1. **(d)** Given $f(x) = \sqrt{x}\log_e(x) - x + 1$

Differentiating with respect to x :

$$f'(x) = \frac{1}{2\sqrt{x}}\log_e(x) + \sqrt{x}\left(\frac{1}{x}\right) - 1$$

$$f'(x) = \frac{\log_e(x)}{2\sqrt{x}} + \frac{1}{\sqrt{x}} - 1$$

Differentiating again with respect to x :

$$f''(x) = \frac{d}{dx}\left(\frac{1}{2}x^{-1/2}\log_e(x) + x^{-1/2} - 1\right)$$

$$f''(x) = \frac{1}{2}\left(-\frac{1}{2}x^{-3/2}\log_e(x) + x^{-1/2}\cdot\frac{1}{x}\right) - \frac{1}{2}x^{-3/2}$$

$$f''(x) = -\frac{\log_e(x)}{4x^{3/2}} + \frac{1}{2x^{3/2}} - \frac{1}{2x^{3/2}}$$

$$f''(x) = -\frac{\log_e(x)}{4x^{3/2}}$$

For $x \in (0,1), \log_e(x) < 0$, which implies $f''(x) > 0$. Thus, $f'(x)$ is strictly increasing in $(0,1)$.
For $x \in (1,\infty), \log_e(x) > 0$, which implies $f''(x) < 0$. Thus, $f'(x)$ is strictly decreasing in $(1,\infty)$.
Therefore, $f'(x)$ attains its maximum value at $x = 1$.
Maximum value of $f'(x) = f'(1) = \frac{\log_e(1)}{2} + 1 - 1 = 0$.
Since the maximum value of $f'(x)$ is 0 , we have $f'(x) \leq 0$ for all $x \in (0,\infty)$, with equality holding only at $x = 1$.
This means $f(x)$ is a strictly decreasing function on $(0,\infty)$.
Hence, $f(x)$ has neither a point of local maximum nor a point of local minimum in the interval $(0,\infty)$.

2. **(c)** For the point P on the parabola $y = x^2$, the slope of the tangent is $\frac{dy}{dx} = 2x$.

Given $2x = 4 \Rightarrow x = 2$.
Substituting $x = 2$ in $y = x^2$, we get $y = 4$.
Thus, the coordinates of P are $(2,4)$.
For the point Q on the circle $x^2 + y^2 = 2$, differentiating with respect to x gives $2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$.
Given $-\frac{x}{y} = -1 \Rightarrow x = y$.
Since Q lies in the first quadrant, substituting $x = y$ in $x^2 + y^2 = 2$ gives $2x^2 = 2 \Rightarrow x = 1, y = 1$.
Thus, the coordinates of Q are $(1,1)$.
For the point R on the ellipse $x^2 + 4y^2 = 8$, differentiating with respect to x gives $2x + 8y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{4y}$.
Given $-\frac{x}{4y} = -\frac{1}{2} \Rightarrow x = 2y$.
Since R lies in the first quadrant, substituting $x = 2y$ in $x^2 + 4y^2 = 8$ gives $4y^2 + 4y^2 = 8 \Rightarrow 8y^2 = 8 \Rightarrow y = 1, x = 2$.
Thus, the coordinates of R are $(2,1)$.
The points are $P(2,4), Q(1,1)$, and $R(2,1)$.
The line segment PR lies on the vertical line $x = 2$, and the line segment QR lies on the horizontal line $y = 1$.
Since PR is perpendicular to QR, the angle $\angle PRQ = 90°$.
Therefore, the triangle PQR is a right-angled triangle with the hypotenuse PQ.
The circumcircle of $\triangle$ PQR has PQ as its diameter.
The length of the diameter is $PQ = \sqrt{(2-1)^2 + (4-1)^2} = \sqrt{1^2 + 3^2} = \sqrt{10}$.
The radius of the circle is $\frac{PQ}{2} = \frac{\sqrt{10}}{2} = \sqrt{\frac{5}{2}}$.

3. **(b)** Any matrix obtained by performing elementary row transformations on the identity matrix is equivalent to the identity matrix, which means it must be non-singular (invertible). Therefore, its determinant must be non-zero.
Evaluating the determinant of the matrix in option (a):

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Evaluating the determinant of the matrix in option (b):

$$\begin{vmatrix}1 & 1 & 1\\2 & 3 & 4\\1 & 2 & 1\end{vmatrix} = 1(3-8) - 1(2-4) + 1(4-3) = -5 + 2 + 1 = -2 \neq 0$$

Evaluating the determinant of the matrix in option (c):

$$\begin{vmatrix}1 & 1 & 1\\2 & 3 & 4\\2 & 5 & 8\end{vmatrix} = 1(24-20) - 1(16-8) + 1(10-6) = 4 - 8 + 4 = 0$$

Evaluating the determinant of the matrix in option (d):

$$\begin{vmatrix}1 & 1 & 1\\-1 & 1 & 2\\0 & 2 & 3\end{vmatrix} = 1(3-4) - 1(-3-0) + 1(-2-0) = -1 + 3 - 2 = 0$$

Since only the matrix in option (b) has a non-zero determinant, it is the only one that can be obtained from the identity matrix by elementary row transformations.

4. **(c)** For the first term, $\cot^{-1}(\cot(-11))$:

The principal value branch of $\cot^{-1}(x)$ is $(0, \pi)$.

Since the cotangent function is periodic with period π, $\cot(-11) = \cot(-11 + 4\pi)$.

Using $\pi = 3.14$, we have $4\pi = 12.56$, so $-11 + 4\pi = 1.56$, which lies in the interval $(0, \pi)$.

Thus, $\cot^{-1}(\cot(-11)) = 4\pi - 11$.

For the second term, $10\sin\left(2\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$:

Since $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$, we get:

$10\sin\left(2 \times \frac{\pi}{4}\right) = 10\sin\left(\frac{\pi}{2}\right) = 10(1) = 10.$

For the third term, $10\sin(2\tan^{-1}(2))$:

Let $\theta = \tan^{-1}(2) \Rightarrow \tan(\theta) = 2$.

Using the multiple angle formula $\sin(2\theta) = \frac{2\tan(\theta)}{1+\tan^2(\theta)}$, we obtain:

$$\sin(2\theta) = \frac{2(2)}{1+2^2} = \frac{4}{5}.$$

Therefore, $10\sin(2\tan^{-1}(2)) = 10 \times \frac{4}{5} = 8$.

Adding the three evaluated terms together:

$(4\pi - 11) + 10 + 8 = 4\pi + 7$.

5. **(a,c)** Total balls in Box I $= 6 + 9 = 15$

Total balls in Box II $= 8 + 12 = 20$

Total balls in the mixture $= 15 + 20 = 35$

Total red balls $= 6 + 8 = 14$

Total green balls $= 9 + 12 = 21$

The probabilities of the given events are:

$$P(E_1) = \frac{15}{35} = \frac{3}{7}$$

$$P(E_2) = \frac{20}{35} = \frac{4}{7}$$

$$P(F_1) = \frac{14}{35} = \frac{2}{5}$$

$$P(F_2) = \frac{21}{35} = \frac{3}{5}$$

The joint probabilities are:

$$P(E_1 \cap F_1) = \frac{6}{35}$$

$$P(E_2 \cap F_2) = \frac{12}{35}$$

For statement (A):

$$P(E_1)P(F_1) = \frac{3}{7} \times \frac{2}{5} = \frac{6}{35} = P(E_1 \cap F_1)$$

Thus, E_1 and F_1 are independent. Statement (a) is TRUE.

