1. (2.35 or 2.36)

Let $\tan\theta = \frac{\pi}{\sqrt{2}} \Rightarrow \frac{\pi}{4} \le \theta \le \frac{\pi}{2} \Rightarrow \pi = \sqrt{2}\tan\theta$

$$\frac{3}{2}\cos^{-1}\sqrt{\frac{2}{2+2\tan^2\theta}} + \frac{1}{4}\sin^{-1}\left(\frac{2\sqrt{2}\sqrt{2}\tan\theta}{2+2\tan^2\theta}\right) + \tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{2}\tan\theta}\right)$$

$$= \frac{3}{2}\cos^{-1}(\cos\theta) + \frac{1}{4}\sin^{-1}(\sin 2\theta) + \tan^{-1}(\cot\theta)$$

$$\frac{3}{2}\theta + \frac{1}{4}(\pi - 2\theta) + \frac{\pi}{2} - \theta = \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$$

2. (0.5)

$$\lim_{x\to\alpha^+} f(g(x)) = f\left(\lim_{x\to\alpha^+} g(x)\right)$$

Now $\lim_{x\to\alpha^+} g(x) = \lim_{x\to\alpha^+} \frac{2\ln\left(\sqrt{x}-\sqrt{\alpha}\right)}{\ln\left(e^{\sqrt{x}} - e^{\sqrt{\alpha}}\right)}\left(\frac{-\infty}{-\infty}\right)$

Apply D'L Hospital

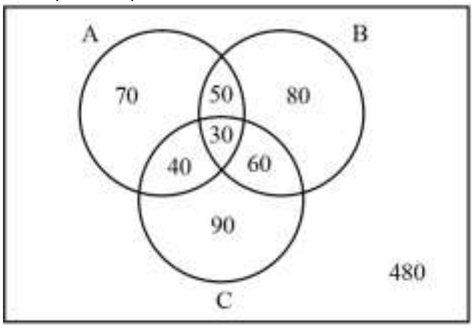
$$\lim_{x\to\alpha^+} \frac{2\left(e^{\sqrt{x}} - e^{\sqrt{\alpha}}\right)}{e^{\sqrt{x}}\left(\sqrt{x}-\sqrt{\alpha}\right)}$$

$$\lim_{x\to\alpha^+} \frac{2e^{\sqrt{\alpha}}\left(e^{\sqrt{x}-\sqrt{\alpha}} - 1\right)}{e^{\sqrt{x}}\left(\sqrt{x}-\sqrt{\alpha}\right)} = 2$$

Now $f(x) = \sin\frac{\pi x}{12}$ given

$$f(2) = \sin\frac{\pi(2)}{12} = \sin\frac{\pi}{6} = \frac{1}{2} = 0.5$$

3. (0.80)



Let A denote the persons having symptoms of fever B denote the persons having symptoms of cough C denote the persons having symptoms of breathing problem

Given that

n(a)=190n(b)=220n(c)=220

$n(A\cap B\cap C) = 30$

$n(A\cup B) = 300\ n(B\cup C) = 350\ n(C\cup A) = 340$

$n(A\cup B) = n(A) + b(B) - n(A\cap B)$

$\Rightarrow 330 = 190 + 220 - n(A\cap B) \Rightarrow n(A\cap B) = 80$

Similarly, $n(B\cap C) = 90$ and $n(C\cap A) = 70$

If we make the Venn diagram

$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(C\cap A) + n(A\cap B\cap C)$

$= 190 + 220 + 220 - 80 - 90 - 70 + 30 = 420$

$\therefore$ Number of persons having at most one symptom

=480+70+80+90=720

$\therefore$ Probability$=\frac{720}{900}=\frac{4}{5}=0.80$

4. (0.50)

$\frac{2+3z+4z^2}{2-3z+4z^2}$ is a real number

$\Rightarrow 1+\frac{6}{\frac{2}{z}-3+4z}\in R \Rightarrow 2z+\frac{1}{z}\in R \Rightarrow 2z+\frac{1}{z}=2\bar{z}+\frac{1}{\bar{z}}$

$\Rightarrow \left(2z\bar{z}-1\right)\left(z-\bar{z}\right)=0$

$\Rightarrow |z|^2=\frac{1}{2}=0.50$

5. (4)

Let $z=x+iy$

$x-iy-x^2+y^2-2ixy=i(x-iy+x^2-y^2+2ixy)$

$(x-x^2+y^2)-i(y+2xy)=(y-2xy)+i(x+x^2-y^2)$

$\Rightarrow \quad x-x^2+y^2=y-2xy \quad \ldots(1)$

$x+x^2-y^2=-y-2xy \quad \ldots(2)$

$(1)+(2) \quad 2x=-4xy$

$\Rightarrow x=-2xy \Rightarrow x(1+2y)=0 \Rightarrow x=0$ or $y=-\frac{1}{2}$

Put $x=0$ in (1) or (2) we get

$y^2=y \Rightarrow y=0, 1$

$\therefore$ 2 complex numbers are possible $0+0i$ and $0+i$

put $y=-\frac{1}{2}$ in (1) or (2) $\quad x-x^2+\frac{1}{4}=-\frac{1}{2}+x$

$\Rightarrow x^2=\frac{3}{4} \quad \Rightarrow x=\pm\frac{\sqrt{3}}{2}$

$\therefore \frac{\sqrt{3}}{2}-\frac{i}{2}$ and $-\frac{\sqrt{3}}{2}-\frac{i}{2}$ are possible

$\therefore$ 4 solutions are possible.

6. (18900)

$A_{51}-A_{50}=1000$

$l_{51}w_{51}-l_{50}w_{50}=100$

$(l_1+50d_1)(w_1+50d_2)-(l_1+49d_1)(w_1+49d_2)=1000$

$l_1w_1+50l_1d_2+50d_1w_1+2500d_1d_2-l_1w_1-49l_1d_2-49d_1w_1-240d_1d_2=1000$

$\Rightarrow l_1d_2+d_1w_1+99d_1d_2=1000$

$\Rightarrow l_1d_2+d_1w_1=10$

$A_{100}-A_{90}=(l_{100}w_{100})-(l_{90}w_{90})$

$=(l_1+99d_1)(w_1+99d_2)-(l_1+89d_1)(w_1+89d_2)$

$=10d_1w_1+10l_1d_2+1880d_1d_2$

$=10\times 10+18800=18900$

7. (569) Number of numbers whose digits are

2   0

—   —   —   —

$6 \times 6 - 7 = 209$

2

—   —   —   —

$5 \times 6 \times 6 = 180$

3   —   —   —

$6 \times 6 \times 6 = 216$

4   —   —   —

$4 \times 6 \times 6 = 144$

Total number are (36-7)+(5x6x6)+(6x6x6)+4x6x6=596

8. (0.83 or 0.84)

