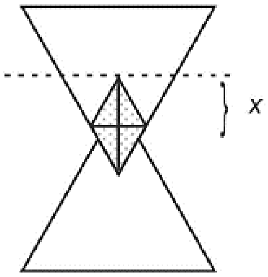
1. (a)

For $x < L$

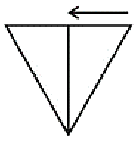


Area $= \frac{x}{2}\frac{x}{2}\tan 30 \times 4 \times \frac{1}{2} = \frac{1}{2}x^2 \tan 30$

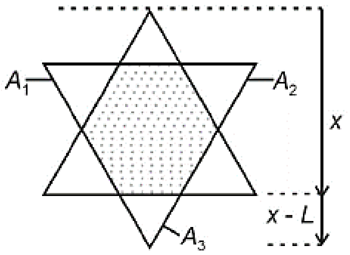
$\phi' = B_0 \times \tan 30V$

$\varepsilon \propto x$

$L\tan 30°$



$x \geq L$



2. (a)

$$\frac{Gmm}{r_0^2} - \frac{3\alpha m}{r_0^4} = \frac{mv^2}{r_0}$$

$$T = \frac{2\pi r_0}{\sqrt{\frac{Gmr_0^2 - 3\alpha}{r_0^3}}}$$

$$T_0^2 = \frac{4\pi^2}{Gm} r_0^3$$

$$\frac{T^3 - T_0^2}{T_1^2} = 1 - \frac{T_0^2}{T_1^2}$$

$$= 1 - \frac{4\pi^2}{Gm}\frac{r_0^3}{4\pi^2 r_0^2}\frac{Gmr_0^2 - 3\alpha}{r_0^3}$$

$$= 1 - 1 + \frac{3\alpha}{Gmr_0^2}$$

$$= \frac{3\alpha}{GMr_0^2}$$

3. (a)

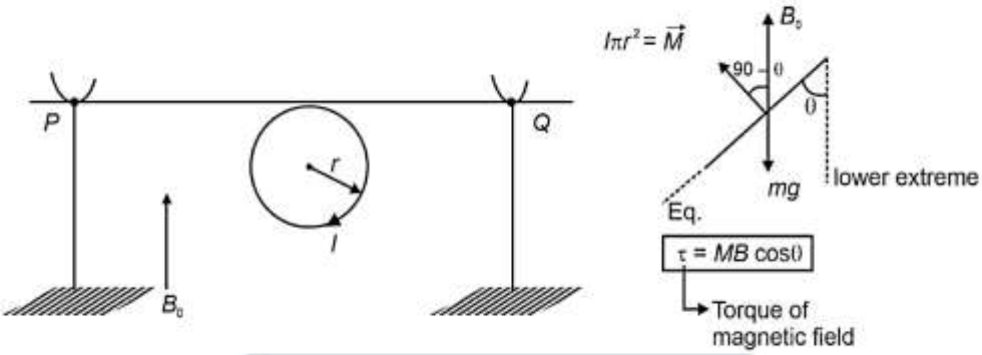
$$\frac{1}{\lambda_\alpha} = \frac{3}{4}R(Z-1)^2 p$$

$$\lambda_{cut} = \frac{hc}{eV}$$

$\Rightarrow$ Ratio $\propto \frac{1}{(Z-1)^2}$ for same beam

$$\frac{Z}{x} = \frac{40^2}{45^2} \Rightarrow x = \frac{45^2}{40^2} \cdot 2 \approx 2.53$$

4. (a)

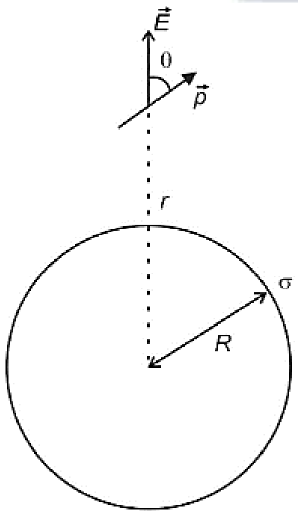


Now $\frac{\text{For equilibrium}}{\tau = mgr\sin\theta}$

$$I\pi r^2 B_0 \cos\theta = mgr\sin\theta$$

$$\tan\theta = \frac{I\pi r B_0}{mg}$$

5. (b, d)



$$\tau = |\vec{p} \times \vec{E}|$$

$$I\alpha = p_0 E \sin\theta$$

$$\alpha = \frac{p.\theta}{l}\left(\frac{1}{4\pi\varepsilon_0}\frac{\sigma 4\pi R^2}{r^2}\right)$$

$\alpha = \left(\frac{p_0 \sigma R^2}{I \varepsilon_0 r^2}\right) \cdot \theta$

$\therefore \omega = \sqrt{\frac{p_0 \sigma R^2}{I \varepsilon_0 r^2}}$

For $r = 2R$

$\omega = \sqrt{\frac{p_0 \sigma}{4 I \varepsilon_0}}$(C is incorrect)

Also, for $r = 10R$

$\omega = \sqrt{\frac{p_0 \sigma}{4/(100)}}$ (D is correct)

It will oscillate for any finite value of $r > R$. (B is correct)

6. (a, b, d) Work done in pushing the ball

$W = (v\rho g)d - (v\sigma g)d$

Where $\rho \rightarrow$ Density of water $\sigma \rightarrow$ Density of ball

$\Rightarrow W = \frac{4}{3}\pi R^3 \times 10 \times 0.7 \left[1000 - \frac{3}{4} \times \frac{10^{-3}}{R^3}\right]$

$W = 0.077 J$ [A is correct]

$\Rightarrow$ When ball is released at bottom same work (i.e.0.077J) is done on ball.

$\therefore \frac{1}{2} mv^2 = 0.077$

$v = \sqrt{\frac{0.077 \times 2}{\frac{22}{7} \times 10^{-3}}}$

$= 7 m/s$ [B is correct]

$\Rightarrow$ also, $H = \frac{v^2}{2g} = \frac{7 \times 7}{2 \times 10} = 2.45 m$ [C is incorrect]

$\Rightarrow$ Net force $F_{net} = v\sigma g - v\sigma g = 0.11 N$

Also, viscous force is maximum when $v = 7 m/s$.