For statement (b):

$P(E_2)P(F_2) = \frac{4}{7} \times \frac{3}{5} = \frac{12}{35} = P(E_2 \cap F_2)$

Thus, E_2 and F_2 are independent. Statement (b) is FALSE.

For statement (c):

$P(F_1 \mid E_1) = \frac{P(E_1 \cap F_1)}{P(E_1)} = \frac{6/35}{15/35} = \frac{2}{5}$

$P(F_1 \mid E_2) = \frac{P(E_2 \cap F_1)}{P(E_2)} = \frac{8/35}{20/35} = \frac{2}{5}$

Since $P(F_1 \mid E_1) = P(F_1 \mid E_2)$, statement (c) is TRUE.

For statement (d):

$P(F_2 \mid E_2) = \frac{P(E_2 \cap F_2)}{P(E_2)} = \frac{12/35}{20/35} = \frac{3}{5}$

Since $\frac{2}{5} < \frac{3}{5}$, $P(F_1 \mid E_1)$ is not greater than $P(F_2 \mid E_2)$. Statement (d) is FALSE.

6. **(a,d)** Let the normal vector to the plane P be $\vec{n}$.

Since the plane P contains the line $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z+2}{1}$, its normal vector $\vec{n}$ is perpendicular to the line's direction vector $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$.

Since P is perpendicular to the plane $x + 2y + 3z = 4$, $\vec{n}$ is perpendicular to its normal vector $\vec{n}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$.

$\vec{n} = (2\hat{i} + 3\hat{j} + \hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k}) = \hat{i}(9-2) - \hat{j}(6-1) + \hat{k}(4-3) = 7\hat{i} - 5\hat{j} + \hat{k}$

Plane P passes through the point $(1,3,-2)$ which lies on the given line.

Equation of plane P is $7(x-1) - 5(y-3) + 1(z+2) = 0 \Rightarrow 7x - 5y + z = -10$

Statement (a) is TRUE.

The plane P_1 is parallel to P and passes through $(4,2,2)$.

Equation of plane P_1 is $7(x-4) - 5(y-2) + 1(z-2) = 0 \Rightarrow 7x - 5y + z = 20$

Distance between parallel planes P and P_1 is $d = \frac{|20-(-10)|}{\sqrt{7^2+(-5)^2+1^2}} = \frac{30}{\sqrt{75}} = 2\sqrt{3}$. Statement (b) is FALSE.

Distance of plane P from the origin is $d_0 = \frac{|-10|}{\sqrt{75}} = \frac{2}{\sqrt{3}}$. Statement (c) is FALSE.

Angle θ between the planes $7x - 5y + z + 10 = 0$ and $2x + 2y + z - 3 = 0$ is given by:

$\cos\theta = \frac{|7(2) + (-5)(2) + 1(1)|}{\sqrt{7^2 + (-5)^2 + 1^2}\sqrt{2^2 + 2^2 + 1^2}} = \frac{5}{\sqrt{75}\sqrt{9}} = \frac{5}{15\sqrt{3}} = \frac{1}{3\sqrt{3}}$

$\theta = \cos^{-1}\left(\frac{1}{3\sqrt{3}}\right)$. Statement (d) is TRUE.

7. **(b,d)** Given $g(x) = xf(x)$ for all $x \in \mathbb{R}$.

For option (a):

Let $f(x) = \frac{1}{x}$ for $x \neq 0$ and $f(0) = 0$.

Then $g(x) = 1$ for $x \neq 0$ and $g(0) = 0$.

$\lim_{x\to 0} g(x) = 1 \neq g(0)$.

Thus, g is not necessarily continuous at $x = 0$. Option (a) is false.

For option (b):

The derivative of g at $x = 0$ is given by:

$g'(0) = \lim_{x\to 0} \frac{g(x) - g(0)}{x - 0} = \lim_{x\to 0} \frac{xf(x) - 0}{x} = \lim_{x\to 0} f(x)$.

If f is continuous at $x = 0$, then $\lim_{x\to 0} f(x) = f(0)$.

Since $f(0)$ is a finite real number, $g'(0)$ exists and g is differentiable at $x = 0$. Option (b) is true.

For option (c):

Let $f(x) = 0$ for $x \neq 0$ and $f(0) = 1$.

Then $g(x) = 0$ for all $x \in \mathbb{R}$.

Here, g is differentiable at $x = 0$ with $g'(0) = 0$.

However, $\lim_{x\to 0} f(x) = 0 \neq f(0)$, so f is not continuous at $x = 0$. Option (c) is false.

For option (d):
If g is differentiable at $x = 0$, then $g'(0)$ exists.
Since $g'(0) = \lim_{x\to 0} \frac{g(x)-g(0)}{x} = \lim_{x\to 0} f(x)$, the limit $\lim_{x\to 0} f(x)$ must exist. Option (d) is true.

8. **(a,c,d)** The characteristic equation of M is given by $\det(M - \lambda I) = 0$:

$$\begin{vmatrix} 2-\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - 2\lambda + 1 = (\lambda - 1)^2 = 0$$

The only eigenvalue is $\lambda = 1$ with algebraic multiplicity 2 .
For the eigenvector, $(M - I)X = 0 \Rightarrow \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x = y$.
Since there is only one linearly independent eigenvector, M is not diagonalizable but can be reduced to its Jordan canonical form $J = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.
Thus, there exists an invertible matrix N such that $M = NJN^{-1} \Rightarrow MN = N\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.
Statement (a) is TRUE.
We can write $M = I + A$, where $A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$.
Notice that $A^2 = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.
Using the binomial theorem (since I and A commute), $M^k = (I + A)^k = I + kA = \begin{bmatrix} 1+k & -k \\ k & 1-k \end{bmatrix}$.
For $k = 26, M^{26} = \begin{bmatrix} 27 & -26 \\ 26 & -25 \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$.
The determinant of M^{26} is $ps - qr = 27(-25) - (-26)(26) = 1$.
Since $\det(M^{26}) = 1$, its inverse has integer entries. Thus, for any integers m and n, the system $\begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} m \\ n \end{bmatrix}$ has a unique integer solution.
Statement (c) is TRUE.
Now, let's find $\sum_{k=1}^{26} M^k = \sum_{k=1}^{26} \begin{bmatrix} 1+k & -k \\ k & 1-k \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

$$a = \sum_{k=1}^{26} (1 + k) = 2 + 3 + \cdots + 27 = \frac{26}{2}(2 + 27) = 13 \times 29 = 377.$$

Statement (b) is FALSE.
We also have:

$$b = \sum_{k=1}^{26} (-k) = -351$$

$$c = \sum_{k=1}^{26} k = 351$$

$$d = \sum_{k=1}^{26} (1 - k) = 26 - 351 = -325$$

For statement (d), the system of equations is $\begin{bmatrix} a+t & b \\ c & d+t \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.
The determinant of the coefficient matrix is:
$\Delta = (a + t)(d + t) - bc = (377 + t)(-325 + t) - (-351)(351)$
$\Delta = t^2 + 52t - 122525 + 123201 = t^2 + 52t + 676 = (t + 26)^2$
For any positive real number $t > 0, \Delta = (t + 26)^2 > 0$, meaning the determinant is non-zero. Hence, the system has a unique solution.
Statement (d) is TRUE.

