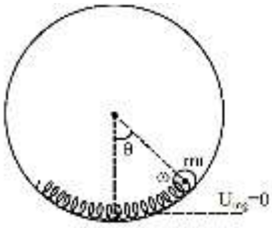
1. (a)

Sol.


$$E=\frac{1}{2}k(R-r)^2\theta^2+mg(R-r)(1-\cos\theta)+\frac{1}{2}mv^2+\frac{1}{2}\frac{mr^2}{2}\omega^2$$

Differentiating wrt t,

$$0=\frac{1}{2}k(R-r)^2\cdot 2\theta\frac{d\theta}{dt}+mg(R-r)\cdot\frac{d}{dt}\left(2\frac{\theta^2}{4}\right)+\frac{1}{2}\ m\cdot 2v\frac{dv}{dt}+\frac{mr^2}{4}\cdot 2\omega\frac{d\omega}{dt}$$

$$\Rightarrow 0=k(R-r)^2\theta\frac{d\theta}{dt}+mg(R-r)\theta\frac{d\theta}{dt}+mv\frac{dv}{dt}+\frac{mr^2}{2}\omega\frac{d\omega}{dt}$$

Also, $\frac{d\theta}{dt}=\frac{V}{(R-r)}\Rightarrow\frac{d^2\theta}{dt^2}=\frac{1}{(R-r)}\frac{dv}{dt}=\frac{1}{R-r}a$

$$\therefore k(R-r)^2\cdot\theta\frac{V}{R-r}+mg(R-r)\theta\frac{V}{R-r}=-mv\alpha r-\frac{mr^2}{2}\frac{V}{r}\alpha$$

$$\Rightarrow k(R-r)+mg\theta=-\frac{3}{2}mr\alpha$$

$$\Rightarrow -[k(R-r)+mg]\theta=\frac{3}{2}\ m(R-r)\frac{d^2\theta}{dt^2}$$

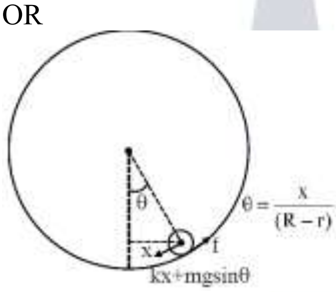
$$\Rightarrow -\frac{2}{3}\left[\frac{k}{m}+\frac{g}{R-r}\right]=\frac{d^2\theta}{dt^2}$$

Compering with standard equation of SHM

$$\omega=\sqrt{\frac{2}{3}\left[\frac{k}{m}+\frac{g}{R-r}\right]}$$

Hence answer is option(A)

OR



$$kx+mg\sin\theta-f=ma$$

$$\Rightarrow kx+mg\frac{x}{(R-r)}-f=ma$$

$$fr=\frac{mr^2}{2}\cdot\alpha\Rightarrow f=\frac{ma}{2}$$

$$\therefore\left(\alpha+\frac{mg}{R-r}\right)x=\frac{3ma}{2}$$

$$\therefore\omega=\sqrt{\frac{2}{3}\left[\frac{k}{m}+\frac{g}{R-r}\right]}$$

2. (d)

$2mv_1=2mv_{1f}\cos\theta+2mv_{2f}\cos\phi$ …..(i)

$2\ m_{1f}\sin\theta=mv_{2f}\sin\phi$ ………(ii)

$$\frac{1}{2}(2m)v_1^2+\frac{1}{2}m(0)^2=\frac{1}{2}(2m)v_{1f}^2+\frac{1}{2}mv_{2f}^2$$

$2v_1^2 = 2v_{1f}^2 + v_{2f}^2$ ....(iii)

From (i), (ii), (iii),

$$3v_{1f}^2-4v_1v_{1f}\cos\theta+v_1^2=0$$

$$(-4v_1\cos\theta)^2-4(3)(v_1^2)\geq 0$$

$$\cos^2\theta\geq\frac{3}{4}$$

$$\cos^2\theta\geq\frac{\sqrt{3}}{2}$$

$$\therefore\theta=\frac{\pi}{6}$$

3. (a)

$$\phi=\mathrm{B_0}\cos\omega t\mathrm{A}\sin\omega t=\frac{\mathrm{B_0\ A}\sin 2\omega t}{2}$$

$$\varepsilon=-\frac{\mathrm{d}\phi}{\mathrm{dt}}=-\mathrm{B_0\ A}\cos 2\omega t=\left(0\leq t\leq\frac{\pi}{\omega}\right)$$

$$\varepsilon=0\ \left(\frac{\pi}{\omega}\leq t\leq\frac{2\pi}{\omega}\right)$$

4. (c)

$10\mathrm{MSD} = 1\ \mathrm{cm}; 1\mathrm{MSD} = 0.1\ \mathrm{cm}$

$7\mathrm{MSD} = 10\mathrm{VSD}$

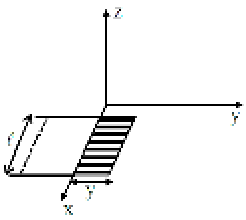
$1\mathrm{VSD} = 0.07\ \mathrm{cm}$

Reading = 2 MSD - VSD

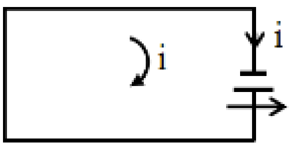
$= 0.2\ \mathrm{cm} - 0.07\ \mathrm{cm} = 0.13\ \mathrm{cm}$

SECTION-2:

5. (b,d)



$$\Rightarrow\frac{-\mathrm{d}\phi}{\mathrm{dt}}=\frac{\mathrm{d}}{\mathrm{dt}}(\mathrm{B_0}\times\ell\times\mathrm{y})=\mathrm{BV}\ell$$



$$\vec{\mathrm{F}}=\mathrm{B}(\hat{\imath})(\ell)(-\hat{\jmath})$$

$$\mathrm{ma}=-\mathrm{B_0}\left[\frac{\mathrm{B_0\ V}\ell}{\mathrm{R}}\right](\ell)$$

$$\mathrm{a}=-\frac{\mathrm{B_0^2}\ell^2\ \mathrm{V}}{\mathrm{mR}}$$

Also $K = \dfrac{B_0^2\ell^2 V}{RM}$

So $[a = -kv]$

$$\frac{dv}{dt} = -kv$$

$$\int_{v_0}^{v}\frac{dv}{dt} = \int_0^t - kdt$$

$$\ell\frac{v}{v_0} = -kt$$

$$[v = v_0e^{-kt}] \; \ldots\ldots(i)$$

$$\frac{dx}{dt} = v_0e^{-kt}\;(x \le \ell)$$

$$\int_0^x dx = \int_0^t v_0e^{-kt}dt$$

$$= \frac{v_0}{k}\left(1 - e^{-kt}\right)$$

When $x = \ell$

$$\ell = \frac{v_0}{k}\left(1 - e^{-kt}\right)$$

Option (D) $(v_0 = 3k\ell)$

$$\ell = \frac{3k\ell}{k}\left(1 - e^{-kt}\right)$$

$$\frac{1}{3} = 1 - e^{-kt}$$

$$f\frac{2}{3} = 2e^{-kt}$$

$$-kt = 8n\left(\frac{2}{3}\right)$$

$$t = \frac{1}{k}\ln\left(\frac{2}{3}\right)$$

Complete loop will enter at $t = \frac{1}{k}\ln\left(\frac{2}{3}\right)$

Option (b)

$$\frac{d\phi}{dt} = 0, \underline{\underline{e}} = 0, i = 0, F = 0$$

6. (d)