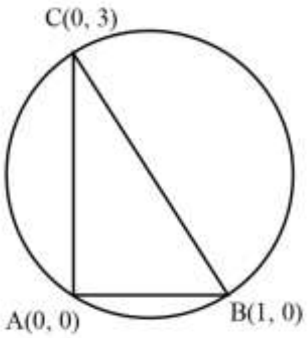
Let A be (0,0), B(1,0) and C(0,3)

$\therefore$ AB lies on x-axis and AC lies on y-axis

$\therefore$ equation of circle touching both x and y-axis is of the form

$$(x-h)^2 + (y-h) = h^2 \ \ (\because h = k = r)$$

It touches the circle $\left(x-\frac{1}{2}\right)^2 + \left(y-\frac{3}{2}\right)^2 = \frac{5}{2}$



$$\therefore c_1c_2 = |r_1 - r_2|$$

$$\sqrt{\left(h-\frac{1}{2}\right)^2 + \left(h-\frac{3}{2}\right)^2} = \left|h - \frac{\sqrt{5}}{\sqrt{2}}\right|$$

$$\Rightarrow h^2 + \frac{1}{4} - h + h^2 + \frac{9}{4} - 3h = h^2 + \frac{5}{2} - \sqrt{10}h$$

$$\Rightarrow h^2 + \left(\sqrt{10} - 4\right)h = 0 \Rightarrow h = 4 - \sqrt{10}$$

$$\therefore r = 4 - \sqrt{10} = 0.8377$$

9. (c, d)

$$\int_1^e \frac{(\ln x)^{1/2}}{x\left(a - (\ln x)^{3/2}\right)^2} dx = 1$$

$$a - (\ln x)^{3/2} = t$$

$$-\frac{3}{2}(\ln x)^{1/2} \times \frac{1}{x} dt = dt$$

$$\int_a^{a-1} \frac{-\frac{2}{3} dt}{t^2} = 1$$

$$\left[-\frac{2}{3} \times -\frac{1}{t}\right]^{a-1} = 1$$

$$\frac{2}{3}\left[\frac{1}{a-1}-\frac{1}{a}\right]=1$$

$$\frac{a-a+1}{a(a-1)}=\frac{3}{2}$$

$$3\left(a^2-a\right)=2$$

$$3a^2-3a-2=0$$

$$a=\frac{3\pm\sqrt{9+24}}{6}=\frac{3\pm\sqrt{33}}{6}$$

10. (b, c)

$$\sum_{n=1}^{n}\left(T_{n+1}-T_n\right)=\sum a_n$$

$$\Rightarrow T_{n+1}-T_1=\sum a_n=\frac{n}{2}\left[2\times 7+(n-1)8\right]$$

$$T_{n+1}=n(4n+3)+T_1$$

$$T_{n+1}=4n^2+3n+3$$

(a) $T_{20}=4\times 19^2+3\times 9+3$

=1444+27+3=1474

(b) $\sum_{k=0}^{19}T_{n+1}=\sum_{k=0}^{19}k(4k+3)+3$

$$=\sum_{k=0}^{19}\left(4k^2+3k+3\right)=10510$$

(c) $T_{30}=29(4\times 29+3)+3=3454$

(d) $\sum_{k=1}^{30}T_k=\sum_{n=0}^{29}T_{n+1}=\sum_{n=0}^{29}n(4n+3)+3$

$$=4\times\left(\frac{29\times 30\times 59}{6}\right)+\frac{3(29\times 30)}{2}+90=35615$$

11. (a,b,d) The line should be either coincident of P_1 or on P_2 or intersect on P_1 and P_2 on different points.

(d) $\frac{x}{1}=\frac{y-4}{-2}=\frac{z}{3}=\lambda$

$\Rightarrow(\lambda,-2\lambda+4,3\lambda)$ lie on P_2

(a) $\frac{x-1}{0}=\frac{y-1}{0}=\frac{z-1}{5}$ intersects P_1 and P_2 on different points

(b) $\frac{x-6}{5}=\frac{y}{2}=\frac{z}{3}$ also intersects P_1 and P_2 on different points

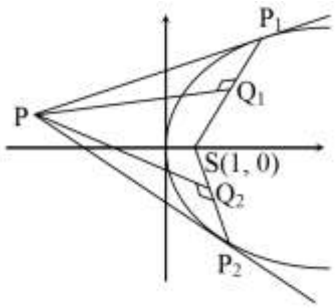
12. (a, b, c) Clearly plane is given by x+y+z=1

Using mirror image formula

$$\Rightarrow\frac{\alpha-10}{1}=\frac{\beta-15}{1}=\frac{\gamma-20}{1}=\frac{-2(10+15+20-1)}{1^2+1^2+1^2}=-\frac{88}{3}$$

$$\Rightarrow\alpha=-\frac{58}{3};\beta=-\frac{43}{3}\text{ and }\gamma=-\frac{28}{3}$$

13. (b, c, d)



Let P_1 and P_2 be $\left(t_1^2, 2t_1\right)$ and $\left(t_2^2, 2t_2\right)$

$\Rightarrow P \equiv (t_1t_2, t_1 + t_2) \equiv (-2, 1)$

$\Rightarrow t_1 = 2, t_2 = -1$ or $t_1 = -1, t_2 = 2$

$\Rightarrow P_1(4, 4)$ and $P_2(1, -2)$

Slope of $SP_1 = \frac{4}{3}$ slope of $SP_2 = \infty$

Equation of SP_1

$$y - 0 = \frac{4}{3}(x - 1) \Rightarrow 4x - 3y - 4 = 0$$

Equation of PQ_1

$$y - 1 = -\frac{4}{3}(x + 2) \Rightarrow 3x + 4y + 2 = 0 \therefore Q_1\left(\frac{2}{5}, -\frac{4}{5}\right)$$

Equation of SP_2 $\quad x = 1$

Equation of PQ_2 $\quad y = 1 \therefore Q_2(1, 1)$

$\therefore SQ_1 = 1, Q_1Q_2 = \frac{3\sqrt{10}}{5}$

$$PQ_1 = \sqrt{\left(-2-\frac{2}{5}\right)^2 + \left(1+\frac{4}{5}\right)^2} = \sqrt{\frac{144}{25} + \frac{81}{25}} = 3$$

and $SQ_2 = \sqrt{(1-1)^2 + (1-0)^2} = 1$

14. (a, c)

$$f(\theta) = \frac{1}{2}\begin{vmatrix} 0 & 0 & 2 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} + \begin{vmatrix} \sin\pi & \cos\left(\theta+\frac{\pi}{4}\right) & \tan\left(\theta-\frac{\pi}{4}\right) \\ \sin\left(\theta-\frac{\pi}{4}\right) & -\cos\frac{\pi}{2} & \log_e\left(\frac{4}{\pi}\right) \\ \cot\left(\theta+\frac{\pi}{4}\right) & \log_e\left(\frac{\pi}{4}\right) & \tan\pi \end{vmatrix}$$

As second determinants is skew symmetric $\therefore$ its value is 0.

