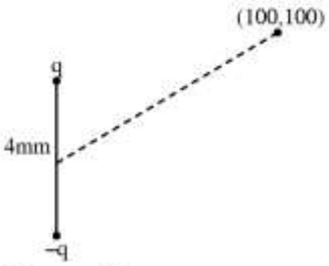
1. (b)

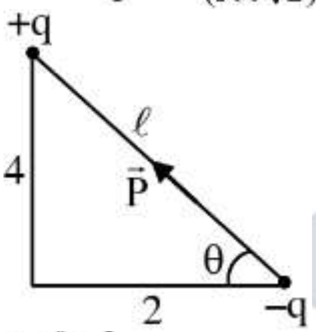


$P_1 = q(4)$

$\vec{P}_1 = P_1\hat{j}$

$\vec{r} = 100(\hat{i}+\hat{j})\text{mm}$

$v_0 = \frac{KP_1.\vec{r}}{r^3} = \frac{K(100P_1)}{(100\sqrt{2})^3}$



$\tan\theta = 2$

$\vec{P}_2 = P_2\left[-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right]$

$\vec{r} = 100(\hat{i}+\hat{j})\text{mm}$

$P_2 = q\ell$

$v = \frac{K\vec{P}_2.\vec{r}}{r^3}$

$v = \frac{K(100P_2)-(-\cos\theta+\sin\theta)}{(100\sqrt{2})^3}$

$\frac{v_0 P_2}{P_1} = (-\cos\theta+\sin\theta)$

$v = v_0\frac{q\ell}{q(4)}[-\cos\theta+\sin\theta]$

$= \frac{v_0}{4}[-2+4] = \frac{v_0}{2}$

2. (a)

$Y = c^{\alpha}h^{\beta}G^{\gamma}$

$ML^{-1}T^{-2} = \left(LT^{-1}\right)^{\alpha}\left(ML^{2}T^{-1}\right)^{\beta}\left(M^{-1}L^{3}T^{-2}\right)^{\gamma}$

$1 = \beta - \gamma$ ...(1)

$-1 = \alpha + 2\beta + 3\gamma$ ...(2)

$-2 = -\alpha - \beta - 2\gamma$ ...(3)

$-3 = \beta + \gamma$

$1 = \beta - \gamma$

$-2 = 2\beta \quad \Rightarrow \beta = -1, \gamma = -2$

$-1 = \alpha - 2 - 6 \quad \therefore \alpha = 7$

3. (a)

$\vec{v} = \alpha\left(yx + 2xy\right)$

$v_x = \alpha y \qquad v_y = 2\alpha x$

$\frac{dv_x}{dt} = \alpha\frac{dy}{dt} = 2\alpha^2 x \qquad \frac{dv_y}{dt} = 2\alpha v_x = 2\alpha^2 y$

$\therefore \vec{F} = m\vec{a} = 2m\alpha^2\left(xx + yy\right)$

4. (c)

$U = \frac{1}{2} fnrT = \frac{frT}{2}$

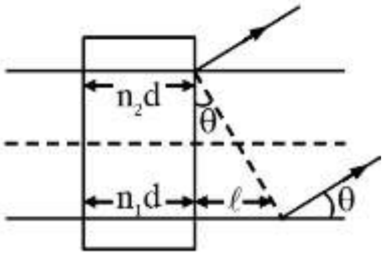
$\therefore$ A and B are wrong.

$v_{sound} = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\left(\frac{2}{f}+1\right)\frac{RT}{M}}$

$\Rightarrow$ more 'f', less 'v'

$\therefore v_5 > v_7$

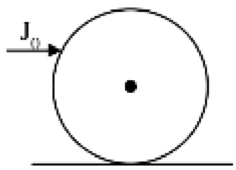
5. (b, d)



$n_1 d + l = n_2 d$

$\tan\theta = \frac{l}{h} = \frac{\left(n_2 - n_1\right)d}{h}$

6. (a, b, c, d)



$J_0 = mv$.....(1)

$J_0 h_{ra} = I_c\omega$......(2)

$v = \omega R$.......(3)

$\Rightarrow h_m = \frac{I_C}{mR}$

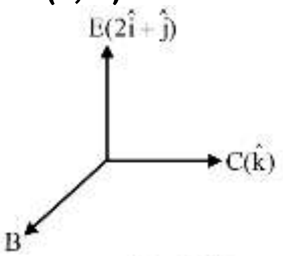
(a) If a = 0 $I_c = \frac{1}{2}mb^2$ & $R = b \ \therefore h_m = \frac{b}{2}$

(b) If a = b $I_c = mb^2$ & $R = b$ $\therefore h_m = b$

(c) $v = \frac{J_0}{m} \Rightarrow 100 = \frac{V}{R} = \frac{J_0}{mR}$

(d) Force is acting on COM $\therefore$ No rotation.

7. (a, d)

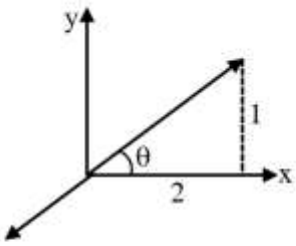


$$C_{\text{medium}} = \frac{5\times10^{14}}{10^7/3} = 1.5\times10^8 \text{ m/s} \quad \therefore \mu = 2$$

$$C_{\text{medium}} = \frac{E}{B} \Rightarrow B = \frac{E}{C_m} = \frac{30\sqrt{5}}{1.5\times10^8} = 2\sqrt{5}\times10^{-7}$$

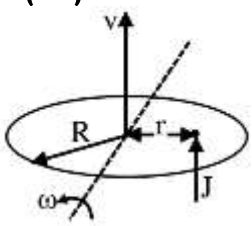
$$\vec{B}_{\text{direction}} = \hat{k}\times\left(2\hat{i}+\hat{j}\right) = \frac{2\hat{j}-\hat{i}}{\sqrt{5}}$$

$$\therefore\ \vec{B} = 2\times10^{-7}\left(-\hat{i}+2\hat{j}\right)\sin\left[27\left(5\times10^{17}t - \frac{10^7}{3}z\right)\right]$$



$\tan\theta = \frac{1}{2}$

8. (30)



$J = mv$ …. (1)

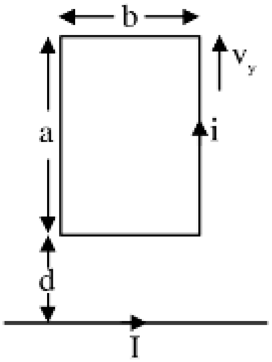
$Jr = I_c\omega$ ..(2)