$L = 10.5$ cm $\rightarrow$ 3 significant digits

$b = 0.05$ cm $\rightarrow$ 1 significant digit

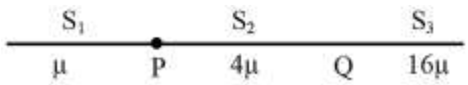
$t = 6.0\mu$ m $\rightarrow$ 2 significant digits

Volume, $V = Lbt$ must have only 1 significant digit

$\Rightarrow V = 10.5 \times 0.05 \times 10^{-1} \times 6.0 \times 10^{-4}\ \text{cm}^3$

$= 3 \times 10^{-5}$cc

7. (a,d)



(a)

$y_1 = y_0\cos(\omega t - kx)$

when wave going from Rarer to Denser,

$$y_r = A_r\cos(\omega t + kx + \pi)$$

$$y_r = a_1y_0\cos(\omega t + kx + \pi)$$

option (a) correct

(b) For transmitted from point P

$y_t = A_t\cos[\omega t - k_1 x]$

$\frac{k_1}{k} = \sqrt{\frac{\mu_1}{\mu}} = \frac{k_1}{k} = \sqrt{\frac{4\mu}{\mu}}$

$k_1 = 2k$

$y_t = a_2 y_0 \cos[\omega t - 2kx]$

option (b) incorrect

(c) when reflected from Q

$y_i = a_2 y_0 \cos[\omega t - 2kx]$

$y_r = a_3 y_0 \cos[\omega t + 2kx + \pi]$

option (c) incorrect

(d) when transmitted from Q

$y_t = a_4 y_0 \cos[\omega t = k_2 x]$

$\frac{k_2}{2k} = \sqrt{\frac{16\mu}{4\mu}} \Rightarrow k_2 = 4k$

$y_t = a_4 y_0 \cos[\omega t - 4kx]$

option (d) correct

SECTION-3:

8. (2.00)

Sol. $y = 8 + 8\sin\frac{2\pi t}{T}$

With respect to elevator, variation in weight will be

$\Delta W = m(\Delta a)_{max}$

$\Delta W = m \times 2\omega^2 A$

Here elevator is performing SHM

$$\Delta W = 2\ m \times \left(\frac{2\pi}{T}\right)^2 \times AN$$

$$\Delta\ W = 2 \times 50 \times \left(\frac{2\pi}{40\pi}\right)^2 \times 8\ N$$

$$\Delta W = 2 \times 50 \times \frac{1}{400} \times 8\ N$$

$$\Delta W = \frac{800}{400}\ N = 2\ N$$

9. (22.98)

Total energy in cube $= 35 \times 10^7 \times hf$

$= 35 \times 10^7 \times 6 \times 10^{-34} \times 10^{15}$

$= 2.1 \times 10^{-10}\ J$

Total energy of EM waves $= \frac{B_0^2}{2\mu_0} \times$ volume

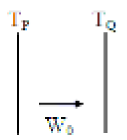
$$B_0^2 = \frac{2.1 \times 10^{-10} \times 8\pi \times 10^{-7}}{1^3}$$

$\Rightarrow B_0 = 22.98 \times 10^{-9}\ T$

10. (3.00)

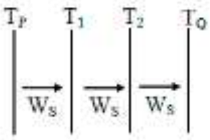
Initially:

$W_0 = \sigma\left(T_P^4 - T_Q^4\right)$



Finally:

Putting heat currents equal in steady state:



$$\sigma(T_P^4 - T_1^4) = \sigma(T_1^4 - T_2^4)$$
$$\sigma(T_1^4 - T_2^4) = \sigma(T_2^4 - T_Q^4)$$

Adding:

$$T_P^4 - T_1^4 = T_2^4 - T_Q^4$$

and

$$\Rightarrow T_1^4 + T_2^4 = T_P^4 + T_Q^4$$
$$\Rightarrow T_1^4 - T_2^4 = T_P^4 - T_1^4$$

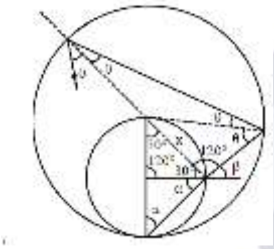
Adding: $T_1^4 = \frac{2\,T_P^4 + T_Q^4}{3}$

So $W_S = \sigma(T_P^4 - T_1^4)$

$$= \sigma\left(T_P^4 - \left(\frac{2\,T_P^4 + T_Q^4}{3}\right)\right) = \sigma\left(\frac{T_P^4 - T_Q^4}{3}\right)$$

hence $\frac{W_S}{W_0} = 3$

11. (0.75)



$$\tan\alpha = \sqrt{3}$$
$$\alpha = 60^\circ$$
$$\sqrt{3}\sin\beta = 1 \times \sin\alpha \Rightarrow \beta = 30^\circ$$
$$\frac{R}{2\sin 30^\circ} = \frac{x}{\sin 120^\circ}$$
$$\frac{R}{\sin 120^\circ} = \frac{R\sqrt{3}}{2 \times \sin\theta} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$
$$\sin\theta = \frac{3}{4}$$

12. (75.60 OR 94.50)

Sol. Solution-1

If we can consider

$$\frac{\Delta b}{b} = \frac{\Delta m}{m} + \frac{\Delta\lambda}{\lambda} + \frac{\Delta D}{D} + \frac{\Delta d}{d}$$
$$\frac{\Delta b}{b} = 0 + 0 + \frac{1\text{ cm}}{1\text{ m}} + \frac{1\text{ mm}}{5\text{ mm}} = 0.21$$
$$b = \frac{m\lambda D}{d} = \frac{3 \times 600 \times 10^{-3} \times 1}{5 \times 10^{-3}}\mu\text{ m} = 360\mu\text{ m}$$
$$\Rightarrow \Delta b = 360 \times 0.21\mu\text{ m} = 75.6\mu\text{ m}$$

However, error in d is too large (20%) for the solution- 1 to be correct. Hence, we propose solution- 2

Solution-2

$$b = \frac{m\lambda D}{d} = 360\mu\text{ m}$$

$$b_{max} = \frac{3 \times 600 \times 10^{-3} \times 1.01}{4 \times 10^{-3}}\mu\text{ m} = 454.5\mu\text{ m}$$

$$b_{min} = \frac{3 \times 600 \times 10^{-3} \times 0.99}{6 \times 10^{-3}}\mu\text{ m} = 297\mu\text{ m}$$