$\Rightarrow f(\theta) = \left(1 + \sin^2\theta\right)$

$\Rightarrow g(\theta) = |\sin\theta| + |\cos\theta| \in \left[1, \sqrt{2}\right]$

$\Rightarrow p(x) = a(x-1)\left(x-\sqrt{2}\right)$ as $p(2) = 2 - \sqrt{2} \Rightarrow a = 1$

$\Rightarrow p(x) = (x-1)\left(x-\sqrt{2}\right)$

and hence $p\left(\frac{3+\sqrt{2}}{4}\right) < 0$ and $p\left(\frac{5\sqrt{2}-1}{4}\right) > 0$

15. (b)

(I) $\cos x + \sin x = 1$

$\Rightarrow \sqrt{2}\cos\left(x-\frac{\pi}{4}\right) = 1$

$\Rightarrow \cos\left(x-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\Rightarrow x-\frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

for $n = 0$, $x = \frac{\pi}{2}, 0$

for $n = 1, 2, 3, \ldots$ no solution in $\left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right]$ $\therefore$ I $\rightarrow$ P

(II) $\tan 3x = \frac{1}{\sqrt{3}} \Rightarrow 3x = n\pi + \frac{\pi}{6} \Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{18}$

for $n = 0$ $x = \frac{\pi}{18}$ for $n = -1$, $x = -\frac{5\pi}{18}$

for $n = 1, 2, 3, \ldots$ no solution $\therefore$ II $\rightarrow$ P

(III) $\cos 2x = \frac{\sqrt{3}}{2} \Rightarrow 2x = 2n\pi \pm \frac{\pi}{6} \Rightarrow x = n\pi \pm \frac{\pi}{12}$

for $n = -1$ $x = -\frac{13\pi}{12}, -\frac{11\pi}{12}$ $\therefore$ III $\rightarrow$ T

(IV) $\cos\left(\frac{\pi}{4}+x\right) = -\frac{1}{\sqrt{2}} \Rightarrow x+\frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4}$

for $n = 0$ $x = \frac{\pi}{2}, -\pi$

for $n = 1$ $x = \pi$ for $n = -1$ $x = -\frac{3\pi}{2}$ $\therefore$ IV $\rightarrow$ R

16. (a)

Let A_1 ; P_1 won the round $\Rightarrow P(A_1) = \frac{{}^6C_2}{6^2} = \frac{6\times 5}{2\times 6\times 6}$

$P(A_1) = \frac{5}{12}$

A_2 ; P_2 won the round $\Rightarrow P(A_2) = \frac{5}{12}$

D ; round ends in draw $\Rightarrow P(D) = \frac{6}{6^2} = \frac{1}{6}$

(i) $P(X_2 \geq Y_2) = P(A_1 \cap A_1) + 2P(A_1 \cap A_1) + P(D \cap D) + 2P(A_1 \cap D)$

$= \frac{5}{12}\cdot\frac{5}{12} + 2\cdot\frac{5}{12}\cdot\frac{5}{12} + \frac{1}{6}\cdot\frac{1}{6} + 2\cdot\frac{5}{12}\cdot\frac{1}{6} = \frac{11}{16}$

$\Rightarrow (1) \rightarrow (Q)$

(ii) $P(X_2 > Y_2) = P(A_1 \cap A_1) + 2P(A_1 \cap D)$

$= \frac{5\times 5}{12\times 12} + 2\cdot\frac{5}{12}\cdot\frac{1}{6} = \frac{25}{144} + \frac{5}{36} = \frac{45}{144} = \frac{5}{16}$

$\Rightarrow (ii) \rightarrow R$

(iii) $P(X_3 = Y_3) = 3P(A_1 \cap D \cap A_2) + P(D \cap D \cap D)$

$$= 6 \times \frac{5}{12} \times \frac{5}{12} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{25}{144} + \frac{1}{216} = \frac{75+2}{432} = \frac{77}{432}$$

$\Rightarrow$ (iii) $\rightarrow$ T

(iv) $P(X_3 > Y_3) = P(A_1 \cap A_1 \cap A_1) + 3P(A_1 \cap A_1 \cap A_2) + 3P((A_1 \cap A_1 \cap D) + 3P(A_1 \cap D \cap D)$

$$= \frac{5}{12} \times \frac{5}{12} \times \frac{5}{12} + 3 \times \frac{5}{12} \times \frac{5}{12} \times \frac{5}{12} + 3 \times \frac{5}{12} \times \frac{5}{12} \times \frac{1}{6} + 3 \times \frac{5}{12} \times \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{710}{12^3} = \frac{355}{144 \times 6} = \frac{355}{864}$$

$\therefore$ IV $\rightarrow$ S

17. (b)

x+y+z=1 ... (1)

10x+100y+1000z=0 ... (2)

$\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 0$ ... (3)

$\frac{1}{p} = A + 9d; \frac{1}{q} = A + 99d; \frac{1}{r} = A + 999d$

$\Rightarrow$ From equation (2) and (3), we get (A-d)x+(A-d)y+(A-d)z=0

$\Rightarrow$ If $A \neq d$, then no solution

If $A = d \Rightarrow \frac{p}{q} = 10, \frac{q}{r} = 10, \frac{p}{r} = 100$ then the equations have infinite solutions

Now, equation (2) and (3) both are same

So, (1) and (2) both equation are satisfying $x = 0, y = \frac{10}{9}, z = \frac{-1}{9}$

18. (c)

Equation of auxiliary circle $x^2 + y^2 = 4$

$\therefore$ Let F be $(2\cos\theta, 2\sin\theta)$

$\therefore$ E is $\left(2\cos\theta, \sqrt{3}\sin\theta\right)$

Equation of tangent at E, $\frac{x\cos\theta}{2} + \frac{y\sin\theta}{\sqrt{3}} = 1$

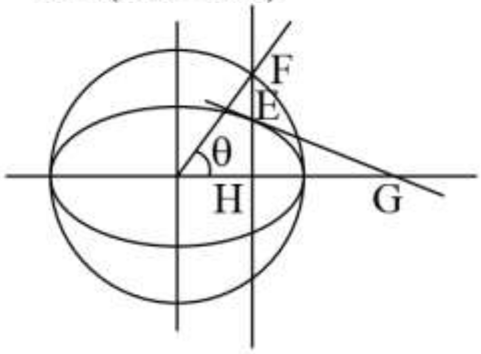
It cuts x-axis at $(2\sec\theta, 0)$

$\therefore$ G is $(2\sec\theta, 0)$

H is $(2\cos\theta, 0)$ and $F(2\cos\theta, 2\sin\theta)$

$\therefore$ Area of ΔFGH is $\frac{1}{2} \times 2\sin\theta(2\sec\theta - 2\cos\theta)$