$\therefore (F_v)_{max} = 6\pi\eta r$

$$= 6 \times \frac{22}{7} \times 10^{-3}\left(\frac{3}{2} \times 10^{-2}\right) \times 7$$
$$= 18 \times 11 \times 10^{-5} N$$

Now,

$$\frac{F_{net}}{(F_v)_{max}} = \frac{500}{9}$$ [D is correct]

7. (a, b)

$x = 2R$

$= 2\frac{mv}{qB}$

$2\frac{\sqrt{2m(e\Delta V)}}{qB}$

For H^+ ion

$x = 3.91cm$
$\simeq 4cm$ (A is correct)

$m = 144(m_P)$
For $= 12(x_{H^+})$
$= 48cm$ (B is correct)

For $1 \le A_M \le 196$
$\Rightarrow (x_1 - x_0)_{min} = 2R_{196} - 2R_1$
$= (14 \times 4) - 4$
$= 52cm$ (C is incorrect)

For $A_M = 196$

$w_{min} = R_{196} = 28cm$ (D is incorrect)

8. (3)

$V = \frac{1}{3}\pi R^2 H$

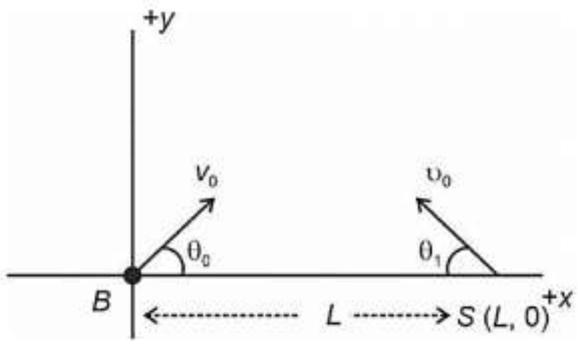
$\Rightarrow \frac{dV}{V} = 2 \cdot \frac{dR}{R} + \frac{dH}{H}$

$\Rightarrow$ % error in measuring volume

$= \left[2 \times \frac{0.2}{20} + \frac{0.2}{20}\right] \times 100$

$= 3$

9. (2)



Let B : Ball

S : Stone

vo: Initial speed of stone.

Since relative acceleration = zero

⇒ Path seen would be straight line

⇒ To meet, $v_0 \sin\theta_0 = v_0 \sin\theta_1$

And $\Delta t = \frac{L}{v_0 \cos\theta_1 + v_0 \cos\theta_0}$

Case I: $v_0 = v_0 \Rightarrow \Delta t_1 = T_1 = \frac{L}{v_0\left[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right]} = \frac{L}{\sqrt{2}v_0}$

Case II : $\sqrt{3}v_0 = v_0 \Rightarrow \Delta t_2 = T_2 = \frac{L}{\sqrt{3}v_0 \cdot \frac{\sqrt{3}}{2} + \frac{v_0}{2}} = \frac{L}{2v_0}$

$\Rightarrow \left(\frac{T_1}{T_2}\right)^2 = (\sqrt{2})^2 = 2$

10. **(3)** For any θ, let us first find the flux inside a cone of half angle θ. we know that for such a cone, solid angle subtended at centre is

$\Omega = 2\pi[1 - \cos\theta]$

⇒ Flux through 1 cone $= \phi_0 = \frac{\Omega}{4\pi} \cdot \frac{Q}{\varepsilon_0} = \frac{Q}{2\varepsilon_0}[1 - \cos\theta]$

⇒ Flux through curved surface

$$= \frac{Q}{\varepsilon_0} - 2\phi_0$$
$$= \frac{Q}{\varepsilon_0} - \frac{Q}{\varepsilon_0}[1 - \cos\theta] = \frac{Q}{\varepsilon_0}\cos\theta$$

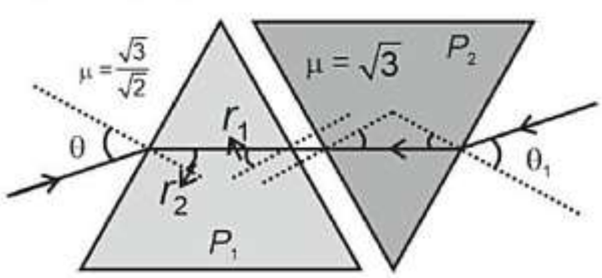
$$\Rightarrow \phi = \frac{Q}{\varepsilon_0} \cdot \frac{\sqrt{3}}{2}$$

And $\frac{\phi}{\sqrt{n}} = \frac{Q}{\varepsilon_0} \cdot \frac{1}{2}$

$\Rightarrow \sqrt{n} = \sqrt{3}$

$\Rightarrow n = 3$

11. (12) By using optical reversibility principle



For prism P_2

→ Minimum deviation

$1 \times \sin\theta_1 = \sqrt{3}\sin r \quad r_1 = r_2 = \frac{A}{2}$

$\sin\theta_1 = \sqrt{3} \times \frac{1}{2} \quad r_1 = r_2 = 30°$

$\Rightarrow i = e = 60°$

For prism P_1

Incident angle will be 60°

$1 \times \sin 60° = \frac{\sqrt{3}}{\sqrt{2}}\sin r_1$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{\sqrt{2}}\sin r_1$

$r_1 + r_2 = 60°$

$\sin r_1 = \frac{1}{\sqrt{2}}$

$r_1 = 45°$

$r_2 = 15°$

$\frac{\sqrt{3}}{\sqrt{2}}\sin(45°) = 1 \times \sin\theta$

$15° = \frac{\pi \times 15}{180}\text{rad} = \frac{\pi}{12}\text{rad}$

$\theta = \sin^{-1}\left[\frac{\sqrt{3}}{\sqrt{2}}\sin\left(\frac{\pi}{12}\right)\right]$

$\beta = 12$

12. (171)

$$E_{\text{Line charge}} = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\Rightarrow \Delta V_{\text{Line charge}} = \int_{0.5}^{2} \frac{\lambda}{2\pi\epsilon_0 r} dr = \frac{\lambda}{2\pi\epsilon_0} \ln 4 \#(i)$$

$$\Delta V_{\text{Sphere}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} - \frac{1}{4\pi\epsilon_0}\frac{Q}{2R}$$

$$\Rightarrow \Delta V_{\text{Net}} = \frac{\lambda}{2\pi\epsilon_0} \ln 4 + \frac{1}{4\pi\epsilon_0}\frac{Q}{2}$$

$= 171$ Volts

13. (96)

Since the situation follow isothermal condition.

$$P_1 V_1 = P_2 V_2$$

$$V_1 = \frac{4}{3}\pi R_1^3, V_2 = \frac{4}{3}\pi R_2^3$$

$$P_1 = P_0 + \Delta P_1, \Delta P_1 = \frac{4T}{R_1}$$

and $P_2 = \frac{8P_0}{27} + \Delta P_2, \Delta P_2 = \frac{4T}{R_2}$

So for isothermal condition

$$(P_0 + \Delta P_1) \times \frac{4}{3}\pi R_1^3 = \left(\frac{8P_0}{27} + \Delta P_2\right) \times \frac{4}{3}\pi R_2^3$$

here $P_0 = 10^5 \text{Pa}$

$\Delta P_1 = 144\text{Pa}$

and $\Delta P_1 \ll P_0$

So $(P_0 + \Delta P_1)\left(\frac{4T}{\Delta P_1}\right)^3 = \left(\frac{8P_0}{27} + \Delta P_2\right)\left(\frac{4T}{\Delta P_2}\right)^3$

$$\frac{P_0}{(\Delta P_1)^3} \approx \frac{8P_0}{27} \times \frac{1}{(\Delta P_2)^3}$$

$$\Delta P_2 = \frac{2}{3}\Delta P_1 = \frac{2}{3} \times (144\text{Pa})$$

$$\Delta P_2 = 96\text{Pa}$$

14. (i) (601.50) (ii) (24.00)

(i)