9. **(2520)**
An equivalence relation on a set S corresponds to a partition of S into disjoint equivalence classes. Let the sizes of these equivalence classes be $a_1, a_2, \ldots, a_k$.
The number of elements in the equivalence relation R is given by the sum of the squares of the sizes of its equivalence classes: $\sum_{i=1}^{k} a_i^2 = 42$
Since the total number of elements in S is 10 , we also have:

$\sum_{i=1}^{k} a_i = 10$

We need to find all possible partitions of 10 such that the sum of their squares is 42 . Let's check the possible sizes of the largest equivalence class, a_1 :

Case 1: $a_1 = 6$

$a_1^2 = 36$. The remaining sum of squares is $42 - 36 = 6$, and the remaining sum of elements is $10 - 6 = 4$.

The only way to partition 4 such that the sum of squares is 6 is 2,1,1 (since $2^2 + 1^2 + 1^2 = 6$).

Thus, one valid partition is $\{6,2,1,1\}$.

Case 2: $a_1 = 5$

$a_1^2 = 25$. The remaining sum of squares is $42 - 25 = 17$, and the remaining sum of elements is $10 - 5 = 5$.

The only way to partition 5 such that the sum of squares is 17 is 4,1 (since $4^2 + 1^2 = 17$).

Thus, another valid partition is $\{5,4,1\}$.

Case 3: $a_1 \leq 4$

The maximum possible sum of squares would be for the partition $\{4,4,2\}$, which gives $4^2 + 4^2 + 2^2 = 36 < 42$.

No other partitions can yield a sum of 42 .

Now, we calculate the number of ways to form these partitions from the 10 elements of S.

For the partition $\{6,2,1,1\}$:

The number of ways to divide 10 elements into groups of sizes 6,2,1,1 is:

$$\frac{10!}{6! \times 2! \times 1! \times 1! \times 2!} = \frac{3628800}{720 \times 2 \times 1 \times 1 \times 2} = \frac{3628800}{2880} = 1260$$

(Note: We divide by 2 ! because there are two groups of identical size 1).

For the partition $\{5,4,1\}$:

The number of ways to divide 10 elements into groups of sizes 5,4,1 is:

$$\frac{10!}{5! \times 4! \times 1!} = \frac{3628800}{120 \times 24 \times 1} = \frac{3628800}{2880} = 1260$$

Total number of equivalence relations in X is:

$1260 + 1260 = 2520$

10. (5)

Let $f(x) = g(x) + h(x)$, where $g(x) = (|x| + |x-1|)\sin x$ and $h(x) = [x \sin x]$.

First, we analyze $h(x) = [x \sin x]$ on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

The function $y = x \sin x$ is an even function. For $x \in \left[0, \frac{\pi}{2}\right)$, $x \sin x$ is strictly increasing.

At $x = 0, x \sin x = 0$.

As $x \to \frac{\pi}{2}, x \sin x \to \frac{\pi}{2} = 1.57$.

Thus, $x \sin x \in [0, 1.57)$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

The value of $[x \sin x]$ will be 0 when $0 \leq x \sin x < 1$, and 1 when $1 \leq x \sin x < \frac{\pi}{2}$.

Let $x_1 \in \left(0, \frac{\pi}{2}\right)$ be the unique point where $x_1 \sin x_1 = 1$. By symmetry, $-x_1$ is the unique point in $\left(-\frac{\pi}{2}, 0\right)$ where $(-x_1)\sin(-x_1) = 1$.

Therefore, $h(x)$ has jump discontinuities at exactly two points: $x = x_1$ and $x = -x_1$.

At $x = 0, x \sin x = 0$ and is non-negative in its neighborhood, so $[x \sin x] = 0$ around $x = 0$, making $h(x)$ continuous and differentiable at $x = 0$.

Next, we analyze $g(x) = (|x| + |x-1|)\sin x$.

The absolute value functions have critical points at $x = 0$ and $x = 1$.

For x in a small neighborhood of 0 (specifically $x \in (-1,1)$):

If $x \in [0,1), g(x) = (x - (x-1))\sin x = \sin x \Rightarrow g'(0^+) = \cos 0 = 1$.

If $x \in (-1,0), g(x) = \big(-x - (x-1)\big)\sin x = (1 - 2x)\sin x \Rightarrow g'(0^-) = -2\sin 0 + 1 \cdot \cos 0 = 1$.

Since $g'(0^+) = g'(0^-), g(x)$ is differentiable at $x = 0$.

For x in a small neighborhood of 1 :

If $x \in (0,1], g(x) = \sin x \Rightarrow g'(1^-) = \cos 1$.

If $x \in \left(1, \frac{\pi}{2}\right), g(x) = (x + (x-1))\sin x = (2x-1)\sin x \Rightarrow g'(1^+) = 2\sin 1 + 1 \cdot \cos 1$.

Since $g'(1^-) \neq g'(1^+), g(x)$ is not differentiable at $x = 1$.

Now, combining the results for $f(x) = g(x) + h(x)$:

(1).Discontinuities: g(x) is continuous everywhere. h(x) is discontinuous at x_1 and $-x_1$. Thus, f(x) is discontinuous at 2 points.
$\Rightarrow \alpha = 2$.
(2).Non-differentiability: f(x) is not differentiable where it is discontinuous (x_1 and $-x_1$). Additionally, at $x = 1, h(x)$ is constant (0) because $1 \cdot \sin 1 \approx 0.84 < 1$, meaning h(x) is differentiable at $x = 1$. However, g(x) is not differentiable at $x = 1$.
Thus, f(x) is not differentiable at $x = 1$.
This gives 3 points of non-differentiability: $-x_1, 1, x_1. \Rightarrow \beta = 3$.
Finally, $\alpha + \beta = 2 + 3 = 5$.