$= 2\sin\theta(\sec\theta - \cos\theta)$



If $\theta=\frac{\pi}{4}$, area $=2\times\frac{1}{\sqrt{2}}\left(\sqrt{2}-\frac{1}{\sqrt{2}}\right)=1$

If $\theta=\frac{\pi}{3}$, area $=2\times\frac{\sqrt{3}}{2}\left(2-\frac{1}{2}\right)=\frac{3\sqrt{3}}{2}$

If $\theta=\frac{\pi}{6}$, area $=2\times\frac{1}{2}\left(\frac{2}{\sqrt{3}}-\frac{\sqrt{3}}{2}\right)=\frac{1}{2\sqrt{3}}$

If $\theta=\frac{\pi}{12}$, area $=2\times\frac{\sqrt{3}-1}{2\sqrt{2}}\left(\frac{2\sqrt{2}}{\sqrt{3}+1}-\frac{\sqrt{3}+1}{2\sqrt{2}}\right)=\left(\frac{\sqrt{3}-1}{8}\right)^4$.

19. (2.30)

$M_B=2M_A \qquad \Rightarrow \rho_B=2\rho_A$

Now after the mass transfer,

$M'_A=\rho_A\frac{4}{3}\times\frac{R^3}{8}$ and $M'_B=G\frac{4}{3}\pi\left(2\rho_A R^3+\frac{7}{8}\rho_A R^3\right)$

and outer radius $R'=\frac{(15)^{1/3}}{2}R$

So, $v_A=\sqrt{\frac{2GM'_A}{R}}=\sqrt{\frac{2}{3}G\rho_A\pi R^2}$

$$v_B=\sqrt{\frac{2G(4/3)\pi\left(2\rho_A R^3+\frac{7}{8}\rho_A R^3\right)}{R'}}$$

$$\frac{v_B}{v_A}=\sqrt{\frac{23}{(15)^{1/3}}}=\sqrt{\frac{(2.3)(10)}{(15)^{1/3}}}$$

So, n=2.30

20. (2.32)

${}^{14}_{7}N+{}^{4}_{2}He\rightarrow{}^{1}_{1}H+{}^{19}_{8}O$

Q=[16.006+4.003-1.008-19.003]x930

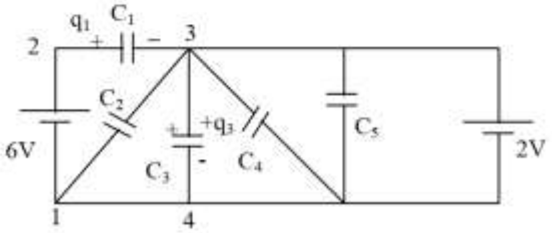
=-1.86MeV

$$E_{th}=\left(1+\frac{m}{M}\right)|Q|\approx\left(1+\frac{4}{16}\right)(1.86)=2.32MeV$$

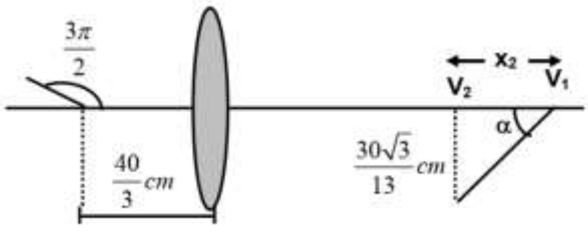
21. (8.00)

In the loop 1234, $+6-\frac{48}{12}-\frac{q_3}{4}=0, q_1=12(6-2)=48\mu C$

$q_3=8\mu C$



22. (6.00)



$u_1 = -\frac{40}{3}$

f=10

$\frac{1}{V_1} + \frac{1 \times 3}{40} = \frac{1}{10}$

$V_1 = 40$

$u_2 = -\frac{43}{3}$

$\frac{1}{V_2} = \frac{1}{10} - \frac{3}{43} = \frac{43-30}{430}$

$V_2 = \frac{430}{13}$

$x_2 = V_1 - V_2 = 40 - \frac{430}{13} = \frac{90}{13}$

$\tan\alpha = \frac{30\sqrt{3}}{13 \times x} = \frac{30\sqrt{3} \times 13}{13 \times 90} = \frac{1}{\sqrt{3}}$

$\alpha = \frac{\pi}{6} = \frac{\pi}{n}$

n=6

23. (0.52)

$\theta = \frac{1}{2}\alpha t^2$

$= \frac{1}{2} \times \frac{2}{3}\pi = \frac{\pi}{3} = 60^0$

$v_{cm} = \alpha t$

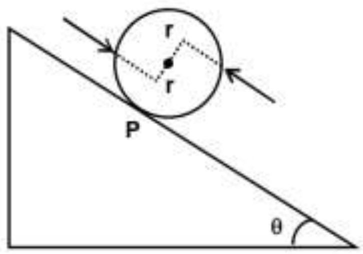
Net velocity of point P is $V = \alpha t$ at an angle 60^0 with horizontal

$u_y = \alpha t \sin 60^0$

$y_{\max} = \frac{1}{2} + \frac{u_y^2}{2g} = \frac{1}{2} + \frac{\alpha^2 t^2}{20}\frac{3}{4} = \frac{1}{2} + \frac{\pi}{60}$

$x = 0.52$

24. (2.85)



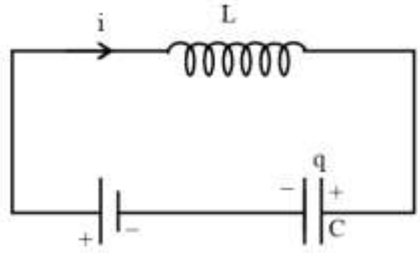
$mg\sin\theta \times R + 1 \times \frac{R}{2} - 1 \times \frac{3R}{2} = I_P \alpha$

$\frac{10}{2} \times 1 + \frac{1}{2} - \frac{3}{2} = \frac{7}{5} \times mR^2\alpha$

$5 - 1 = \frac{7}{3}\alpha$

$a_{cm} = R\alpha = \frac{20}{7}$

25. (4.00)



$V=\beta A$

$\phi=(B_0+\beta t)A$

$|\varepsilon_{ind}|=\dfrac{d\phi}{dt}=\beta A$

Applying kVL in the equivalent circuit diagram of the loop.