As central bright fringe position is not changing, the two slits are oscillating with a phase diff of π. For 8^{th} bright fringe

$$y=\frac{8\lambda D}{(0.8+0.04\sin\ \omega t)}\times 10^{3}$$
$$=\frac{8\times 6000\times 10^{-10}\times 10^{3}}{(0.8+0.04\sin\ \omega t)}$$
$$y=\frac{48\times 10^{-4}}{(0.8+0.04\sin\ \omega t)}$$
d varies from 0.84mm to 0.76mm
$$\Delta y=6.015\times 10^{-4}$$
$$=601.50\mu\text{m}$$

(ii)

Finding speed

$$\frac{\delta y}{\delta t}=\frac{\delta}{\delta t}\left(\frac{8\lambda D}{d}\right)$$
$$=-\frac{8\lambda D}{d^2}\frac{\delta d}{(\delta t)}$$
$$v=-\frac{8\lambda D}{d^2}(0.04\omega\cos\ \omega t)\times 10^{-3}$$
$$v_{\max}=\frac{8\lambda D}{d^2}\times 4\omega\times 10^{-5}$$
$$=\frac{8\times 6\times 10^{-7}\times 1\times 4\times 8\times 10^{-7}}{64\times 10^{-8}}$$
$$=24\times 10^{-6}$$
$$=24\mu m/s$$

15. (i) **(00.75)** (ii) **(04.25)**

(i)

At $t=0,2$ is at mean position

$\therefore u_2=$ aw towards left after collision, velocity will exchange

$\therefore v_2=\frac{a\omega}{2}$ towards right

$u_1=a\omega$ towards right

$$\therefore v_{cm}=\frac{3a\omega}{4}$$

$$\frac{v_{cm}}{a\omega}=\frac{3}{4}=0.75$$

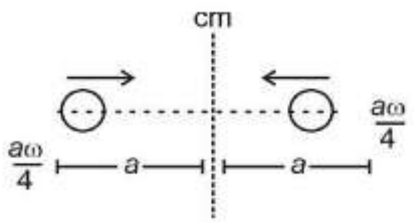
At $t=\frac{\pi}{2\omega}, u_2=0$

After collision, $v_2=\frac{a\omega}{2}$ towards right

$u_1=0$

$\therefore v_{cm} = \frac{a\omega}{4}$ towards right

w.r.t.centre of mass



$v = \omega\sqrt{A^2 - x^2}$

$\frac{a\omega}{4} = \omega\sqrt{A^2 - a^2}$

$\frac{a^2}{16} + a^2 = A^2$

$\frac{17}{16}a^2 = A^2 = b^2$

$\therefore b^2 = \frac{17}{16}a^2 \frac{4b^2}{a^2} = \frac{17}{4}$

(ii)

At $t = 0,2$ is at mean position

$\therefore u_2 = a\omega$ towards left after collision, velocity will exchange

$\therefore v_2 = \frac{a\omega}{2}$ towards right

$u_1 = a\omega$ towards right

$\therefore v_{cm} = \frac{3a\omega}{4}$

$\frac{v_{cm}}{a\omega} = \frac{3}{4} = 0.75$

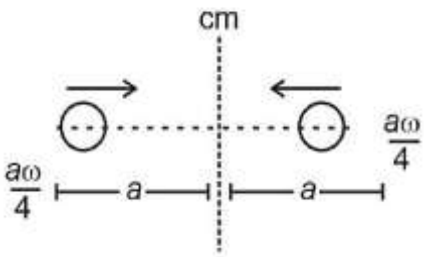
At $t = \frac{\pi}{2\omega}, u_2 = 0$

After collision, $v_2 = \frac{a\omega}{2}$ towards right

$u_1 = 0$

$\therefore v_{cm} = \frac{a\omega}{4}$ towards right

w.r.t.



$v = \omega\sqrt{A^2 - x^2}$

$\frac{a\omega}{4} = \omega\sqrt{A^2 - a^2}$

$\frac{a^2}{16} + a^2 = A^2$

$\frac{17}{16}a^2 = A^2 = b^2$

$\therefore b^2 = \frac{17}{16}a^2$

$\frac{4b^2}{a^2} = \frac{17}{4}$

$= 4.25$

16. (b)

K.E. of electron in n^{th} Bohr's orbit,

K.E. $= 13.6\frac{Z^2}{n^2}$ eV/ atom

$n = 1(\text{H-atom}) \rightarrow \text{K.E.} \propto \frac{1^2}{1^2} = 1$

$n = 1(\text{He}^+\text{ion}) \rightarrow \text{K.E.} \propto \frac{2^2}{1^2} = 4$

$n = 2(\text{He}^+\text{ion}) \rightarrow \text{K.E.} \propto \frac{2^2}{2^2} = 1$

$n = 2(\text{Li}^{2+}\text{ ion}) \rightarrow \text{K.E.} \propto \frac{3^2}{2^2} = \frac{9}{4}$

Highest for $\rightarrow n = 1$ of He^+ion.

17. (d)

MxY_2O_4

$M^{+2} = \frac{X}{3}, M^{+3} = \frac{2X}{3}$

So, total of O.N. of all atoms

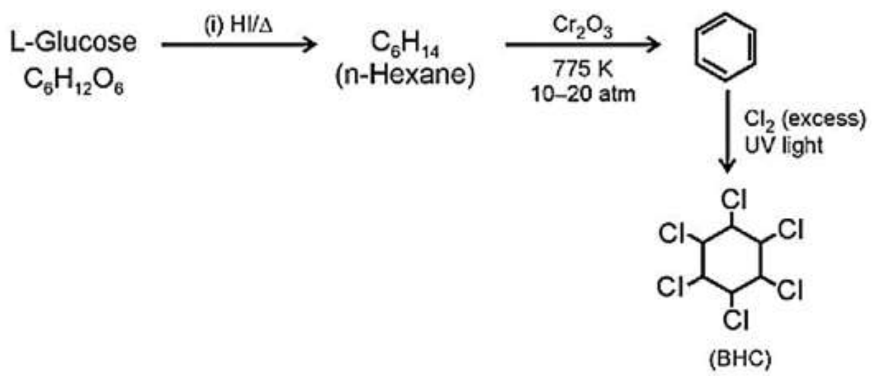
$$\frac{2X}{3} + 3\left(\frac{2X}{3}\right) + 2(+3) + 4(-2) = 0$$

$$\frac{2X}{3} + 2X + 6 - 8 = 0$$

$$\frac{8X}{3} = 2$$

$$X = \frac{6}{8} = \frac{3}{4} = 0.75$$

18. (d)



19. (b) If PCl5 is fluorinated in a polar solvent, ionic isomers are formed. e.g.: -

$[PCl4_4]^{[}[PClLF_4]^-$(colourless crystals)

and $[PCl_4]^+[PF_6]^-$(white crystals)

20. (a, c)

$$\underset{\substack{\text{Oxidation}\\ \text{(Anode)}}}{\overset{-2}{N_2H_4}} + \underset{\substack{\text{Reduction}\\ \text{(Cathode)}}}{\overset{0}{O_2}} \longrightarrow \overset{0}{N_2} + H_2O^{-2}$$