Maximum value of b gives error, $\Delta b_1 = 94.5\mu$ m

Minimum value of by gives error, $\Delta b_2 = 63\mu$ m

∴ We always report the largest error, hence correct answer should be 94.5μ m

13. (72.00)

$$\frac{mv^2}{r} = \frac{KZe^2}{r^2}$$

$$mv^2r = \frac{1}{4\pi\epsilon_0}Ze^2 \qquad \ldots (1)$$

$$mvr = \frac{nh}{2\pi} \qquad \ldots (2)$$

(1)/ (2) gives

$$v = \frac{\frac{Ze^2}{4\pi\epsilon_0}}{\frac{nh}{2\pi}} = \frac{Ze^2}{2\epsilon_0 nh}$$

$$\frac{h}{mv} = \frac{h}{\sqrt{2\,m_N \cdot K_B\,T}}$$

$$T = \frac{m^2Z^2e^4}{8\epsilon_0^2 n^2\,h^2\,m_N\,K_B}$$

$$n = 3 \Rightarrow T = \frac{m^2Z^2e^4}{72\epsilon_0^2\,h^2\,m_N\,K_B}$$

$$\frac{(1)}{(2)^2} \Rightarrow \frac{1}{mr} = \frac{\frac{Ze^2}{4\pi\epsilon_0}}{\frac{n^2\,h^2}{4\pi^2}}$$

$$r = \frac{n^2\,h^2\epsilon_0}{\pi Ze^2 \cdot m} \Rightarrow a_0 = \frac{h^2\epsilon_0}{\pi e^2\,m}$$

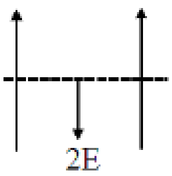
$$a_0^2 = \frac{h^4\epsilon_0^2}{\pi^2e^4\,m^2}$$

$$Ta_0^2 = \frac{m^2Z^2e^4}{72\epsilon_0\,h^2\,m_Nk_B} \cdot \frac{h^4\epsilon_0^2}{\pi^2e^4\,m^2}$$

$$T = \frac{h^2Z^2}{72\pi^2a_0^2\,m_Nk_B} \Rightarrow \alpha = 72$$

14. (c)

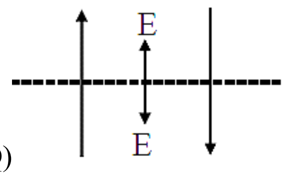
Sol. (P)



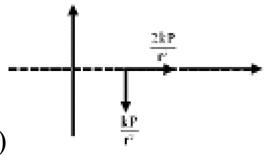
$$E_{net} = \frac{-2kP}{r^3}\hat{j}$$

$$E_{net} = \frac{-P\hat{j}}{2\pi\epsilon_0 r^3}$$

(Q)



$E_{net} = 0$

(R)  $\frac{2P\hat{i}}{4\pi\epsilon_0 r^3} - \frac{P\hat{j}}{4\pi\epsilon_0 r^3}$

(S) $\longrightarrow \xrightarrow{2E} \longrightarrow$ $E_{net} = \frac{4kP\hat{i}}{r^3}$

$P \to 2, Q \to 1, R \to 4, S \to 5$

15. (a)

For P

$i = \frac{V}{R} = 10\sin 400t \Rightarrow (3)$

For Q

$X_L = \omega L = 400 \times 100 \times 10^{-3} = 40\Omega$

$\therefore Z = 50\Omega$

$\therefore i = \frac{300}{50}\sin(400t - 53°)\left[\text{current will lag by } \tan^{-1}\frac{X_L}{R}\right] \Rightarrow (5)$

For R

$X_C = \frac{10^6}{400 \times 50}\Omega = 50\Omega$ and $X_L = 400 \times 25 \times 10^{-3} = 10\Omega$

$\therefore Z = 50\Omega$

$\therefore i = \frac{300}{50}\sin(400t + 53°)\left[\text{Current will lead by } \tan^{-1}\frac{X_C - X_L}{R}\right] \Rightarrow (2)$

For S

$X_C = 50\Omega$ and $X_L = 400 \times 125 \times 10^{-3} = 50\Omega$

$R = 60\Omega$

$\therefore i = \frac{300}{60}\sin(400t)\ X_L = X_C \Rightarrow$ Resonance $\Rightarrow (1)$

16. (c)

(P) Energy of H-like atom is

$E = -13.6\frac{Z^2}{n^2}$ So

$E \propto Z^2$

$P \to (5)$

(Q) Energy of characteristic X-ray by moseley's correction

$E = -13.6(Z-1)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ So

$E \propto (Z-1)^2$

$Q \to (1)$

(R) Electrostatics binding energy is proportional to $Z(Z-1)$

$R \to (2)$

(S) For stable nuclei with mass no. in range 30 to 170. Binding energy per nucleon is constant & graph is straight line

$S \to (4)$

PART-II: CHEMISTRY

17. (a)

$NH_4NO_2 \xrightarrow[60-70°C]{\Delta} N_2 + 2H_2O$

$NH_4NO_3 \xrightarrow[200-250°C]{\Delta} N_2O + 2H_2O$

18. (a)

$\Delta_O \propto \frac{1}{\lambda}$

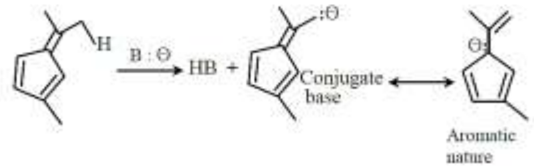
$\therefore$ The absorb wave length order is

$[Co(CN)_6]^{3-} < [Co(NH_3)_6]^{3+} < [Co(NH_3)_5H_2O]^{3+} < [Co(NH_3)_5Cl]^{2+}$

19. (c)

$I^- + 2MnO_4^- + H_2O \xrightarrow{\text{Neutral solution}} 2MnO_2 + IO_3^- + 2OH^-$

20. (b)



Base HB + Bature

B is most acidic: It forms most stable conjugate base.

21. (b,c)

Sol. (i) $Ne_2 \Rightarrow (\sigma 1s^2)(\sigma^* 1s^2)(\sigma 2s^2)(\sigma^* 2s^2)(\sigma 2p_z^2)(\pi 2p_x^2 = \pi 2p_y^2)(\pi^* 2p_x^2 = \pi^* 2p_y^2)(\sigma^* 2p_z^2)$

B.O. $= \frac{6-6}{2} = 0$

(ii) $F_2 \Rightarrow (\sigma 1\ s^2)(\sigma^* 1\ s^2)(\sigma 2\ s^2)(\sigma^* 2\ s^2)(\sigma 2p_z^2)(\pi 2p_x^2 = \pi 2p_y^2)(\pi^* 2p_x^2 = \pi^* 2p_y^2)$

HOMO

(iii) $O_2^{\oplus} \Rightarrow (\sigma 1\ s^2)(\sigma^* 1\ s^2)(\sigma 2\ s^2)(\sigma^* 2\ s^2)(\sigma 2p_z^2)(\pi 2p_x^2 = \pi 2p_y^2)(\pi^* 2p_x^1 = \pi^* 2p_y)$