$\beta A-\dfrac{Ldi}{dt}-\dfrac{q}{C}=0$ ...(i)

Also, $i=\dfrac{dq}{dt}$ ...(ii)

From (i) and (ii) $L\dfrac{d^2i}{dt^2}=-\dfrac{i}{c}$

$i=i_m\sin\omega t\ \ \omega=\dfrac{1}{\sqrt{LC}}$

$\int_0^q dq=i_m\int_0^t \sin(\omega t)dt$

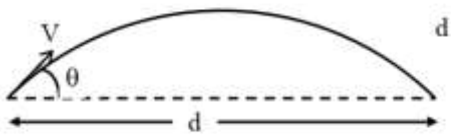
$q=\dfrac{i_m}{\omega}(1-\cos\omega t)$

When $i=i_m\Rightarrow\sin\omega t=1\Rightarrow\cos\omega t=0$

$q=\dfrac{i_m}{\omega}=\beta AC$

$I_m=\omega\beta AC=\beta A\sqrt{\dfrac{C}{L}}=4mA$

26. (0.95)



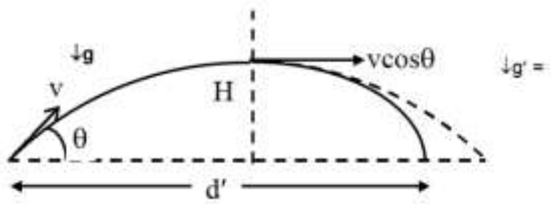
$d=\dfrac{2V^2\sin\theta\cos\theta}{g}$

$$d'=\frac{d}{2}+\sqrt{\frac{2H}{g'}}v\cos\theta=\frac{d}{2}+\sqrt{\frac{2}{g'}\frac{v^2\sin^2\theta}{2g}}v\cos\theta$$

$$d'=\frac{d}{2}+\frac{V^2\sin\theta\cos\theta}{\sqrt{g\dfrac{g}{0.81}}}=\frac{d}{2}+\frac{d}{2}\left(\frac{9}{10}\right)=\frac{19d}{20}$$

$d'=0.95d$

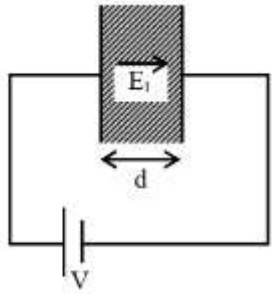
$n=0.95$



27. (b) Refer to figure (a)

$V=E_1d$

$$C_1 = \frac{k\varepsilon_0 A}{d}$$



Figure(a)

Refer to figure (b)

$$V = E_3\frac{d}{2} + E_2\frac{d}{2} + E_3\frac{d}{2}$$

Also $E_3 = E_2 K$

$$V = E_2 d + KE_2 d$$

$$V = (K+1)E_2 d$$

$$C_2 = \frac{\varepsilon_0 A}{d + \frac{d}{K}} = \frac{K}{K+1}\frac{\varepsilon_0 A}{d}$$

Now $\frac{E_1}{E_2} = K+1$

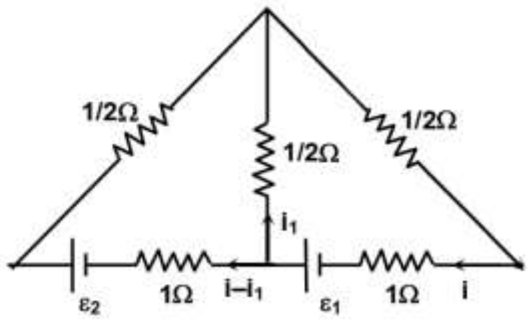
$$\frac{C_1}{C_2} = (K+1)$$

$$W_{ext} + W_{battery} = \Delta U$$

$$W_{ext} + (C_2 - C_1)V^2 = (C_2 - C_1)\frac{V^2}{2}$$

$$W_{ext} + (C_2 - C_1)V^2 = (C_2 - C_1)\frac{V^2}{2}$$

28. (a, b, c, d)



$$-\left(i-i_1\right)+6-\frac{1}{2}\left(i-i_1\right)+\frac{1}{2}i_1=0$$

$$-\frac{3}{2}\left(i-i_1\right)+\frac{i_1}{2}+6=0$$

$$-\frac{3}{2}i+\frac{3}{2}i_1+\frac{i_2}{2}+6=0$$

$$-\frac{3}{2}i+2i_1+6=0$$

$$-\frac{i_1}{2}-\frac{i}{2}-i+12=0$$

$$-\frac{i_1}{2}-\frac{3}{2}i+12=0$$

$$\frac{5}{2}i_1-6=0$$

$$i_1=\frac{6\times 2}{5}=\frac{12}{5}=2.4$$

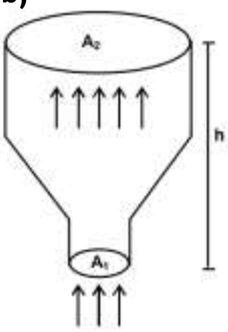
$$-1.2-1.5i+12=0$$

$$1.5i=10.8$$

$$i=\frac{10.8}{1.5}$$

$$i=7.2$$

29. (b)



$P^{1-\gamma}T^{\gamma}=$ Const.

$P_2=150\text{Pa}$

$$\frac{dm}{dt}=\rho_1A_1v_1$$

$v_1=40\text{m/s}$

$$\rho=\frac{PM}{RT}$$

$\rho_2=0.1\text{Kg/m}^3$

$\rho_1A_1v_1=\rho_2A_2v_2$

$v_2=20\text{m/s}$

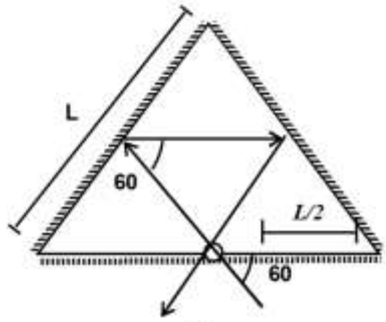
From work energy theorem,

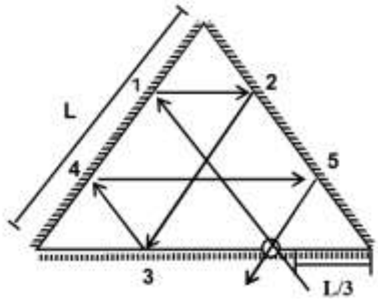
$$P_1A_1v_1dt-\rho_2A_2v_2dt+\rho_1A_1v_1dtg(0)-\rho_2A_2v_2dtg(h)$$

$$=\frac{1}{2}\rho_2A_2v_2dtv_2^2-\frac{1}{2}\rho_1A_1v_1dtv_1^2+\frac{1}{\gamma-1}\left[P_2A_2v_2dt-P_1A_1v_1dt\right]$$

30. (a, b)

For option ‘A’ there will be normal incidence and ray retrace its path. For option ‘B’





31. (a, b, c)
When x=q, electric field at O is zero.
When x=-q, electric field at O is,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{\left(\sqrt{3}a\right)^2} = \frac{q}{6\pi\varepsilon_0 a^2}$$