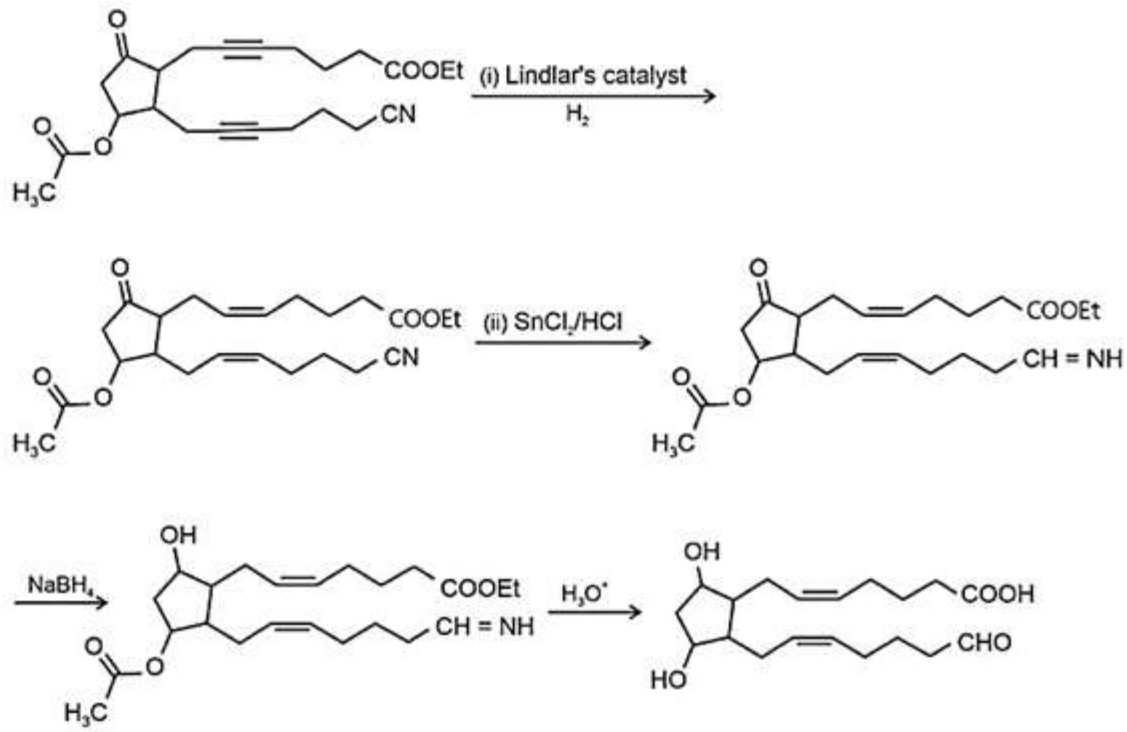
At anode: $N_2H_4 + 4OH^- \longrightarrow N_2 + 4H_2O + 4e^-$

At cathode: $O_2 + 2H_2O + 4e^- \longrightarrow 4OH^-$

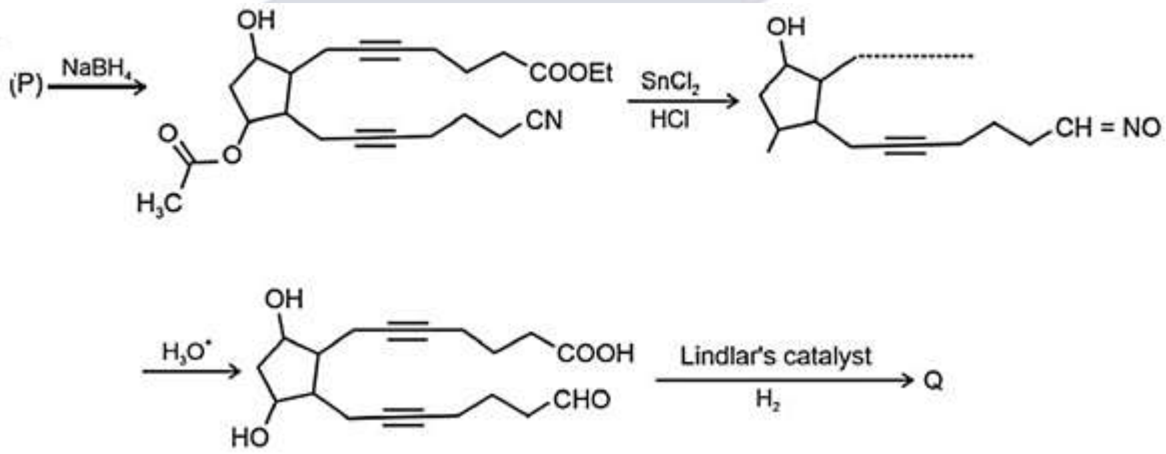
Complete reaction: $N_2H_4 + O_2 \longrightarrow N_2 + 2H_2O$

Statements (a) and (c) are correct.

21. (a, c, d)



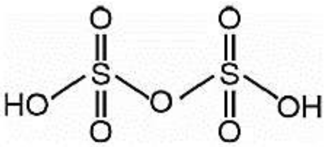
(a)

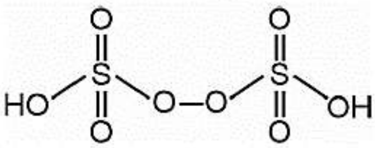


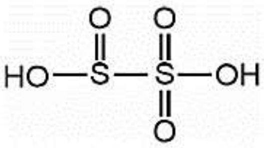
(c)

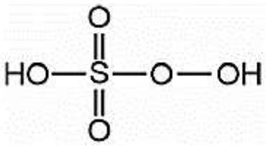
(d) $$(P) \xrightarrow{\text{(i) Lindlar's catalyst, } H_2;\ \text{(ii) } NaBH_4;\ \text{(iii) } SnCl_2/HCl;\ \text{(iv) } H_3O^+} (Q)$$

22.(b, d)

. $H_2S_2O_7$ 

$H_2S_2O_8$ 

$H_2S_2O_5$ 

H_2SO_5 

23.(2500)

Number of moles of unabsorbed $CH_3COOH = \frac{40\times1}{1000} = 4\times10^{-2}\text{mol}$

Number of moles of adsorbed $CH_3COOH = \frac{100\times0.5}{1000} - 4\times10^{-2}$

$= 10^{-2}\text{mol}$

Surface area occupied by one molecule of

$CH_3COOH = \dfrac{1.5\times10^2}{10^{-2}\times6\times10^{23}} = \dfrac{150\times10^2\times10^{-23}}{6}$

$= 2500\times10^{-23}\text{m}^2$

$\therefore$ As per question $P = 2500$

24.(150)

Vessel-I

$(\Delta T_B)_1 = i_x\dfrac{w_2}{M_x}\cdot\dfrac{1}{w_1}\times1000\times K_b$

$Mx =$ Molar mass of ' X '

Vessel-II

$(\Delta T_B)_{II} = i_Y\dfrac{w_2}{M_Y}\cdot\dfrac{1}{w_1}\times1000\times K_B$

$M_Y =$ Molar mass of ' Y '

$$\frac{(\Delta T_b)_1}{(\Delta T_b)_{III}} \times 100 = \frac{i_X}{i_Y} \cdot \frac{M_Y}{M_X} \times 100$$
$$= 1.2 \times \frac{100}{80} \times 100$$
$$= 150\%$$

25.(41)