11. (206)

Let r_i and b_i be the number of red and blue pens given to the i-th person, where $i = 1,2,3,4$.
Given that each person receives exactly 6 pens, we have:
$r_i + b_i = 6 \Rightarrow b_i = 6 - r_i$
Since the number of blue pens must be non-negative ($b_i \geq 0$), we get $r_i \leq 6$. Also, $r_i \geq 0$.
The total number of red pens is 10 , so:
$r_1 + r_2 + r_3 + r_4 = 10$
The number of ways to distribute the pens is the number of non-negative integer solutions to this equation subject to $0 \leq r_i \leq 6$
This is equivalent to finding the coefficient of x^{10} in the expansion of $(1 + x + x^2 + \cdots + x^6)^4$.

$$(1 + x + x^2 + \cdots + x^6)^4 = \left(\frac{1 - x^7}{1 - x}\right)^4 = (1 - x^7)^4 (1 - x)^{-4}$$

Expanding both terms:
$(1 - x^7)^4 = 1 - 4x^7 + 6x^{14} - \cdots$

$$(1 - x)^{-4} = \sum_{k=0}^{\infty} {}^{k+3}C_3 x^k$$

We need the coefficient of x^{10} in the product $(1 - 4x^7 + \cdots) \sum_{k=0}^{\infty} {}^{k+3}C_3 x^k$.
Coefficient of $x^{10} = 1 \times$ (coefficient of x^{10} in $(1 - x)^{-4}$) $-4 \times$ (coefficient of x^3 in $(1 - x)^{-4}$)
$= {}^{10+3}C_3 - 4 \times {}^{3+3}C_3$
$= {}^{13}C_3 - 4 \times {}^{6}C_3$
$= \frac{13 \times 12 \times 11}{3 \times 2 \times 1} - 4 \times \frac{6 \times 5 \times 4}{3 \times 2 \times 1}$
$= 286 - 4 \times 20$
$= 286 - 80 = 206$

12. (4)

Let the given expression be α. First, we simplify the angles in the product.

$$\cos\left(\frac{27\pi}{11}\right) = \cos\left(2\pi + \frac{5\pi}{11}\right) = \cos\left(\frac{5\pi}{11}\right)$$

$$\cos\left(\frac{81\pi}{11}\right) = \cos\left(7\pi + \frac{4\pi}{11}\right) = -\cos\left(\frac{4\pi}{11}\right) = \cos\left(\pi - \frac{4\pi}{11}\right) = \cos\left(\frac{7\pi}{11}\right)$$

Thus, the angles are $\frac{\pi}{11}, \frac{3\pi}{11}, \frac{5\pi}{11}, \frac{7\pi}{11}, \frac{9\pi}{11}$, which can be written as $\theta_k = \frac{(2k-1)\pi}{11}$ for $k = 1,2,3,4,5$.
The expression becomes:

$$\alpha = \prod_{k=1}^{5} (1 - 2\cos\theta_k)$$

Using the trigonometric identity $1 - 2\cos\theta = 1 - 2(2\cos^2(\theta/2) - 1) = 3 - 4\cos^2(\theta/2)$.
Multiplying and dividing by $\cos(\theta/2)$, we get:

$$1 - 2\cos\theta = \frac{3\cos(\theta/2) - 4\cos^3(\theta/2)}{\cos(\theta/2)} = -\frac{\cos(3\theta/2)}{\cos(\theta/2)}$$

Applying this identity to the product:

$$\alpha = \prod_{k=1}^{5} \left(-\frac{\cos(3\theta_k/2)}{\cos(\theta_k/2)}\right) = (-1)^5 \frac{\prod_{k=1}^{5} \cos(3\theta_k/2)}{\prod_{k=1}^{5} \cos(\theta_k/2)}$$

The terms in the denominator are $\cos\left(\frac{\pi}{22}\right), \cos\left(\frac{3\pi}{22}\right), \cos\left(\frac{5\pi}{22}\right), \cos\left(\frac{7\pi}{22}\right), \cos\left(\frac{9\pi}{22}\right)$.

The terms in the numerator are $\cos\left(\frac{3\pi}{22}\right), \cos\left(\frac{9\pi}{22}\right), \cos\left(\frac{15\pi}{22}\right), \cos\left(\frac{21\pi}{22}\right), \text{co}\left(\frac{27\pi}{22}\right)$.

We can simplify the larger angles in the numerator:

$$\cos\left(\frac{15\pi}{22}\right) = \cos\left(\pi - \frac{7\pi}{22}\right) = -\cos\left(\frac{7\pi}{22}\right)$$

$$\cos\left(\frac{21\pi}{22}\right) = \cos\left(\pi - \frac{\pi}{22}\right) = -\cos\left(\frac{\pi}{22}\right)$$

$$\cos\left(\frac{27\pi}{22}\right) = \cos\left(\pi + \frac{5\pi}{22}\right) = -\cos\left(\frac{5\pi}{22}\right)$$

Substituting these back into the numerator product:

$$\prod_{k=1}^{5} \cos(3\theta_k/2) = \cos\left(\frac{3\pi}{22}\right)\cos\left(\frac{9\pi}{22}\right)\left(-\cos\left(\frac{7\pi}{22}\right)\right)\left(-\cos\left(\frac{\pi}{22}\right)\right)\left(-\cos\left(\frac{5\pi}{22}\right)\right)$$

$$\prod_{k=1}^{5} \cos(3\theta_k/2) = -\left[\cos\left(\frac{\pi}{22}\right)\cos\left(\frac{3\pi}{22}\right)\cos\left(\frac{5\pi}{22}\right)\cos\left(\frac{7\pi}{22}\right)\cos\left(\frac{9\pi}{22}\right)\right]$$

Notice that the term in the brackets is exactly the denominator product. Therefore, the ratio of the numerator product to the denominator product is -1.

$\alpha = (-1)^5 \times (-1) = (-1) \times (-1) = 1$

We need to find the value of $5 - \alpha^2$:

$5 - (1)^2 = 5 - 1 = 4$

13. **(c)** For (P):

Since α and β are roots of $x^2 + x + 1 = 0$, we have $\alpha^2 + \alpha + 1 = 0 \Rightarrow \alpha + 1 = -\alpha^2$ and $\alpha^3 = 1$.

$$\frac{1}{(\alpha+1)^{2026}} = \frac{1}{(-\alpha^2)^{2026}} = \frac{1}{\alpha^{4052}}$$

Since $4052 = 3 \times 1350 + 2, \alpha^{4052} = (\alpha^3)^{1350} \cdot \alpha^2 = \alpha^2$.

Thus, $\frac{1}{\alpha^2} = \frac{\alpha^3}{\alpha^2} = \alpha$.

Similarly, $\frac{1}{(\beta+1)^{2026}} = \beta$.

The quadratic equation with roots α and β is the original equation $x^2 + x + 1 = 0$.

So, (P) $\rightarrow$ (1).

For (Q) :

$$\frac{1}{(\alpha+1)^{2027}} = \frac{1}{(-\alpha^2)^{2027}} = -\frac{1}{\alpha^{4054}}$$

Since $4054 = 3 \times 1351 + 1, \alpha^{4054} = \alpha$.

Thus, $-\frac{1}{\alpha} = -\beta$.

Similarly, the other root is $-\alpha$.

Sum of roots $= -\alpha - \beta = -(\alpha + \beta) = -(-1) = 1$.

Product of roots $= (-\alpha)(-\beta) = \alpha\beta = 1$.

The quadratic equation is $x^2 - x + 1 = 0$.

So, (Q) $\rightarrow$ (2).

For (R) :

Since γ and δ are roots of $x^2 - x + 1 = 0$, we have $\gamma^2 - \gamma + 1 = 0 \Rightarrow \gamma - 1 = \gamma^2$ and $\gamma^3 = -1$.

$$\frac{1}{(\gamma-1)^{2026}} = \frac{1}{(\gamma^2)^{2026}} = \frac{1}{\gamma^{4052}}$$

$\gamma^{4052} = (\gamma^3)^{1350} \cdot \gamma^2 = (-1)^{1350} \cdot \gamma^2 = \gamma^2$

Thus, $\frac{1}{\gamma^2} = \frac{\gamma}{\gamma^3} = -\gamma$

Similarly, the second term is $-\delta$.