When x=2q,potential at O is,

$$V = \frac{7q}{4\sqrt{3}\pi\varepsilon_0 a}$$

When x=-3q, potential at O is,

$$V = \frac{2q}{4\sqrt{3}\pi\varepsilon_0 a}$$

32. (a, b, d)

33. None The magnitude of n mentioned in List-I of the equation is not 1, it is $\frac{1}{\sqrt{2}}$

34. (c) ω is same, therefore angle between velocity vectors remains same

$$V_{rel} = \sqrt{1^2+1^2} = \sqrt{2}m/s$$

$$\Rightarrow \vec{v}_1 = \frac{5\pi}{2}\hat{i} + \frac{5\pi}{3}j$$

$$\vec{v}_1 = -\frac{5\pi}{2}\hat{i} + \left(\frac{5\pi}{3}+1\right)j$$

$$v_{rel} = \left|\vec{v}_2 - \vec{v}_1\right| = \sqrt{25\pi^2+1}$$

35. (c)

$$W = P\Delta V = 0.1kJ$$

$$Q = mL = 2.25kJ$$

$$\Delta U = Q - W = 2.15kJ$$

$$\Rightarrow \frac{V}{500} = \frac{3V}{T} \Rightarrow T = 1500$$

$$\Delta U = nC_v\Delta T = 4kJ$$

$$\Rightarrow W = \frac{PV - (32P)(V/8)}{1-5/3} = 3 \quad \left[PV = P'(V/8)^{\gamma}; P' = 32P\right]$$

$\Rightarrow \frac{\Delta U}{\Delta Q}=\frac{nC_v\Delta T}{nC_P\Delta T}=\frac{1}{v}$ (f=6-vibration included)

$\Delta U = 9\times(3/4)\approx 7$

36. (a) $\frac{1}{v}=\frac{1}{f}+\frac{1}{u}(concave\,lens)$

$\Rightarrow v=-10$

$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}(convex\,lens)$

$\frac{1}{v}=\frac{1}{10}+\frac{1}{-15}$

$\Rightarrow v=+30$

37. $\Delta T = 312.8-298=14.8$

Molar heat capacity of calorimeter = $20kJK^{-1}$

Heat released by combustion of 2 moles of Hg(g)

=-20x14.8

=-296.kJ

$\Delta U_{combustion} = -\frac{296}{2} = -148 kJmol^{-1}$

$Hg(g)+\frac{1}{2}O_2(g) \longrightarrow HgO(s)$

$\Delta n_g = 0-\left(1+\frac{1}{2}\right)=-\frac{3}{2}$

$\Delta H_{combustion} = \Delta U_{combustion} + \Delta n_g RT$

$= -148+\left(-\frac{3}{2}\right)\times 8.3\times 10^{-3}\times 298$

. $= -148-3.710$

$= -151.710 kJmol^{-1}$

$Hg^{-}(\ell)+\frac{1}{2}O_2(g) \longrightarrow HgO(s)$

$\Delta H^\circ_{combustion} = \Delta H^\circ_f(HgO) - \Delta H_{Hg^\circ_{g(\ell)}\to Hg_{g(g)}} + \frac{1}{2}\Delta H^\circ_f O_2$

$-151.710 = \Delta H^\circ_f(HgO) - 61.32 + 0$

$\Delta H^\circ_f(HgO) = -151.710 + 61.32 = 90.39$

$\Delta H^\circ_f = 90.39$

38.(i) $MnO_4^-(aq)+8H^+ + 7e^- \longrightarrow Mn(s)+4H_2O(\ell); \Delta G^\circ_1 = -7\times F\times E^\circ_1$

(ii) $MnO_4^- + 4H^+ + 3e^- \longrightarrow MnO_2(s)+2H_2O(\ell); \Delta G^\circ_2 = -3\times F\times 1.68$

(iii) $MnO_2(s)+4H^+ + 2e^- \longrightarrow Mn^{2+}(aq)+2H_2O(\ell); \Delta G^\circ_3 = -2\times F\times 1.21$

(iv) $Mn^{2+}(aq)+2e^- \longrightarrow Mn(s); \Delta G^\circ_4 = -2\times F\times(-1.03)$

Now,

(i) = (ii) + (iii) + (iv)

$\Delta G^\circ_1 = \Delta G^\circ_2 + \Delta G^\circ_3 + \Delta G^\circ_4$

$-7\times F\times E^\circ_1 = -3\times F\times 1.68 - -2\times F\times 1.21 + 2\times F\times 1.03$

$-7E^\circ_1 = 3\times 1.68 + 2\times 1.21 - 2\times 1.03$

$E^\circ_1 = \frac{3\times 1.68+2\times 1.21-2\times 1.03}{7}$

$E^\circ_1 = \frac{5.04+2.42-2.06}{7}$

$= 0.7714$

$= \mathbf{0.77V}$

39.

	H_2CO_3	+ $NaOH \longrightarrow$	$NaHCO_3$	+ H_2O
Initial mol	0.01	0.01	0.01	
After neutralization	0	0	$0.01+0.01$ $=0.02$	

Now, mixture contains 0.01 mole Na_2CO_3 and 0.02 mol of $NaHCO_3$ so it is a buffer.

$$pH = pK_{a_2} + \log\frac{[CO_3^{2-}]}{[HCO_3^-]}$$
$$pH = 10.32 + \log\frac{0.01}{0.02}$$
$$= 10.32 + \log\frac{1}{2}$$
$$= 10.32 - 0.30$$
$$= 10.02$$

40.

$$2Cu(NO_3)_2 + 4KI \longrightarrow 2CuI\downarrow + I_2 + 4KNO_3$$
$$I_2 + KI \longrightarrow KI_3$$
$$2Cu(NO_3)_2 + 5KI \longrightarrow 2CuI\downarrow + KI_3 + 4KNO_3$$
$$\text{Mole } = \frac{3.74}{187} = 0.02 \quad \frac{0.02}{2} = 0.01$$
$$KI_3 + H_2S \longrightarrow S\downarrow + 2HI + KI$$

0.01 mol

0.01 mol

(X)

$$\text{Mass of } 'S' = 0.01 \times 32 = 0.32$$

41. Mole of $P_4 = \frac{1.24}{124} = 0.01$

$$P_4 = 3NaOH + 3H_2O \rightarrow 3NaH_2PO_2 + \underset{(Q)}{PH_3}$$

mole 0.01 0.01

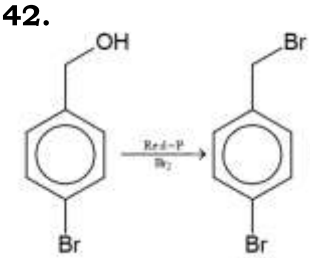
$$2\underset{(Q)}{PH_3} + 3CuSO_4 \rightarrow Cu_3P_2 + 3H_2SO_4$$

mole $0.01 \quad \frac{3}{2}\times 0.01$

$\therefore$ Mole of $CuSO_4$ required $= 0.01\times\frac{3}{2}$

Weight of $CuSO_4 = 0.01\times\frac{3}{2}\times 159 = 2.385g$

42.