A	G	T	C	A	C	G	T	A	A	G	T	C
‖	‖‖	‖	‖‖	‖	‖‖	‖‖	‖	‖	‖	‖‖	‖	‖‖
T	C	A	G	T	G	C	A	T	T	C	A	G
(1)	(i)	(2)	(ii)	(3)	(iii)	(iv)	(4)	(5)	(6)	(v)	(7)	(vi)

Total energy $= [BEH\text{-bond } A - T \times \text{No. of } A = T \text{ pair} \times 2] + [BEH\text{-bond G} - \text{C} \times \text{No. of G} \equiv \text{C pair} \times 3]$

$= [1 \times 7 \times 2] + [1.5 \times 6 \times 3]$

$= 14 + 27$

$= 41\text{kcal}$

26.(143)

Life of sample → t years

[A]0 ∝ Initial mole of U-238

[A]t ∝ Final mole of U-238

$$\frac{[A]_0}{[A]_t} = \frac{\frac{1}{238} + \frac{7}{206}}{\frac{1}{238}}$$
$$= \frac{0.0042 + 0.0340}{0.0042}$$
$$= 9.1$$
$$= \frac{2.303\log 2 \times t}{4.5 \times 10^9} = 2.303\log 9.1$$

$t = 14.27 \times 10^9$ years

$= 142.7 \times 10^9$ years

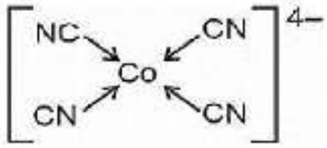
$P = 142.7$

$P \simeq 143$

27. (3) $[Co(CN)_4]^4 \Rightarrow Co^0 \Rightarrow 3d^7 4s^2$

Due to SFL, CN^-pairing and transference of electron takes place and hybridisation is dsp^2

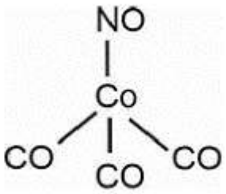
Geometry ⇒ Square planer



$[Co(CO)_3NO]$

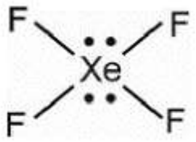
$Co^{-1} \Rightarrow 3d^{10}$ due to SFL CO and NO sp^3 hybridisation

Geometry = Tetrahedral

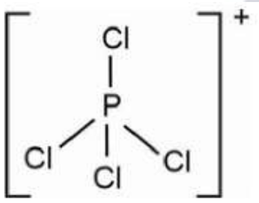


$XeF_4 \Rightarrow 4bp + 2lp \Rightarrow sp^3d^2$

Square planer

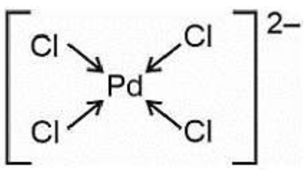


$PCl_4 + \Rightarrow 4pb + 0lp$
$sp^3 \Rightarrow$ tetrahedral



$[PdCl_4]^{2-} \Rightarrow Pd^{2+}, Cl^{-}$behaves as SFL

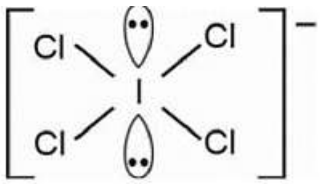
$Pd^{2+} \Rightarrow 4d^8 \Rightarrow dsp^2 \Rightarrow$ square planer



$ICl_4^- \Rightarrow 4bp + 2Ip$

sp^3d^2

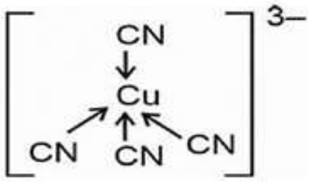
square planer



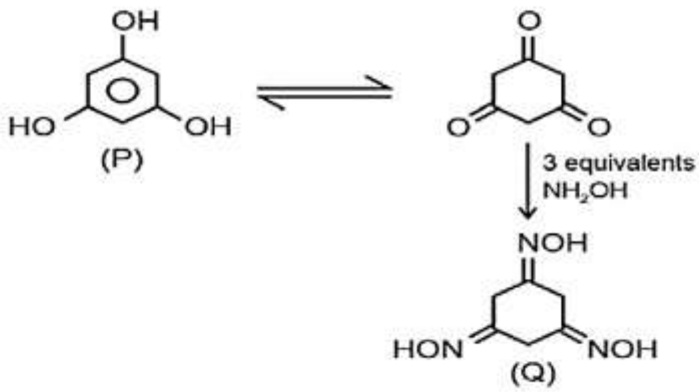
$[Cu(CN)_4]^{3-} \Rightarrow Cu^{+1} \Rightarrow 3d^{10}$

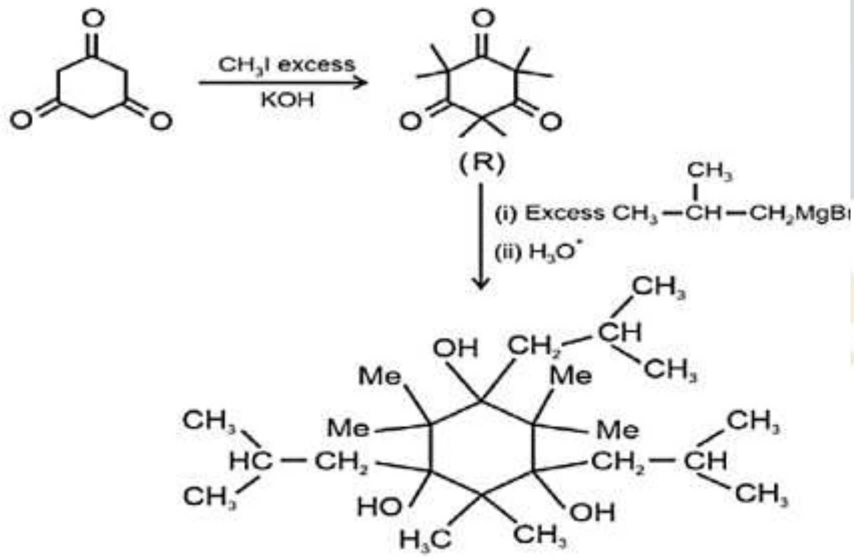
$\Rightarrow sp^3$

Tetrahedral



28.(12)