$J_C = \frac{1}{4}mR^2$ … (3)

$t = \frac{2v}{g}$ … (4)

$\theta = 2\pi N = \omega t$ … (5) $\therefore N = 30$

9. (4)



R = 0.1 Ω

ε = (Bi - B_2)bv_y

$$i = \frac{\varepsilon}{R} = \frac{\mu_0 I}{2\pi R}\left(\frac{1}{d} - \frac{1}{d+a}\right)bv_y$$

$$\Rightarrow 10^{-5} = \frac{2\times10^{-7}\times10}{0.1}\left[\frac{1}{4} - \frac{1}{8}\right]\times 2.v_y$$

$$\therefore v_y = 2$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{1}{\sqrt{3}}$$

$$\therefore v_x = 2\sqrt{3}$$

$$\therefore v = \sqrt{v_x^2 + v_y^2} = 4$$

10. (5)

$$f' = \frac{P}{2l}\sqrt{\frac{T}{\mu}}$$

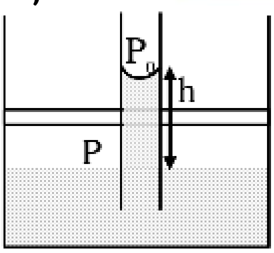
$$750 = \frac{P}{2}\sqrt{\frac{T}{\mu}} \;.... \;(1)$$

$$1000 = \frac{P+1}{2}\sqrt{\frac{T}{\mu}} \;...\; (2)$$

$$\frac{4}{3} = \frac{P+1}{P} \;\therefore P = 3$$

$$\Rightarrow 1000 = \frac{4}{2}\sqrt{\frac{T}{2\times10^{-5}}} \;\therefore T = 5N$$

11. (25)



$$h_0 = \frac{2T\cos\theta}{\rho g r} = \frac{2\times0.075\times1}{10^3\times10\times10^{-4}} = 15cm$$

$$P_0V_0 = P\frac{100V_0}{101} \Rightarrow P = \frac{101}{100}P_0$$

$$P_0 - \frac{2T\cos\theta}{r} + \rho g h = P = \frac{101}{100}P_0$$

$$\Rightarrow -\rho g h_0 + \rho g h = \frac{P_0}{100}$$

$$\Rightarrow h = h_0 + \frac{P_0}{100\rho g}$$

$$= 15cm + \frac{10^5}{100 \times 10^3 \times 10} = 25cm$$

12. (16)

	S_1	S_2
$t = 0$	A_0	A_0
$t = \tau$	A_1	A_2

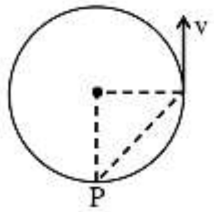
$$\frac{A_1}{A_2} = \frac{A_0 (0.5)^{t/(t_{1/2})_1}}{A_0 (0.5)^{t/(t_{1/2})_2}} = \frac{(0.5)^3}{(0.5)^7} = 2^4 = 16$$

13. (2)

$$\frac{W_I}{W_{II}} = \frac{4P_0V_0 + 8P_0V_0 \ln 2 - 6P_0V_0 - 0}{4P_0V_0 \ln 2 - 0 - P_0V_0 + 0}$$

$$= \frac{8\ln 2 - 2}{4\ln 2 - 1} = 2$$

14. (648)



$$f' = \frac{C}{C + v\cos 45^0} f$$

$$= \frac{324}{324 + 4\sqrt{2} \times \frac{1}{\sqrt{2}}} \times 656 = 648Hz$$

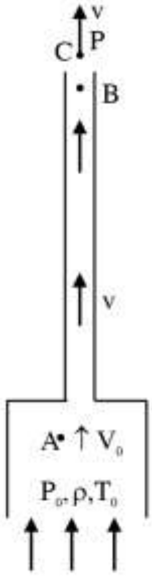
15. (8.2)

$f_{P\ from\ S2}$=656Hz

$f_{pfrom S_1} = \frac{C}{C-V} f = \frac{656 \times 324}{324 - 4}$ =664.2

Δ f= 664.2-656 = 8.2Hz

16. (60.80,60.81)



$\rho_0 T_0 = \rho T$

$\Rightarrow 1.2 \times 300 = \rho(360) \therefore \rho = 1$

Between A & B

$$P_0 + \frac{1}{2}\rho V_0^2 = P + \frac{1}{2}\rho V^2 + \rho g h \quad ....(1)$$

$$\frac{\pi D^2}{4} V_0 = \frac{\pi d^2}{4} V \quad ....(2)$$

Between B & C

$$P + \frac{1}{2}\rho V^2 = P_0 - \rho_0 g(H+h) + \frac{1}{2}\rho V^2 \quad .....(3)$$

from (1) & (2) :

$$\Rightarrow P_0 + \frac{1}{2}\rho\left(V\frac{d^2}{D^2}\right)^2 = P + \frac{1}{2}\rho V^2 + \rho g h$$

$$\Rightarrow \rho_0 g(H+h) = \frac{1}{2}\rho V^2\left[1 - \frac{d^4}{D^4}\right] + \rho g h$$

$$\Rightarrow V^2 \simeq \frac{2\rho_0}{\rho} g(H+h) - 2gh$$

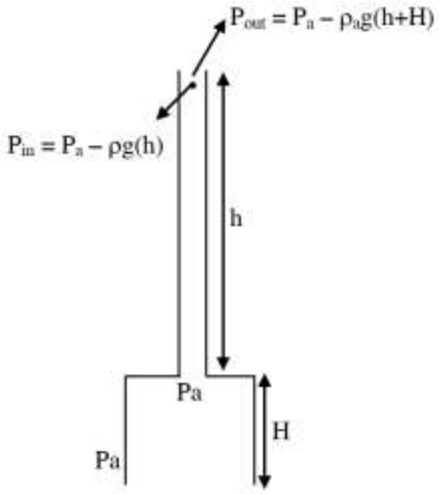
$= 2 \times 1.2 \times 10 \times 10 - 2 \times 10 \times 9$

$= 240 - 180 = 60 \quad \therefore V = \sqrt{60} m/s$

$$Q_m = \rho\frac{\pi d^2}{4} V = 1 \times \frac{\pi}{4} \times 10^{-2} \times \sqrt{60} \simeq 60.80$$

Ans. 60.80 to 60.81

17. (30)



P = constant

$\Rightarrow \rho_a T_a = \rho T$

1.2 x 300 = ρ x 360

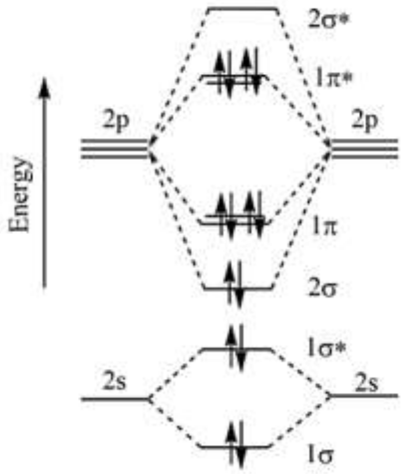
ρ = 1 kg/m^3

$\Delta P = \rho_a g\left(h+H\right) - \rho gh$

= 1.2x10x 10-1 x 10x9

= 120 - 90 = 30 N/m^2

18. (c) $F_2(18e^-)$



Naming of molecular orbitals are as per preference of formation of σ & π bonds respectively.

