## PART-I PHYSICS

**1. (b)**

$S =$ emf per unit temperature difference $\sigma =$ Electrical conductivity

k = Thermal conductivity

$[S] = [ML^2\, T^{-3}\, T^{-1}\, K^{-1}]$

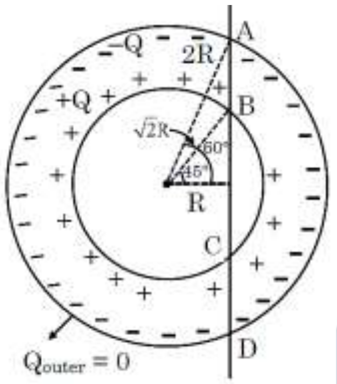
$[\sigma] = [M^{-1}\, L^{-3}\, T^3\, T^2]$

$[K] = [M^1\, L^1\, T^{-3}\, K^{-1}]$

$[Z] = \dfrac{S^2\sigma}{K} = \dfrac{[M^1\, L^1\, T^{-3}\, K^{-2}]}{[M^1\, L^1\, T^{-3}\, K^{-1}]}$

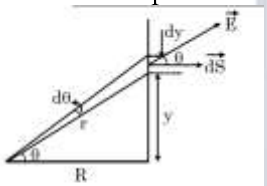
$[Z] = [K^{-1}]$

**2. (c)**




**Sol.**

Here we are assuming that " $\ell$ " is very large just for the sake of symmetry. Outside cylinder will have zero electric field inside, so the flux generated on the plate will be due to inner cylinder only in sections AB and CD, as section be will be at that place where electric field is zero.




flux through element will be

$d\phi = \vec{E} \cdot \vec{dS}$

$d\phi = \dfrac{2k\lambda}{r} dy \cdot \ell \cdot \cos\theta$ ... (i)

from figure we can say that

$\cos\theta = \dfrac{R}{r} \Rightarrow r = R\sec\theta$

$\tan\theta = \dfrac{y}{R} \Rightarrow y = R\tan\theta$

$\Rightarrow dy = R\sec^2\theta d\theta$

$d\phi = \dfrac{2k\lambda}{R\sec\theta} R \cdot \sec^2\theta \cdot l \cdot \cos\theta d\theta$

$d\phi = 2k\lambda\ell dq$

$$\int_0^{\phi_{AB}} d\phi = 2k\lambda\ell\int_{\pi/4}^{\pi/3} d\theta$$

$$\phi_{AB} = 2k\lambda\ell\left[\frac{\pi}{3}-\frac{\pi}{4}\right]$$

$$\phi_{AB} = 2kQ\left[\frac{\pi}{12}\right]$$

$$\phi_{AB} = 2\times\frac{1}{4\pi\epsilon_0\epsilon_r}Q\left[\frac{\pi}{12}\right]$$

$$\phi_{AB} = \frac{Q}{120\,\epsilon_0}$$

$$\phi_{plate} = \phi_{AB} + \phi_{BC} + \phi_{CD}\ (\phi_{CD} = \phi_{AB})$$

$$\phi_{plate} = \frac{Q}{60\,\epsilon_0}$$

**3. (c)**

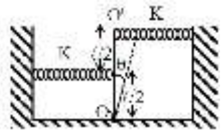


Fig. 1

$$\frac{ML^2}{3}\ddot{\theta} + K\cdot\frac{\ell}{2}\theta\cdot\frac{\ell}{2} + K\cdot\ell\cdot\theta\cdot\ell = 0$$

$$\ddot{\theta} + \frac{15}{4}\frac{k}{M}\theta = 0$$

$$\ddot{\theta} = -\left(\frac{15}{4}\frac{k}{M}\right)\theta$$

$$\omega_1 = \sqrt{\frac{15\,K}{4M}}$$

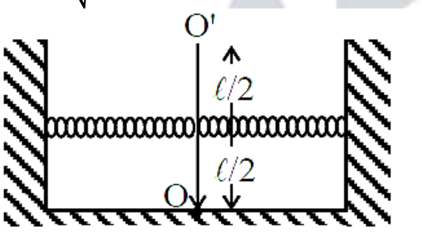



Fig. 2

$$\frac{1}{3}ML^2\ddot{\theta} + 2\,K\frac{L}{2}\theta\cdot\frac{L}{2} = 0$$

$$\ddot{\theta} + \frac{3}{2}\frac{K}{M}\theta = 0$$

$$\ddot{\theta} = -\frac{3}{2}\frac{K}{M}\theta$$

$$\omega_2 = \sqrt{\frac{3}{2}\frac{K}{M}}$$

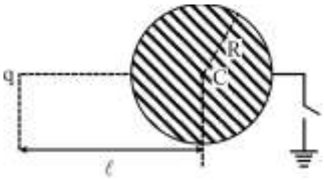
$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{15}{4}\times\frac{2}{3}} = \sqrt{\frac{5}{2}}$$

**4. (b)**

$$m_2\omega^2 r = \frac{Gm_1\,m_2}{r^2}$$

$$\omega = \sqrt{\frac{Gm_1}{r^3}}$$
$$L = m_2\omega r^2$$
$$= m_2\sqrt{\frac{Gm_1}{r^3}}r^2$$
$$L = m_2\sqrt{Gm_1 r} = \text{ const.}$$
$$\ell\text{nL} = \ell\text{nm}_2 + \ell\text{nG} + \frac{1}{2}\ell\text{ nm}_1 + \frac{1}{2}\ell\text{nr}$$
$$0 = \frac{dm_2}{m_2} + \frac{1}{2}\frac{dm_1}{m_1} + \frac{1}{2}\frac{dr}{r}$$
$$\frac{dr}{rdt} = -\frac{2dm_2}{m_2dt} - \frac{dm_1}{m_1dt} \approx -\frac{2\gamma}{m_2} \text{ Here } \left(\frac{dm}{dt} = \gamma\right)$$

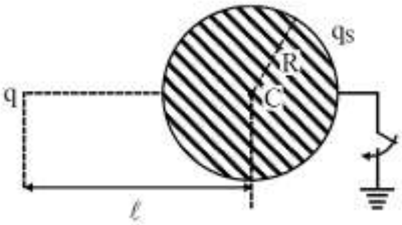
**5. (a, b,c)**




Before grounding

$$\text{V}_{\text{sphere}} = (\text{V}_c)_{net} = (\text{V}_\text{c})_\text{q} + (\text{V}_\text{c})_\text{ind}$$
$$\text{V}_{sphere} = \frac{\text{kq}}{\ell} + 0 = \frac{9\times10^9\times10^{-8}}{0.2} - \frac{90}{0.2} - \frac{900}{2} = 450 \text{ volt}$$