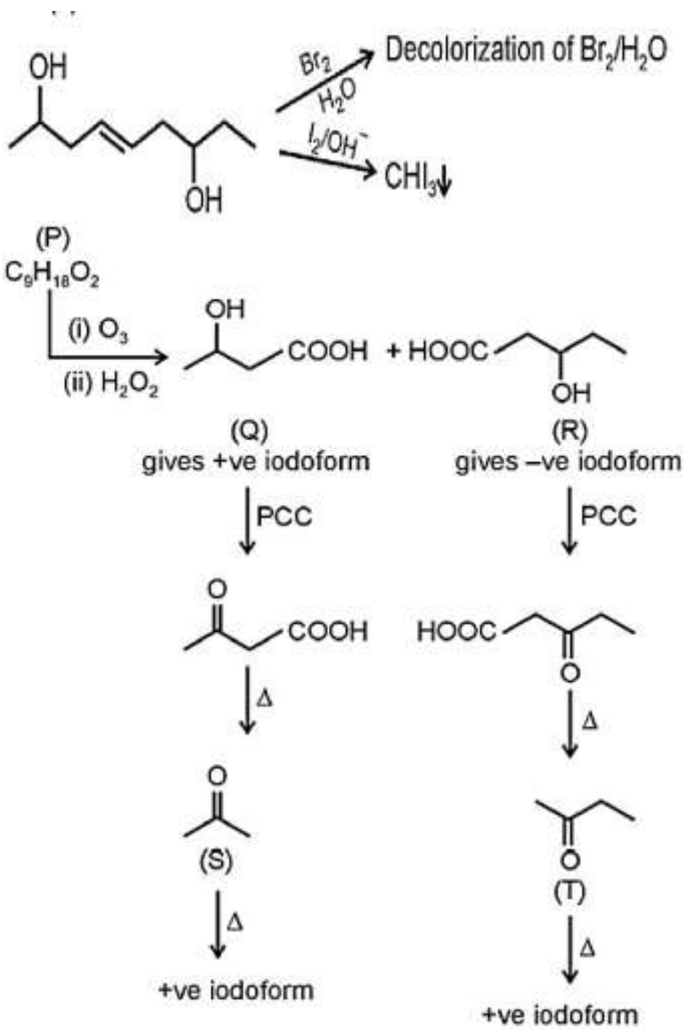




Number of CH_3 groups = 12

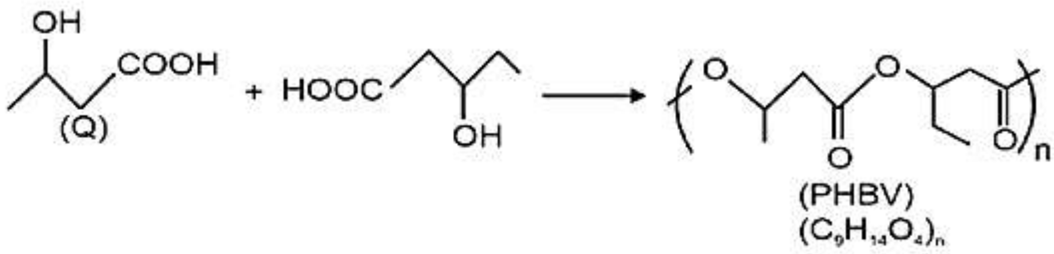
29. (i) **(2)** (ii) **(102018)**

(i)



Sum of number of O-atoms in **S** and **T** $= 1 + 1 = 2$

(ii)



Mol. wt. of polymer $= (104 \times 500) + (118 \times 500) - 18 \times 499$

$= 52000 + 59000 - 8982$
$= 102018\text{g}$

30. (i) **(2)** (ii) **(3)**

(i) From this equation we need 2 mol of KI

$2KI + 2K_3[Fe(CN)_6] \rightarrow I_2 + 2K_4[Fe(CN)_6]$
$2K_4[Fe(CN)_6] + 3ZnCl_2 \rightarrow K_2Zn_3[Fe(CN)_6]_2 + 6KCl$

(ii) From this equation we need 2 mol of KI

$$2KI + 2K_3[Fe(CN)_6] \rightarrow I_2 + 2K_4[Fe(CN)_6]$$
$$2K_4[Fe(CN)_6] + 3ZnCl_2 \rightarrow K_2Zn_3[Fe(CN)_6]_2 + 6KCl$$

31. (b)

$$\tan\left(\sin^{-1}\left(\frac{3}{5}\right) - 2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$$

Let $\sin^{-1}\frac{3}{5} = \alpha, 2\cos^{-1}\frac{2}{\sqrt{5}} = \beta \Rightarrow \cos\frac{\beta}{2} = \frac{2}{\sqrt{5}}$

$$\because \sin\alpha = \frac{3}{5} \Rightarrow \tan\alpha = \frac{3}{4} \tan\beta = \frac{2\tan\frac{\beta}{2}}{1 - \tan^2\frac{\beta}{2}} = \frac{2 \times \frac{1}{2}}{1 - \frac{1}{4}} = \frac{4}{3}$$

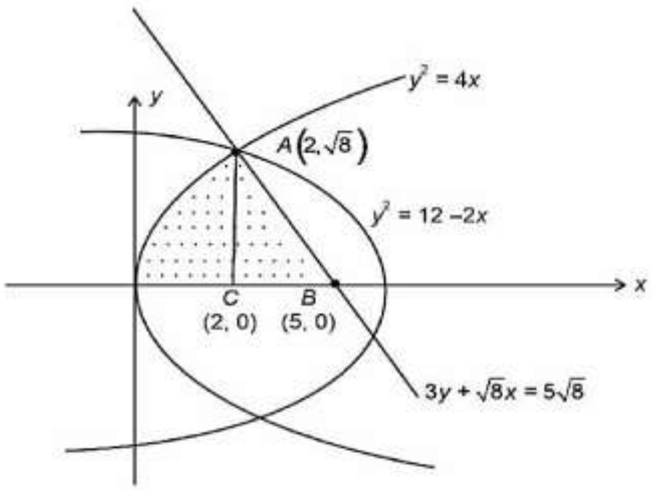
$$\Rightarrow \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} = \frac{\frac{3}{4} - \frac{4}{3}}{1 + 1} = -\frac{7}{24}$$

32.(b)

$y^2 = 4x, y^2 = 12 - 2x \Rightarrow x = 2, y = \sqrt{8}$

$$A = \int_0^2 2\sqrt{x}dx + \frac{1}{2} \times 3 \times \sqrt{8}$$
$$= \left[2 \times \frac{2}{3}x^{\frac{3}{2}}\right]_0^2 + 3\sqrt{2} = \frac{4}{3} \times 2\sqrt{2} + 3\sqrt{2} = \frac{17}{3}\sqrt{2}$$
$$\because A = \alpha\sqrt{2} \Rightarrow \alpha = \frac{17}{3}$$

Option (b) is correct.



33.(b)

$I = \lim_{x\to 0^+} (\sin(\sin kx) + \cos x + x)^{\frac{2}{x}} = e^6$

$\Rightarrow \ln I = \lim_{x\to 0^+} \frac{2}{x}(\sin(\sin kx) + \cos x + x - 1)$

$$\Rightarrow \ln I = \lim_{x\to 0^+} 2\left(\frac{\sin(\sin kx)}{\sin kx}\cdot\frac{\sin kx}{kx}\cdot\frac{kx}{x} + 1 - \frac{(1-\cos x)}{x^2}\cdot x\right)$$

$$\Rightarrow \ln I = 2(k+1) \Rightarrow I = e^{2(k+1)} = e^6$$

$$k + 1 = 3 \Rightarrow k = 2$$

34.(d)

$$f(x) = \begin{cases} x^2\sin\left(\frac{\pi}{x^2}\right), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

$$f(x) = 0 \Rightarrow \sin\left(\frac{\pi}{x^2}\right) = 0$$

$$\Rightarrow \frac{\pi}{x^2} = n\pi$$

$$\Rightarrow x^2 = \frac{1}{n}$$

$$\Rightarrow x = \frac{1}{\sqrt{n}}$$

If $x \in \left[\frac{1}{10^{10}}, \infty\right)$	If $x \in \left[\frac{1}{\pi}, \infty\right)$	If $x \in \left(0, \frac{1}{10^{10}}\right)$
$\frac{1}{\sqrt{n}} \in \left[\frac{1}{10^{10}}, \infty\right)$	$\frac{1}{\sqrt{n}} \in \left[\frac{1}{\pi}, \infty\right)$	n infinite
$\sqrt{n} \in (0, 10^{10}]$	$\sqrt{n} \in (0, \pi]$	If $x \in \left(\frac{1}{\pi^2}, \frac{1}{\pi}\right)$
$n \in (0, (10^{10})^2]$	$n \in (0, \pi^2]$	$\sqrt{n} \in (\pi, \pi^2)$
Finite values of n	$n = 1,2,3 \ldots 9$	$n \in (\pi^2, \pi^4)$
		$n \in (9.8, 97.2 \ldots)$