19. (a)

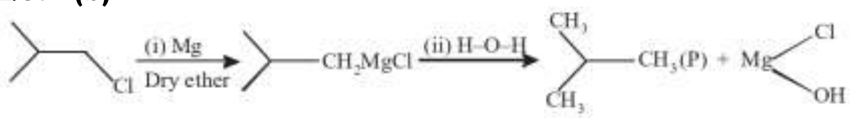
(I) As in Lyophobic colloids there is no interaction between dispersed phase and dispersion medium, special methods are used for preparation, simple mixing will not form colloid.

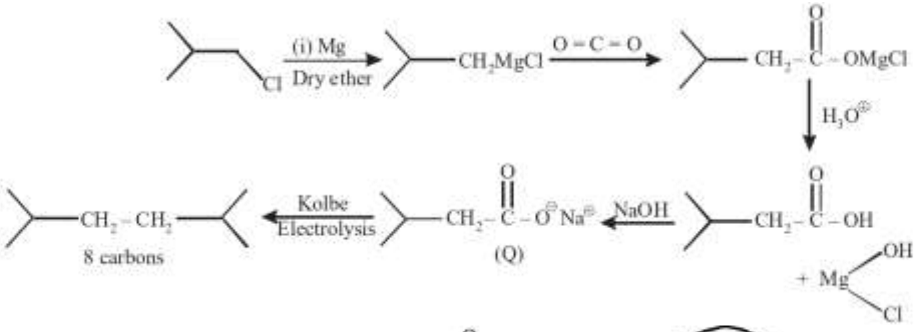
(II) Emulsions are liquid in liquid type colloids.

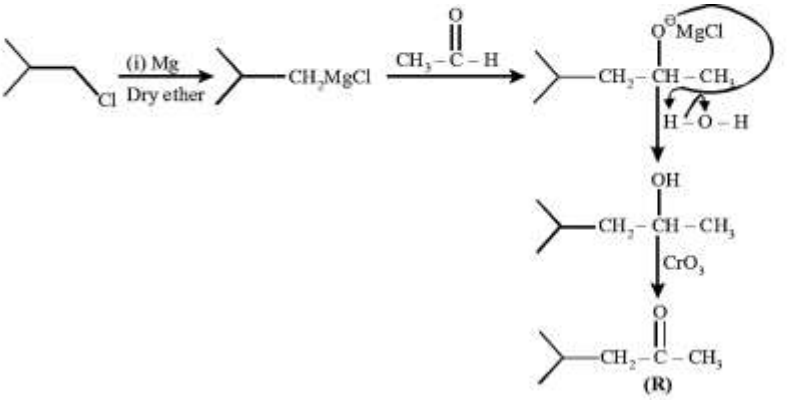
(III) Dissolving surfactant in a proper solvent will only form micelles at temperature above Kraft's temperature.

(IV) For Tyndall effect there must be a large difference in refractive index between dispersed phase and dispersion medium in order to have diffraction of light.

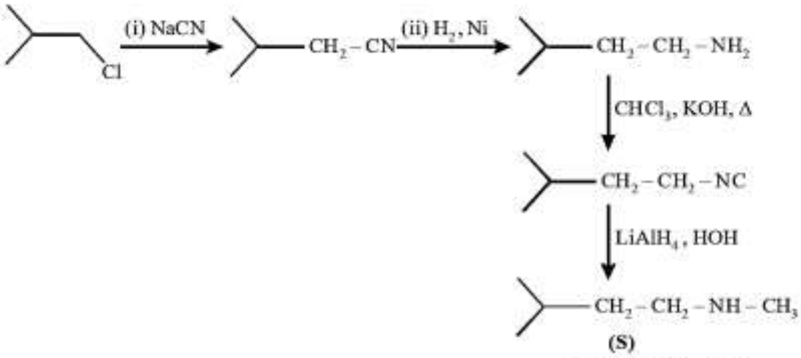
20. (b)







It does not give Cannizaro reaction



It's secondary amine

21. (a)

Sucrose $\xrightarrow{H_3O^+}$ Glucose + Fructose

Specific rotation + 52.5° -92° (mixture of products is laevorotatory)

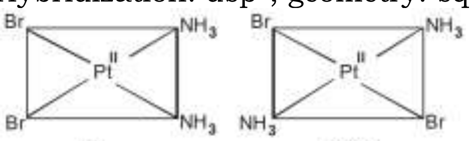
Sucrose $\xrightarrow{Br_2+H_2O}$ No reaction

BCD $\Rightarrow$ reducing sugars, will get oxidized by Br_2 + H_2O

22. (c, d)

$\overset{+2}{\left[Pt(NH_3)_2Br_2\right]}$

Hybridization: dsp^2, geometry: square planar

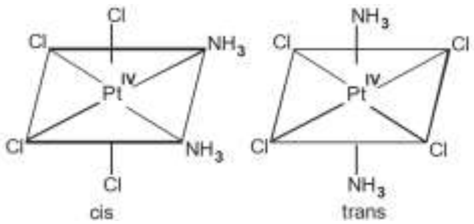


cis trans

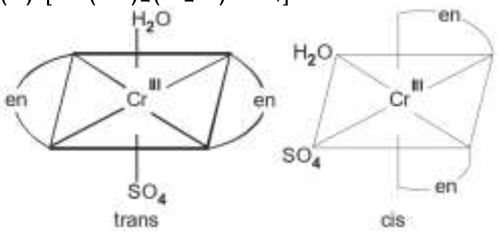
(a) $[Pt(en)(SCN)_2]$: square planar, cis-trans not possible

(b) $[Zn(NH_3)_2Cl_2]$: tetrahedral, cis-trans not possible

(c) $[Pt(NH_3)_2Cl_4]$: octahedral, cis-trans possible



(d) $[Cr(en)_2(H_2O)SO_4]^+$: Octahedral



23. (a, b, d)