After grounding



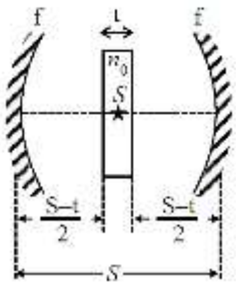

$$\frac{\text{kQ}}{\ell} + \frac{\text{kg}_\text{s}}{\text{R}} - \text{V}'_{\text{sphere}} = 0$$
$$\text{q}_\text{s} = -\frac{\text{R}}{\ell}\text{q} - -\frac{1}{2}\times10^{-\text{s}} = -5\times10^{-9}$$
$$\text{q}_\text{s} = -5\times10^{-9}\text{ Coulomb}$$

Charge flower from sphere to ground $= 5\times10^{-9}$ Coulomb

After grounding is removed

$$(\text{V}_{\text{sphere}})_{\text{final}} - \frac{\text{kq}}{\ell'} + \frac{\text{kq}_\text{s}}{\text{R}}$$
$$-\frac{9\times10^9\times10^2\times10^{-8}}{30} - \frac{9\times10^9\times5\times10^{-9}\times10^2}{10}$$
$$-\frac{9\times1000}{30} - 450 = 300 \text{ volt } - 450 \text{ volt } = -150 \text{ volt}$$

**6. (a,b)**




$$\frac{S-t}{2}+\frac{t}{2n_0}=2f$$

$$S+t\left(\frac{1}{n_0}-1\right)=4f$$

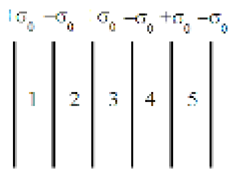
$$S=4f+\left(1-\frac{1}{n_0}\right)t$$

Also

$$\frac{S-t}{2}+\frac{t}{2n_0}=f$$

$$S=2f+\left(1-\frac{1}{n_0}\right)t$$

**7. (a)**




Configuration I

**Sol.**

$$(E_4)_I=\frac{\sigma_0}{2\epsilon_0}[1-1+1-1-1+1]=0$$

$$(V_{First})_I=\frac{-\sigma_0}{2\epsilon_0}[-1+2-3+4-5]d$$

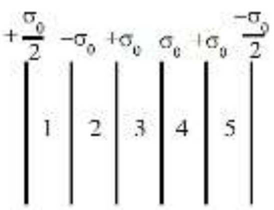
$$=\frac{-\sigma_0}{2\epsilon_0}[-3]d=\frac{\sigma_0 3\,d}{2\epsilon_0}$$

$$(V_{Last})_I=\frac{-\sigma_0}{2\epsilon_0}[1-2+3-4+5]$$

$$=\frac{\sigma_0}{2\epsilon_0}[-3\,d]$$

$$(V_{First}-V_{Last})_I=\frac{3\sigma_0\,d}{\epsilon_0}$$

$$=\frac{3\times 9\times 10^{-6}\times 1\times 10^{-6}}{9\times 10^{-12}}=3\text{volt}$$




Configuration II

$$(E_3)_{II} = \frac{\sigma_0}{2\epsilon_0}\left[\frac{1}{2} - 1 + 1 + 1 - 1 + \frac{1}{2}\right] = \frac{\sigma_0}{2\epsilon_0}$$

$$= \frac{-\sigma_0}{2\epsilon_0}\left[-1 + 2 - 3 + 4 - \frac{5}{2}\right] d$$

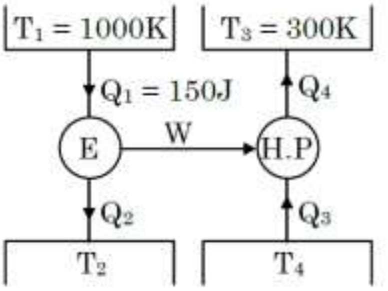
$$= \frac{-\sigma_0}{2\epsilon_0}[2 - 2.5] d = \frac{\sigma_0 d}{4\epsilon_0}$$

$$(V_{Last})_{II} = \frac{-\sigma_0}{2\epsilon_0}\left[1 - 2 + 3 - 4 + \frac{5}{2}\right] d$$

$$= \frac{-\sigma_0}{2\epsilon_0}[6.5 - 6] d = \frac{-\sigma_0 d}{4\epsilon_0}$$

$$(V_{First} - V_{Last})_{II} = \frac{\sigma_0 d}{2\epsilon_0} \neq 0$$

**8. (a,b,c)**




$\eta = 0.4$

$(COP)_{HP} = 10$

$W = \eta \times Q_1 = 0.4 \times 150 = 60$ J

$$COP = \frac{\text{Dissered Effect}}{\text{Work input}}$$

$$10 = \frac{Q_3}{W}$$

$Q_3 = 600$ J

Also

$$\eta = 0.4 = 1 - \frac{T_2}{T_1}$$

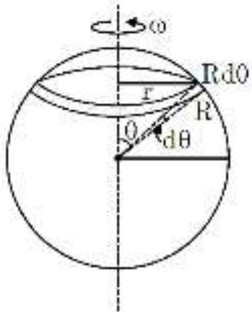
$T_2 = 600$ K

$$COP = \frac{T_3}{T_3 - T_4} = 10$$

$$= \frac{300}{300 - T_4} = 10$$

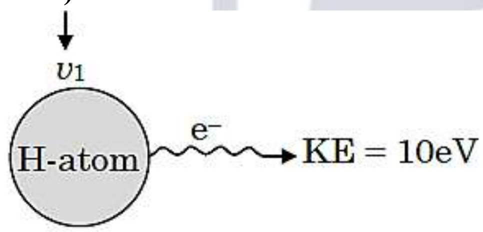
$T_4 = 270$ K

**9. (1.66 to 1.67)**



$$dM = dIA$$
$$A = \pi r^2 = \pi(R\sin\theta)^2$$
$$dI = \frac{da}{T} = \frac{\sigma(2\pi r)(Rd\theta)\omega}{2\pi}$$
$$dI = \frac{\sigma 2\pi R^2\omega\sin\theta d\theta}{2\pi}$$
$$\mathrm{dI} = \sigma \mathrm{R}^2\omega\sin\theta\ \mathrm{d}\theta$$
Magnetic dipole moment:
$$M = \int dM = \int_0^{\pi} \sigma R^2\omega\pi R^2\sin^3\theta d\theta$$
$$M = \sigma R^4\omega\pi\int_0^{\pi}\sin^3\theta d\theta \left(\because \int_0^{\pi}\sin^3\theta d\theta = \frac{4}{3}\right)$$
$$M = \left(\frac{Q}{4\pi R^2}\right)R^4\omega\pi\left(\frac{4}{3}\right)$$
Magnetic dipole moment
$$\mathrm{M} = \frac{\mathrm{QR}^2\omega}{3}$$
Angular momentum
$$\mathrm{L} = \left(\frac{2}{5}\mathrm{MR}^2\right)\omega$$
$$\frac{\mathrm{M}}{\mathrm{L}} = \frac{\mathrm{QR}^2\omega}{3\times\frac{2}{5}\mathrm{MR}^2\omega} = \frac{\mathrm{Q}}{2\mathrm{M}}\left(\frac{5}{3}\right)$$
$$\alpha = \frac{5}{3} = 1.67$$