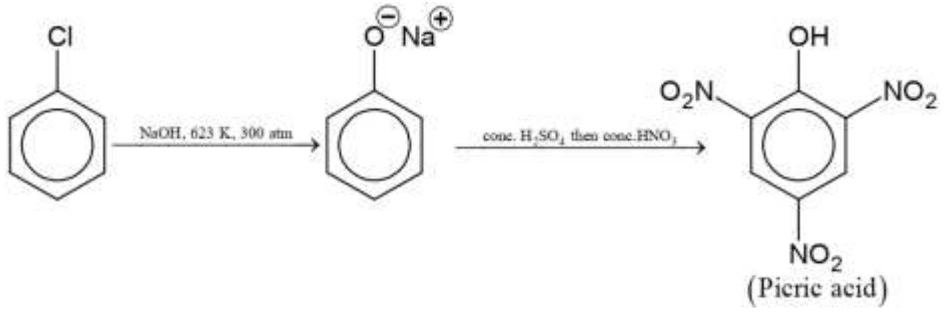
$(C_7H_6Br_2)$

(R)

Molecular weight of (R)=7x12+1x6+80x2=250g/mol
Mole of AgBr formed = 2xmole of (R)

$$=2\times\frac{1}{250}\times 188=8\times 10^{-3}\times 188=1.504g$$

43.



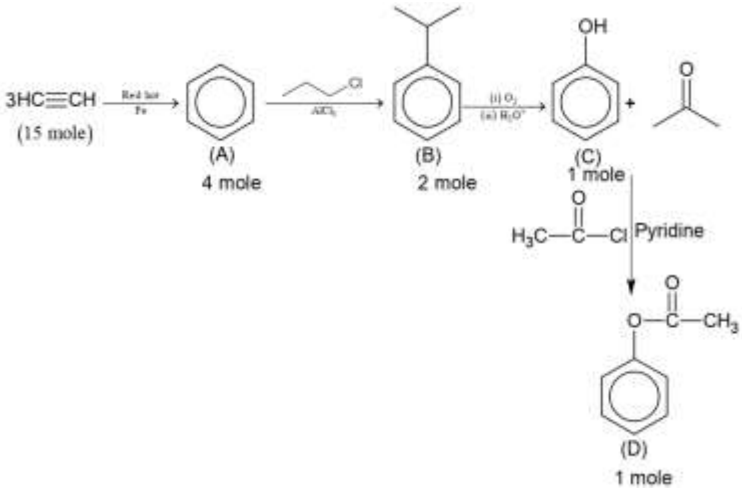
$C_6H_3O_7N_3$

Molecular weight = 12x6+1x3+16x7+14x3

=72+3+112+42=229gm/mole

$\therefore$ Weight percentage of $H=\frac{3}{229}\times 100=1.3100\%$

44.



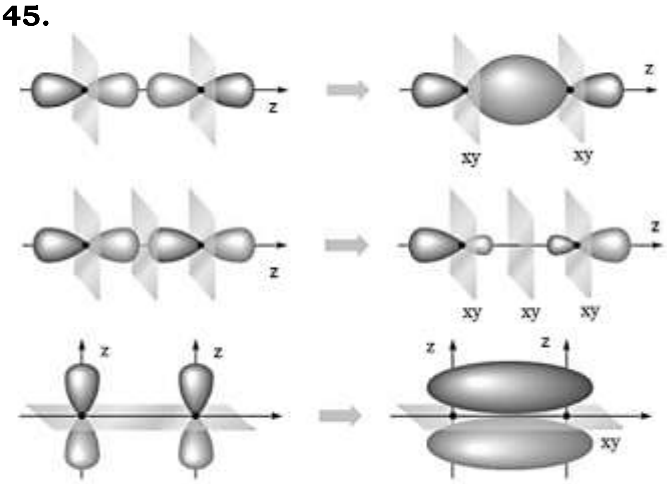
Molecular weighty = $C_8H_8O_2$

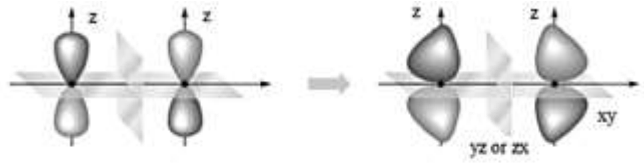
=12x8+1x8+16x2=96+8+32=136

Moles of product (d) formed

$=\frac{15}{3}\times 0.8\times 0.5\times 0.5=\frac{3}{3}=1$ $\therefore$ Weight of product (d) = 1x136=136g

45.





46. $\rightarrow$Chemisorption is unimolecular layer and exothermic.
$\rightarrow$ The enthalpy change in physiorption is 20-40kJ/mol
$\rightarrow$Physicorption decreases with increase of temperature

47. $2NaAlO_2 + CO_2 + 3H_2O \rightarrow 2Al(OH)_3 \downarrow + Na_2CO_3$

$2Al(OH)_3 \xrightarrow{\Delta} Al_2O_3 + 3H_2O$

$\rightarrow Al_2O_3 + 2NaOH \rightarrow 2NaAlO_2 + H_2O$

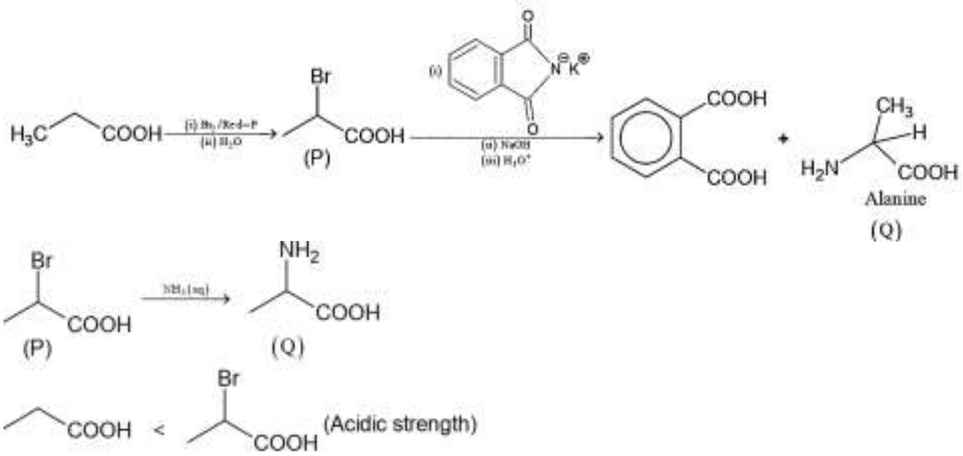
$\rightarrow$During electrolysis of alumina, cryolite (Na_3AlF_6) and fluorspar (CaF_2) are added to decrease the melting point of alumina.