The sum is $-\gamma - \delta = -(\gamma + \delta) = -(1) = -1$.

So, (R) $\rightarrow$ (4).

For (S):

Since p and r are roots of $x^2 + x - 1 = 0$, we have $x^2 + x = 1 \Rightarrow x(x+1) = 1 \Rightarrow x + 1 = \frac{1}{x}$.

$\frac{1}{(p+1)^3} + \frac{1}{(r+1)^3} = p^3 + r^3$

We know $p + r = -1$ and $pr = -1$.

$p^3 + r^3 = (p+r)^3 - 3pr(p+r) = (-1)^3 - 3(-1)(-1) = -1 - 3 = -4$.

So, (S) → (5).

14. **(b)** For (P):

$\sin^6 x + \cos^4 x = 1$

$\sin^6 x + (1 - \sin^2 x)^2 = 1$

$\sin^6 x + 1 - 2\sin^2 x + \sin^4 x = 1$

$\sin^2 x(\sin^4 x + \sin^2 x - 2) = 0$

$\sin^2 x(\sin^2 x - 1)(\sin^2 x + 2) = 0$

This gives $\sin^2 x = 0$ or $\sin^2 x = 1$.

In the interval $[-\pi, \pi]$, the solutions are $x \in \left\{-\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi\right\}$.

Number of solutions = 5.

For (Q):

$\sin^2 x + \cos^6 x = 1$

$1 - \cos^2 x + \cos^6 x = 1$

$\cos^2 x(\cos^4 x - 1) = 0$

This gives $\cos^2 x = 0$ or $\cos^2 x = 1$.

In the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the solutions are $x \in \left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$.

Number of solutions = 3.

For (R) :

$\cos^2\left(\frac{x}{2}\right) - \sin^2 x = \frac{1}{2}$

$\frac{1+\cos x}{2} - (1 - \cos^2 x) = \frac{1}{2}$

$1 + \cos x - 2 + 2\cos^2 x = 1$

$2\cos^2 x + \cos x - 2 = 0$

Solving the quadratic equation for cosx, we get:

$\cos x = \frac{-1 \pm \sqrt{17}}{4}$

Since $\cos x \in [-1, 1]$, we reject the negative root. Thus, $\cos x = \frac{\sqrt{17}-1}{4}$.

Since $0 < \frac{\sqrt{17}-1}{4} < 1$, there are exactly 2 solutions in $[-\pi, \pi]$.

Number of solutions = 2.

For (S):

$6\sin^2\left(\frac{x}{2}\right) - \cos 3x = 3$

$3(1 - \cos x) - (4\cos^3 x - 3\cos x) = 3$

$3 - 3\cos x - 4\cos^3 x + 3\cos x = 3$

$-4\cos^3 x = 0 \Rightarrow \cos x = 0$

In the interval $[-2\pi, 2\pi]$, the solutions are $x \in \left\{-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$.

Number of solutions = 4.

Therefore, the correct matching is (P) → (5), (Q) → (3), (R) → (2), (S) → (4).

15. **(a)** Since $MM^T = I$, M is an orthogonal matrix.

The columns of an orthogonal matrix form an orthonormal basis for R^3. The given vectors $\vec{u}, \vec{v}, \vec{w}$ are exactly the columns of M . Therefore, $\vec{u}, \vec{v}, \vec{w}$ are mutually orthogonal unit vectors.

For (P) : The rows of M are also orthonormal. The third row is $[\gamma \quad \delta \quad \mu]$, so $\gamma^2 + \delta^2 + \mu^2 = 1$.

The third column $\vec{w}$ is a unit vector, so $|\vec{w}|^2 = \left(-\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \mu^2 = 1$.

$\frac{1}{2}+\frac{1}{3}+\mu^2=1 \Rightarrow \mu^2=\frac{1}{6}$

Substituting μ^2 into the row equation gives $\gamma^2+\delta^2=1-\frac{1}{6}=\frac{5}{6}$. Thus, (P) → (5).

For (Q): Given $x\vec{u}+y\vec{v}+z\vec{w}=\hat{j}$. Taking the dot product with $\vec{u}$ on both sides gives $x(\vec{u}\cdot\vec{u})+y(\vec{v}\cdot\vec{u})+z(\vec{w}\cdot\vec{u})=\hat{j}\cdot\vec{u}$.

Since the vectors are orthonormal, $\vec{u}\cdot\vec{u}=1$ and $\vec{v}\cdot\vec{u}=\vec{w}\cdot\vec{u}=0$.

Thus, $x=\hat{j}\cdot\vec{u}$. From the definition of $\vec{u}$, the $\hat{j}$ component is $\frac{1}{\sqrt{3}}$, so $x=\frac{1}{\sqrt{3}}$. Thus, (Q → (4).

For (R): The scalar triple product $\vec{u}\cdot(\vec{v}\times\vec{w})$ represents the determinant of the matrix formed by these column vectors, which is M.

Since M is orthogonal, $\det(M)=\pm1$. Thus, $|\vec{u}\cdot(\vec{v}\times\vec{w})|=|\det(M)|=1$. Thus, (R) → (2).

For (S): Using the vector triple product expansion, $\vec{u}\times(\vec{v}\times\vec{w})=(\vec{u}\cdot\vec{w})\vec{v}-(\vec{u}\cdot\vec{v})\vec{w}$.

Since $\vec{u},\vec{v},\vec{w}$ are mutually orthogonal, $\vec{u}\cdot\vec{w}=0$ and $\vec{u}\cdot\vec{v}=0$.

Thus, $\vec{u}\times(\vec{v}\times\vec{w})=\vec{0}$, and its magnitude is 0 . Thus, (S) → (1).

16. (b) For (P):

The radius of the circle is the perpendicular distance from $(1,2)$ to $3x+4y-1=0$.

$r=\frac{|3(1)+4(2)-1|}{\sqrt{3^2+4^2}}=\frac{10}{5}=2$

The equation of the circle is $(x-1)^2+(y-2)^2=4$.

Checking the given points, $(3,2)$ satisfies the equation.

Thus, (P) → (3).

For (Q):

The equation of a tangent to $y^2=8x$ with slope m is $y=mx+\frac{2}{m}$.

Since it is also a tangent to $x^2+y^2=2$, the perpendicular distance from $(0,0)$ to the line $mx-y+\frac{2}{m}=0$ is equal to the radius $\sqrt{2}$.

$\frac{\left|\frac{2}{m}\right|}{\sqrt{m^2+1}}=\sqrt{2}$

$\frac{4}{m^2}=2(m^2+1)$

$m^4+m^2-2=0$

$(m^2+2)(m^2-1)=0$

Since m is real and positive, $m=1$.

The equation of the common tangent is $y=x+2$.

Checking the given points, $(7,9)$ satisfies the equation.

Thus, (Q) → (2).

For (R) :

The equation of the ellipse is $\frac{x^2}{16}+\frac{y^2}{12}=1$.

Here $a^2=16$ and $b^2=12$.

Eccentricity $e=\sqrt{1-\frac{12}{16}}=\frac{1}{2}$.

