi|dx=2\int_{\pi/8}^{\pi/4}(\pi-4x)dx$$

$$S_2=\left(\pi x-2x^2\right)\Big|_{\pi/8}^{\pi/4}=\pi\left(\frac{\pi}{4}-\frac{\pi}{8}\right)-2\left(\frac{\pi^2}{16}-\frac{\pi^2}{64}\right)$$

$$S_2=\frac{4\pi^2}{8}-\frac{3\pi^2}{32}=\frac{\pi^2}{32}=\frac{48S_2}{\pi^2}=\frac{48}{\pi^2}\times\frac{\pi^2}{32}=\frac{3}{2}$$

$$\frac{48S_2}{\pi^2}=1.5$$

51. (d)



$C_1 \to (0,0), r = 1$

$C_2 \to (1,0), r = 1/2$

$C_3 \to (3/2,0), r = 1/4$

$\vdots\ C_n\left(2\left(1-\frac{1}{2^{n-1}}\right),0\right), r = \frac{1}{2^{n-1}}$

$\Rightarrow 2\left(1-\frac{1}{2^{n-1}}\right)+\frac{1}{2^{n-1}} < r$

$2-\frac{1}{2^{n-1}} < r$

$\Rightarrow 2-\frac{1}{2^{n-1}} < \frac{1025}{513}$

$\Rightarrow \frac{1}{2^{n-1}} > \frac{1}{513}$

$\Rightarrow 2^{n-1} < 513 \Rightarrow n-1 \le 9$

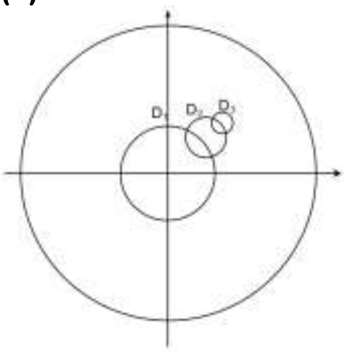
$\Rightarrow n \le 10 \Rightarrow k = 10$

Also, no two by C_1, C_3, C_5, C_7, C_9 intersect each other. And no two of C_2, C_4, C_6, C_8, C_{10} intersect each other

For both, we get 1 = 5

$\Rightarrow$ 3k + 21 = 40

52. (b)



$\sqrt{2}S_{n-1} + a_n < r$

$\Rightarrow \sqrt{2}\left(2\left(1-\frac{1}{2^{n-1}}\right)\right)+\frac{1}{2^{n-1}} < \left(\frac{2^{199}-1}{2^{198}}\right)\sqrt{2}$

$\Rightarrow \sqrt{2}\left(1-\frac{1}{2^{n-1}}\right)+\frac{1}{2^{n}} < \left(1-\frac{1}{2^{199}}\right)\sqrt{2}$

$\Rightarrow \frac{1}{\left(\sqrt{2}\right)^{2n}} - \frac{1}{\left(\sqrt{2}\right)^{2n-3}} < \frac{-1}{\left(\sqrt{2}\right)^{397}}$

$$\Rightarrow \frac{2\sqrt{2}-1}{\left(\sqrt{2}\right)^{2n}} > \frac{1}{\left(\sqrt{2}\right)^{397}}$$

$$\Rightarrow \left(\sqrt{2}\right)^{2n-397} < 2\sqrt{2}-1$$

$$\Rightarrow 2n-397 \le 1 \Rightarrow n \le 199$$

53. (c)

$\psi_2(x) = x^2 - 2x - 2e^{-x} + 2, x \ge 0$

$\psi'_2(x) = 2x - 2 + 2e^{-x} = 2\left(x + e^{-x} - 1\right)$

$\psi'_2(\beta) = 2\left(\psi_1(\beta) - 1\right)$

Since $\psi_2(x)$ is a continuous and differentiable function $\forall x \in [0, x]$

$\psi_2(0) = 0, \psi_2(x) = x^2 - 2x - 2e^{-x} + 2$

Hence according to LMVT there exist at least one $\beta \in (0, x)$ such that

$$\left(\frac{\psi_2(x) - \psi_2(0)}{x}\right) = \psi'_2(\beta)$$

$$= \frac{\psi_2(x)}{x} = 2\left(\psi_1(\beta) - 1\right)$$

$$\psi_2(x) = 2x\left(\psi_1(\beta) - 1\right)$$

54. (d) $\psi'_1(x) = 1 - e^{-x} > 0$

$\psi_2(x) = (x-1)^2 + 1 - 2e^{-x} > 0$ for x=1

$$\because e^{-t} = 1 - t + \frac{t^2}{2} - \frac{t^3}{3} + \ldots$$

$$\sqrt{t}e^{-t} = \sqrt{t} - t^{3/2} + \frac{1}{2}t^{5/2} \ldots.$$

So, $\sqrt{t}e^{-t} < \sqrt{t} - t^{3/2} + \frac{1}{2}t^{5/2}$ for $t \in (0,1)$

$$\int_0^{x^2} \sqrt{t}e^{-t}\,dt < \int_0^{x^2} \left(\sqrt{t} - t^{3/2} + \frac{1}{2}t^{5/2}\right)dt$$

$$= \frac{2}{3}x^3 - \frac{2}{5}x^5 + \frac{1}{7}x^7$$

55. (214) Let A ={1,2, 3, ….,2000}

Let E_1 = 3m, $1 \le m \le 666$, $m \in N$

E_2 = 7k, $1 \le k \le 285, k \in N$

= $n(E_1 \cup E_2)$ = 666 + 285 - 95 = 856

= $P(E_1 \cup E_2)$ = 856 / 2000 = 500 β = 856 / 4 = 214

56. (4) M(P, Q) is mid-point of PQ

M(P, Q') is mid-point of PQ'

In a A PQQ' since M(P, Q) & M(P, Q') are mid-point of PQ & PQ' hence line joining M(P,Q), M(P,Q') is parallel to QQ' and half of it.

$\Rightarrow$ max distance = $\frac{1}{2}$ QQ' = $\frac{1}{2}$ (8) = 4

57. (182)

$$I = \int_0^{10} \left[\sqrt{\frac{10x}{x+1}}\right]dx$$

$$\left[\sqrt{\frac{10x}{x+1}}\right]=n\Rightarrow\frac{n^2}{10-n^2}\le x<\frac{(n+1)^2}{10-(n+1)^2}$$ where n $\in$ I

For n = 0, $0 \le x < 1/9$

n=1; $1/9 \le x < 2/3$

n = 2; $2/3 \le x < 9$, n = 3, $x \ge 9$

$$\Rightarrow I=\int_0^{1/9}0.dx+\int_{1/9}^{2/3}1.dx+\int_{2/3}^{9}2.dx+\int_9^{10}3.dx$$

$$=\left(\frac{2}{3}-\frac{1}{9}\right)+2\left(9-\frac{2}{3}\right)+3(10-9)=\frac{182}{9}=9I=182$$