Element	X	Y	Z
Packing	FCC	BCC	Primitive
Edge	L_x	L_y	L_z
Relation between edge length and radius	$L_x = 2\sqrt{2}r_x$	$L_y = \frac{4}{\sqrt{3}}r_y$	$L_z = 2r_z$
Packing fraction	$\frac{\pi}{3\sqrt{2}}$	$\frac{\sqrt{3}\pi}{8}$	$\frac{\pi}{6}$

Now, $r_y = \frac{8}{\sqrt{3}}r_x$ & $r_z = \frac{\sqrt{3}}{2}r_y = \frac{\sqrt{3}}{2}\times\frac{8}{\sqrt{3}}r_x \Rightarrow r_z = 4r_x$

So, $L_x = 2\sqrt{2}r_x$, $L_y = \frac{4}{\sqrt{3}}\times\frac{8}{\sqrt{3}}r_x$, $L_z = 8r_x$

$L_x = 2\sqrt{2}r_x$, $L_y = \frac{32}{3}r_x$, $L_z = 8r_x$

So $L_y > L_z > L_x$

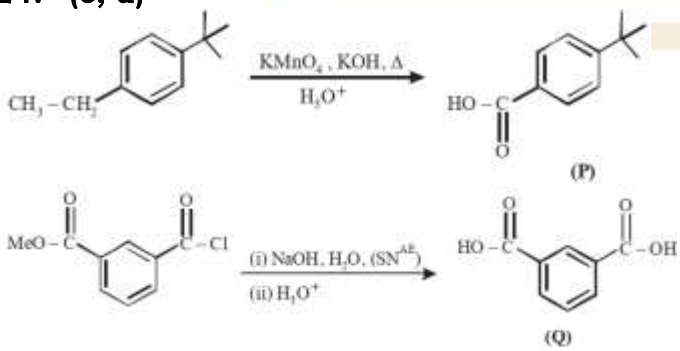
Density $\frac{4M_x}{L_x^3}, \frac{2\times M_y}{L_y^3}$

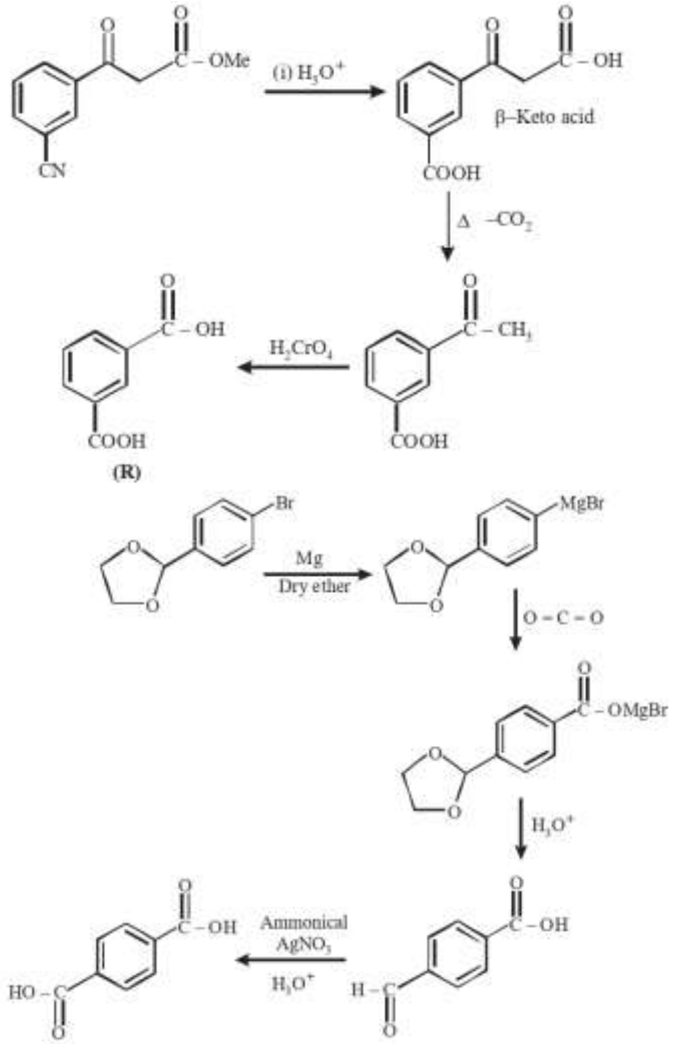
Now, $3M_x = \frac{3M_y}{2}$ or $M_x \times 2 = M_y$

$$\frac{\text{density}(x)}{\text{density}(y)} = \frac{4M_x}{2M_y}\times\frac{L_y^3}{L_x^3} = \frac{4M_x}{4M_x}\times\frac{\left(\frac{32}{3}\right)^3}{\left(2\sqrt{2}\right)^3}$$

Hence $d(x) > d(y)$

24. (c, d)





25. (18)

$$2K\overset{+7}{Mn}O_4 + 5H_2S + 3H_2SO_4 \rightarrow K_2SO_4 + 2\overset{+2}{Mn}SO_4 + 5S + 8H_2O$$

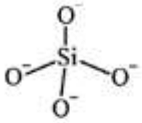
x = 8 (moles of H_2O produced)

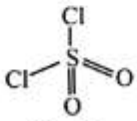
y = 14-4 = 10 (number of electrons involved)

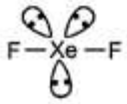
x + y= 10 + 8= 18

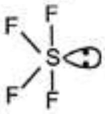
26. (5)

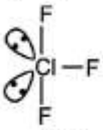
I_3^+ :  : sp^3

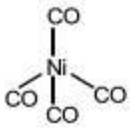
SiO_4^{4-} :  : sp^3

SO_2Cl_2 :  : sp^3

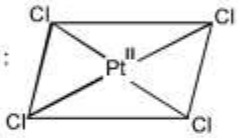
XeF_2 :  : sp^3d

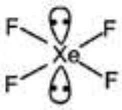
SF_4 :  : sp^3d

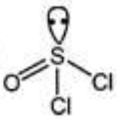
ClF_3 :  : sp^3d

$[Ni(CO)_4]$:  : sp^3

XeO_2F_2 :  : sp^3d

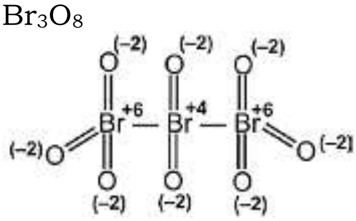
$[PtCl_4]^{2-}$:  : dsp^2

XeF_4 :  : sp^3d^2

$SOCl_2$:  : sp^3

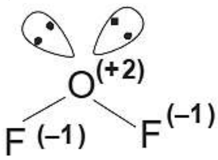
27. (6)