The end point of the latus rectum in the first quadrant is $\left(ae,\frac{b^2}{a}\right)=\left(4\times\frac{1}{2},\frac{12}{4}\right)=(2,3)$.

The equation of the normal at $(2,3)$ is $\frac{16x}{2}-\frac{12y}{3}=16-12$.

$8x-4y=4 \Rightarrow 2x-y=1$

Checking the given points, $(1,1)$ satisfies the equation.

Thus, (R) → (1).

For (S):

The centre is $(0,0)$. The focus is $(ae,0)=(5,0) \Rightarrow ae=5$.

The directrix is $x=-\frac{16}{5} \Rightarrow \frac{a}{e}=\frac{16}{5}$.

Multiplying the two equations gives $a^2=16$.

Dividing the two equations gives $e^2=\frac{25}{16}$.

$b^2 = a^2(e^2 - 1) = 16\left(\frac{25}{16} - 1\right) = 9$

The equation of the hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

Checking the given points, $(8, 3\sqrt{3})$ satisfies the equation.

Thus, (S) → (5).

17. (c) Let the radius of the large disk be R and the radius of the small disks be r. We are given $r = R/50$, which implies $R = 50r$.

The centers of the small disks move along a circular path of radius $R + r = 50r + r = 51r$.

When the two small disks are in contact, the distance between their centers is $2r$. The angular separation $\Delta\theta$ between their centers is given by the arc length (which is approximately the chord length for small angles):

$(R + r)\Delta\theta = 2r$

$(51r)\Delta\theta = 2r \Rightarrow \Delta\theta = \frac{2}{51}\text{rad}$

The small disks roll without slipping on the stationary large disk. For a disk of radius r rolling with angular velocity ω, the velocity of its center is $v = \omega r$.

The angular velocity of the center of the first small disk about the center of the large disk is:

$\Omega_1 = \frac{v_1}{R + r} = \frac{\omega r}{51r} = \frac{\omega}{51}$

Similarly, for the second small disk rolling with angular velocity 2ω, the angular velocity of its center is:

$\Omega_2 = \frac{v_2}{R + r} = \frac{2\omega r}{51r} = \frac{2\omega}{51}$

Since they move in opposite directions, their relative angular velocity is:

$\Omega_{rel} = \Omega_1 + \Omega_2 = \frac{\omega}{51} + \frac{2\omega}{51} = \frac{3\omega}{51}$

The disks are initially in contact, meaning their centers are separated by an angle $\Delta\theta$. They will be in contact again when their centers are once again separated by $\Delta\theta$ after crossing each other on the opposite side of the large disk.

The total angular distance that the two centers must cover together is:

$\theta_{total} = 2\pi - 2\Delta\theta$

Substituting $\Delta\theta = \frac{2}{51}$:

$\theta_{total} = 2\pi - 2\left(\frac{2}{51}\right) = 2\pi - \frac{4}{51}$

The time τ taken for them to meet again is:

$\tau = \frac{\theta_{total}}{\Omega_{rel}} = \frac{2\pi - \frac{4}{51}}{\frac{3\omega}{51}}$

$\tau = \frac{51}{3\omega}\left(2\pi - \frac{4}{51}\right)$

18. (c) Let the larger coil be coil 1 and the smaller coil be coil 2 .

The self-inductance of the larger coil is given by:

$L_1 = \mu_0 n_1^2 A_1 l_1 = \mu_0 N^2 S d = L$

The self-inductance of the smaller coil is:

$L_2 = \mu_0 n_2^2 A_2 l_2 = \mu_0 (2N)^2 \left(\frac{S}{2}\right)\left(\frac{d}{2}\right) = \mu_0 (4N^2)\left(\frac{Sd}{4}\right) = \mu_0 N^2 S d = L$

The mutual inductance between the two coils is:

$M = \mu_0 n_1 n_2 A_{common}\, l_{common}$

Since the smaller coil is completely inside the larger coil, the common area is $S/2$ and the common length is $d/2$.

$M = \mu_0 (N)(2N)\left(\frac{S}{2}\right)\left(\frac{d}{2}\right) = \frac{1}{2}\mu_0 N^2 S d = \frac{L}{2}$

Let i_1 be the current in the larger coil and i_2 be the current in the smaller coil. The voltage across the larger coil is:

$V = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt}$

Since the smaller coil is short-circuited, the net voltage across it is zero:

$0 = L_2 \frac{di_2}{dt} + M\frac{di_1}{dt} \Rightarrow \frac{di_2}{dt} = -\frac{M}{L_2}\frac{di_1}{dt}$

Substituting this into the first equation, we get the effective inductance L_{eq} :

$$V = L_1\frac{di_1}{dt} - \frac{M^2}{L_2}\frac{di_1}{dt} = \left(L_1 - \frac{M^2}{L_2}\right)\frac{di_1}{dt}$$

$$L_{eq} = L_1 - \frac{M^2}{L_2} = L - \frac{(L/2)^2}{L} = L - \frac{L}{4} = \frac{3L}{4}$$

The resonant angular frequency of the circuit is:

$$\omega = \frac{1}{\sqrt{L_{eq}C}} = \frac{1}{\sqrt{\frac{3L}{4}C}} = \frac{2}{\sqrt{3LC}}$$

19. (b) Let the mass of the cylinder be m and its radius be R.

When the cylinder is rolling on the horizontal surface, its kinetic energy is:

$$K_i = \frac{1}{2}mv_0^2 + \frac{1}{2}I_{cm}\omega_0^2$$

Since it rolls without slipping, $\omega_0 = \frac{v_0}{R}$ and $I_{cm} = \frac{1}{2}mR^2$.

$$K_i = \frac{1}{2}mv_0^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v_0}{R}\right)^2 = \frac{3}{4}mv_0^2$$

Given $v_0 = \sqrt{\frac{gR}{3}}$, we have $v_0^2 = \frac{gR}{3}$.

$$K_i = \frac{3}{4}m\left(\frac{gR}{3}\right) = \frac{1}{4}mgR$$

Let the corner be the reference level for potential energy. The initial total mechanical energy of the cylinder just as it reaches the comer is:

$$E_i = K_i + U_i = \frac{1}{4}mgR + mgR = \frac{5}{4}mgR$$

As the cylinder rotates around the corner, let θ be the angle the line connecting the corner to the center of mass makes with the vertical. Let v be the speed of the center of mass at this angle.

The total mechanical energy at angle θ is:

$$E_f = \frac{3}{4}mv^2 + mgR\cos\theta$$

By conservation of mechanical energy, $E_q = E_f$:

$$\frac{5}{4}mgR = \frac{3}{4}mv^2 + mgR\cos\theta$$

The forces acting on the cylinder along the radial direction (towards the corner) are the component of gravity $mg\cos\theta$ and the normal force N. The equation of motion is:

$$mg\cos\theta - N = \frac{mv^2}{R}$$

The cylinder loses contact with the corner when the normal force becomes zero ($N = 0$):

$$mg\cos\theta = \frac{mv^2}{R} \Rightarrow mgR\cos\theta = mv^2$$

Substituting this into the energy conservation equation:

$$\frac{5}{4}mgR = \frac{3}{4}mv^2 + mv^2$$

$$\frac{5}{4}mgR = \frac{7}{4}mv^2$$

Solving for v :

$$v^2 = \frac{5}{7}gR$$

$$v = \sqrt{\frac{5gR}{7}}$$

20. (a) The focal length f of a lens immersed in a medium of refractive index n_L is given by the Lens Maker's formula:

$$\frac{1}{f} = \left(\frac{n_g}{n_L} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Given that the lens is double convex, the radii of curvature are $R_1 = +20\ \text{cm} = +0.2\ \text{m}$ and $R_2 = -20\ \text{cm} = -0.2\ \text{m}$. The refractive index of the glass is $n_g = 1.5$.