More than 25 solutions

35.(b, c)

$$\lim_{x\to\infty} \frac{\sin(x^2)\sin\left(\frac{1}{x^2}\right)(\ln x)^{\alpha}}{x^{\alpha\beta}(\ln(1+x))^{\beta}} = 0$$

$$= \lim_{x\to\infty} \frac{(\sin x^2)\sin\left(\frac{1}{x^2}\right)\frac{1}{x^2}}{\left(\frac{1}{x^2}\right)x^{\alpha\beta}(\ln(1+x))^{\beta}} = 0$$

It is possible if $\alpha\beta + 2 > 0$

$\alpha\beta > -2$

(a) $\alpha\beta = -3$

(b) $\alpha\beta = -1$

(c) $\alpha\beta = -1$

(d) $\alpha\beta = -2$

36.(a, c)

Equation of line parallel to $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-6}{1}$ through P(1,3,2) is $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z-2}{1} = \lambda$ (let)

Now, putting any point $(\lambda + 1, 2\lambda + 3, \lambda + 2)$ in L_1

$\lambda = 1$

$\Rightarrow$ Point Q(2,5,3)

Equation of line through Q(2,5,3) perpendicular to L_1 is

$$\frac{x-2}{1} = \frac{y-5}{-1} = \frac{z-3}{3} = \mu \text{ (Let)}$$

Putting any point $(\mu + 2, -\mu + 5, 3\mu + 3)$ in L_2

$\mu = -1$

$\Rightarrow$ Point R(1,6,0)

(a) $PQ = \sqrt{1+4+1} = \sqrt{6}$

(b) R(1,6,0)

(c) Centroid $\left(\frac{4}{3}, \frac{14}{3}, \frac{5}{3}\right)$

(d) $PQ + QR + PR = \sqrt{6} + \sqrt{11} + \sqrt{13}$

37. (a , c)

Let $A_1 = (2t_1^2, 4t_1)$ and $B_1 = (2t_2^2, 4t_2)$

$C \equiv (-4,0) \equiv \left(2t_1t_2, 2(t_1 + t_2)\right)$

$\Rightarrow t_2 = -t_1$ and $t_1(-t_1) = -2$

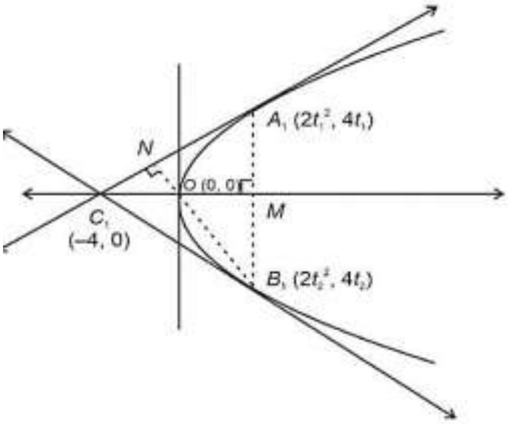
$t_1 = \sqrt{2}, t_2 = -\sqrt{2}$

$A_1 \equiv (4, 4\sqrt{2}), B_1 \equiv (4, -4\sqrt{2})$

$\therefore OA_1 = \sqrt{4^2 + (4\sqrt{2})^2} = 4\sqrt{3}$

$A_1B_1 = 8\sqrt{2}$

Altitude $C_1M: y = 0$



Altitude B_1N: $\sqrt{2}x + y = 0$

$\therefore$ Orthocentre $\equiv (0,0)$

38.(51)

$f(x + y) = f(x) + f(y)$

$\Rightarrow f(x) = kx$

$f\left(\frac{-3}{5}\right) = 12 \Rightarrow k = -20$

$\therefore f(x) = -20x$

$g(x + y) = g(x)g(y) \Rightarrow g(x) = a^x$

$g\left(\frac{-1}{3}\right) = 2 \Rightarrow a = \frac{1}{8}$

$\therefore g(x) = \left(\frac{1}{8}\right)^x$

$\left(f\left(\frac{1}{4}\right) + g(-2) - 8\right) g(0) = (-5 + 64 - 8) \times 1 = 51$

39.(11)

N Balls $= 3W + 6G + (N - 9)B$

$P(W_1 \cap G_2 \cap B_3) = \frac{2}{5N}$

$\Rightarrow \frac{3}{N} \times \frac{6}{N-1} \times \frac{N-9}{N-2} = \frac{2}{5N}$

$\Rightarrow N^2 - 48N + 407 = 0$

$\Rightarrow N = 11$ or 37

$$P(B_3 \mid W_1 \cap G_2) = \frac{2}{9}$$

$$\Rightarrow \frac{P(W_1 \cap G_2 \cap B_3)}{P(W_1 \cap G_2)} = \frac{2}{9}$$

$$\Rightarrow \frac{\frac{2}{5N}}{\frac{3}{N} \times \frac{6}{N-1}} = \frac{2}{9}$$

$$\Rightarrow \frac{N-1}{45} = \frac{2}{9}$$

$$\Rightarrow N = 11$$

40. (01)

$(x) = 0$

$$\Rightarrow \frac{x^{2023} + 2024x + 2025}{(x^2 - x + 3)}\left[\frac{\sin x + 2}{e^{\pi x}}\right] = 0$$

$$\Rightarrow x^{2023} + 2024x + 2025 = 0$$

Let $g(x) = x^{2023} + 2024x + 2025$

$g'(x) = 2023x^{2022} + 2024 > 0 \ \forall x \in \mathbb{R}$

$\therefore f(x) = 0$ has only one solution

41. (2)

$$2\vec{p} + \vec{q} = 5\hat{\imath} + \hat{\jmath} + 7\hat{k}$$

$$\vec{p} - 2\vec{q} = 0\hat{\imath} + 3\hat{\jmath} + \hat{k}$$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \hat{\imath}(4) - \hat{\jmath}(-1) + \hat{k}(-3)$$

$$= 4\hat{\imath} + \hat{\jmath} - 3\hat{k}$$

$$15\hat{\imath} + 10\hat{\jmath} + 6\hat{k} = \alpha(5\hat{\imath} + \hat{\jmath} + 7\hat{k}) + \beta(3\hat{\jmath} + \hat{k}) + \gamma(4\hat{\imath} + \hat{\jmath} - 3\hat{k})$$

$\therefore 15 = 5\alpha + 4\gamma$

$10 = \alpha + 3\beta + \gamma$

$6 = 7\alpha + \beta - 3\gamma$

$\therefore \alpha = \frac{7}{5}, \beta = \frac{11}{5}, \gamma = 2$

$\therefore \gamma = 2$

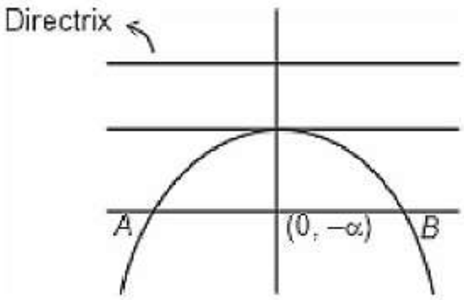
42.(12)

$x^2 = -4ay$

Equation of normal

$$y = mx - 2a - \frac{a}{m^2}$$

$$-\alpha = -2a - \frac{a}{\frac{1}{6}} = -8a$$



$\Rightarrow \alpha = 8a$

Equation of required line

$y = -\alpha$

$\Rightarrow y = -8a$, solving with $x^2 = -4ay$

$\Rightarrow x^2 = 32a^2$

$\Rightarrow x = \pm 4\sqrt{2}a$

$= \pm \dfrac{\alpha}{\sqrt{2}}$

$A\left(\dfrac{\alpha}{\sqrt{2}}, -\alpha\right), B\left(\dfrac{-\alpha}{\sqrt{2}}, -\alpha\right) \Rightarrow AB = \sqrt{2}\alpha$

$\Rightarrow \dfrac{r}{s} = \dfrac{4a}{2\alpha^2} = \dfrac{1}{16} \Rightarrow \dfrac{4a}{2 \times 64a^2} = \dfrac{1}{16}$

$\Rightarrow a = \dfrac{1}{2}$

$\Rightarrow 24a = 12$

43.(5)

$$f(t) = \left(\frac{(2n+1)-t}{2}\right)(-1)^{n+1}2 + \left(\frac{t-(2n-1)}{2}\right)(-1)^{n+2}2, t \in (2n-1, 2n+1)$$

$$\Rightarrow f(t) = 2(-1)^{n+1}(2n-t), t \in (2n-1, 2n+1)$$

$$\Rightarrow g(x) = \int_1^x f(t)dt, x \in (1,8]$$

$$= \begin{cases} \int_1^x 2(2-t)dt, 1 < x \le 3, n = 1 \\ \int_1^3 2(2-t)dt + \int_3^x (2t-8)dt, 3 < x \le 5, n = 2 \\ \int_1^3 2(2-t)dt + \int_3^5 (2t-8)dt + \int_5^x 2(6-t)dt, 5 < x \le 7, n = 3 \\ 1 \\ \int_1^3 2(2-t)dt + \int_3^5 (2t-8)dt + \int_5^7 2(6-t)dt + \int_7^x (2t-16)dt, x \in (7,8], n = 4 \end{cases}$$

$$= \begin{cases} -x^2 + 4x - 3, 1 < x \le 3, \\ x^2 - 8x + 15, 3 < x \le 5 \\ -x^2 + 12x - 35, 5 < x \le 7 \\ x^2 - 16x + 63, 7 < x \le 8 \end{cases} = \begin{cases} -(x-1)(x-3), 1 < x \le 3 \\ (x-3)(x-5), 3 < x \le 5 \\ -(x-5)(x-7), 5 < x \le 7 \\ (x-7)(x-9), 7 < x \le 8 \end{cases}$$

$\Rightarrow g(x) = 0 \Rightarrow x = 3,5,7 \Rightarrow \alpha = 3$

$$\beta = \lim_{x \to 1^+} \left(\frac{g(x)}{x-1}\right) = \lim_{x \to 1^+} -\frac{(x-1)(x-3)}{x-1} = 2$$

$\Rightarrow \alpha + \beta = 5$

44. (i) **(20)** (ii) **(36)**

(i)

$S = \{1,2,3,4,5,6\} R: S \to S$

Number of elements in $R = 6$

and for each $(a,b) \in R; |a-b| \ge 2$

$X \to$ set of all relation $R: S \to S$

$$\left.\begin{array}{ll} \text{If } a = 1, b = 3, 4, 5, 6 & \to 4 \\ a = 2, b = 4, 5, 6 & \to 3 \\ a = 3, b = 1, 5, 6 & \to 3 \\ a = 4, b = 1, 2, 6 & \to 3 \\ a = 5, b = 1, 2, 3 & \to 3 \\ a = 6, b = 1, 2, 3, 4 & \to 4 \end{array}\right] \text{Total number of ordered pairs } (a, b) \text{ s.t. } |a-b| \ge 2 = 20$$

$\therefore$ $n(X)$ = number of elements in X

$= {}^{20}C_6$ $\therefore$ $m = 20$

(ii)

$S = \{1,2,3,4,5,6\}$ $R: S \to S$

Number of elements in $R = 6$

and for each $(a,b) \in R; |a-b| \ge 2$

$X \to$ set of all relation $R: S \to S$

$$\left.\begin{array}{lll} a = 1 & b = 3,4,5,6 & \to 4 \\ a = 2 & b = 4,5,6 & \to 3 \\ a = 3 & b = 1,5,6 & \to 3 \\ a = 4 & b = 1,2,6 & \to 3 \\ a = 5 & b = 1,2,3 & \to 3 \\ \text{If } a = 6 & b = 1,2,3,4 & \to 4 \end{array}\right\}$$

Total number of ordered pairs (a,b) s. t. $|a-b| \ge 2 = 20$

$\therefore n(X) =$ number of elements in X

$= {}^{20}C_6 \therefore m = 20$

$Y = \{R \in X$: The range of R has exactly one element $\}$

From above, if range of R has exactly one element, then maximum number of elements in R will be 4 .

$\therefore n(Y) = 0$

$Z = \{R \in X : R$ is a function from S to $S\}$

$n(Z) = \ {}^4C_1 \times \ {}^3C_1 \times \ {}^3C_1 \times \ {}^3C_1 \times \ {}^3C_1 \times \ {}^4C_1$

$= (36)^2$

$n(y) + n(z) = 0 + (36)^2 = k^2$

$\Rightarrow |k| = 36$

45. (0) $f(x) = \sin^2 x, g(x) = \sqrt{\frac{\pi}{2}x - x^2}$

Here $f\left(\frac{\pi}{2} - x\right) = \cos^2 x, g\left(\frac{\pi}{2} - x\right) = g(x)$

Let $I_1 = 2\int_0^{\frac{\pi}{2}} f(x)g(x) = 2\int_0^{\frac{\pi}{2}} \sin^2 x \cdot g(x)dx$.........(i)

as $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

$\Rightarrow I_1 = 2\int_0^{\frac{\pi}{2}} \cos^2 xg(x)dx$.................(ii)

(1) + (2)

$\Rightarrow 2I_1 = 2\int_0^{\frac{\pi}{2}} g(x)dx$

$\Rightarrow I_1 = \int_0^{\frac{\pi}{2}} g(x)dx$

$\Rightarrow 2\int_0^{\frac{\pi}{2}} f(x)g(x) - \int_0^{\frac{\pi}{2}} g(x)dx = 0$