Br_3O_8



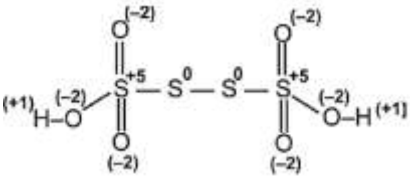
Number of atoms with zero oxidation state = 0

F_2O



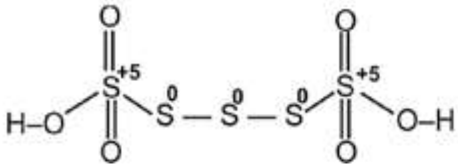
Number of atom with zero oxidation state = 0

$H_2S_4O_6$



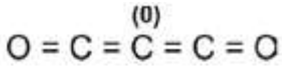
Number of atoms with zero oxidation state = 2

$H_2S_5O_6$



Number of atoms where zero oxidation state = 3

C_3O_2



Number of atoms with zero oxidation state = 1

28. (30)

For single electron system

r = 52.9 x $\frac{n^2}{Z}$ pm

Given Z = 2 for He^+

r_2 = 105.8 pm

So, 105.8 = 52.9 x $\frac{n_2^2}{2}$

n_2 = 2

r_1 = 26.45

So, 26.45 = 52.9 x $\frac{n_1^2}{2}$

n_1 = 1

So, transition is from 2 to 1.

Now $\frac{hc}{\lambda} = R_H Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

So λ = 30 x 10^{-9} m = 30 nanometer.

Here 'R_H' is given in terms of energy value.

29. (682)

Weight of 50 ml 0.2 molal urea = V x d = 50 x 1.012 = 50.6 gm

Given 0.2 molal implies

1000 gm solvent has 0.2 moles urea

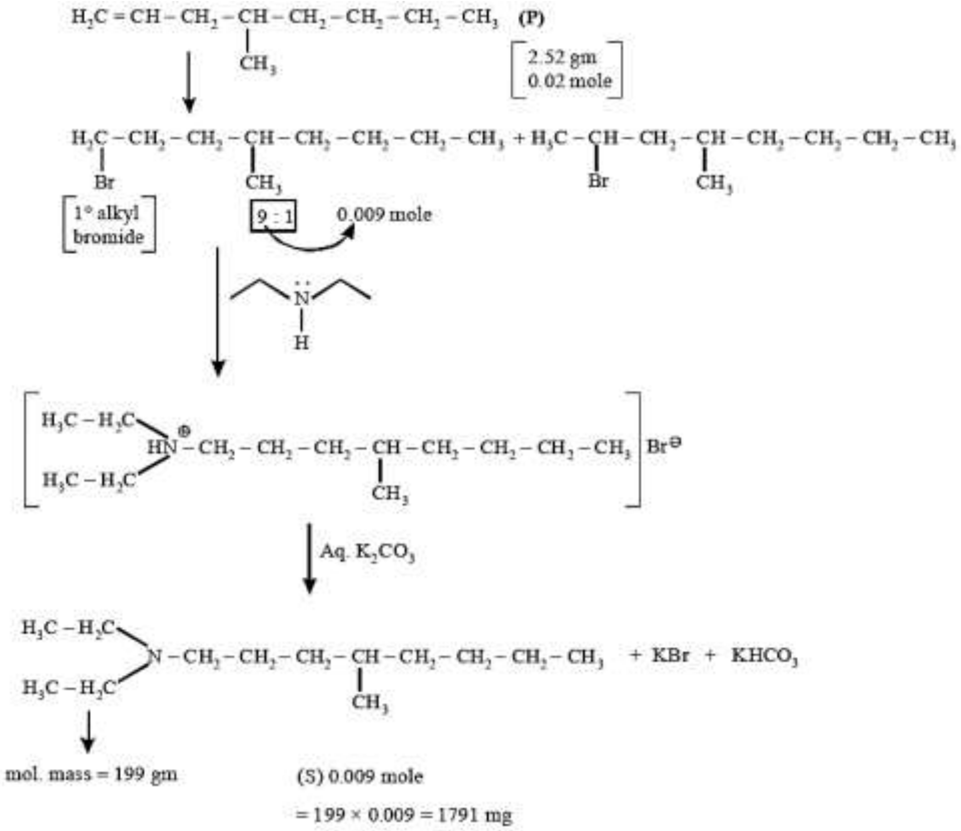
So, weight of solution = 1000 + 0.2 x 60 = 1012 gm.

So, wt. of urea in 50.6 gm solution = $\frac{12 \times 50.6}{1012}$ = 0.6 gm

Total urea = 0.6 + 0.06 = 0.66 gm Total volume = 300 ml

Now, osmotic pressure $\pi = C \times R \times T = \frac{0.66 \times 62 \times 300}{60 \times 0.3}$ = 682 Torr.

30. (1791)



31. (0.31)

At 1 bar

$\alpha \longrightarrow \beta$

$$S^o_{\alpha(600)} = S^o_{\alpha(300)} + C_{P(\alpha)} \ln \frac{600}{300}$$

$$S^o_{\beta(600)} = S^o_{\beta(300)} + C_{P(\beta)} \ln \frac{600}{300}$$

$$S^o_{\beta(600)} - S^o_{\alpha(600)} = S^o_{\beta(300)} - S^o_{\alpha(300)} + \left(C_{P(\beta)} - C_{P(\alpha)}\right) \ln 2$$

$$6 - 5 = S^o_{\beta(300)} - S^o_{\alpha(300)} + 1 \times \ln 2$$

$$1 = S^o_{\beta(300)} - S^o_{\alpha(300)} + 0.69$$

$$\text{So } {}^{o}_{\beta(300)} - S^o_{\alpha(300)} = 0.31$$

32. (300)