**10. (11.80)**




**Sol.**

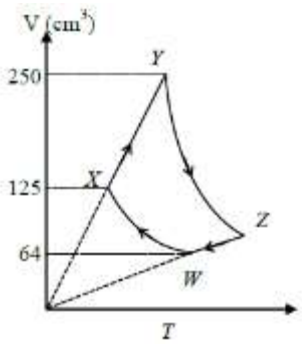
$h\nu_1 = 13.6 + 10 = 23.6\text{eV}$ …. (1)

Energy of positronium in ground state
$$= -13.6\frac{\mu}{m}\left(\frac{z}{n}\right)^2 \text{ eV}$$
$$= -13.6\times\frac{1}{2}\text{eV} = -6.8\text{eV}$$
So to make positronium 6.8 eV must release & 5 eV is the KE of COM. So total energy of photon released ($h\nu_2$) will be:

$h\nu_2 = (10-5) + 6.8 = 11.8\text{eV}$ …… (2)

$\therefore$ Difference in energy $= 23.6 - 11.8 = 11.8\text{eV}$

**11. (1.60)**



$nRT_W = P_W V_W = 1\ J$

$P_W = \frac{1}{64} \times 10^6\ Pa$

For WX process

$P_X V_X^{y} = P_W V_W^{y}$

$\Rightarrow P_X = P_W \left(\frac{V_W}{V_X}\right)^{y}$

amount of heat absorbed in XY process

$Q = nC_P \Delta T = n \times \frac{5}{2} R \times [T_Y - T_X]$ [For monoatomic gas $C_P = \frac{5R}{2}$]

$Q = \frac{5}{2}[nRT_Y - nRT_X]$

$= \frac{5}{2}[P_Y V_Y - P_X V_X]$

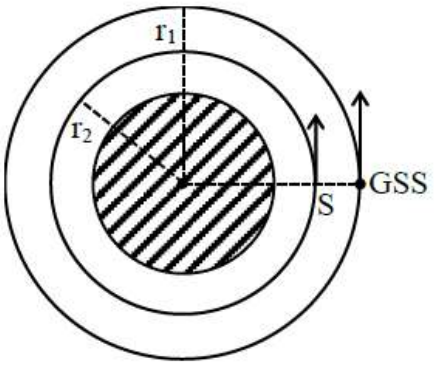
$= \frac{5}{2} P_X [V_Y - V_X]$ [$\because P_X = P_Y$; Isobaric process ]

$= \frac{5}{2} \times P_W \times \left[\frac{V_W}{V_X}\right]^{y} [V_Y - V_X]$

Putting values:

$Q = 1.6$ Joule

**12. (2.33)**




$T \propto r^{3/2}$

$\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2}$

$\frac{\omega_2}{\omega_1} = \left(\frac{r_1}{r_2}\right)^{3/2} = (1.21)^{3/2}$

$\omega_2 = \omega_1(1.331)$ ... (i)

$(\omega_2 + \omega_1) t_0 = 2\pi$ ... (ii)

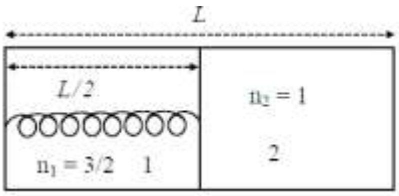
$t_0 = \frac{2\pi}{\omega_2 + \omega_1} = \frac{2\pi}{\left(\frac{4}{3} + 1\right)\omega_1} = \frac{6\pi}{7\omega_1}$

$t_0 = \frac{6\pi}{2\pi} \frac{(T_{GSS})}{(7)}$

$t_0 = \frac{3 \times 24}{7} \text{hours} = \frac{24}{p}$ hours

$p = \frac{7}{3} = 2.33$

**13. (0.20)**




Extension in spring

$x = 0.5\text{ L} - 0.4\text{ L}$

$= 0.1\text{ L}$

FBD of piston

$\text{kx} + \text{P}_2\text{ A} = \text{P}_1\text{ A}$

$\text{P}_2\text{ A} = \text{P}_1\text{ A} - \text{kx}$

$\text{P}_2 = \text{P}_1 - \frac{\text{kL}}{\text{A}(10)} \ldots.(\text{i})$

$P_1V = n_1RT$

$P_2V = n_2RT$

$\frac{P_1}{P_2} = \frac{n_1}{n_2} = \frac{3}{2}$

$P_1 = \frac{3}{2}P_2 \ldots\ldots(\text{i})$

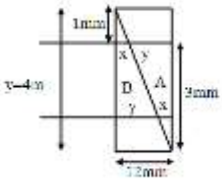
$P_2 = \frac{3}{2}P_2 - \frac{kL}{10A}$

$\frac{P_2}{2} = \frac{kL}{10A}$

$P_2 = \frac{kL}{5A} = \frac{kL}{A}\alpha$

$\alpha = \frac{1}{5} = 0.2$

**14. (1.20)**




$x + y = 12\mu m$

$\frac{4}{12} = \frac{1}{x}$

$x = 3\mu m$

$y = 6\mu m$

$\therefore \Delta = \frac{yd}{D} - (\mu_B - 1)\text{x} - (\mu_A - 1)\text{y} + (\mu_B - 1)\text{y} + (\mu_A - 1)\text{x}$

$\frac{-yd}{D} = -0.5 \times 3 - 0.7 \times 9 + 0.5 \times 9 + 0.7 \times 3$

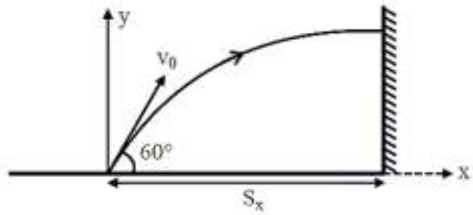
$\frac{-yd}{D} = -0.5 \times 6 - 0.7 \times 6$

$\Rightarrow \frac{-yd}{D} = -1.2\mu\text{ m}$

$\Rightarrow y = \frac{1.2 \times D}{d} = \frac{1.2 \times 2}{2 \times 10^{-3}} \times 10^{-6}$