(CaF_2) Al metal is obtained at cathode while CO_2 releases at anode.

48. $3PbS + 8HNO_3 \rightarrow 3Pb(NO_3)_2 + 3S + 2NO \uparrow + 4H_2O$

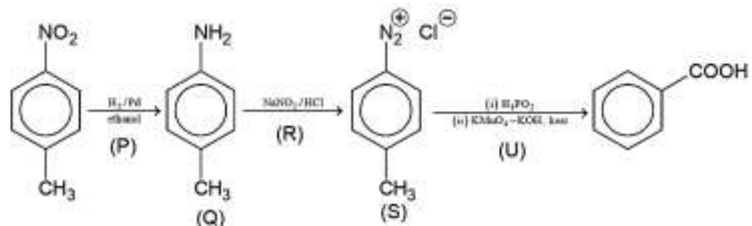
$NO \rightarrow$ paramagnetic, neutral oxide and colourless gas

49.

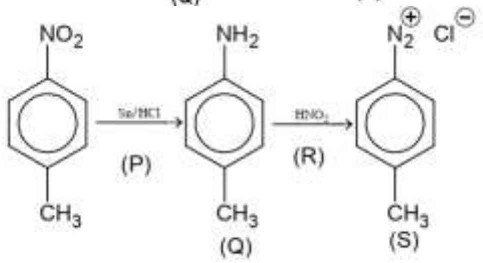


50.

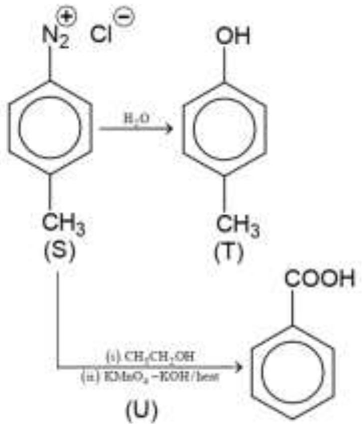
(A)



(B)

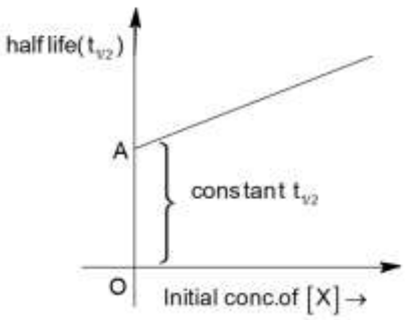


(C)



51. $I \rightarrow Rate = \frac{k[X]}{X_s + [X]}$

If [X] is low, then it follows first order kinetic ie. $t_{1/2}$ is constant (OA part in graph)
If [X] is high, then it follows zero order kinetics. So half-life ($t_{1/2}$) varies linearly with [X] as shown in graph.



$I \rightarrow P$

$II \rightarrow Rate = \frac{k[X]}{X_s + [X]}$

If [X] is less than X_s, then reaction follows I^{st} order kinetics as shown by the graph Q and T
$II \rightarrow Q, T$

$III \rightarrow Rate = \frac{k[X]}{X_s + [X]}$

If [X] >> X_s, then reaction follows zero order kinetics, as show by the graph (S)
$III \rightarrow S$

$IV \rightarrow Rate = \frac{k[X]^2}{X_s + [X]}$

If [X] >> [X_s], then reaction follows first order kinetics, as show by the graph (Q), (T)
$IV \rightarrow Q, T$

52. $Mg(HCO_3)_2 + 2Ca(OH)_2 \rightarrow Mg(OH)_2\downarrow + 2CaCO_3\downarrow + 2H_2O$

$BaO_2 + H_2SO_4 \rightarrow BaSO_4\downarrow + H_2O_2$

$Ca(OH)_2 + MgCl_2 \rightarrow Mg(OH_2)\downarrow + CaCl_2$

$BaO_2 + 2HCl \rightarrow BaCl_2 + H_2O_2$

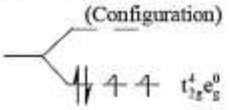
$Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow 2CaCO_3\downarrow + H_2O$

I→Q,S; II →P,R;III →S;IV →P,T

53.

$\left[Cr(CN)_6\right]^{4-}$ $Cr^{2+} \longrightarrow 4s^0 3d^4$ (Configuration)

$I \longrightarrow P, T, R$

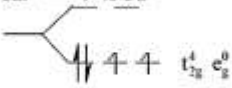


Low – spin

$n = 2$
$\mu = \sqrt{2(2+2)}$
$= \sqrt{8}$ B.M

$[RuCl_6]^{2-}$ $Ru^{4+} \longrightarrow 4s^0 3d^4$

$II \longrightarrow P, R, S, T$

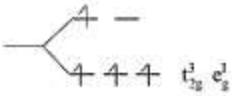


Low – spin

$\mu = \sqrt{8}$ B.M

$\left[Cr(H_2O)_6\right]^{2+}$ $Cr^{2+} \longrightarrow 4s^0 3d^4$

$III \longrightarrow Q, T$

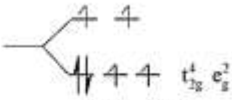


High – spin

$\mu = \sqrt{24}$ B.M

$\left[Fe(H_2O)_6\right]^{2+}$ $Cr^{2+} \longrightarrow 4s^0 3d^6$

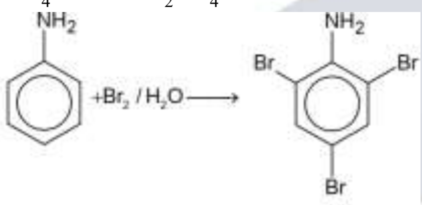
$IV \longrightarrow P, T$



High – spin

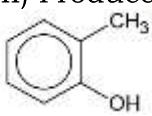
$\mu = \sqrt{4(4+2)} = \sqrt{24}$ B.M

54. (i) Since aniline contains C and N so its sodium extract will give Prussian Blue colour with $FeSO_4 / Conc.H_2SO_4$



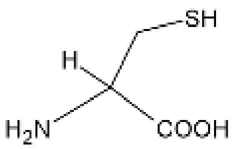
white ppt

(II) Produces violet colour with neutral $FeCl_3$



(III)It contains – COOH, so it give effervescence with $NaHCO_3$

It also contains C,N,S, so its Na-extract will give blood red colour with $FeCl_3 / H_2SO_4$



(IV) Caprolectum, it contains C and N, so its sodium extract will give Prussian blue colour