Substituting these values into the formula gives:

$$\frac{1}{f} = \left(\frac{1.5}{n_L} - 1\right)\left(\frac{1}{0.2} - \frac{1}{-0.2}\right)$$

$$\frac{1}{f} = \left(\frac{1.5}{n_L} - 1\right)(5+5) = 10\left(\frac{1.5}{n_L} - 1\right)$$

The power P of the lens is defined as the reciprocal of its focal length in meters:

$$P = \frac{1}{f} = \frac{15}{n_L} - 10$$

This equation represents a hyperbola, indicating that the graph of P versus n_L is a curve, not a straight line.

Evaluating the power at specific values of n_L yields $P = 5D$ at $n_L = 1.0$, $P = 0D$ at $n_L = 1.5$, and $P = -2.5D$ at $n_L = 2.0$

The second derivative $\frac{d^2P}{dn_L^2} = \frac{30}{n_L^3}$ is positive for $n_L > 0$, meaning the curve is concave upwards.

The correct plot shows a concave upwards curve passing through $(1.0,5)$, $(1.5,0)$, and $(2.0,-2.5)$.

21. (a,c) From Bohr's quantization condition, the angular momentum of an electron in the k^{th} orbit is given by:

$$mv_k r_k = \frac{kh}{2\pi}$$

The kinetic energy of the electron in the k^{th} orbit is:

$$K_k = \frac{1}{2}mv_k^2 = \frac{1}{2}(mv_k r_k)\frac{v_k}{r_k}$$

Substituting the value of angular momentum:

$$K_k = \frac{1}{2}\left(\frac{kh}{2\pi}\right)\frac{v_k}{r_k} = \frac{khv_k}{4\pi r_k}$$

The magnitude of change in kinetic energy for a transition from the n^{th} orbit to the 1^{st} orbit is:

$$|\Delta K| = |K_n - K_1| = \frac{h}{4\pi}\left|\frac{nv_n}{r_n} - \frac{v_1}{r_1}\right|$$

This makes statement (A) correct.

Since the total energy $E_k = -K_k$, the magnitude of change in total energy is equal to the magnitude of change in kinetic energy:

$$|\Delta E| = |\Delta K| = \frac{h}{4\pi}\left|\frac{nv_n}{r_n} - \frac{v_1}{r_1}\right|$$

This makes statement (d) incorrect.

The electrostatic force provides the necessary centripetal force:

$$\frac{mv_k^2}{r_k} = \frac{e^2}{4\pi\epsilon_0 r_k^2} \Rightarrow K_k = \frac{1}{2}mv_k^2 = \frac{e^2}{8\pi\epsilon_0 r_k}$$

The total energy is $E_k = -K_k = -\frac{e^2}{8\pi\epsilon_0 r_k}$. The energy of the emitted photon during the transition is:

$$\Delta E = E_n - E_1 = -\frac{e^2}{8\pi\epsilon_0 r_n} - \left(-\frac{e^2}{8\pi\epsilon_0 r_1}\right) = \frac{e^2}{8\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_n}\right)$$

The frequency of the emitted radiation is $\nu = \frac{\Delta E}{h}$:

$$\nu = \frac{e^2}{8\pi\epsilon_0 h}\left(\frac{1}{r_1} - \frac{1}{r_n}\right)$$

This makes statement (c) correct.

The de Broglie wavelength of the electron is $\lambda_k = \frac{h}{mv_k}$. Using Bohr's quantization $mv_k = \frac{kh}{2\pi r_k}$, we get: $\lambda_k = \frac{2\pi r_k}{k}$

From the kinetic energy relation $r_k = \frac{e^2}{8\pi\epsilon_0 K_k}$, substituting r_k gives:

$$\lambda_k = \frac{2\pi}{k}\left(\frac{e^2}{8\pi\epsilon_0 K_k}\right) = \frac{e^2}{4\epsilon_0 k K_k}$$

The magnitude of change in de Broglie wavelength is:

$$|\Delta\lambda| = |\lambda_n - \lambda_1| = \frac{e^2}{4\epsilon_0}\left|\frac{1}{nK_n} - \frac{1}{K_1}\right|$$

This makes statement (b) incorrect.

22. (a,b) The equation of trajectory of a projectile is given by:

$$y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$$

The particle passes through point P(5,1). Substituting $x = 5$ and $y = 1$:

$$1 = 5\tan\theta - \frac{25g}{2v^2\cos^2\theta}$$

For option (a), substituting $\theta = 45°$:

$$1 = 5\tan45° - \frac{25g}{2v^2\cos^2 45°}$$

$$1 = 5(1) - \frac{25g}{2v^2(1/2)}$$

$$1 = 5 - \frac{25g}{v^2}$$

$$\frac{25g}{v^2} = 4 \Rightarrow v^2 = \frac{25g}{4} \Rightarrow v = \frac{5\sqrt{g}}{2}\ ms^{-1}$$

Thus, option (a) is correct.

For option (b), the horizontal distance to the maximum height is $x_m = \frac{R}{2} = \frac{v^2\sin(2\theta)}{2g}$.

For $\theta = 45°$ and $v^2 = \frac{25g}{4}$:

$$x_m = \frac{\left(\frac{25g}{4}\right)\sin90°}{2g} = \frac{25}{8} = 3.125\ m$$

Since $x_m = 3.125\ m < 5\ m$, the particle reaches its maximum height before it reaches P. Thus, option (b) is correct.

For option (c), substituting $\theta = 30°$ into the trajectory equation:

$$1 = 5\tan30° - \frac{25g}{2v^2\cos^2 30°}$$

$$1 = \frac{5}{\sqrt{3}} - \frac{25g}{2v^2(3/4)} \Rightarrow \frac{50g}{3v^2} = \frac{5}{\sqrt{3}} - 1$$

$$v^2 = \frac{50\sqrt{3}g}{3(5-\sqrt{3})}$$

The horizontal distance to the maximum height is:

$$x_m = \frac{v^2\sin60°}{2g} = \frac{v^2\sqrt{3}}{4g} = \frac{50\sqrt{3}g}{3(5-\sqrt{3})} \times \frac{\sqrt{3}}{4g} = \frac{25}{2(5-\sqrt{3})} = 3.82\ m$$

Since $x_m < 5\ m$, the particle reaches its maximum height before reaching P. Thus, option (c) is incorrect.