$= 1.2\text{ mm}$

**15. (170.00)**




$$\vec{F}_{\text{net}} = m\frac{d\vec{v}}{dt}$$

$$m\vec{g} + \vec{F} = \frac{md\vec{v}}{dt}$$

$$m\vec{g} - C\vec{v} = \frac{md\vec{v}}{dt}$$

Horizontal direction

$$-\mathrm{C}\mathrm{v}_{\mathrm{x}} = \frac{\mathrm{m}\mathrm{d}\mathrm{v}_{\mathrm{x}}}{\mathrm{dt}}$$

$$-\frac{C}{m}\int_0^t dt = \int_{v_{0x}}^{v_x}\frac{dv_x}{v_x}$$

$$-\frac{t}{2} = \ell n\frac{v_x}{v_{0x}}$$

$$\frac{dx}{dt} = v_x = v_{0x}e^{-t/2}$$

$$\int_0^{s_x} dx = v_{0x}\int_0^t e^{-t/2}dt$$

$$S_x = 2v_{0x}\left(1 - e^{-t/2}\right)$$

$$att = 2\mathrm{sec}$$

$$S_x = 2 \times 270 \times \cos 60^\circ\left[1 - \frac{1}{e}\right]$$

$$S_x = 270\left(1 - \frac{1}{2.7}\right)$$

$$\mathrm{S_x} = 270\left(1 - \frac{1}{2.7}\right)$$

$$= \frac{270}{2.7} \times (1.7)$$

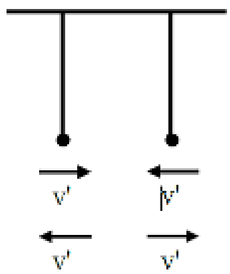
$$= 170\ \mathrm{m}$$

$$\mathrm{S_x} = 170\ \mathrm{m}$$

**16. (31.19 to 32.27)**

$$\cos\theta_0 = 1 - \frac{\theta_0^2}{2} = 0.9$$

$$\frac{\theta_0^2}{2} = 0.1 \Rightarrow \theta_0 = 10.2 = \frac{1}{\sqrt{5}}$$



$$f_{\max}=\frac{v+v'}{v-v'}f$$

$$f_{\min}=\frac{v-v'}{v+v'}f$$

$$\Delta f_{\max}=f_{\max}-f_{\min}=\frac{v+v'}{v-v'}f-\frac{v-v'}{v+v'}f$$

$$=\frac{(v+v')^2-(v-v')^2}{v^2-v'^2}f$$

$$\Delta f_{\max}=\frac{4vv'}{v^2-v'^2}f\ \ldots\ldots\text{(i)}$$

Here, $v'=\ell\Omega_{\max}$

$$=\ell\cdot\theta_0\cdot\omega\ (\omega=\text{ angular frequency })$$

$$=\ell\theta_0\sqrt{\frac{g}{\ell}}$$

$$v'=\theta_0\sqrt{g\ell}$$

$$v'=\frac{1}{\sqrt{5}}\sqrt{10\times 8}$$

$$v'=4$$

Put in equation (i)

$$\Delta f_{\max}=\frac{4\times 330\times 4\times 660}{330^2-4^2}$$

$$\approx\frac{16\times 330\times 660}{330}\approx 32$$

## PART-II CHEMISTRY

**17. (a)**

$$Na_2[Fe(CN)_5(NO)]+Na_2\,S\rightarrow Na_4[Fe(CN)_5(NOS)]$$

Sodium nitroprusside purple solution

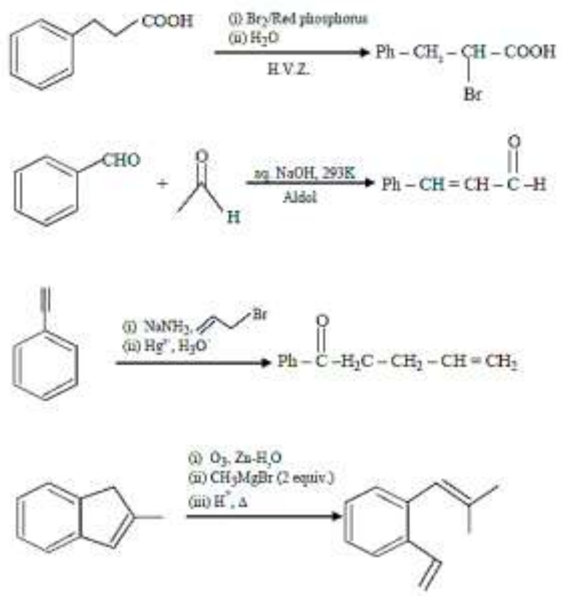
**18. (a)**

$$ICl\xrightarrow{H_2O}\underset{IO^{\ominus}}{HIO}+HCl$$

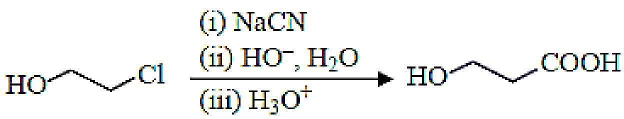
$$ClF_3\xrightarrow{H_2O}\underset{\substack{\downarrow\\ ClO_2^-}}{HClO_2}+3HF$$

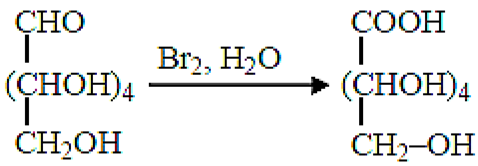
$$BrF_5\xrightarrow{H_2O}\underset{\substack{\downarrow\\ BrO_3}}{HBrO_3}+5HF$$

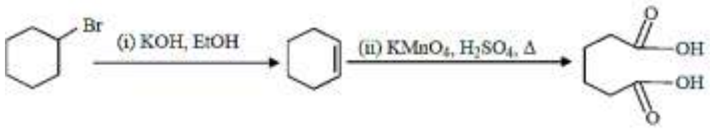
**19. (d)**

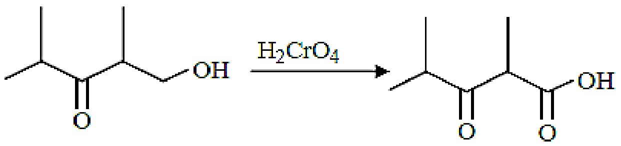


**20. (c)**

(a) 

(b) 

(c) 

(d) 

**21. (c, d)**

(i) Ion - Ion → Interaction energy $\propto \frac{1}{r}$

Ion - dipole → Interaction energy $\propto \frac{1}{r^2}$

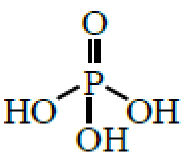
Ion - dipole Interaction energy approaches zero more rapidly as ' $r$ ' increases

(ii) Rotating Polar molecules → Interaction energy $\propto \frac{1}{r^6}$.