As the phase transition temperature is 600 K

So at 600 K $\Delta G^\circ_{rxn} = 0$

So $\Delta H^0_{reaction\ (600)} = T \Delta S^\circ_{reaction\ (600)}$

$\Delta H^\circ_{(600)} = 600 \times 1 = 600$ Joule/mole

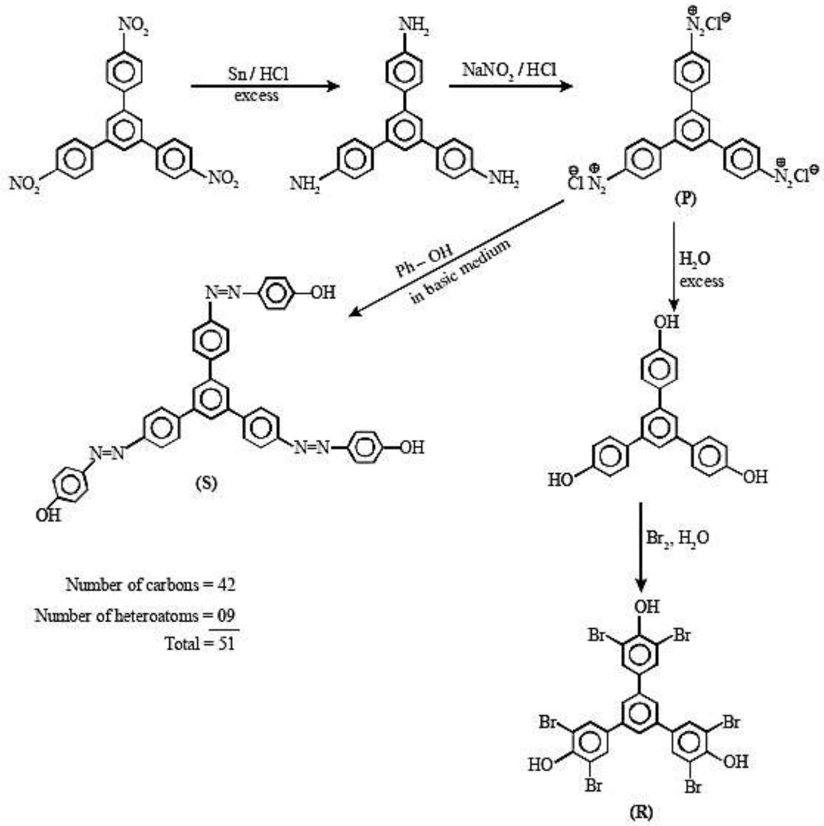
So $\Delta H_{600} - \Delta H_{300} = \Delta C_P(T_2 - T_1)$

$\Delta H_{600} - \Delta H_{300} = 1 \times 300$

$\Delta H_{300} = \Delta H_{600} - 300 = 600 - 300 = 300$ Joule/mole.

33. (9)

34. (51)



Number of carbons = 42
Number of heteroatoms = 09
Total = 51

Number of hetero atoms in R is 9

35. (c)

Diff. w.r.t 'x'

$3f(x) = f(x) + xf'(x) - x^2$

$$\frac{dy}{dx} - \left(\frac{2}{x}\right)y = x$$

$$IF = e^{-2\ln x} = \frac{1}{x^2}$$

$$y\left(\frac{1}{x^2}\right) = \int x.\frac{1}{x^2}\,dx$$

$$y = x^2 \ln x + cx^2$$

$$\therefore y(1) = \frac{1}{3} \Rightarrow c = \frac{1}{3}$$

$$y(e) = \frac{4e^2}{3}$$

36. (b)

$P(H) = \frac{1}{3}; P(T) = \frac{2}{3}$

Req. prob = P(HH or HTHH or HTHTHH or.......)
+ P(THH or THTHH or THTHTHH or ....)

$$= \frac{\frac{1}{3}.\frac{1}{3}}{1 - \frac{2}{3}.\frac{1}{3}} + \frac{\frac{2}{3}.\frac{1}{3}.\frac{1}{3}}{1 - \frac{2}{3}.\frac{1}{3}} = \frac{5}{21}$$

37. (c)

Case-I: $y \in (-3,0)$

$$\tan^{-1}\left(\frac{6y}{9-y^2}\right)+\pi+\tan^{-1}\left(\frac{6y}{9-y^2}\right)=\frac{2\pi}{3}$$

$$2\tan^{-1}\left(\frac{6y}{9-y^2}\right)=-\frac{\pi}{3}$$

$$y^2-6\sqrt{3}y-9=0\Rightarrow y=3\sqrt{3}-6\left(\because y\in(-3,0)\right)$$

Case-1: $y\in(0,3)$

$$2\tan^{-1}\left(\frac{6y}{9-y^2}\right)=\frac{2\pi}{3}\Rightarrow\sqrt{3}y^2+6y-9\sqrt{3}=0$$

$y=\sqrt{3}$ or $y=-3\sqrt{3}$ (rejected)

sum $=\sqrt{3}+3\sqrt{3}-6=4\sqrt{3}-6$

38. (b)

$$P\left(\hat{i}+2j-5k\right)=P\left(\vec{a}\right)$$

$$Q\left(3\hat{i}+6\hat{j}+3\hat{k}\right)=Q\left(\vec{b}\right)$$

$$R\left(\frac{17}{5}\hat{i}+\frac{16}{5}\hat{j}+7\hat{k}\right)=R\left(\vec{c}\right)$$

$$S\left(2\hat{i}+\hat{j}+\hat{k}\right)=S\left(\vec{d}\right)$$

$$\frac{\vec{b}+2\vec{d}}{3}=\frac{7\hat{i}+8\hat{j}+5\hat{k}}{3}$$

$$\frac{5\vec{c}+4\vec{a}}{9}=\frac{21\hat{i}+24\hat{j}+15\hat{k}}{9}$$

$$\Rightarrow\frac{\vec{b}+2\vec{d}}{3}=\frac{5\vec{c}+4\vec{a}}{9}$$

so [B] is correct, option -D

$$\left|\vec{b}\times\vec{d}\right|^2=\left|\vec{b}\right|\left|\vec{d}\right|^2-\left(\vec{b}.\vec{d}\right)^2$$

$= (9 + 36 + 9)(4 + 1 + 1) - (6 + 6 + 3)^2 = 54\text{x}6-(15)^2 = 324-225 = 99$

39. (b, c)

$$M=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}=\begin{bmatrix}1&1&1\\1&0&1\\0&1&0\end{bmatrix}$$

$|M| = -1 + 1 = 0 \Rightarrow$ M is singular so non-invertible

$$\text{(B) } M\begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix}=\begin{bmatrix}-a_1\\-a_2\\-a_3\end{bmatrix}\Rightarrow\begin{bmatrix}1&1&1\\1&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix}=\begin{bmatrix}-a_1\\-a_2\\-a_3\end{bmatrix}$$

$$\left.\begin{aligned}a_1+a_2+a_3&=-a_1\\a_1+a_3&=-a_2\\a_2&=-a_3\end{aligned}\right\}\Rightarrow a_1=0 \text{ and } a_2+a_3=0 \text{ infinite solutions exists [B] is correct.}$$

Option (D)

$$M - 2I = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$$

$|M - 2I| = 0 \Rightarrow$ [D] is wrong

Option (C) :

$$MX = 0 \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x + y + z = 0$

$x + z = 0$

$y = 0$

$\therefore$ Infinite solution

[C] is correct

40. (a, b)

$$f(x) = \begin{cases} 0 & ; \ 0 < x < \frac{1}{4} \\ \left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right) & ; \ \frac{1}{4} \le x < \frac{1}{2} \\ 2\left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right) & ; \ \frac{1}{2} \le x < \frac{3}{4} \\ 3\left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right) & ; \ \frac{3}{4} \le x < 1 \end{cases}$$

f(x) is discontinuous at x = $\frac{3}{4}$ only

$$f'(x) = \begin{cases} 0 & ; \ 0 < x < \frac{1}{4} \\ 2\left(x - \frac{1}{4}\right)\left(x - \frac{1}{2}\right) + \left(x - \frac{1}{4}\right)^2 & ; \ \frac{1}{4} < x < \frac{1}{2} \\ 4\left(x - \frac{1}{4}\right)\left(x - \frac{1}{2}\right) + 2\left(x - \frac{1}{4}\right)^2 & ; \ \frac{1}{2} < x < \frac{3}{4} \\ 6\left(x - \frac{1}{4}\right)\left(x - \frac{1}{2}\right) + 3\left(x - \frac{1}{4}\right)^2 & ; \ \frac{3}{4} < x < 1 \end{cases}$$

f(x) is non-differentiable at x = $\frac{1}{2}$ and $\frac{3}{4}$

minimum values of f(x) occur at x = $\frac{5}{12}$ whose value is $-\frac{1}{432}$

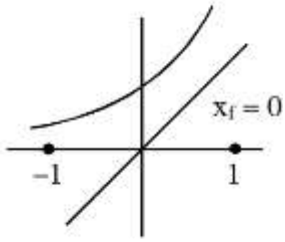
41. (a, b, c)

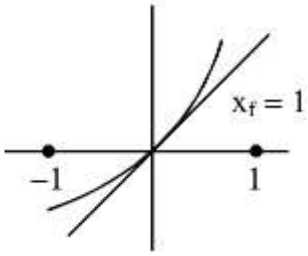
S = Set of all twice differentiable functions f : R $\to$ R

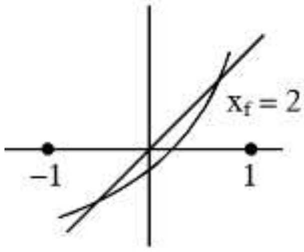
$\frac{d^2 f}{dx2}$ >0 in (-1, 1) dx

Graph 'f' is Concave upward.

Number of solutions of f(x) = x $\to$ x_f

(1)


(2)


(3)


$\Rightarrow$ Graph of y = f(x) can intersect graph of y = x at at-most two points $\Rightarrow 0 \le x_f \le 2$

<u>Aliter</u>

$\frac{d^2 f(x)}{dx^2} > 0$

Let $\phi(x) = f(x) - x$

$\phi''(x) > 0$

$\therefore \phi'(x) = 0$ has at-most 1 root in x ∈ (-1, 1)

$\therefore \phi(x) = 0$ has at-most 2 roots in x ∈ (-1, 1)

$\therefore x_f \le 2$

42. (0)

$$f(x) = \int_0^{x\tan^{-1}x} \frac{e^{t-\cos t}}{1+t^{2023}}\,dt$$

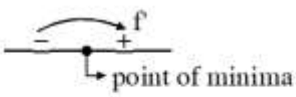
$$f'(x) = \frac{e^{x\tan^{-1}x - \cos(x\tan^{-1}x)}}{1+(x\tan^{-1}x)^{2023}} \cdot \left(\frac{x}{1+x^2} + \tan^{-1}x\right)$$

For $x < 0$, $\tan^{-1}x \in \left(-\frac{\pi}{2}, 0\right)$

For $x \ge 0$, $\tan^{-1}x \in \left[0, \frac{\pi}{2}\right)$

$\Rightarrow x\tan^{-1}x \ge 0\ \forall\ x \in R$

And $\frac{x}{1+x^2} + \tan^{-1}x = \begin{cases} >0 & \text{For } x > 0 \\ <0 & \text{For } x < 0 \\ 0 & \text{For } x = 0 \end{cases}$



Hence minimum value is $f(0) = \int_0^0 = 0$

43. (16)

$$\frac{dy}{dx} - \frac{2x}{x^2-5}y = -2x\left(x^2-5\right)$$

$$IF = e^{-\int \frac{2x}{x^2-5}dx} = \frac{1}{\left(x^2-5\right)}$$

$$y.\frac{1}{x^2-5} = \int -2x.dx + c$$

$$\Rightarrow \frac{y}{x^2-5} = -x^2 + c$$

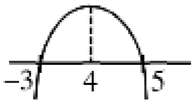
x=2, y=7

$$\frac{7}{-1} = -4 + c \Rightarrow c = -3$$

$y = -(x^2-5)(x^2 + 3)$

put $x^2 = t > 0$

$y = -(t-5)(t+3)$



y_{max}= 16 when x^2=1

y_{max}= 16

44. (31)

No. of elements in X which are multiple of 5

$$\left.\begin{array}{l}
\underbrace{_\ _\ _\ _}_{1,2,2,2}\ \underbrace{0}_{\text{fixed}} \to \frac{|\underline{4}}{|\underline{3}} = 4 \\
\underbrace{_\ _\ _\ _}_{1,4,2,2}\ \underbrace{0}_{\text{fixed}} \to \frac{|\underline{4}}{|\underline{2}} = 12 \\
\underbrace{_\ _\ _\ _}_{4,2,2,2}\ \underbrace{0}_{\text{fixed}} \to \frac{|\underline{4}}{|\underline{3}} = 4 \\
\underbrace{_\ _\ _\ _}_{2,2,4,4}\ \underbrace{0}_{\text{fixed}} \to \frac{|\underline{4}}{|\underline{2}|\underline{2}} = 6 \\
\underbrace{_\ _\ _\ _}_{1,2,4,4}\ \underbrace{0}_{\text{fixed}} \to \frac{|\underline{4}}{|\underline{2}} = 12
\end{array}\right\}\text{Total} = 38$$

Among these 38 elements, let us calculate when element is not divisible by 20

$$\left.\begin{array}{l}
\underbrace{_\ _\ _}_{2,2,2}\ \underbrace{1\ \ 0}_{\text{fixed}} \to \frac{|\underline{3}}{|\underline{3}} = 1 \\
\underbrace{_\ _\ _}_{2,2,4}\ \underbrace{1\ \ 0}_{\text{fixed}} \to \frac{|\underline{3}}{|\underline{2}} = 3 \\
\underbrace{_\ _\ _}_{2,4,4}\ \underbrace{1\ \ 0}_{\text{fixed}} \to \frac{|\underline{3}}{|\underline{2}} = 3
\end{array}\right\}\text{Total} = 7$$

$$\therefore\ p = \frac{38-7}{38} \quad \therefore\ 38p = 31$$

45. (512)

$z^8 - 2^8 = (z - 2)(z - \alpha)(z - \alpha^2)\ldots(z - \alpha^7)$

Put $z = 2e^{i\theta}$

$2^8\left(e^{i8\theta}-1\right)=\left(2e^{i\theta}-2\right)\left(2e^{i\theta}-\alpha\right)....\left(2e^{i\theta}-\alpha^7\right)$

Take mod

$2^8|e^{i8\theta}-1| = PA_1PA_2...PA_8$

$2^8|2\sin 4\theta| = PA_1PA_2...PA_8$

$(PA_1.PA_2...PA_8)_{max} = 512$

46. (3780)

Let us calculate when $|R| = 0$

Case-I ad = be = 0

Now ad = 0

$\Rightarrow$Total - (When none of a & d is 0)

$= 8^2 - 1 = 15$ ways

Similarly, be = 0 $\Rightarrow$ 15 ways

$\therefore$ 15 x 15 = 225 ways of ad = bc = 0

Case-II ad = be $\neq$ 0

either a = d = b = c OR a $\neq$ d, b $\neq$ d but ad = bc

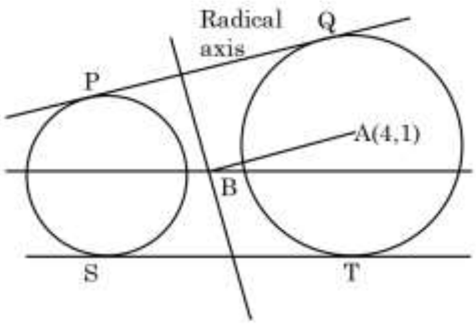
${}^7C_1 = 7$ ways ${}^7C_2 \times 2 \times 2 = 84$ ways

Total 91 ways

$\therefore$ $|R| = 0$ in 225 + 91 = 316 ways

$|R| \neq O$ in 8^4-316 = 3780

47. (2)



Let $C_2\left(x-4\right)^2+\left(y-1\right)^2=r^2$

radical axis $8x+2y-17=1-r^2$

$8x+2y=18-r^2$

$B\left(\frac{18-r^2}{8},0\right)A\left(4,1\right)$

$AB=\sqrt{5}$

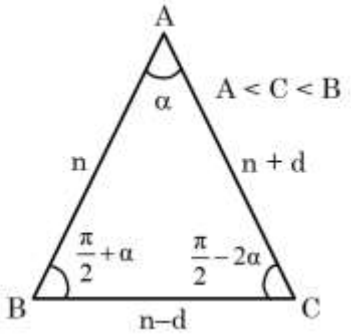
$\sqrt{\left(\frac{18-r^2}{8}-4\right)^2+1}=\sqrt{5}$

$r^2=2$

$\Rightarrow n=\sin\alpha+\cos\alpha$

48. (1008.00)

A



$n-d=2\sin\alpha$ ..(1)

$n + d = 2\sin\left(\frac{\pi}{2}+\alpha\right)$

$\Rightarrow n + d = 2\cos\alpha$ ....(2)

$n = 2\sin\left(\frac{\pi}{2}-2\alpha\right)$

$\Rightarrow n=2\cos 2\alpha$ ...(3)

$\Rightarrow$ $2\cos 2\alpha = \sin\alpha + \cos\alpha$

$\Rightarrow$ $2(\cos\alpha - \sin\alpha) = 1$

$\Rightarrow \sin 2\alpha=\frac{3}{4}$

Then, $a=\frac{1}{2}.n.(n+d).\sin\alpha=\frac{1}{2}.2\cos 2\alpha.2\cos\alpha\sin\alpha$

$=\sin 2\alpha.\cos 2\alpha$

$=\frac{3}{4}\times\frac{\sqrt{7}}{4}=\frac{3\sqrt{7}}{16}$

$(64a)^2=\left(64\times\frac{3\sqrt{7}}{16}\right)=16\times 9\times 7=1008$

49. (0.25)

From above equation in Ques. 14

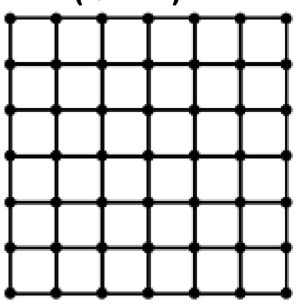
$r=\frac{\Delta}{s}=\frac{1}{2}\frac{n(n+d)\sin\alpha}{\left(\frac{3n}{2}\right)}$

$=\frac{(n+d).\sin\alpha}{3}$

$=\frac{2\cos\alpha.\sin\alpha}{3}$ (from (2))

$r=\frac{\sin 2\alpha}{3}=\frac{1}{4}$

50. (24.00)



P_i = Probability that randomly

selected points has friends
$P_0 = 0$ (0 friends)
$P_1 = 0$ (exactly 1 friends)

$P_2 = \frac{{}^4C_1}{{}^{49}C_1} = \frac{4}{9}$ (exactly 2 friends)

$P_3 = \frac{{}^{20}C_1}{{}^{49}C_1} = \frac{20}{49}$ (exactly 3 friends)

$P_4 = \frac{{}^{25}C_1}{{}^{49}C_1} = \frac{25}{49}$ (exactly 4 friends)

x	0	1	2	3	4
P(x)	0	0	$\frac{4}{49}$	$\frac{20}{49}$	$\frac{25}{49}$

Mean = E(x) = $\sum x_i P_i = 0 + 0 + \frac{8}{49} + \frac{60}{49} + \frac{100}{49} = \frac{168}{49}$

$7(E(x)) = \frac{168}{49} \times 7 = 24$

51. (0.50)
Total number of ways of selecting 2 persons = ${}^{49}C_2$
Number of ways in which 2 friends are selected = 6x7x2 = 84

$7P = \frac{84 \times 2}{49 \times 48} \times 7 = \frac{1}{2}$