B.O. $= \frac{6-1}{2} = 2.5$

$O_2 \Rightarrow (\sigma 1\ s^2)(\sigma^* 1\ s^2)(\sigma 2\ s^2)(\sigma^* 2\ s^2)(\sigma 2p_z^2)(\pi 2p_x^2 = \pi 2p_y^2)(\pi^* 2p_x^1 = \pi^* 2p_y^1)$

B.O. $\frac{6-2}{2} = 2$ (Bond order increases, Bond strength increases)

(iv) Size of atom increases, Bond length increases

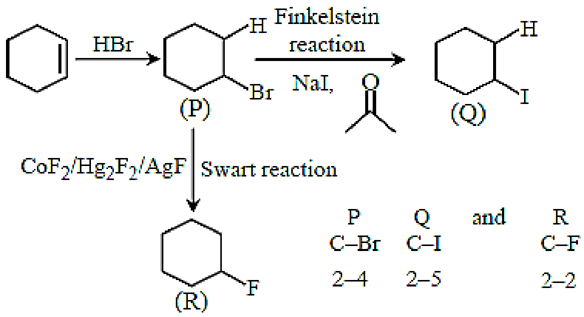
Size of Li > B

So, Bond length of $Li_2 > B_2$

22. (a,b)

$La^{+3} \rightarrow [\,_{54}Xe]4f^0$	diamagnetic
$Yb^{+2} \rightarrow [\,_{54}Xe]4f^{14}$	diamagnetic
$Lu^{+3} \rightarrow [\,_{54}Xe]4f^{14}$	diamagnetic
$La^{+2} \rightarrow [\,_{54}Xe]5\ d^1$	paramagnetic
$Ce^{+4} \rightarrow [\,_{54}Xe]4f^0$	diamagnetic
$Ce^{+3} \rightarrow [\,_{54}Xe]4f^1$	paramagnetic
$Yb^{+3} \rightarrow [\,_{54}Xe]4f^{13}$	paramagnetic
$Lu^{+2} \rightarrow [\,_{54}Xe]4f^{14}5\ d^1$	paramagnetic

23. (b)



Period no. ↑ Size ↑ B.L. ↑ B.O. ↓ B.E. ↓

(a) Wrong: bond length order Q > P > R

(b) Correct: Bond enthalpy R > P > Q

(c) Wrong: S_N2 reactivity Q > P > R

(d) Wrong: pKa(HI < HBr < HF) R > P > Q(pKa)

SECTION-3:

24. (100.00)

For reduction of dichromate, balanced reaction is:

$Cr_2O_7^{-2}(aq) + 6e^- + 14H^+(aq) \rightarrow 2Cr^{3+}(aq) + 7H_2O(l)$

3 mol 1 mol

Number of Farads required $= 3$ mol

Let current $= I$ amperes

$\Rightarrow \frac{I \times 48.25 \times 60}{96500} = 3$

$I = 100$ A

25. (2.23 or 2.24)

Sol. Because concentration of H + from weak acid is less we need to consider self ionization of H_2O also.

$HX(aq) \rightleftharpoons H^+ + X^-(aq)$

$10^{-3} - x\ x + y\ x$

$H_2O(l) \rightleftharpoons H^+(aq) + OH^-(aq)$

$x + y\ y$

Approximation : $(10^{-3} - x) \simeq 10^{-3}$

$\Rightarrow \frac{x(x+y)}{10^{-3}} = K_a = 4 \times 10^{-11} \ldots \ldots (1)$

$\Rightarrow y(x+y) = Kw = 10^{-14} \ldots \ldots \ldots .. (2)$

Add (1) + (2)

$\Rightarrow (x+y)^2 = 5 \times 10^{-14}$

$\Rightarrow x + y = [H^+] = \sqrt{5} \times 10^{-7}$

$\Rightarrow x = \sqrt{5} = 2.236$

26. (-7.10)

$\left(P + \frac{a}{V_m^2}\right)(V_m - b) = RT$

$PV_m - bP + \frac{a}{V_m} - \frac{ab}{V_m^2} - RT = 0$

$\Rightarrow PV_m^2 - (bP + RT)V_m^2 + aV_m - ab = 0$

Coefficient of $V_m^2 = -(bP + RT)$

Coefficient of $V_m = a$

Ratio $= -\frac{(bP + RT)}{a} = -\left[\frac{0.06 \times 300 + 24.6}{6}\right] = -7.1$

27. (29.88)

Sol. $H_2O(l) \rightarrow H_2(g) + \frac{1}{2}O_2(g)$

144 g $-$ $-$

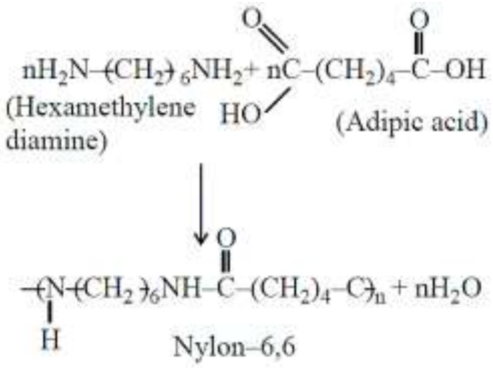
8 mol 8 mol 4 mol

$W = -P\Delta V = -(\Delta n)RT$

Change in gaseous moles = 12

$\Rightarrow W = -\frac{12 \times 8.3 \times 300}{1000}$ kJ $= -29.88$ kJ.

28. (280.00)



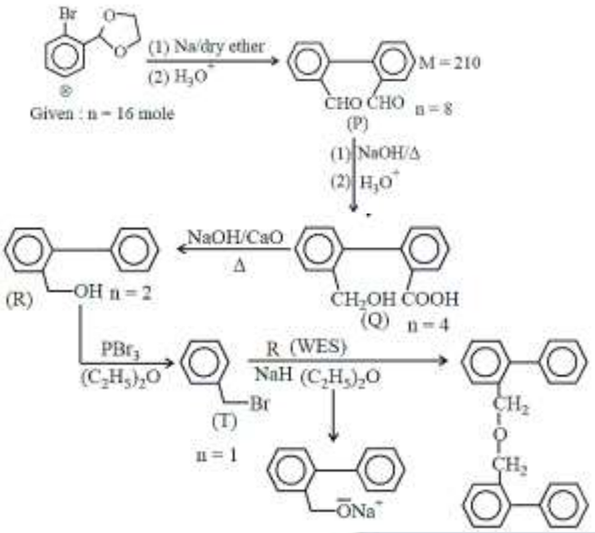
Hexamethylene diamine series give (+) carbylamines test.

$C_xH_yN_z + (2x + 1/2)CuO \rightarrow xCO_2 + \frac{y}{2}H_2O + \frac{z}{2}\ N_2$ (Dumas method)

$C_6H_{16}\ N_2$ then $\frac{z}{2}\ N_2 = \frac{2}{2}\ N_2 = 1$ mole

Given 10 mole in this reaction so $w_{N_2} = 10 \times 28 = 280gm$

29. (175.00)



$M = (C_6H_4)_2(C_6H_5)_2C_2H_4O$

$= 350$

Weight of product of $S = 350 \times 1/2 = 175gm$ Ans.

* WES: Williamson ether synthesis

30. (a)

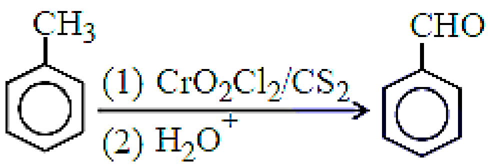
$Mn^{+2} \xrightarrow{H_2\ S+NH_4OH} \underset{\text{Pink/buff ppt.}}{MnS\downarrow}$

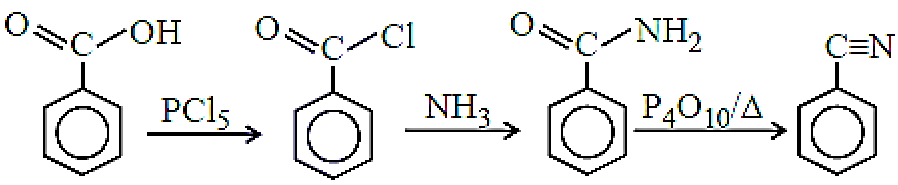
$Ba^{+2} \xrightarrow{(NH_4)_2CO_3+NH_4OH} \underset{\text{White ppt.}}{BaCO_3 \downarrow}$

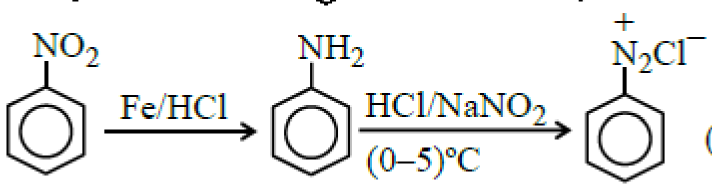
$Al^{+3} \xrightarrow{NH_4Cl+NH_4OH} \underset{\text{White ppt.}}{Al(OH)_3 \downarrow}$

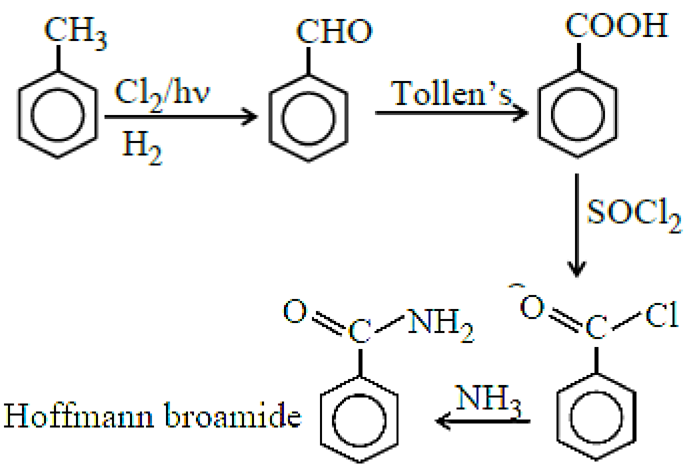
$Cu^{+2} \xrightarrow{H_2\ S+HCl(\text{ dil })} \underset{\text{Black ppt.}}{CuS \downarrow}$

31. (b)

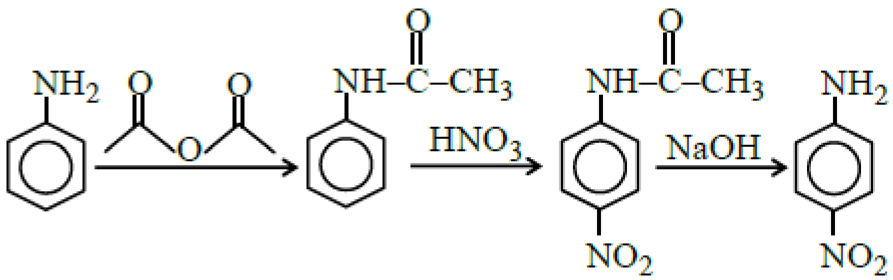
(1)  (S) Cannizaro

(2)  (P) Stephen

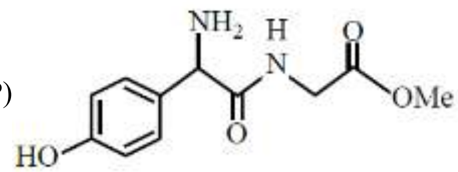
(3)  (Q) Sandymeyer

(4) 

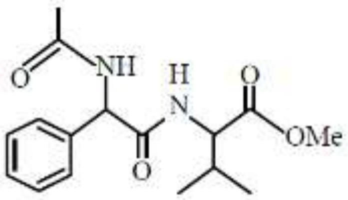
(R)

(5) 

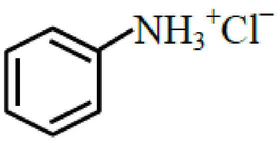
32. (b)

(P) 

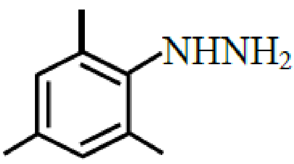
Gives $FeCl_3$ test because of presence of phenolic group and ninhydrin test.

(Q) 

Amino acid obtained gives (+) Ninhydrin test but not phthalein dye test.

(R) 

Reaction with phenyl diazonium salt gives yellow dye.

(S) 

Compound will form hydrazone derivative with aldehydic group of glucose

PART-II: MATHEMATICS

33. (c)

$h(x) = f(x+1) - g(x+2)$

$= a_1 + 10(x+1) + a_2(x+1)^2 + a_3(x+1)^3 + (x+1)^4 - b_1 - 3(x+2) - b_2(x+2)^2 - b_3(x+2)^3 - (x+2)^4$

Coeff. of x^3 in $h(x)$

$= a_3 - b_3 - 4$

$f(x) - g(x) \neq 0\ \forall x \in R$

$\Rightarrow a_1 + 10x + a_2x^2 + a_3x^3 + x^4 - b_1 - 3x - b_2x^2 - b_3x^3 - x^4 \neq 0$

$\Rightarrow x^3(a_3 - b_3) + x^2(a_2 - b_2) + 7x + (a_1 - b_1) \neq 0$

Cubic Eq. will become zero at atleast are value of x

So it will be quadratic

$\Rightarrow \mathrm{a}_1 - \mathrm{b}_1 = 0$

34. (a)

$$\mathrm{P(U)} = 1 - \mathrm{P}(\mathrm{S}_1' \cap \mathrm{S}_2' \cap \mathrm{S}_3') = \frac{1}{2}$$

$$\Rightarrow \mathrm{P}(\mathrm{S}_1' \cap \mathrm{S}_2' \cap \mathrm{S}_3') = \frac{1}{2}; \mathrm{P}(\mathrm{S}_1') \cdot \mathrm{P}(\mathrm{S}_2') \cdot \mathrm{P}(\mathrm{S}_3') = \frac{1}{2}$$

$$\Rightarrow (1 - \mathrm{P}(\mathrm{S}_1))(1 - \mathrm{P}(\mathrm{S}_2))(1 - \mathrm{P}(\mathrm{S}_3)) = \frac{1}{2} \ldots\ldots (1)$$

$$\mathrm{P(V)} = \frac{\mathrm{P}(\mathrm{S}_1 \cap \mathrm{S}_2' \cap \mathrm{S}_3')}{\mathrm{P}(\mathrm{S}_2' \cap \mathrm{S}_3')} = \frac{1}{10}$$

$$\Rightarrow \mathrm{P}(\mathrm{S}_1) \cdot \mathrm{P}(\mathrm{S}_2')\mathrm{P}(\mathrm{S}_3') = \frac{1}{10}\mathrm{P}(\mathrm{S}_2') \cdot \mathrm{P}(\mathrm{S}_3')$$

$$\Rightarrow \mathrm{P}(\mathrm{S}_1) = \frac{1}{10}$$

$$\mathrm{P(W)} = \mathrm{P}(\mathrm{S}_2 \cap \mathrm{S}_3') = \frac{1}{12}$$

$$\mathrm{P}(\mathrm{S}_2) \cdot \mathrm{P}(\mathrm{S}_3') = \frac{1}{12}$$

$\mathrm{P}(\mathrm{S}_2)(1 - \mathrm{P}(\mathrm{S}_3)) = \frac{1}{12} \ldots\ldots (2)$

Eq. (1)

$$\left(1 - \frac{1}{10}\right)(1 - \mathrm{P}(\mathrm{S}_2))(1 - \mathrm{P}(\mathrm{S}_3)) = \frac{1}{2}$$

$(1 - P(S_2))(1 - P(S_3)) = \frac{5}{9} \ldots (3)$

$$\frac{\text{Eq. (2)}}{\text{Eq. (3)}} \Rightarrow \frac{P(S_2)}{1 - P(S_2)} = \frac{1}{12} \times \frac{9}{5}$$

$$P(S_2) = \frac{3}{23}$$

Put in Eq. (2)

$$\frac{3}{23}(1 - \mathrm{P}(\mathrm{S}_3)) = \frac{1}{12}$$

$$1 - \mathrm{P}(\mathrm{S}_3) = \frac{23}{36}$$

$$\mathrm{P}(\mathrm{S}_3) = \frac{13}{36}$$

35. (c)

Sol. (a) RHD at $\mathrm{x} = 0$: $\lim_{\mathrm{h} \to 0} \frac{\left(2 - 2\mathrm{h}^2 - \mathrm{h}^2 \sin\frac{1}{\mathrm{h}}\right) - 2}{\mathrm{h}} = 0$

Similarly LHD at $\mathrm{x} = 0$ is also equal to 0 .

$\therefore$ Differentiable at $\mathrm{x} = 0$

(b) $f'(x) = -4x - 2x\sin\frac{1}{x} - x^2\left(\cos\frac{1}{x}\right)\left(\frac{-1}{x^2}\right)$

$$f'(x) = -\left(4x + 2x\sin\frac{1}{x}\right) + \cos\frac{1}{x}$$

$$f'(x) = -\left(2x\left(4 - \sin\frac{1}{x}\right)\right) + \cos\frac{1}{x}$$

for $x \in (0, \delta)$

$\Rightarrow$ We can't say $f(x)$ is decreasing on $(0, \delta)$ as $\cos\frac{1}{x}$ oscillates.

(c) for $x \in (-\delta, 0)$, for any $\delta > 0$

$\Rightarrow f(x)$ is not increasing on $(-\delta, 0)$ as $\cos\frac{1}{x}$ oscillates from -1 to 1 .

(d) $f(0) = 2$

$f(0+h) < 2$

$f(0-h) < 2$

$\therefore x = 0$ is local maxima

36. (c)

$Q^{-1} = Q^T \Rightarrow Q^T = I$

Q is orthogonal matrix

Let $Q = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

$PQ = QP \Rightarrow \begin{bmatrix} 2a_1 & 2\,b_1 & 2c_1 \\ 2a_2 & 2\,b_2 & 2c_2 \\ 3a_3 & 3\,b_3 & 3c_3 \end{bmatrix} = \begin{bmatrix} 2a_1 & 2\,b_1 & 3c_1 \\ 2a_2 & 2\,b_2 & 3c_2 \\ 2a_3 & 2\,b_3 & 3c_3 \end{bmatrix}$

$c_1 = 0, c_2 = 0, a_3 = 0, b_3 = 0$

$Q = \begin{bmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix}$

$a_1 a_2 + b_1 b_2 = 0$

$a_1{}^2 + b_1{}^2 = 1, a_2{}^2 + b_2{}^2 = 1, c_3{}^2 = 1$

a_1	b_1	a_2	b_2	c_3
1	0	0	$1, -1$	$+1, -1$
-1	0	0	$1, -1$	$1, -1$
0	1	$1, -1$	0	$1, -1$
0	-1	$1, -1$	0	$1, -1$

Total 16 matrices

SECTION-2:

37. (a,c)

Sol. Let $L_1: \vec{r} = \vec{a} + t\vec{b}$

$\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{vmatrix} = \hat{i}(1) - \hat{j}(1) + \hat{k}(1)$

Dr's of $L_1: < 1, -1, 1 >$

$L_1: \frac{x+1}{1} = \frac{y-0}{-1} = \frac{z-6}{1}$

$\& L_2: \frac{x-2}{1} = \frac{y+1}{-1} = \frac{z-3}{1} = \lambda$

$M: 2x + y - 2z - 6 = 0$

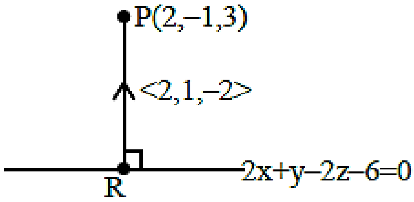
Let point on $L_2(\lambda+2, -\lambda-1, \lambda+3)$

$2(\lambda+2) - \lambda - 1 - 2\lambda - 6 - 6 = 0$

$2\lambda + 4 - 3\lambda - 13 = 0$

$\lambda = -9$

$\therefore Q(-7, 8, -6)$



Line $PR: \frac{x-2}{2}=\frac{y+1}{1}=\frac{z-3}{-2}=\mu$

$R(2\mu+2, \mu-1, -2\mu+3)$

Put in plane

$2(2\mu+2)+\mu-1-2(-2\mu+3)-6=0$

$4\mu+4+\mu-1+4\mu-6-6=0$

$9\mu-9=0 \Rightarrow \mu=1$

$R(4,0,1)$

$P(2,-1,3) \& Q(-7,8,-6)$

$PQ=\sqrt{81+81+81}=9\sqrt{3}$

$Q(-7,8,-6) \& R(4,0,1)$

$QR=\sqrt{121+64+49}=\sqrt{234}$

Area ($\triangle$ PQR)

$$=\frac{1}{2}|\overrightarrow{QP}\times\overrightarrow{QR}|$$

$$=\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 9 & -9 & 9\\ 11 & -8 & 7\end{vmatrix}$$

$$=\frac{3}{2}\sqrt{234}$$

$\overrightarrow{PQ}=-9\hat{i}+9\hat{j}-9\hat{k}=-9(\hat{i}-\hat{j}+\hat{k})$

$\overrightarrow{PR}=2\hat{i}+\hat{j}-2\hat{k}$

$$\cos\theta=\left|\frac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{PQ\cdot PR}\right|$$

$$=\frac{9}{9\sqrt{3}\times 3}=\frac{1}{3\sqrt{3}}$$

$$\theta=\cos^{-1}\left(\frac{1}{3\sqrt{3}}\right)$$

38. (a,d)

$$f(n)=\begin{cases}(n+1)/2 & \text{if } n \text{ is odd}\\ (4-n)/2 & \text{if } n \text{ is even}\end{cases}$$

$f(n)=\{(1,1),(2,1),(3,2),(4,0),(5,3),(6,-1),\ldots\}$

$\therefore f(n)$ is many one and onto function

$$g(n)=\begin{cases}3+2n & \text{if } n\geq 0\\ -2n & \text{if } n<0\end{cases}$$

$g(n)=\{(-3,6),(-2,4),(-1,2),(0,3),(1,5),(2,7),(3,9),(4,15),\ldots\}$

$\therefore g(n)$ is one-one and into function

$f(g(n))=2+n, n\in N$

fog is one-one and into

$$g(f(n))=\begin{cases}4+n & \text{if } n \text{ is odd natural number}\\ 7-n & \text{if } n=2,4\\ n-4 & \text{if } n \text{ is even natural number and } n\geq 6\end{cases}$$

$g(f(2))=g(f(1))=5$

$\therefore$ gof is many one and into

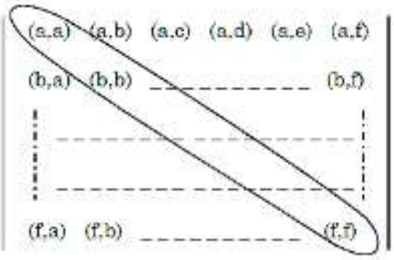
39. (a,d)

$|x+iy-1-2i| = 2|x+iy-3i|$
$\Rightarrow\ (x-1)^2+(y-2)^2 = 4(x^2+(y-3)^2)$
$\Rightarrow\ 3x^2+3y^2+2x-20y+31=0$
$\Rightarrow\ x^2+y^2+\frac{2x}{3}-\frac{20y}{3}+\frac{31}{3}=0$

$\therefore$ S is a circle with centre $\left(-\frac{1}{3},\frac{10}{3}\right)$ and radius $= \sqrt{\frac{1}{9}+\frac{100}{9}-\frac{31}{3}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

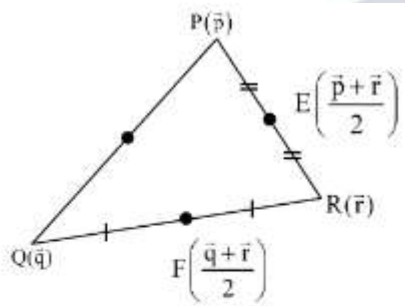
SECTION-3 :

40. (105)

Sol.


For relation to be reflexive all the diagonal elements must be taken and out of remaining 30 elements there are 15 pairs and we need 2 pairs such that R contains exactly 10 elements and is both reflexive and symmetric.
$\therefore$ number of ways $= {}^{15}C_2 = 105$

41. (1.20)



$\because\ \overrightarrow{SP}+5\overrightarrow{SQ}+6\overrightarrow{SP}=\vec{0}$
(Let P.V. of P be $\vec{p}$, Q be $\vec{q}$ and R be $\vec{r}$)
$\Rightarrow (\vec{p}-\vec{s})+5(\vec{q}-\vec{s})+6(\vec{r}-\vec{s})=\vec{0}$
$\Rightarrow \vec{S}=\frac{\vec{p}+5\vec{q}+6\vec{r}}{12}$
$\Rightarrow \overrightarrow{EF}=\left(\frac{\vec{q}-\vec{p}}{2}\right)$
$\Rightarrow \overrightarrow{ES}=\left(\frac{5\vec{q}-5\vec{p}}{12}\right)=\frac{5}{12}(\vec{q}-\vec{p})$
$\therefore \frac{|\overrightarrow{EF}|}{|\overrightarrow{ES}|}=\frac{6}{5}=1.2$

42. (762)

Let A → " 0 " appear exactly twice.
and B → "1" appear exactly twice.
$\therefore$ A ∩ B → "0" and " 1 " both appears exactly twice.

n(A) = $\_\ \ \underbrace{\_\ \_\ \_\ \_\ \_\ \_}$

$= 6_{C^2}\ \underset{placing\ zero}{(1)}\ (2)^5 = \frac{6\times 5}{2}\times 2^5 = 480$

for n(B)
C-I: 1 at first place

$1\_ \; \underbrace{\_\;\_\;\_\;\_\;\_\;\_}$

Number of ways $= 6_{C^1} \underbrace{(1)}_{placing\ 1} (2)^5 = 192$

C-II: 2 at first place

$2.\_ \; \underbrace{\_\;\_\;\_\;\_\;\_\;\_}$

Number of ways $= 6_{C^2} \underbrace{(1)}_{placing\ 1} (2)^4 = \frac{6\times 5}{2} \times 2^4 = 240$

$n(B) = 240 + 192$

for $n(A \cap B)$

$\_ \; \underbrace{\_\;\_\;\_\;\_\;\_\;\_}$

for $n(A \cap B) = \underbrace{{}^6C_2}_{\text{placing zero}} (1) \times \underbrace{{}^5C_2}_{\text{placing 1}} (1) \times \underbrace{(1)}_{\text{2at rest places1}} = \frac{6\times 5}{2} \times \frac{5\times 4}{2} = 150$

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 480 + (192 + 240) - 150$

$= 762$

43. (2.40)

$$\lim_{x\to 0} \frac{\frac{\alpha}{2}\int_0^x \frac{1}{1-t^2}dt + \beta x\cos x}{x^3}$$

$$= \lim_{x\to 0} \frac{\frac{\alpha}{2}\left(\frac{1}{1-x^2}\right) + \beta\cos x - \beta x\sin x}{3x^2}$$

$$= \frac{\frac{\alpha}{2}(1-x^2)^{-1} + \beta\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!}\ldots\right) - \beta x\left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\ldots\right)}{3x^2}$$

$$= \frac{\frac{\alpha}{2}(1 + x^2 + x^4\ldots) + \beta\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!}\ldots\right) - \beta\left(x^2 - \frac{x^4}{3!}\ldots\right)}{3x^2}$$

$$= \frac{\left(\frac{\alpha}{2} + \beta\right) + x^2\left(\frac{\alpha}{2} - \frac{\beta}{2} - \beta\right) + x^4()\ldots}{3x^2} = 2(\text{Given})$$

$$\therefore \frac{\alpha}{2} + \beta = 0 \text{ and } \frac{\alpha - 3\beta}{6} = 2$$

$\Rightarrow \alpha = -2\beta$ and $\alpha = 12 + 3\beta$

$\Rightarrow \beta = -\frac{12}{5}$ and $\alpha = \frac{24}{5}$

$\therefore \alpha + \beta = \frac{12}{5} = 2.40$

44. (96)

$\because f(x+y) = f(x)\cdot f(y)$

$\Rightarrow f(x) = k^x\ (f(x) > 0 \forall x \in R)$

$\because f(a_{31}) = 64 f(a_{25})$

$\Rightarrow k^{(a+30d)} = 64 \cdot k^{(a+2d)}$
$\Rightarrow k^{6d} = 64$
$\Rightarrow k^{d} = 2$

$\sum_{i=1}^{50} f(a_i) = f(a_1) + f(a_2) + \cdots \ldots + f(a_{50})$

$= k^a + k^{a+d} + \cdots + k^{a+49d} = \frac{k^a\left(k^{50d}-1\right)}{k^d - 1}$

$= k^a(2^{50}-1) = 3(2^{25}+1)(\text{ Given })$

$\Rightarrow$

$k^a = \frac{3}{2^{25}-1}$

$\therefore \sum_{i=6}^{30} f(a_i) = k^{a+5d} + k^{a+6d} + \cdots + k^{a+29d}$

$= k^{a+5d}\frac{\left(k^{25d}-1\right)}{k^d-1} = k^a \cdot \left(k^d\right)^5(2^{25}-1)$

$= \frac{3}{2^{25}-1} \cdot 2^5(2^{25}-1) = 96$

45. (2.00)

$$\frac{dy_1}{y_1} + \frac{dy_2}{y_2} + \frac{dy_3}{y_3} = \left(\sin^2 x + \cos^2 x + \frac{2-x^3}{x^3}\right)dx$$

$$\ln(y_1y_2y_3) = \frac{-1}{x^2} + C$$

$$\ln(y_1(x)y_2(x)y_3(x)) = \frac{-1}{x^2} + C$$

$$\ln\left(5 \cdot \frac{1}{3} \cdot \frac{3}{5e}\right) = \frac{-1}{x^2} + C$$

$\therefore C = 0$

$$y_1(x)y_2(x)y_3(x) = e^{\frac{-1}{x^2}}$$

$$\lim_{x\to 0} \frac{e^{\frac{-1}{x^2}} + 2x}{e^{3x}\sin x}$$

$$\lim_{x\to 0} \frac{1}{e^{3x+\frac{1}{x^2}}\sin x} + \lim_{x\to 0}\frac{2x}{e^{3x}\sin x}$$

$$\lim_{x\to 0} \frac{1}{e^{3x} \cdot \frac{\sin x}{x} \cdot \frac{e^{\frac{1}{x^2}}}{\frac{1}{x}}} + 2$$

$$= 0 + 2\left\{\lim_{y\to\infty} \frac{e^{y^2}}{y} = \frac{e^{y^2}\cdot 2y}{1} = \infty\right\}$$

$= 2$

SECTION-4:

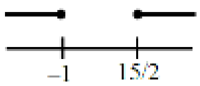
46. (c)

X_i	f_i	$\bar{x} = 7$ $d_i = \lvert x_i - \bar{x}\rvert$	$M = 6$ $e_i = \lvert x_i - M\rvert$	$\sum f_i\, d_i$	$\sum f_i e_i$	$\sum f_i\, d_i^2$
4	5	3	2	15	10	45
5	4	2	1	8	4	16
6	1	1	0	1	0	1
8	3	1	2	3	6	3
9	2	2	3	4	6	8
11	3	4	5	12	15	48

12	1	5	6	5	6	25
				48	47	146

$f_1 = 4$
$f_2 = 3$
$\alpha = \frac{48}{19}, \beta = \frac{47}{19}, \sigma^2 = \frac{146}{19}$

47. (b)
$P(x) = 10x^3 - 45x^2 + 60x + 35$
$P'(x) = 30(x-1)(x-2)$
$P(x)$ decreases in $[1,2]$
$\Rightarrow$ Range of $P(x) = [55,60]$
$f(x) = \left[\frac{P(x)}{n}\right]$ min value of $n = 9$
(Q) For $g(x)$ to be increasing
$2n^2 - 13n - 15 \geq 0$



$n \in N$
$\Rightarrow n = 8$
(R) $h(x) = (x^2-9)^n(x^2+2x+3)$
$h'(x) = (x^2-9)^n(2x+2) + (x^2+2x+3)n(x^2-9)^{n-1} \cdot 2x$
$= (x^2-9)^{n-1}[2(x^2-9)(x+1) + 2nx(x^2+2x+3)]$
$= (x+3)^{n-1}(x-3)^{n-1} \cdot q(x)$
Derivative must change sign at $x = 3$
$\because n - 1 = \text{odd}$
$n = \text{even}$
$n = 6$
(S) $\cos\left|x - k + \frac{1}{2}\right|$ is differentiable everywhere
$\Rightarrow \sin|x-k|$ is NOT diff. at $k = 0,1,2,3,4$

48. (a)
$\vec{v} \times \vec{w} = \vec{u}\ \vec{v} \perp \vec{w}$
$\vec{w} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$
$\vec{u} = \alpha\hat{\imath} + \beta\hat{\jmath} + \gamma\hat{k}$
$\vec{u}.\vec{w} = 0 = \alpha + \beta - 2\gamma$
$(\vec{v}.\vec{w}) \times \vec{v} = \vec{u} \times \vec{v}$
$\vec{w}|\vec{v}|^2 - 0 = \vec{w}$
$|\vec{v}| = 1(P)$

$\vec{u} \times \vec{v} = \vec{w}\ \vec{u} \perp \vec{v}\ \vec{u} \perp \vec{w}$
$$\begin{vmatrix} -t & 1 & 1 \\ 1 & -t & 1 \\ 1 & 1 & -t \end{vmatrix} = 0$$
$t = -1$ or 2
$t = -1$
$\alpha + \beta + \gamma = 0$
$t = 2$
$\alpha = \beta = \gamma$

Also $\vec{u} \times \vec{v} = \vec{w}$
$|\vec{u}||\vec{v}| = |\vec{w}|$
$\Rightarrow |\vec{u}| = |\vec{w}| = \sqrt{6}$
Case-1
$t = 2$
$\alpha = \beta = \gamma$
$\alpha^2 + \beta^2 + \gamma^2 = 6$
$a = \sqrt{2}, -\sqrt{2}$
(S) $t + 3 = 5$
Case-2

$t = -1$
$\alpha + \beta + \gamma = 0$
$\alpha + \beta - 2\gamma = 0$
$\gamma = 0, \alpha = -\beta$
(Q) $\alpha = \sqrt{3}, \gamma^2 = 0, \beta = -\sqrt{3}$
(R) $\alpha = \sqrt{3}(\gamma + \beta)^2 = \beta^2 = 3$
(P) $\rightarrow$ (2), (Q) $\rightarrow$ (1), (R) $\rightarrow$ (4), (S) $\rightarrow$ (5)