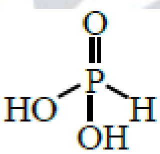
(iii) Dipole - induced dipole forces are independent of temperature.

(iv) Non-polar species show London dispersion forces.

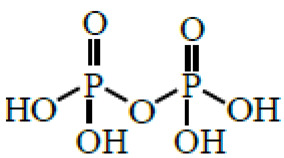
**22. (b, d)**



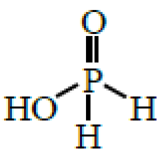
(a) $H_3PO_4$



(b) $H_3PO_3$

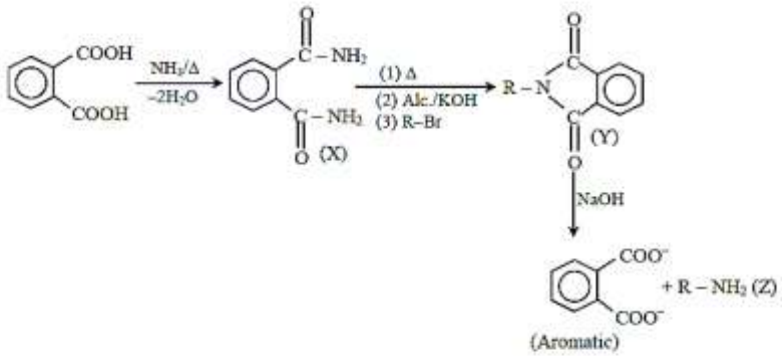


(c) $H_4P_2O_7$

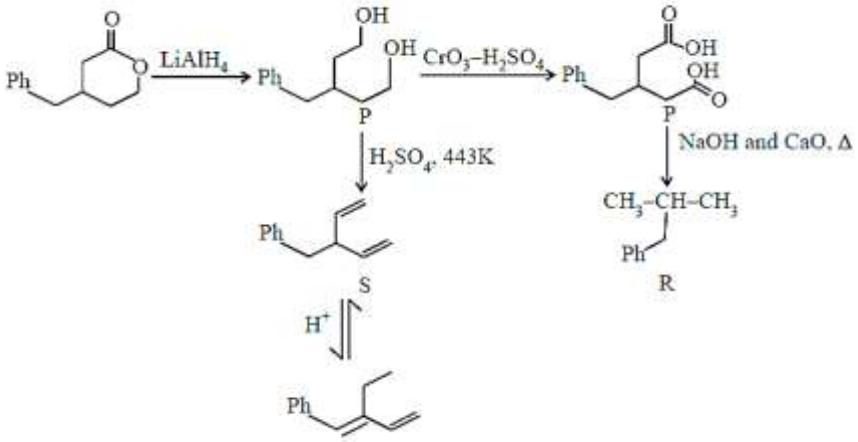


(d) $H_3PO_2$

**23. (a, c)**



**24. (b,c)**



**25. (11.00)**

CCP :

$(Z=4, a=400\text{pm}, M=105.6\ \text{g/mol})$

$$\text{Density } = \frac{Z \times M}{a^3 \times N_A}$$

$$= \frac{4 \times 105.6}{(400 \times 10^{-10})^3 \times 6 \times 10^{23}} = 11.00\ \text{g/cm}^3$$

**26. (3.95)**

$Ba(NO_3)_2(aq) + 2NaIO_3(aq) \rightarrow Ba(IO_3)_2(s) + 2NaNO_3(aq)$

2mmol 10mmol

LR

\- 6 mmol 4mmol

$[NaIO_3] = \frac{6}{300} = 2 \times 10^{-2}\text{M}$

$Ba(IO_3)_2(s) \rightleftharpoons Ba^{2+} + 2IO_3^-$

s $(2 \times 10^{-2} + 2\text{ s})$

$K_{sp} = [Ba^{+2}][IO_3^-]^2$

$1.58 \times 10^{-9} = [Ba^{+2}] \times (2 \times 10^{-2})^2$

$[Ba^{+2}] = \text{s} = 3.95 \times 10^{-6}\text{M}$

$X = 3.95$

**27. ( 15.62 or 16.00 )**

$\frac{x}{m} = K \times C^{1/n}$

$$\log\left(\frac{x}{m}\right) = \log K + \frac{1}{n}\log C$$

$$\log 4 = \log K + \frac{1}{n}\log 10$$

$$0.6 = \log K + \frac{1}{n} \ldots\ldots\ldots (1)$$

$$\log 10 = \log K + \frac{1}{n}\log 16$$

$$1 = \log K + \frac{1}{n} \times 1.2 \ldots\ldots (2)$$

Equation (2) − equation (1)

$$0.4 = \frac{1}{n} \times (0.2) \Rightarrow n = 0.5$$

and $\log K = -1.4$

$$\log\frac{x}{m} = \log K + \frac{1}{n} \times \log C$$

$$= -1.4 + 2 \times \log 20$$

$$= -1.4 + 2.6 = 1.2$$

$$\frac{x}{m} = 10^{+1.2} = 16$$

$(\log 2 = 0.3, 4\log 2 = 1.2, 16 = 10^{+1.2})$

OR

$$\frac{x}{m} = K \times C^{1/n}$$

$$4 = K(10)^{\frac{1}{n}} \ldots\ldots (1)$$

$$10 = K(16)^{\frac{1}{n}} \ldots\ldots (2)$$

$$x = K(20)^{\frac{1}{n}} \ldots\ldots (3)$$

On solving equation (1) and (2)

$$\frac{1}{n} = 2$$

On solving equation (1) and (3)

$$\frac{4}{X} = \left(\frac{10}{20}\right)^2$$

$$X = 16$$

On solving equation (2) and (3)

$$\frac{10}{X} = \left(\frac{16}{20}\right)^2$$

$$X = 15.625$$

**28.(4.16 or 4.17)**

$A + R \rightarrow$ Product

$t = 0 \quad A_0 \quad 10\ A_0$

$t = t \quad 0.6\ A_0 \quad 9.6\ A_0$

Rate $= k[A][R]$

Rate $_1 = k(0.6\ A_0) \times 9.6\ A_0$

$A + R \rightarrow$ Product

$t = 0 \quad A_0 \quad 10\ A_0$ (excess)

$t = t \quad 0.6\ A_0 \quad 10\ A_0$

Rate $= k'[A], k' = k[R]$

Rate $_2 = (k \times 10\ A_0) \times (0.6\ A_0)$

$$100 \times \frac{\Delta \text{ Rate}}{\text{Rate}_1} = \frac{(0.6 \times 10 - 0.6 \times 9.6}{0.6 \times 9.6} \times 100$$

$= 4.1666$

**29. (2.49)**

$\pi = \rho g h = 10^3 \times 10 \times 2 \times 10^{-2}$ Pascal $= 200$ Pascal

$\pi = \mathrm{CRT}$

$200 = \dfrac{2}{\mathrm{M}} \times 1000 \times 8.3 \times 300$

$\mathrm{M} = 24900 = 2.49 \times 10^4$ g/mol

$\mathrm{X} = 2.49$

**30. (105.50)**

$$C_4H_{10}(g) + \frac{13}{2}O_2(g) \rightarrow 4CO_2(g) + 5H_2O(l)$$

$$\Delta_r G^\circ = 4\Delta_f G^\circ_{CO_2} + 5\Delta_f G^\circ_{H_2O} - \Delta_f G^\circ_{C_4H_{10}}$$

$$= 4 \times (-394) + 5(-237) + 18$$

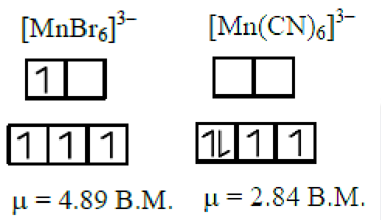
$$= -2743 \text{ kJ/mol}$$

$$\Delta_r G^\circ = -nFE^\circ$$

$$-2743 \times 1000 = -26 \times FE^\circ$$

$$E^\circ = \frac{105.5}{F} \times 10^3 = 105.50$$

**31. (7.70 to 7.73)**




Sum of spin magnetic moments of both complexes is 7.70 to 7.73 B.M.

**32. (2.00)**

$$\underset{\text{M.M.}=1024}{\text{Octa saccharide}} + \underset{\text{M.M.}=126}{7H_2O} \longrightarrow \text{Ribose} + 2\text{deoxyribose} + \text{glucose}$$

Total mass $= 1024 + 126 = 1150$

$$58.26 = \frac{134 \times n}{1150} \times 100$$

$$\frac{66.999}{100} = 134n \; n = 4.99 = 5$$

5 units of 2-Deoxyribose

$1150 = (5 \times 150) + (x \times 150) + (y \times 180)$

$$1150 = \underbrace{750}_{5\text{ unit}} + \underbrace{150x}_{\substack{300\\2\text{ unit}}} + \underbrace{180y}_{\substack{180\\1\text{ unit}}}$$

$n = 2.00$

**33. (c)**

$$e^{x_0} + x_0 = 0$$

$$g(x) = \frac{3xe^x + 3x - \alpha e^x - \alpha x}{3(e^x + 1)}$$

$$g(x) = x - \frac{\alpha(e^x + x)}{3(e^x + 1)}, g(x) + e^{x_0} = x + e^{x_0} - \frac{\alpha}{3}\left(\frac{e^x + x}{e^x + 1}\right)$$

$$g(x) + e^{x_0} = (x - x_0) - \frac{\alpha}{3}\left(\frac{e^x + x}{e^x + 1}\right)$$

$\Rightarrow$ As very clear $g(x_0) + x_0 \Rightarrow 0$

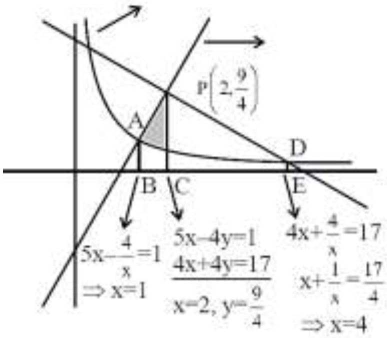
$$\lim_{x\to x_0}\left|\frac{g(x)+e^{x_0}}{x-x_0}\right|\frac{0\uparrow}{0\uparrow}=\lim_{x\to x_0}\left|\frac{g'(x)}{1}\right|=|g'(x_0)|$$

$$g'(x)=1-\frac{\alpha}{3}\left(\frac{(e^x+1)(e^x+1)-(e^x+x)e^x}{(e^x+1)^2}\right)$$

$$g'(x_0)=1-\frac{\alpha}{3}\left(\frac{(e^{x_0}+1)^2}{(e^{x_0}+1)^2}\right)=1-\frac{\alpha}{3}$$

$$\Rightarrow \lim_{x\to x_0}\left|\frac{g(x)+e^{x_0}}{x-x_0}\right|=|g'(x_0)|=\left|1-\frac{\alpha}{3}\right|$$

**34. (b)**




$$\text{Area } =\underset{\substack{\downarrow\\ \text{Trapezium APBC}}}{\frac{1}{2}\left(1+\frac{9}{4}\right)}+\underset{\substack{\downarrow\\ \text{Trapezium PDCE}}}{\frac{1}{2}\left(\frac{9}{4}+\frac{1}{4}\right)\cdot 2}-\int_1^4\frac{dx}{x}$$

$$\text{Area } =\frac{13}{8}+\frac{20}{8}-\log_e 4=\left(\frac{33}{8}-\log_e 4\right)$$

**35. (c)**

**Sol.** Let $\alpha=\frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9+\tan^2\theta}\right)$

$\theta=\tan^{-1}(2\tan\theta)-\alpha$

$\theta+\alpha=\tan^{-1}(2\tan\theta)$

$\tan(\theta+\alpha)=2\tan\theta$

$\frac{\tan\theta+\tan\alpha}{1-\tan\alpha\tan\theta}=2\tan\theta$……..(i)

$$\sin 2\alpha=\frac{6\tan\theta}{9+\tan^2\theta}=\frac{2\tan\alpha}{1+\tan^2\alpha}$$

$3\tan\theta+3\tan\theta\tan^2\alpha=9\tan\alpha+\tan\alpha\tan^2\theta$

$3(\tan\theta-3\tan\alpha)=\tan\alpha\tan\theta(\tan\theta-3\tan\alpha)$

$\tan\theta=\frac{3}{\tan\alpha}$ or $\tan\theta=3\tan\alpha$

**Case-I:** $\tan\theta=3\tan\alpha$

$$\frac{\tan\theta+\frac{\tan\theta}{3}}{1-\frac{\tan^2\theta}{3}}=2\tan\theta$$

$$\tan\theta=0,\frac{2}{3}=1-\frac{\tan^2\theta}{3}\ \Rightarrow \tan\theta=1,-1$$

$$\tan\theta=0,-1,1\ \Rightarrow \boxed{\theta=\frac{\pi}{4},-\frac{\pi}{4},0}$$

**Case-II:** $\tan\theta=\frac{3}{\tan\alpha}$

$$\frac{\tan\theta+\frac{3}{\tan\theta}}{-2}=2\tan\theta$$

$$\tan\theta + \frac{3}{\tan\theta} = -4\tan\theta \Rightarrow \underset{\substack{\downarrow \\ \text{No Solution}}}{\frac{3}{\tan\theta} = -5\tan\theta}$$

**36. (a)**

$$\left.\begin{aligned} 4x - 3y &= 12\alpha \\ 4x + 3y &= \frac{12}{\alpha} \end{aligned}\right\} \rightarrow 16x^2 - 9y^2 = 144$$

$$\frac{x^2}{9} - \frac{y^2}{16} = 1 \rightarrow \text{ Curve : } S$$

$$T: y = mx \pm \sqrt{9m^2 - 16}$$

$$m = \frac{4\sqrt{2}}{3}$$

$$y = \frac{4\sqrt{2}x}{3} \pm \sqrt{32 - 16}$$

$$3y = 4\sqrt{2}x \pm 12$$

as $q > 0$

$$3y = 4\sqrt{2}x + 12$$

$$p = -\frac{3}{\sqrt{2}} \ \&q = 4$$

$$pq = -6\sqrt{2}$$

**37. $(\boldsymbol{a}, \boldsymbol{b})$**

Sol. $QR = RP$

$$P = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \quad Q = \begin{pmatrix} x & y \\ z & 4 \end{pmatrix}$$

$$\begin{pmatrix} x & y \\ z & 4 \end{pmatrix}\begin{pmatrix} r_1 & r_2 \\ r_3 & r_4 \end{pmatrix} = \begin{pmatrix} r_1 & r_2 \\ r_3 & r_4 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$

$$\left.\begin{aligned} xr_1 + yr_3 &= 2r_1 \\ zr_1 + 4r_3 &= 2r_3 \end{aligned}\right\} \rightarrow \frac{x-2}{-y} = \frac{r_3}{r_1} = \frac{z}{-2}$$

$$\boxed{2x - 4 = yz}$$

$$\left.\begin{aligned} xr_2 + yr_4 &= 3r_2 \\ zr_2 + 4r_4 &= 3r_4 \end{aligned}\right\} \rightarrow \frac{x-3}{-y} = \frac{r_4}{r_2} = -z$$

$$\boxed{x - 3 = yz}$$

$$\Rightarrow 2x - 4 = x - 3 \ \Rightarrow \boxed{x = 1 \& yz = -2}$$

$$\Rightarrow Q - \lambda I = \begin{pmatrix} x - \lambda & y \\ z & 4 - \lambda \end{pmatrix}$$

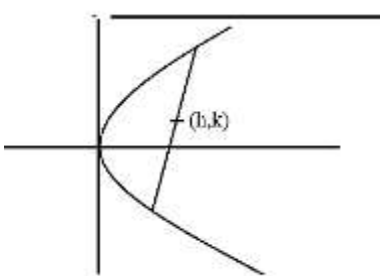
$$|Q - \lambda I| = (\lambda - x)(\lambda - 4) - yz$$

$$= \lambda^2 - (x + 4)\lambda + 4x - yz$$

$$|Q - \lambda I| = \lambda^2 - 5\lambda + 6$$

Now verify the option

**38. (a,c)**



$T = S_1$

$ky - \left(\frac{x+h}{2}\right) = k^2 - h$

$x - 2ky + 2k^2 - h = 0$

$k^2 - h < 0 \Rightarrow h - k^2 > 0$

For Area : interchange x&y

$y - 2kx + 2k^2 - h = 0 \& y = x^2$

$x^2 - 2kx + (2k^2 - h) = 0 <_{\beta}^{\alpha}$

$|\alpha - \beta| = \sqrt{4k^2 - 4(2k^2 - h)} = \sqrt{4h - 4k^2}$

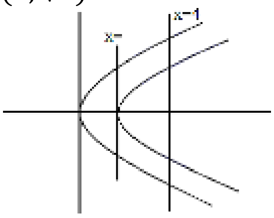
$A = \int_{\alpha}^{\beta} \left((2kx + h - 2k^2) - x^2\right) dx$

$A = \frac{(4h - 4k^2)^{3/2}}{6} = \frac{4}{3}$

$(4h - 4k^2)^{3/2} = 8 \Rightarrow (4h - 4k^2) = 4$

$h - k^2 = 1$

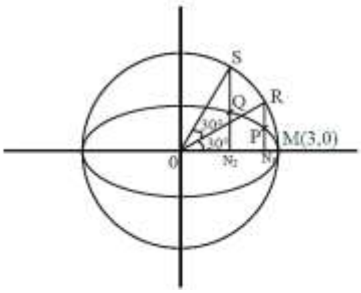
$(4, \sqrt{3}) \in S$




$A = \int_1^4 (\sqrt{x} - \sqrt{x-1}) dx = \frac{2}{3}\left(x^{3/2} - (x-1)^{3/2}\right)_1^4$

$A = \frac{2}{3}(8 - 3\sqrt{3} - 1) = \frac{2}{3}(7 - 3\sqrt{3}) = \frac{14}{3} - 2\sqrt{3}$

**39. (a,c)**




$P \equiv (3\cos 30°, 2\sin 30°) \equiv \left(\frac{3\sqrt{3}}{2}, 1\right)$

$Q \equiv (3\cos 60°, 2\sin 60°) \equiv \left(\frac{3}{2}, \sqrt{3}\right)$

$R\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right), S\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

Slope of PQ $= m_{PQ} = \frac{\sqrt{3}-1}{\frac{3}{2} - \frac{3\sqrt{3}}{2}} = -\frac{2}{3}$

Equation of line PQ

$y - \sqrt{3} = -\frac{2}{3}\left(x - \frac{3}{2}\right)$

$\Rightarrow 2x + 3y = 3(\sqrt{3} + 1)$ option (a) is correct

Now

if $N_2 = (x_2, 0) = \left(\frac{3}{2}, 0\right)$

$|N_2Q| = \sqrt{3}$ and $|N_2 S| = \frac{3\sqrt{3}}{2}$

$\Rightarrow 3|N_2Q| = 2|N_2S|$ option (c) is correct

Now, if $N_1 = (x_1, 0) \Rightarrow N_1 = \left(\frac{3\sqrt{3}}{2}, 0\right)$

$\Rightarrow |N_1P| = 1, |N_1R| = \frac{3}{2}$ option (d) is incorrect

**40. (b,c,d)**

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = 3$

since $3 > \frac{7}{3} \Rightarrow 3 > f(0)$

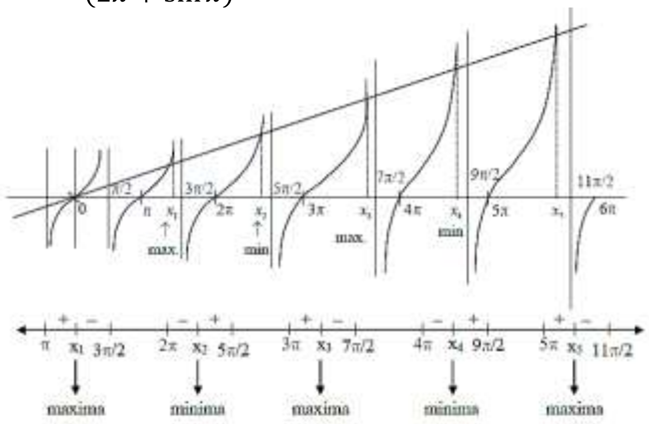
$\Rightarrow x = 0$ is local minima ....option (B) is correct

Now,

$$f(x) = \frac{6x + \sin x}{2x + \sin x} = 1 + \frac{4x}{2x + \sin x}$$

$$f'(x) = \frac{4[(2x + \sin x) \cdot 1 - x(2 + \cos x)]}{(2x + \sin x)^2} = \frac{4(\sin x - x\cos x)}{(2x + \sin x)^2}$$

$$= \frac{4\cos x(\tan x - x)}{(2x + \sin x)^2}$$




Sign of f '(x)

**41. (0.75)**

Put y = vx $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

D.E.

$$x^2\left(v + x\frac{dv}{dx}\right) + x^2 v = x^2(1 + v^2)$$

$$\Rightarrow v + x\frac{dv}{dx} + v = 1 + v^2$$

$$\Rightarrow x\frac{dv}{dx} = 1 + v^2 - 2v$$

$$\Rightarrow \int \frac{dv}{(v-1)^2} = \int \frac{dx}{x}$$

$$\Rightarrow -\frac{1}{v-1} = \ln|x| + C$$

$$\Rightarrow \frac{x}{x-y} = \ln|x| + C = \ln x + C$$

(Since $x > \frac{1}{e}$)

Given $y(1) = 0$

$\Rightarrow C = 1$

So $\frac{x}{x-y} = \ln(ex)$

Now $y = (e) = \frac{e}{2}$ and $y(e^2) = \frac{2e^2}{3}$

$$\therefore \frac{2(y(e))^2}{y(e^2)} = \frac{2 \cdot \frac{e^2}{4}}{\frac{2e^2}{3}} = \frac{3}{4} = 0.75$$

**42. (6.00)**

For x = 1

$$\left(1+\frac{2}{5}\right)^{23} = a_0 + a_1 + a_2 + \cdots + a_{23}$$

for numerically greatest term

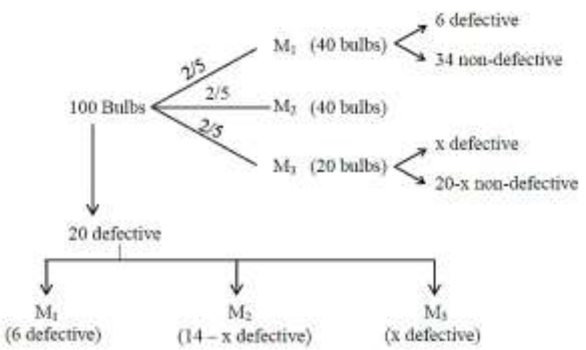
$$\frac{n+1}{1+\left|\frac{a}{b}\right|} = \frac{23+1}{1+\frac{5}{2}} = \frac{48}{7}$$

$$\Rightarrow \left[\frac{48}{7}\right] = 6 = m \quad \text{(where [.] greatest integer function)}$$

so, T, is numerical greatest term

Hence $r = 6$

**43. (0.30)**




Now given probability

$$P\left(\frac{\text{Produced by } M_2}{\text{defective}}\right) = \frac{\frac{40}{100} \times \frac{14-x}{40}}{\frac{20}{100}} = \frac{2}{5}$$

$$\Rightarrow \frac{14-x}{40} = \frac{1}{5}$$

$$\Rightarrow 14 - x = 8$$

$$\Rightarrow x = 6$$

So, the required probability

$$= \frac{6}{20} = 0.3$$

**44. (-2.00)**

$[\vec{x}\vec{y}\vec{z}] = 0$

$$\Rightarrow \begin{vmatrix} \alpha & \beta & -1 \\ -1 & \alpha & \beta \\ \beta & -1 & \alpha \end{vmatrix} \underbrace{\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}}_{\neq 0} = 0$$

$$\Rightarrow (\alpha^3 + \beta^3 - 1) - (-\alpha\beta - \alpha\beta - \alpha\beta) = 0$$

$$\Rightarrow \alpha^3 + \beta^3 + 3\alpha\beta = 1$$

$$\Rightarrow \alpha^3 + \beta^3 + (-1)^3 = 3(\alpha)(\beta)(-1)$$

$$\Rightarrow \alpha + \beta - 1 = 0$$

So$\boxed{, \alpha + \beta - 3 = -2}$

**45. (-2.00)**

$$\alpha = \arg(-\omega + \omega^2 - \omega^3 + \cdots \ldots \ldots + (-\omega)^{2025})$$

$$\alpha = \arg\left(\frac{-\omega((-\omega)^{2025} - 1)}{-\omega - 1}\right)$$

$$\alpha = \arg\left(\frac{-\omega}{-\omega - 1}(-2)\right)$$

$$\alpha = \arg\left(\frac{-2\omega}{\omega + 1}\right)$$

$$\alpha = \arg\left(\frac{-2\omega}{-\omega^2}\right)$$

$$\alpha = \arg\left(\frac{2}{\omega}\right)$$

$$\alpha = \arg(2\omega^2)$$

$$\alpha = \frac{-2\pi}{3}$$

$$\frac{3\alpha}{\pi} = -2$$

**46. (0.25)**

Let $h(x) = f(g^{-1}(x))$ and $g(0) = 2$

$\mathrm{h}'(\mathrm{x}) = \mathrm{f}'(\mathrm{g}^{-1}(\mathrm{x})) \cdot (\mathrm{g}^{-1}(\mathrm{x}))'$

$\mathrm{h}'(2) = \mathrm{f}'(\mathrm{g}^{-1}(2) \cdot (\mathrm{g}^{-1})'(2)$

$\quad = \mathrm{f}'(0) \cdot (\mathrm{g}^{-1})'(2)$

Now

$f(x) = \log_e(x^2 + 2x + 4)$

$$f'(x) = \frac{2x + 2}{x^2 + 2x + 4}$$

$$f'(0) = \frac{1}{2}$$

$$g(x) = \frac{4}{1 + e^{-2x}, g(0) = 2}$$

$g^{-1}(g(x)) = x$

$((g^{-1})'(g(x)))g'(x) = 1$

$$(g^{-1})'(2) = \frac{1}{g'(0)} = \frac{1}{2}$$

So $h'(2) = \frac{1}{4} = 0.25$

$$g(x) = \frac{4}{1+e^{-2x}}$$

$$g'(x) = \frac{8e^{-2x}}{(1+e^{-2x})^2}$$

$$g'(0) = \frac{8}{4} = 2$$