For option (d), substituting $\theta = \tan^{-1}\left(\frac{1}{5}\right)$, so $\tan\theta = \frac{1}{5}$:

$$1 = 5\left(\frac{1}{5}\right) - \frac{25g}{2v^2\cos^2\theta} \Rightarrow 1 = 1 - \frac{25g}{2v^2\cos^2\theta}$$

$$\frac{25g}{2v^2\cos^2\theta} = 0$$

This is only possible if $v \to \infty$. Thus, option (d) is incorrect.

23. (a,b,c) For the monoatomic ideal gas, $\gamma = \frac{5}{3}$.

Process ab is isothermal, so $T_a = T_b$ and $V_a = V_1, V_b = V_2$.

Process bc is isochoric, so $V_b = V_c = V_2$.

Process ca is adiabatic, so $T_c V_c^{\gamma-1} = T_a V_a^{\gamma-1}$.

$$T_c V_2^{2/3} = T_a V_1^{2/3} \Rightarrow T_c = T_a\left(\frac{V_1}{V_2}\right)^{2/3}.$$

If $\frac{V_2}{V_1} = 8, T_c = T_a\left(\frac{1}{8}\right)^{2/3} = \frac{T_a}{4} \Rightarrow T_a = 4T_c$. Thus, statement (c) is correct.

Heat absorbed in isothermal process ab is $Q_{ab} = nRT_a\ln\left(\frac{V_2}{V_1}\right)$.

For $\frac{V_2}{V_1} = 8, Q_{ab} = nRT_a \ln(8) = 3nRT_a \ln 2 = 3 \times 0.7nRT_a = 2.1nRT_a$.

Heat released in isochoric process bc is $Q_{bc} = nC_v(T_b - T_c) = n\left(\frac{3}{2}R\right)\left(T_a - \frac{T_a}{4}\right) = \frac{9}{8}nRT_a = 1.125nRT_a$.

Since $1.125nRT_a < 2.1nRT_a$, the heat released in bc is smaller than the heat absorbed in ab. Thus, statement (a) is correct.

The efficiency of the cycle is $\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{Q_{bc}}{Q_{ab}}$.

$$\eta = 1 - \frac{n\left(\frac{3}{2}R\right)T_a\left(1 - \left(\frac{V_1}{V_2}\right)^{2/3}\right)}{nRT_a\ln\left(\frac{V_2}{V_1}\right)} = 1 - \frac{3\left(1 - \left(\frac{V_1}{V_2}\right)^{2/3}\right)}{2\ln\left(\frac{V_2}{V_1}\right)}.$$

This expression depends only on the volume ratio $\frac{V_2}{V_1}$ and is independent of the temperature T_a. Thus, statement (b) is correct.

For the isothermal process ab, $P_aV_a = P_bV_b \Rightarrow P_aV_1 = P_bV_2 \Rightarrow P_a = 8P_b$. Thus, statement (d) is incorrect.

24. (a,c) The given electric field is $\vec{E} = E_0\sin(3y + 4z + \omega t)\hat{i}$.

Comparing the phase $\phi = 3y + 4z + \omega t$ with the standard wave equation phase $\vec{k} \cdot \vec{r} - \omega t$, we can rewrite it as $-(-3y - 4z - \omega t)$.

Thus, the wave vector is $\vec{k} = -3\hat{j} - 4\hat{k}$.

The direction of wave propagation is given by the unit vector $\hat{n}$:

$$\hat{n} = \frac{\vec{k}}{|\vec{k}|} = \frac{-3\hat{j} - 4\hat{k}}{\sqrt{(-3)^2 + (-4)^2}} = -\frac{1}{5}(3\hat{j} + 4\hat{k})$$

The magnitude of the wave vector is $|\vec{k}| = 5\ m^{-1}$.

The angular frequency ω is:

$\omega = c|\vec{k}| = (3 \times 10^8) \times 5 = 1.5 \times 10^9 rad\ s^{-1}$

The magnetic field $\vec{B}$ is given by:

$$\vec{B} = \frac{1}{c}(\hat{n} \times \vec{E}) = \frac{1}{c}\left[-\frac{1}{5}(3\hat{j} + 4\hat{k})\right] \times [E_0\sin(3y + 4z + \omega t)\hat{i}]$$

$$\vec{B} = -\frac{E_0}{5c}\sin(3y + 4z + \omega t)[3(\hat{j} \times \hat{i}) + 4(\hat{k} \times \hat{i})]$$

$$\vec{B} = -\frac{E_0}{5c}\sin(3y + 4z + \omega t)(-3\hat{k} + 4\hat{j}) = \frac{E_0}{5c}\sin(3y + 4z + \omega t)(-4\hat{j} + 3\hat{k})$$

25. (1.73)

Let A be the cross-sectional area of the rod.

Mass of the rod, $M = \rho AL$.

Weight of the rod, $W = \rho ALg$, acting at a distance $L/2$ from the hinge.

When the rod is displaced by a small angle θ, the restoring torque is provided by the buoyant forces from the two liquids.

Buoyant force due to the lower liquid, $F_{B1} = (6\rho)A(L/2)g = 3\rho ALg$.

This force acts at the center of the immersed part in the lower liquid, which is at a distance $L/4$ from the hinge.

Torque due to F_{B1} is $\tau_{B1} = F_{B1}(L/4)\theta = (3\rho ALg)(L/4)\theta = \frac{3}{4}\rho AL^2g\theta$.

Buoyant force due to the upper liquid, $F_{B2} = (2\rho)A(L/2)g = \rho ALg$.

This force acts at the center of the immersed part in the upper liquid, which is at a distance $L/2 + L/4 = 3L/4$ from the hinge.

Torque due to F_{B2} is $\tau_{B2} = F_{B2}(3L/4)\theta = (\rho ALg)(3L/4)\theta = \frac{3}{4}\rho AL^2g\theta$.

Destabilizing torque due to gravity is $\tau_g = W(L/2)\theta = (\rho ALg)(L/2)\theta = \frac{1}{2}\rho AL^2g\theta$.

Net restoring torque, $\tau_{net} = \tau_{B1} + \tau_{B2} - \tau_g = \left(\frac{3}{4} + \frac{3}{4} - \frac{1}{2}\right)\rho AL^2g\theta = \rho AL^2g\theta$.

Moment of inertia of the rod about the hinge, $I = \frac{ML^2}{3} = \frac{\rho AL^3}{3}$.

The equation of motion is $I\alpha = -\tau_{net}$

$$\frac{\rho AL^3}{3}\alpha = -\rho AL^2g\theta$$

$\alpha = -\frac{3g}{L}\theta$

This represents simple harmonic motion with $\omega^2 = \frac{3g}{L}$.

Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3}}\sqrt{\frac{L}{g}}$.

26. (33)

Let Q_0 be the heat absorbed by the first engine and Q_1 be the heat rejected.

The efficiency of the first engine is $\eta = 1 - \frac{Q_1}{Q_0} \Rightarrow Q_1 = Q_0(1-\eta)$

Similarly, for the second engine, the heat rejected is $Q_2 = Q_1(1-\eta) = Q_0(1-\eta)^2$

Following this pattern, for the 5^{th} engine, the heat rejected is $Q_5 = Q_0(1-\eta)^5$

The total work done by all five engines is

$W = W_1 + W_2 + W_3 + W_4 + W_5 = (Q_0 - