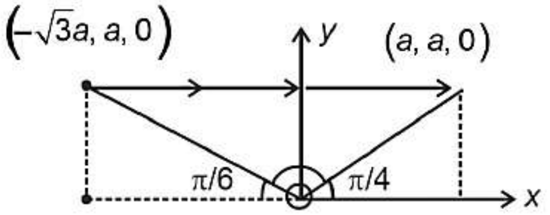
1. (a) $[AT]^{\alpha}[M^{-1}L^{-3}T^{4}A^{2}]^{\beta}[ML^{2}T^{-1}]^{\gamma}[LT^{-1}]^{\delta}=0$

$$\begin{aligned}&\alpha+2\beta=0\\&-\beta+\gamma=0\\&-3\beta+2\gamma+\delta=0\\&\alpha+4\beta-\gamma-\delta=0\end{aligned}\quad\Rightarrow\quad\begin{aligned}\gamma&=-2\beta\\\gamma&=\beta\\\delta&=\beta\end{aligned}$$

$(-2\beta,\beta,\beta,\beta)$

2. (a)

$(-\sqrt{3}a, a, 0)$



$$\theta=\pi-\frac{\pi}{4}-\frac{\pi}{6}$$

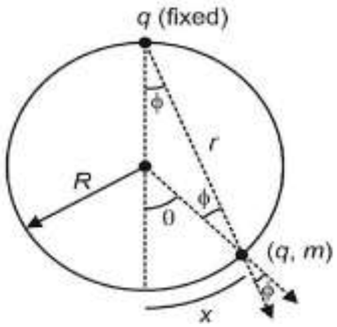
$$\Rightarrow\ \theta=\frac{12\pi-3\pi-2\pi}{12}=\frac{7\pi}{12}$$

So, $\int\vec{B}\cdot\vec{dl}$ along the line is

$$\int\vec{B}\cdot\vec{dl}=-\frac{\mu_0(I)}{2\pi}\cdot\theta=\frac{\mu_0 I}{2\pi}\cdot\frac{7\pi}{12}$$

$$\Rightarrow\left|\int\vec{B}\cdot\vec{dl}\right|=\frac{7\mu_0 I}{24}$$

3. (b) As the hoop mass is not given so it must not move or else its inertia must have some effect.



Here $r=2R\cos\phi$

Also $\theta=2\phi$

$$\Rightarrow\theta=\frac{\phi}{2}$$

And $\theta=\frac{x}{R}$

If θ is the small angular displacement of free charge, then $F(\phi) = \frac{Kq^2}{r^2}$

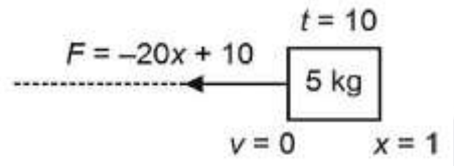
So, restoring force towards mean position is $F_{(R)} = \frac{Kq^2}{r^2} \sin\phi$

$$a_R = \frac{F_{(R)}}{m} = \frac{-Kq^2}{mr^2} \cdot \sin\phi = \frac{-Kq^2 \cdot \sin\phi}{m4R^2\cos^2\phi}$$

$$\Rightarrow a_R = -\frac{Kq^2}{4mR^2\cos^2\left(\frac{\phi}{2}\right)} \cdot \sin\left(\frac{\theta}{2}\right) \approx \frac{-Kq^2}{4mR^2}\frac{1}{2} \cdot \frac{x}{R} = \omega^2 \cdot x$$

So, $\omega^2 = \frac{q^2}{32\pi\varepsilon_0 mR^3}$

4. (c)



Now,

$$a = -\frac{20x + 10}{5} = -4x + 2$$

$$a = \frac{vdv}{dx} = -4x + 2$$

Hence $\int_0^v vdv = \int_1^x (-4x + 2)dx \rightarrow \frac{v^2}{2} = [-2x^2 + 2x]_1^x$

$v = -2\sqrt{x - x^2}$ (as particle starts moving in-ve x-direction)

$$\Rightarrow \frac{dx}{dt} = -2\sqrt{x - x^2} \rightarrow \int_{x=1}^{x=x} \frac{dx}{\sqrt{x - x^2}} = -2\int_0^{\pi/4} dt$$

$$\sin^{-1}[2x - 1]_1^x = -\frac{\pi}{2}$$

$$x = \frac{1}{2} = 0.5 \text{ m}$$

Also, momentum $= mV = 5(V)_{x=\frac{1}{2}} = -5 \text{ kg} - \text{m/s}$

5. (a, b, c)

$L = mvr = nh$, also, $\frac{mv^2}{r} = kr$

$$mv^2 = kr^2$$
$$m^2v^2 = mkr^2$$
$$mv = \sqrt{mkr^2}$$
$$mvr = r^2\sqrt{mk}$$
$$nh = r^2\sqrt{mk}$$
$$\frac{nh}{\sqrt{mk}} = r^2$$

Option (A) is correct

Also, $v^2 = \frac{kr^2}{m}$

$$= \frac{nh}{\sqrt{mk}} \cdot \frac{k}{m}$$

$$v^2 = nh\sqrt{\frac{k}{m^3}}$$

Option (b) is correct

Now,

$$T.E = E = \frac{1}{2}kr^2 + \frac{1}{2}kr^2$$
$$E = kr^2$$

$$= k\frac{nh}{\sqrt{mk}}$$

$$= nh\sqrt{\frac{k}{m}}$$

Option (d) is incorrect

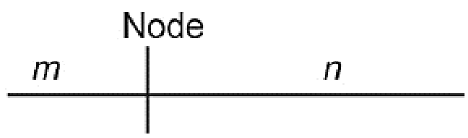
$$\Rightarrow \frac{L}{mr^2} = \frac{h\sqrt{mk}}{mnh}$$

$$\frac{L}{mr^2} = \sqrt{\frac{k}{m}}$$

Option (c) is correct

6. (a, c, d)

With node at O



$v = \sqrt{\frac{T}{\mu}}\ v' = \sqrt{\frac{T}{4\mu}} = \frac{1}{2}v$

$\Rightarrow m\frac{1}{2/}\sqrt{\frac{T}{\mu}} = n\frac{1}{2(2/)}\sqrt{\frac{T}{4\mu}}$

$\Rightarrow m = \frac{n}{2} \times \frac{1}{2}$

$\Rightarrow \frac{m}{n} = \frac{1}{4}$

$\therefore m = 1, n = 4$

With antinode at O

$m\frac{1}{4'}\sqrt{\frac{T}{\mu}} = n\frac{1}{4(2l)}\sqrt{\frac{T}{4\mu}}$

$\Rightarrow \frac{m}{n} = \frac{1}{2} \times \frac{1}{2}$

$\Rightarrow \frac{m}{n} = \frac{1}{4}$

$\therefore m = 1\ f_{min} = 1\frac{1}{4l}\sqrt{\frac{T}{\mu}} = \frac{v_0}{2}$

(B is wrong)

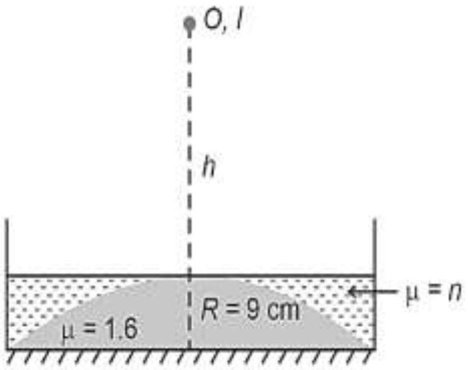
Also, when node at O.

Total nodes = 6

(C is correct)

A, C, D are correct

7. (a, b)



For image to coincide with object

$-h = 2(f_{net})$

$$\Rightarrow -\frac{1}{f_{net}} = 2\left(\frac{1}{f_{liq}}\right) + 2\left(\frac{1}{f_{lens}}\right) + \left(\frac{-1}{f_{mirror}}\right) \ldots\ldots\ldots\ldots (i)$$

$$-\frac{1}{f_{net}} = 2\left(\frac{n-1}{-9}\right) + 2\left(\frac{0.6}{9}\right) + \left(-\frac{1}{\infty}\right) \ldots\ldots\ldots.. (ii)$$

From (i) and (ii)

$$h = \frac{9}{(1.6-n)}$$

For $n = 1.42, h = 50$ cm (A is correct)

For $n = 1.35, h = 36$ cm (B is correct)

For $n = 1.45, h = 60$ cm (C is incorrect)

For $n = 1.48, h = 75$ cm (D is incorrect)

8. (25000)

$$C = \frac{dQ/m}{dT}$$

$$\Rightarrow dQ = m \cdot C \cdot dT$$

$$= 1 \cdot kTdT$$

$$Q = \int_{200}^{300} kTdT = \frac{k}{2}[300^2 - 200^2]$$

$$= \frac{10^4}{2} \cdot k \cdot 5$$

$$= 25000k$$

$$\Rightarrow n = 25000$$

9. (12) Conserving angular momentum of the system (bigger disc + smaller disc) about the symmetric axis of bigger disc:

$$\frac{MR^2}{2}\omega' + M \cdot R\omega' \cdot R + \frac{M(R/2)^2}{2}\omega = 0$$

Where ω' : required angular speed.

$$\Rightarrow \frac{3}{2}MR^2\omega' = \frac{-MR^2\omega}{8}$$

$$\Rightarrow \omega' = -\frac{\omega}{12}$$

$$\Rightarrow n = 12$$

10. (8) $I \propto \frac{1}{l^2}$

Where I : distance from point source.

$\Rightarrow \frac{I_A}{I_B} = \frac{I_B^2}{I_A^2} = 2$

$\Rightarrow I_B = \sqrt{2} I_A$.........(i)

Also, due to polaroids:

$I_B{}' = \frac{I_B}{2}\cos^2 45^\circ = \frac{I_B}{4}$

$\Rightarrow I_B{}' = \frac{I_B}{4}$..............(ii)

$\Rightarrow$ Ratio becomes 4 times.

$\Rightarrow r_{new} = 8$

11. (200)

For Case 1:

$\xrightarrow{S} f \xrightarrow{v} 0$

$f_{app} = f\left(\frac{c+v}{c-v}\right) \Rightarrow 288 = 240\left(\frac{c+v}{c-v}\right)$

For Case 2 :

$\underset{S}{\overset{V}{\rightarrow}} \overset{v}{V}\ V$

$f_{app} = f\left(\frac{c-v}{c+v}\right)$

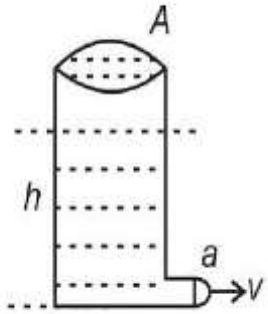
From (i) and (ii)

$288 \times f_{app} = 240\left(\frac{c+v}{c-v}\right) \times 240\left(\frac{c-v}{c+v}\right)$

$288 f_{app} = 240 \times 240$

$f_{app} = 200\ \text{Hz}$

12. (3) In general case



$a\sqrt{2gh} = -A\frac{dh}{dt}$

$$dt = -\frac{A}{a}\frac{dh}{\sqrt{2gh}}$$

$$T = \int dt = \frac{-2A}{a\sqrt{2g}}\left(\sqrt{h_f} - \sqrt{h_i}\right)$$

$$T = \frac{2A}{a\sqrt{2g}}\left(\sqrt{h_i} - \sqrt{h_f}\right)$$

For tank 1 :

$h_i = h, h_f = 0$

$$T_1 = \frac{2A}{a\sqrt{2g}}(\sqrt{h})$$

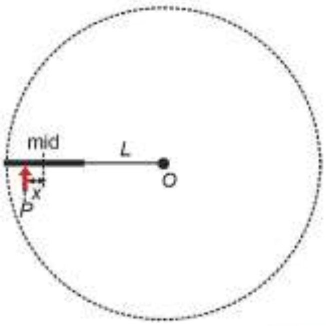
For tank 2 :

$$h_f = \frac{16h}{9}, h_i = h + H = \frac{25h}{9}$$

$$T_2 = \frac{2A\sqrt{h}}{a\sqrt{2g}}\left(\frac{5}{3} - \frac{4}{3}\right) = \frac{2A\sqrt{h}}{a\sqrt{2g}} \times \frac{1}{3}$$

$$\frac{T_1}{T_2} = 3$$

13. (18)



M.I. of rod about centre $(O) = \frac{mL^2}{12} + m\left(L + \frac{L}{2}\right)^2$

$$I_0 = \frac{7mL^2}{3}$$

Since rod is in pure rotation about ' O '

So, angular impulse $= P\left(x + \frac{3L}{2}\right) = I_0\omega_0$..............(i)

and linear impulse $= mv_c$................(ii)

where $V_c = \frac{3L}{2}\omega_0$...............(iii)

(20m (i)) $m\left(\frac{3L}{2}\omega_0\right)\left(x+\frac{3L}{2}\right)=\frac{7}{3}mL^2\cdot\omega_0$

$$x+\frac{3L}{2}=\frac{14}{9}L$$

$$x=\frac{L}{18}$$

14. (b) From M → J, isothermal

$W = nRT\ln 2$

$= RT_0\ln 2$

$\Delta U = 0$

From J → K, isobaric

$W = nR(3T_0 - T_0)$

$W = 2RT_0$

$\Delta U = nC_v\Delta T$

$=\frac{3R}{2}\times 2T_0 = 3RT_0$

From K → L, isothermal

$W = -nR(3T_0)\ln 2$

$W = -3RT_0\ln 2$

$\Delta U = 0$
$Q = -3RT_0\ln 2$

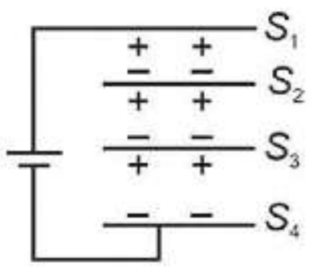
From L → M, isobaric

$W = nR(T_0 - 3T_0)$
$W = -2RT_0$
$\Delta U = nC_v\Delta T$
$= n\times\frac{3R}{2}\times(-2T_0)$
$= -3RT_0$

For P :-

$W_{net} = RT_0\ln 2 + 2RT_0 - 3RT_0\ln 2 - 2RT_0$
$= -2RT_0\ln 2$
P → 4
for Q → 3
for R → 5
for S → 2

15. (c) For P

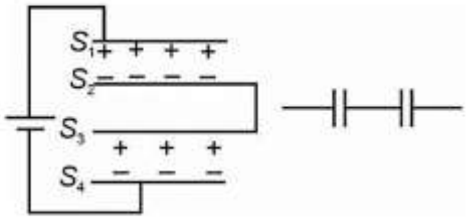


All are in series

$C_{eq} = \frac{C_0}{3}$

P → (3)

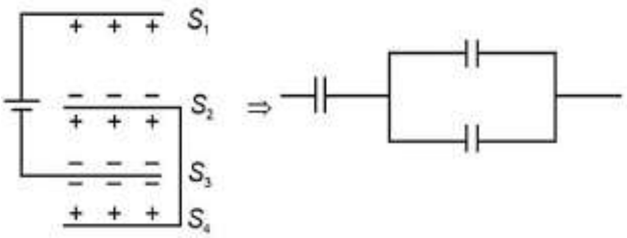
For Q



$C_{eq} = \frac{C_0}{2}$

Q → (2)

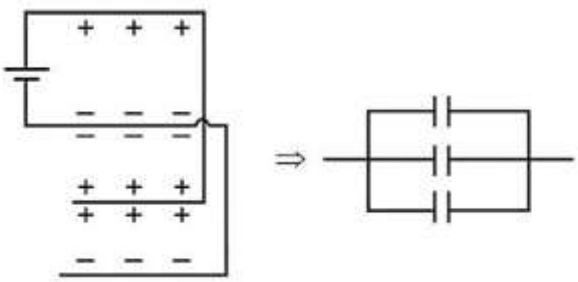
For R



$C_{eq} = \frac{2C}{3}$

R → (4)

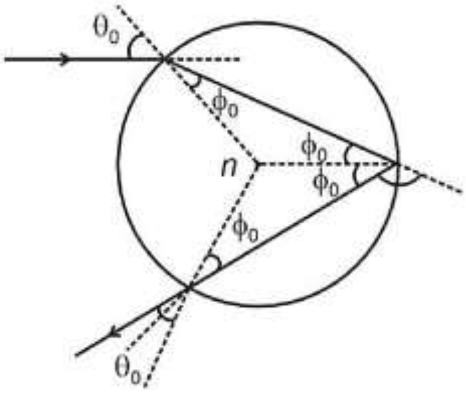
For S



$\Rightarrow C_{eq} = 3C_0$

$S \rightarrow (1)$

16. (a)



$\alpha = (\theta_0 - \phi_0) + (\pi - 2\phi_0) + (\theta_0 - \phi_0)$

$\alpha = \pi + 2\theta_0 - 4\phi_0$..............(i)

$\sin\theta_0 = n\sin\phi_0$.........(ii)

For (P)

$n = 2, \alpha = 180$

if $\alpha = \pi, 2\theta_0 - 4\phi_0 = 0$

$\theta_0 = 2\phi_0$

$\sin\theta_0 = 2\sin\left(\frac{\theta_0}{2}\right)$

$P \rightarrow (5)$

For Q, $n = \sqrt{3}, \alpha = 180$

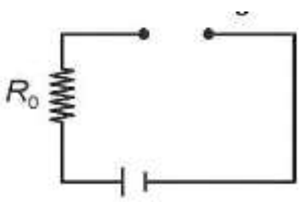
$\theta_0 = 2\phi_0$

$Q \rightarrow (2)$

$\sin\theta_0 = \sqrt{3}\sin\left(\frac{\theta_0}{2}\right)$

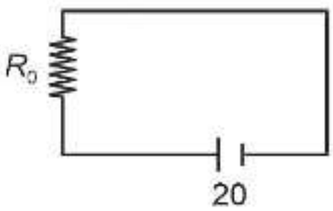
$\theta_0 = 60, 0°$

17. (a) Just after closing K_1



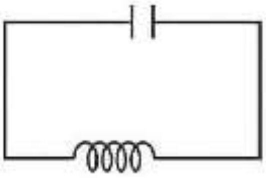
$i_1 = 0,$ $\qquad$ $P \rightarrow (1)$

After long time



$i_2 = \frac{20}{5} = 4$ A, $\quad Q \to (3)$

After opening K_1 & closing K_2



$$\omega = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{25 \times 10^{-3} \times 10 \times 10^{-6}}}$$

$\omega = 2000 \text{rad/s}$

$\omega = 2 \text{krad/s}$

$R \to (2)$

$i_0 = 4$

$$Q_0 = \frac{i_0}{\omega} = \frac{4}{2 \times 10^3} = 2 \text{mC}$$

$$\therefore V_0 = \frac{Q_0}{C} = \frac{2 \times 10^3}{10} = 200$$

18. (d) Given,

$W_X = 10$ g
$P_x = 2$ atm
$W_Y = 80$ g
$P_Y = P_{total} - P_x$
$\Rightarrow 6 - 2 = 4$ atm

As $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$\frac{(V_{rms})_X}{(V_{rms})_Y} = \sqrt{\frac{M_Y}{M_X}}$$

As we know,

$PV = nRT$

Volume and temperature remains same.

$$P_X V = \frac{W_X}{M_X} RT$$

$$P_Y V = \frac{W_Y}{M_Y} RT$$

$$M_X \propto \frac{W_X}{P_X}$$

$$M_Y \propto \frac{W_Y}{P_Y}$$

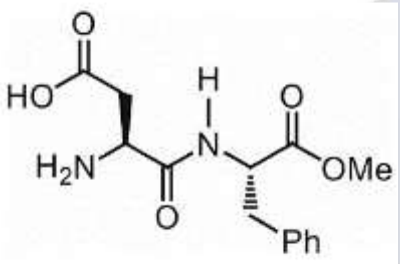
$$\frac{(V_{rms})_X}{(V_{rms})_Y} = \sqrt{\frac{W_Y}{P_Y} \cdot \frac{P_X}{W_X}} = \sqrt{\frac{80}{4} \times \frac{2}{10}} = \sqrt{4} = \frac{2}{1} = 2:1$$

19. (a)

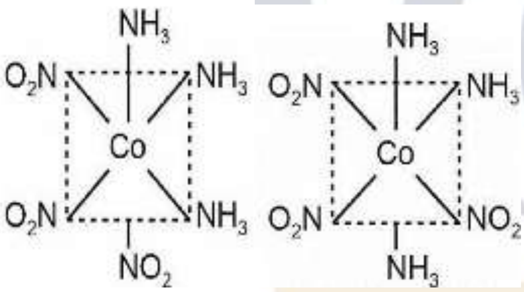
$$3HNO_{+3} \rightleftharpoons \underset{+5}{HNO_3} + 2\underset{+1}{NO} + H_2O$$

H_3O^+, NO_3^- and NO

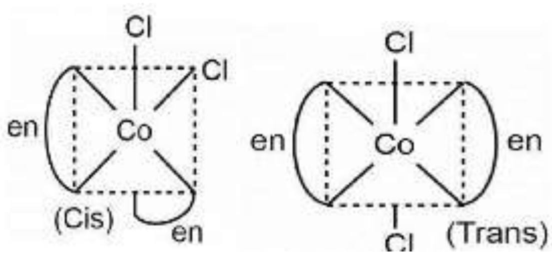
20. (b) Aspartame is



21. (c) Set-I:



(Facial) (Meridional)



Set-II: $[Co(NH_3)_5Cl]SO_4$ and $[Co(NH_3)_5SO_4]Cl$ are ionisation isomers.

22. (a, b, c)

(a) Uncertainty principle rules out existence of definite paths or trajectories of electron and other similar particles. So, option (a) is correct.

(b) Shell or orbit more near to nucleus has less energy than faraway. So, option (b) is also correct.

(c) $E = -13.6\frac{z^2}{n^2}\text{eV/ atom}$

So, $n = 1$ has most negative energy.

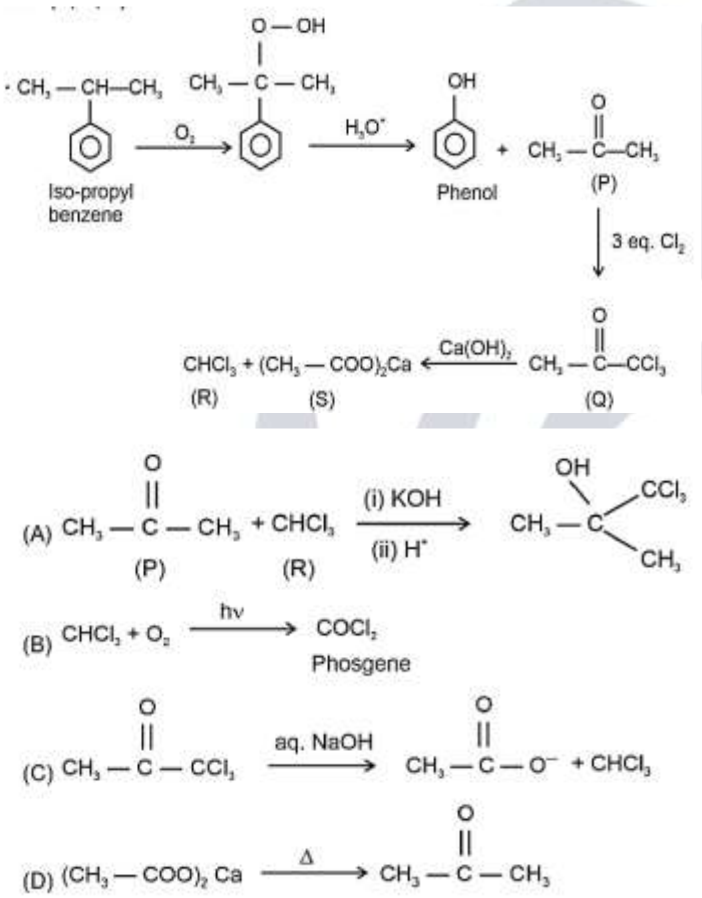
So, option (c) is also correct.

(d) $V = V_0 \times \frac{Z}{n}$

when n increases velocity decreases.

So, option (d) is incorrect.

23. (a, b, d)



24. (a, d)

(a) CO_2, C_2H_4 and HCl follow octet rule.

(b) O_3 and HCl and follow octet rule.

(c) None of them follow octet rule.

(d) CO_2, O_3 and C_2H_4 follow octet rule.

Correct answer is (a) and (d)

25. (8120)

$X \rightarrow Y$ is an isothermal process an ideal gas:

$\Delta H = 0$

$Y \rightarrow Z$ is an isochoric process

$\therefore w = 0$

$\Delta U = nC_{V,m}(T_2 - T_1)$

$= 5 \times 12(415 - 335)$

$= 4800\ J$

$\Delta H = \Delta U + \Delta(PV)$

$= \Delta U + nR\Delta T$

$= 4800 + 5 \times 8.3 \times (415 - 335)$

$= 8120\ J$

26. (3) Rate of reaction (according to slowest step)

$\Rightarrow r = k_2[\ N_2O_2][H_2]$

Now for intermediate $[N_2O_2]$,

$$\frac{k_1}{k_{-1}} = \frac{[N_2O_2]}{[NO]^2}$$

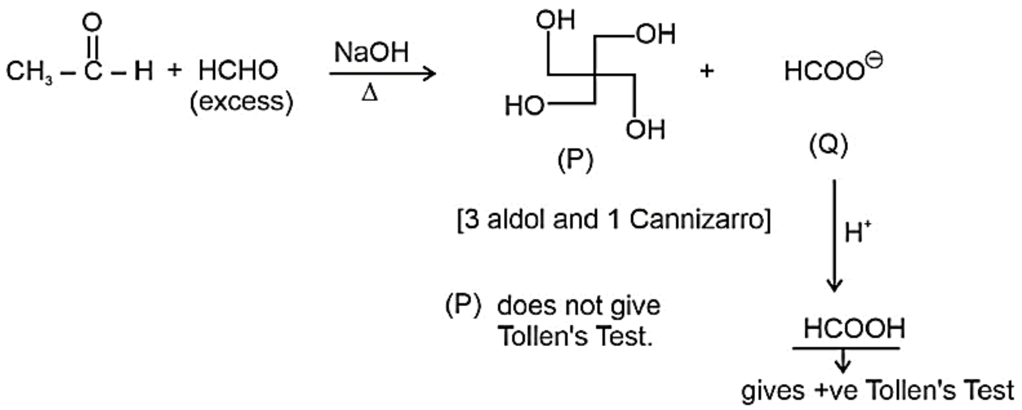
$$\Rightarrow\ [N_2O_2] = \frac{k_1}{k_{-1}}[NO]^2$$

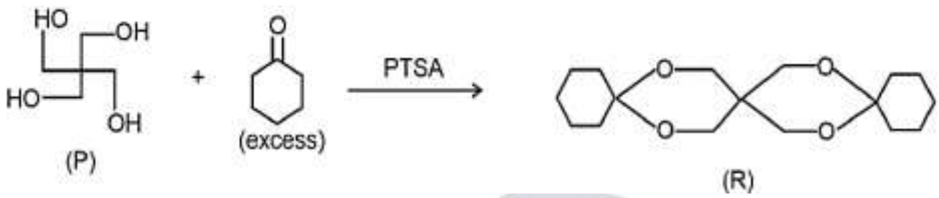
from equation (1) and (2)

$$r = \frac{k_2 k_1}{k_{-1}}[NO]^2[H_2]$$

Overall order of reaction $= 2 + 1 = 3$

27. (18)





Number of CH_2 groups in R = 14

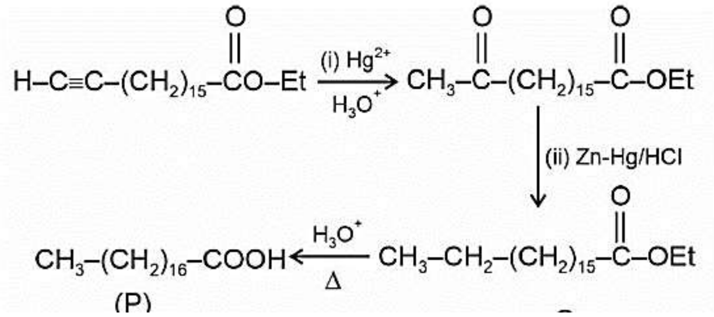
Number of O-atoms = 4

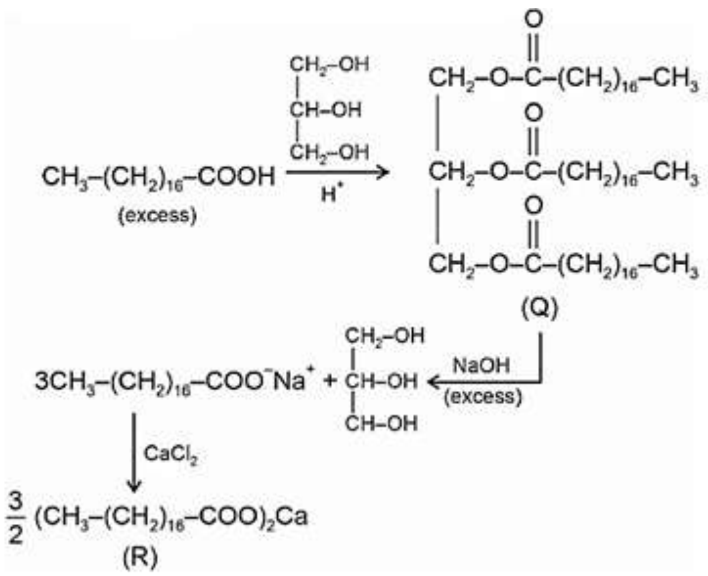
Required Answer = 14 + 4 = 18

28. (1) Total number of electron in $Ni(CO)_4$ = 84

Species		Total electron
$V(CO)_6$	-	107
$Cr(CO)_5$	-	94
$Cu(CO)_3$	-	71
$Mn(CO)_5$	-	95
$Fe(CO)_5$	-	96
$[Co(CO)_3]^{3-}$	-	72
$[Cr(CO)_4]^{4-}$	-	84
$Ir(CO)_3$	-	119

29. (909)





1 mole of Q will give 1.5 mole of R.

So, mass of R produced $= 606\text{ g} \times 1.5$

$= 909\text{ g}$

30. (1)

$[Mn(NH_3)_6]^{3+}$: Paramagnetic

$[MnCl_6]^{3-}$: Paramagnetic

$[FeF_6]^{3-}$: Paramagnetic

$[CoF_6]^{3-}$: Paramagnetic

$[Fe(NH_3)_6]^{3+}$: Paramagnetic

$[\text{ Co(en) }3]^{3+}$: Diamagnetic

Only 1 complex is diamagnetic.

31. (c)

(P) $KCl + AgNO_3 \longrightarrow AgCl \downarrow + KNO_3$

Cl^- is replaced by NO_3^-

Conductance will first decrease and then after equivalence point, it will increase

$P \longrightarrow 3$

(Q) $AgNO_3 + KCl \longrightarrow AgCl + KNO_3$

$Ag^+$is replaced by K^+

Conductance will first increase slightly and then will increase further

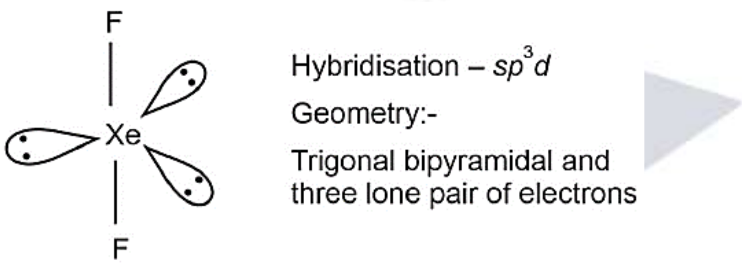
(R) $NaOH + HCl \longrightarrow NaCl + H_2O$

$OH^-$is replaced by Cl^-

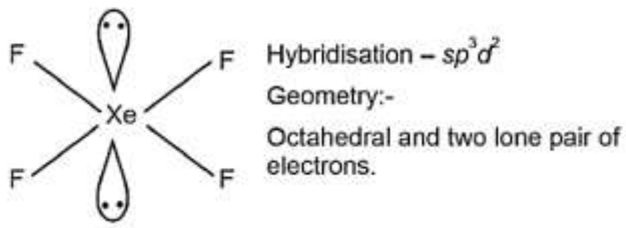
(S) $NaOH + CH_3COOH \longrightarrow CH_3CCONa + H_2OOH^-$is replaced by CH_3COO^-conductance will first decrease and them become almost constant due to buffer formation.

32. (b)

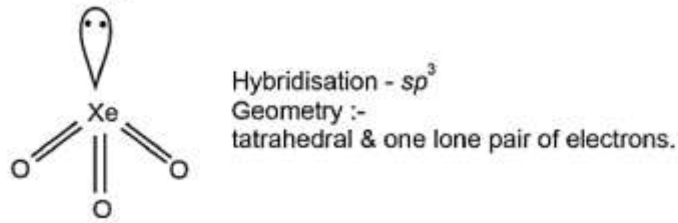
Sol. XeF_2 :



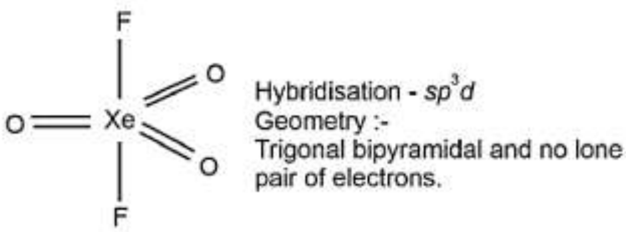
XeF_4 :



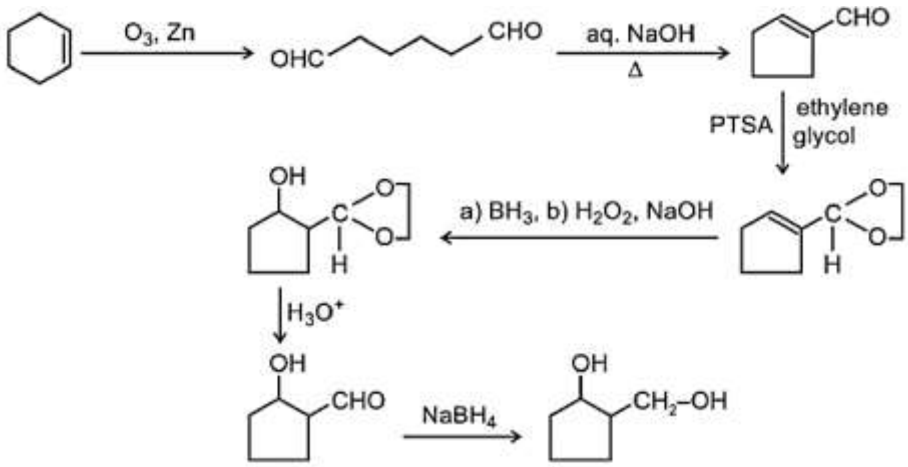
XeO_3 :



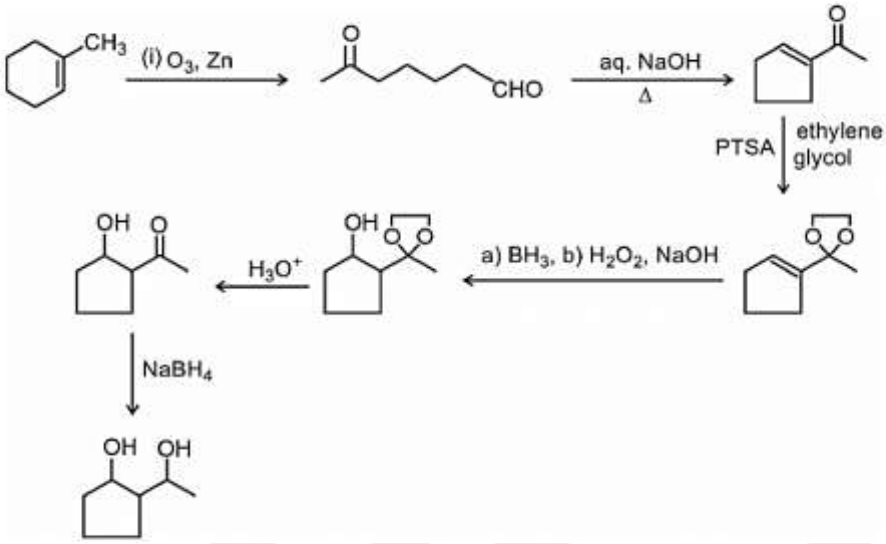
XeO_3F_2 :



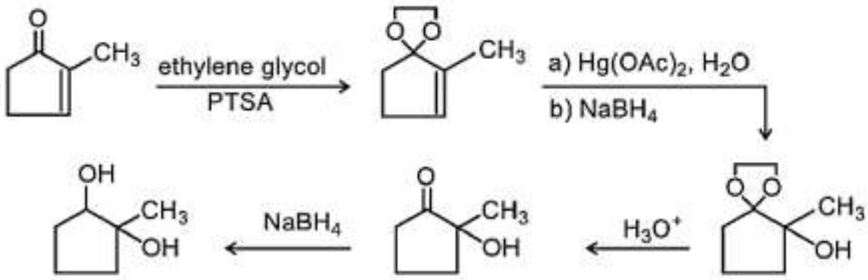
33. (a)

(P) 

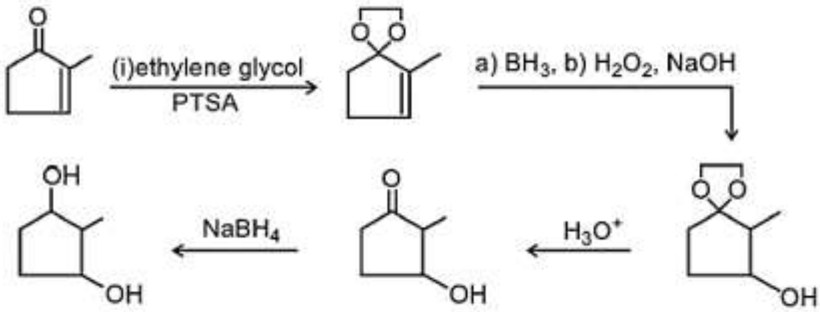
P-3

(Q) 

Q-5

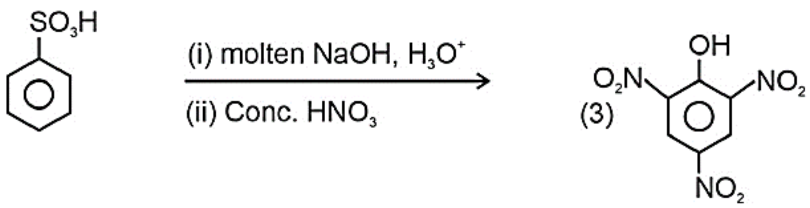
(R) 

R-4

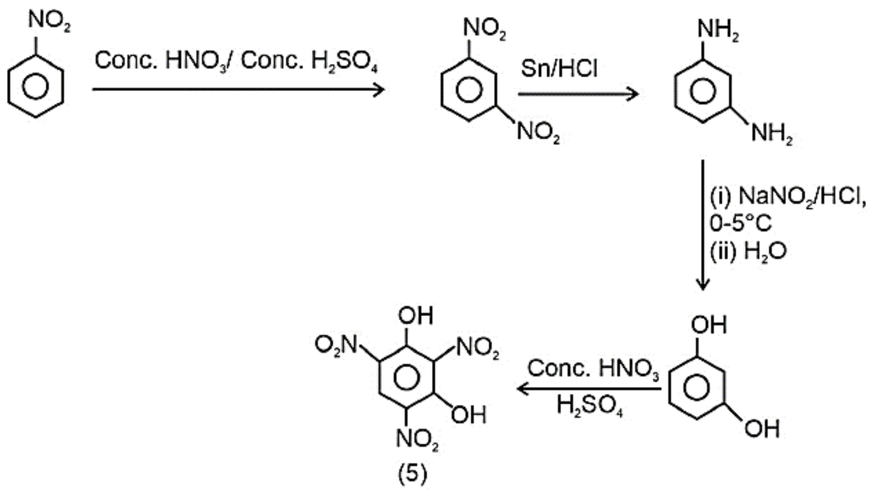
(S) 

S-1

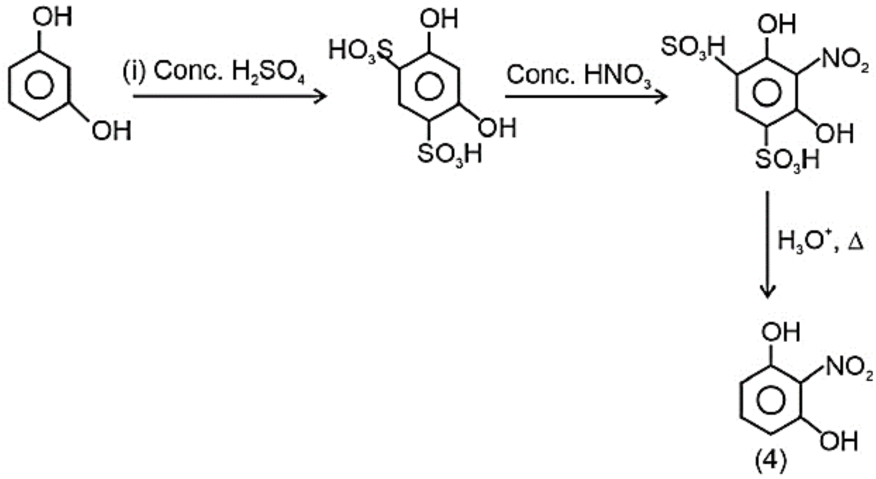
34. (c)


SO_3H
(i) molten NaOH, H_3O^+
(ii) Conc. HNO_3
OH
O_2N
NO_2
(3)
NO_2

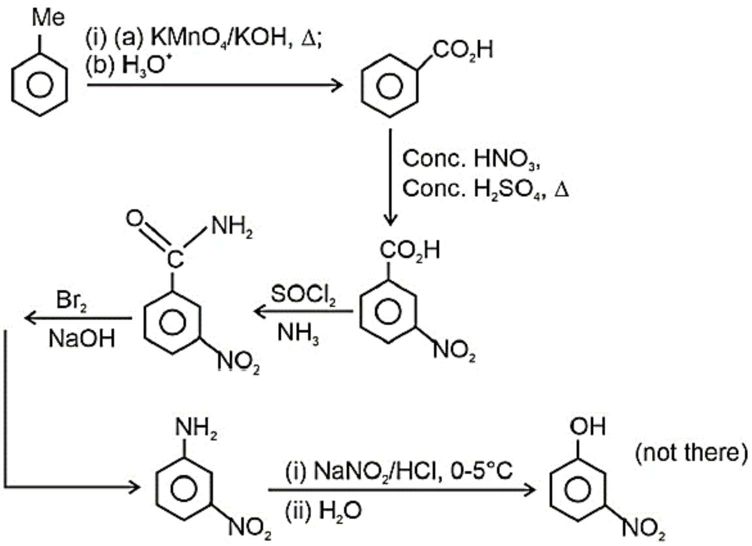
(Q)


NO_2
Conc. HNO_3/ Conc. H_2SO_4
NO_2
NO_2
Sn/HCl
NH_2
NH_2
(i) $NaNO_2$/HCl, 0-5°C
(ii) H_2O
OH
OH
Conc. HNO_3
H_2SO_4
OH
O_2N
NO_2
OH
NO_2
(5)

(R)


OH
OH
(i) Conc. H_2SO_4
HO_3S
OH
OH
SO_3H
Conc. HNO_3
SO_3H
OH
NO_2
OH
SO_3H
H_3O^+, Δ
OH
NO_2
OH
(4)

(S)


Me
(i) (a) $KMnO_4$/KOH, Δ;
(b) H_3O^+
CO_2H
Conc. HNO_3,
Conc. H_2SO_4, Δ
CO_2H
NO_2
$SOCl_2$
NH_3
O
NH_2
C
NO_2
Br_2
NaOH
NH_2
NO_2
(i) $NaNO_2$/HCl, 0-5°C
(ii) H_2O
OH
NO_2
(not there)

35. (b) $\lim\limits_{t\to x}\frac{t^{10}f(x)-x^{10}f(t)}{t^9-x^9}=1$

$$\text{limit}_{t\to x}\frac{10t^9f(x)-f'(t)x^{10}}{9t^8}=1$$

$$\Rightarrow 10x^9f(x)-f'(x)x^{10}=9x^8$$

$$\Rightarrow f'(x)-\frac{10}{x}f(x)=-\frac{9}{x^2}$$

$$\text{IF}=e^{-\int\frac{10}{x}dx}=\frac{1}{x^{10}}$$

$\therefore$ Soln

$$\frac{y}{x^{10}}=\int-\frac{9}{x^{10}}\times\frac{1}{x^2}dx$$

$$=-9\int x^{-12}dx$$

$$\frac{y}{x^{10}}=\frac{9}{11}x^{-11}+C$$

$$\because y(1)=2\Rightarrow C=\frac{13}{11}$$

$$\Rightarrow y=\frac{9}{11x}+\frac{13}{11}x^{10}$$

36. (c) Let P (knows answer) $=k$

P (guesses) $=1-k$

$$P\left(\frac{\text{correct ans}}{\text{guessed}}\right)=\frac{1}{2}$$

$$P\left(\frac{\text{guessed}}{\text{correct answer}}\right)=\frac{P\text{ (guessed) }P\left(\frac{\text{correct ans}}{\text{guessed}}\right)}{P\text{ (guessed) }P\left(\frac{\text{correct ans}}{\text{guessed}}\right)+P(\text{ knows })P\left(\frac{\text{correct ans}}{\text{knows}}\right)}$$

$$=\frac{(1-k)\left(\frac{1}{2}\right)}{(1-k)\left(\frac{1}{2}\right)+k(1)}=\frac{1}{6}$$

$$\Rightarrow(3-3k)=\frac{1}{2}+\frac{k}{2}$$

$$\Rightarrow\frac{5}{2}=\frac{7k}{2}\Rightarrow k=\frac{5}{7}$$

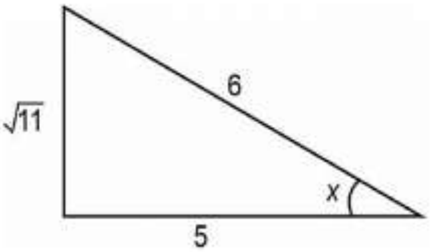
37. (b) Let $E=\sin 6x\cos\frac{11x}{2}-\cos 6x\sin\frac{11x}{2}+\cos 6x\cos\frac{11x}{2}+\sin 6x\sin\frac{11x}{2}$

$$E=\sin\frac{x}{2}+\cos\frac{x}{2}$$

Now, $E^2 = 1 + \sin x$

$\because \cot x = \dfrac{-5}{\sqrt{11}}$

$= 1 + \dfrac{\sqrt{11}}{6}$

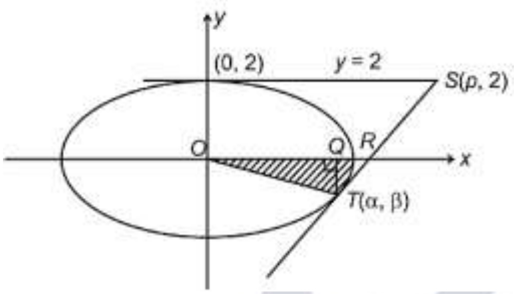


$\therefore\ E = \sqrt{\dfrac{6+\sqrt{11}}{6}}$

$= \sqrt{\dfrac{12+2\sqrt{11}}{12}}$

$= \dfrac{\sqrt{11}+1}{2\sqrt{3}}$

38. (a)



$q = 2$

Area (ORT) $= \dfrac{3}{2}$

$\Rightarrow \left|\dfrac{1}{2} \times \text{OR} \times \text{QT}\right| = \dfrac{3}{2}$

$\Rightarrow \left|\dfrac{1}{2} \times 3 \times \beta\right| = \dfrac{3}{2}$

$\Rightarrow \beta = -1$

$\therefore \dfrac{\alpha^2}{9} + \dfrac{\beta^2}{4} = 1$

$\Rightarrow \dfrac{\alpha^2}{9} = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

$\Rightarrow \alpha^2 = \dfrac{27}{4} \Rightarrow \alpha = \dfrac{3\sqrt{3}}{2}$

Tangent at T

$$T = 0$$

$$\frac{x \cdot \frac{3\sqrt{3}}{2}}{9} + \frac{y(-1)}{4} = 1|_{(p,2)}$$

$$\Rightarrow \frac{p\sqrt{3}}{6} - \frac{1}{2} = 1 \Rightarrow \frac{p\sqrt{3}}{6} = \frac{3}{2} \Rightarrow p = 3\sqrt{3}$$

$$\therefore p = 3\sqrt{3}, q = 2$$

39. (a, c, d)

$S = \{a + b\sqrt{2} : a, b \in Z\}$

For $b = 0; Z \subset S$

$T_1 = \{(-1+\sqrt{2})^n : n \in N\}$ and $T_2 = \{(1+\sqrt{2})^n : n \in N\}$

For $n \in N$ elements of T_1 and T_2 are of the form $a + b\sqrt{2}$

Hence $Z \cup T_1 \cup T_2 \subset S$

Now, $-1 + \sqrt{2} < 1$ and its higher powers decreases

$\Rightarrow (-1+\sqrt{2})^n < 1$ and can be made in $\left(0, \frac{1}{2024}\right)$ for some higher n.

$1 + \sqrt{2} > 1$ and its higher power increases

$\Rightarrow (1+\sqrt{2})^n$ can be made in $(2024, \infty)$ for some higher n.

$\cos \pi(a + b\sqrt{2}) + i\sin \pi(a + b\sqrt{2}) \in Z$ if

$a + b\sqrt{2}$ is an integer $\Rightarrow b = 0$

40. (b) $ax^2 + 2bxy + cy^2 > 0$

$$y, x \in \mathbb{R} - \{(0,0)\}$$

$$\Rightarrow c\left(\frac{y}{x}\right)^2 + 2b\left(\frac{y}{x}\right) + a > 0$$

$$\Rightarrow c > 0, D < 0$$

$$4b^2 - 4ac < 0$$

$$\Rightarrow b^2 < ac$$

(a) $\left(2, \frac{7}{2}, 6\right)$

$\left(\frac{7}{2}\right)^2 > 2 \times 6$

$\therefore$ option A is incorrect

(b) If $\left(3, b, \frac{1}{12}\right) \in S$

$\Rightarrow b^2 < 3 \cdot \frac{1}{12}$
$\Rightarrow b^2 < \frac{1}{4}$
$\Rightarrow 4b^2 < 1$
$\Rightarrow |2b| < 1$ option B is correct

(c) $ax + by = 1$

$bx + cy = -1$

$D = \begin{vmatrix} a & b \\ b & c \end{vmatrix} = ac - b^2 \neq 0$

$\therefore$ unique solution option C is correct.

(d) $(a+1)x + by = 0$

$bx + (c+1)y = 0$

$D = \begin{vmatrix} (a+1) & b \\ b & (c+1) \end{vmatrix}$

$= (a+1)(c+1) - b^2$

$\Rightarrow ac - b^2 + a + c + 1$

$b^2 < ac \Rightarrow ac$ is +ve

$\Rightarrow$ a and c are positive then $(ac - b^2) + a + c + 1 > 0 \therefore$ unique solution

$\therefore$ option D is correct.

41. (a, b, c)

$S: \{((x-1)^2 + (y-2)^2 + (z-3)^2) - ((x-4)^2 + (y-2)^2 + (z-7)^2) = 50\}$

$\Rightarrow S: \{6x + 8z - 105 = 0\}$

Similarly $T = \{6x + 8z - 5 = 0\}$

S represents a plane. So it will contain a triangle of area 1 . So (a) is correct.

T represents a plane. So (B) is correct.

$S\&T$ are two parallel planes at a distance of 10 units from each other.

$\therefore$ (C) is correct and (D) is incorrect.

42. (8)

$a = 3\sqrt{2} \Rightarrow a^2 = 18$

Notice that $1080 = 5 \cdot 6^3 \Rightarrow$

$5^{\frac{1}{6}} \cdot 6^{\frac{1}{2}} = (1080)^{\frac{1}{6}} = \frac{1}{b} \Rightarrow 1080^{\frac{1}{2}} = \frac{1}{b^3}$

$\Rightarrow 3x + 2y = \log_a (a^2)^{\frac{5}{4}} = \frac{5}{2}$..........(i)

$2x - y = \log_b \frac{1}{b^3} = \log_b b^{-3} = -3$..........(ii)

$\Rightarrow$ Solving (i) & (ii)

$\Rightarrow x = \frac{-1}{2}, y = 2 \Rightarrow 4x + 5y = 8$

43. (20)

$\because f(1) = -9 \Rightarrow 1 + a + b + c = -9$......... (i)

$4x^3 + 3ax^2 + 2bx = 0$

$\Rightarrow x = 0,\ 4x^2 + 3ax + 2b = 0$............... (ii)

$\Rightarrow \sqrt{3}i$ and $-\sqrt{3}i$ are roots of (2)

$\Rightarrow \sqrt{3}i - \sqrt{3}i = \frac{-3a}{4}, \sqrt{3}i(-\sqrt{3}i) = \frac{2b}{4}$

$\Rightarrow a = 0, b = 6, c = -16$

$\Rightarrow f(x) = 0 \Rightarrow x^4 + 6x^2 - 16 = 0$

$\Rightarrow\ x^2 = \frac{-6 \pm \sqrt{36 + 64}}{2} = -3 \pm 5 = 2, -8$

$x = -\sqrt{2}, +\sqrt{2}, -2\sqrt{2}i, 2\sqrt{2}i$

$\Rightarrow |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2 = 20$

44. (16)

$|A| = -(e - d) + c(b - a) = \pm 1$

Case (i) : $c = 0 \Rightarrow (e, d) = (1,0), (0,1) \rightarrow 2$ ways

b and a can be each 2 ways

$\Rightarrow$ Total $= 8$ ways

Case (ii): $c \neq 0 \Rightarrow c = 1$

$\Rightarrow d - e + b - a = \pm 1$

$$\left.\begin{matrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}\right\} \rightarrow 4 \times 2 = 8 \text{ ways}$$

Total $= 16$ ways

45. (665) Number of required ways

$= \frac{9!}{2!3!4!} - \left(n(s_1 \in X) + n(s_2 \in Y) - n(s_1 \in X \text{ and } s_2 \in Y)\right)$

$= \frac{9!}{2!\,3!\,4!} - \left(\frac{8!}{1!\,3!\,4!} + \frac{8!}{2!\,2!\,4!} - \frac{7!}{1!\,2!\,4!}\right)$

$= 665$

46. (5)

$(\overrightarrow{OP} \times \overrightarrow{OQ}) \cdot \overrightarrow{OR} = 0$

$$\begin{vmatrix} \frac{\alpha-1}{\alpha} & 1 & 1 \\ 1 & \frac{\beta-1}{\beta} & 1 \\ 1 & 1 & \frac{1}{2} \end{vmatrix} = 0 \Rightarrow \frac{\alpha-1}{\alpha}\left(\frac{\beta-1}{2\beta} - 1\right) - \left(\frac{1}{2} - 1\right) + 1\left(1 - \frac{\beta-1}{\beta}\right) = 0$$

$\frac{\alpha-1}{\alpha}\left(\frac{-\beta-1}{2\beta}\right) + \frac{1}{2} + \frac{1}{\beta} = 0$

$\Rightarrow \frac{\beta+2}{2\beta} = \frac{\alpha\beta+\alpha-\beta-1}{2\alpha\beta}$

$\Rightarrow \alpha\beta + 2\alpha = \alpha\beta + \alpha - \beta - 1 \Rightarrow \alpha + \beta + 1 = 0$…….. (i)

Now $(\alpha, \beta, 2)$ lies on $3x + 3y - z + I = 0$

$\Rightarrow 3(\alpha + \beta) - 2 + I = 0$………………(ii)

$\Rightarrow -3 - 2 + I = 0 \Rightarrow I = 5$

47. (42)

$\sum_{x=0}^{4} xP(x) = \frac{5}{2}$

$\sum_{x=0}^{4} x^2P(x) = ?$

$(0, P(0)), (1, P(1)), (2, P(2)), (3, P(3)), (4, P(4))$

$K = P(1) - P(0) = P(2) - P(1) = P(3) - P(2) = P(4) - P(3)$

$P(1) = K + P(0)$

$P(2) = 2K + P(0)$

$P(3) = 3K + P(0)$

$P(4) = 4K + P(0)$

$P(0) + P(1) + P(2) + P(3) + P(4) = 1$

$\Rightarrow 5P(0) + 10K = 1$

$K + P(0) + 4K + 2P(0) + 9K + 3P(0) + 16K + 4P(0) = \frac{5}{2}$

$30K + 10P(0) = \frac{5}{2}$

$\therefore\ 10K = \frac{1}{2}$

$K = \frac{1}{20}, P(0) = \frac{1}{10}$

$P(1) = \frac{3}{20}, P(2) = \frac{4}{20}, P(3) = \frac{5}{20}, P(4) = \frac{6}{20}$

$\sum_{x=0}^{4} x^2 P(x) = 8$

$\therefore\ \text{Variance} = 8 - \frac{25}{4} = \frac{32-25}{4} = \frac{7}{4}$

$\therefore\ 24\alpha = \frac{24 \times 7}{4} = 42$

48. (c)

$x^2 + x - 1 = 0 \rightarrow$ roots are α and β

$\alpha + \beta = -1\ \alpha\beta = -1$

Set $T = \{1, \alpha, \beta\}$ $M = (a_{ij})3 \times 3$

$R_i = a_{i1} + a_{i2} + a_3\ C_j = a_{1j} + a_{2j} + a_{3j}$

(P) $R_i = C_j = 0$ for all i, j

$\alpha + \beta = -1\ T = \{1, \alpha, \beta\}$

Number of matrices

$= \lfloor 3 \times 2 \times 1 = 12$

Number of ways to arrange $1, \alpha, \beta$ in R_1

Number of ways to arrange $1, \alpha, \beta$ in R_2

$\begin{bmatrix} 1 & \underline{\alpha} & \underline{\beta} \end{bmatrix}$

(Q) Number of symmetric matrices = ?

$C_j = 0 \forall j$

Number of symmetric matrices

$= \lfloor 3 \times 1 = 6 \begin{bmatrix} 1 & \alpha & \beta \\ \alpha & \beta & 1 \\ \beta & 1 & \alpha \end{bmatrix}$

(R) $M \rightarrow$ skew symmetric of 3×3

$|M| = 0\ a_{ij} \in T$ for $i > j$

$$M\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}a_{12}\\0\\-a_{23}\end{pmatrix}$$

$$\begin{bmatrix}0 & -a_{21} & -a_{31}\\a_{21} & 0 & -a_{32}\\a_{31} & a_{32} & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}a_{12}\\0\\-a_{23}\end{bmatrix}$$

As $x, y, z \in R$ and $a_{12} \& a_{23} \in R \& |M| = 0$

$\therefore$ System has infinite solutions

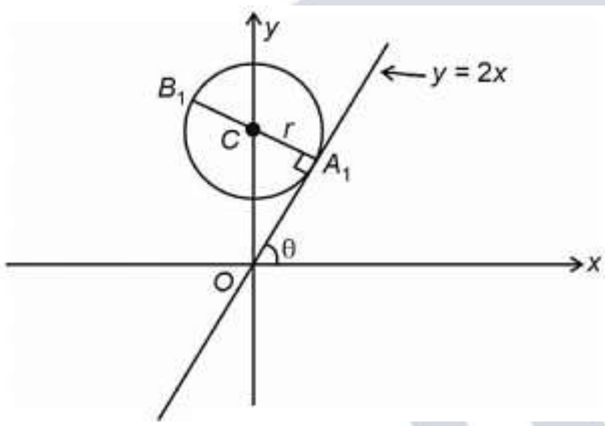
(S) $R_i = 0 \forall i$

$$M=\begin{bmatrix}1 & \alpha & \beta\\ \alpha & \beta & 1\\ \beta & 1 & \alpha\end{bmatrix}$$

$$C_1 \to C_1 + C_2 + C_3 |M| = \begin{vmatrix}1+\alpha+\beta & \alpha & \beta\\1+\alpha+\beta & \beta & 1\\1+\alpha+\beta & 1 & \alpha\end{vmatrix} = 0$$

(P) → (2)(Q) → (4)(R) → (3)(S) → (5)

49. (c)



Slope of line $= 2 \Rightarrow \tan\theta = 2$

$C(0, \alpha)\ \alpha > 0$
$\alpha + r = 5 + \sqrt{5}$................(i)

Line $y = 2x$ is tangent to the circle

$$\therefore \left|\frac{0-\alpha}{\sqrt{4+1}}\right| = r$$

$\Rightarrow |-\alpha| = r\sqrt{5}$

$\Rightarrow \alpha = r\sqrt{5}$ as $\alpha > 0$

From equation (1) $r\sqrt{5} + r = 5 + \sqrt{5}$

$\Rightarrow\ r(\sqrt{5}+1) = \sqrt{5}(\sqrt{5}+1)$

$\Rightarrow r = \sqrt{5}$

And $\alpha = r\sqrt{5} = \sqrt{5} \times \sqrt{5} = 5$

Centre $C(0,5)$

$OC = 5\ A_1C = \sqrt{5}$

$\therefore\ OA_1 = \sqrt{25-5} = \sqrt{20} = 2\sqrt{5}$

$\tan\theta = 2$ (from figure)

$\cos\theta = \frac{1}{\sqrt{5}}\ \sin\theta = \frac{2}{\sqrt{5}}$

$A_1(0 + OA_1\cos\theta, 0 + OA_1\sin\theta)$

$A_1\left(2\sqrt{5} \times \frac{1}{\sqrt{5}}, 2\sqrt{5} \times \frac{2}{\sqrt{5}}\right)$

$A_1(2,4)$

Let $B_1(x_1, y_1)$

$\therefore\ \frac{x_1+2}{2} = 0$ and $\frac{y_1+4}{2} = 5$

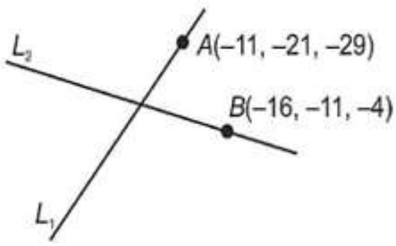
$x_1 = -2\ y_1 = 6$

$B_1(-2,6)$

$\alpha = 5\ r = \sqrt{5}\ A_1(2,4)$

$B_1(-2,6)$

50. **(c)** Vector parallel to the line L_1 (say $\vec{b}_1$) $== \hat{\imath} + 2\hat{\imath} + 3\hat{k}$



Normal vector of plane $(\vec{n})$ containing L_1 and L_2 will be perpendicular to both $\vec{b}_1$ and $\overrightarrow{AB}$

$\Rightarrow\ \vec{n} = p(\overrightarrow{AB} \times \vec{n}) = p(5\hat{\imath} - 10\hat{\jmath} - 25\hat{k}) \times (i + 2\hat{\jmath} + 3\hat{k})$

$= p(20\hat{\imath} - 40\hat{\jmath} + 20\hat{k})$

$\Rightarrow \hat{n} = \frac{1}{\sqrt{6}}\hat{\imath} - \frac{2}{\sqrt{6}}\hat{\jmath} + \frac{1}{\sqrt{6}}\hat{k}$

Now, vector parallel to L_2 (say $\vec{b}_2$) is perpendicular to $\vec{n} \Rightarrow \vec{b}_2 \cdot \vec{n} = 0$

$(3\hat{\imath} + 2\hat{\jmath} + \gamma\hat{k}) \cdot p(20\hat{\imath} - 40\hat{\jmath} + 20\hat{k}) = 0$
$\Rightarrow \gamma = 1$

Now, for point of intersection (POI)

$L_1: \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3} = \lambda$ and $L_2: \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{\gamma} = u$

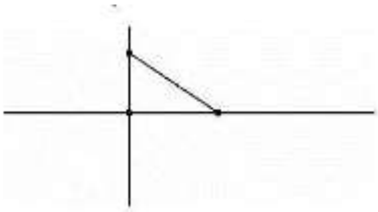
Comparing x and y coordinates, $-11 + \lambda = -16 + 3u$ and $-21 + 2\lambda = -11 + 2u$

$\Rightarrow \lambda = 10, u = 5$

$\Rightarrow$ POI i.e., $\overrightarrow{OR_1}: (-\hat{i} - \hat{j} + \hat{k})$ and $\overrightarrow{OR} \cdot \hat{n} = \sqrt{\frac{2}{3}}$

51. (c)

$$g(x) = \begin{cases} 1 - 2x, & 0 \le x \le \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$



$$g\left(\frac{1}{2} - x\right) = \begin{cases} 1 - 2\left(\frac{1}{2} - x\right), & 0 \le \frac{1}{2} - x \le \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} 2x, & 0 \le x \le \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$g(x) + g\left(\frac{1}{2} - x\right) = \begin{cases} 1, & 0 \le x \le \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

Now, option (P), at $b = 1$

$h(x) = g(x) + g\left(\frac{1}{2} - x\right)$ has range $\{0,1\}$

(P) → (5)

Option (Q) at $a = 1, h(x) = f(x)$

$$f'(0^+) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{h^2 \sin\frac{1}{h}}{h}$$
$$f'(0^-) = \lim_{h\to 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h\to 0} h \sin h = 0$$
$$\Rightarrow f'(0^+) = f'(0^-)$$

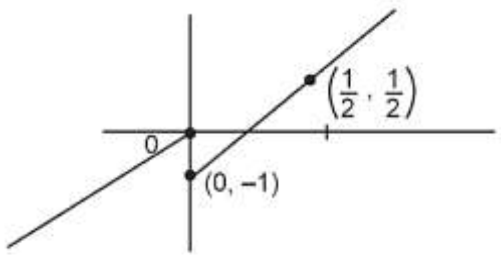
as $f'(0^+) = f(0^-)$

$\Rightarrow h(x) = f(x)$ is differentiable at $x = 0$ and all other points $f(x) = h(x)$ is differentiable as product of two differentiable functions

(Q) → (3)

Option (R)

$$h(x) = x - g(x) = \begin{cases} 3x - 1, & 0 \le x \le \frac{1}{2} \\ x, & \text{otherwise} \end{cases}$$



by graph $h(x)$ has range R and onto (R) → (2)

(S) at $d = 1 h(x) = g(x)$ has range $[0,1]$(S) → (4)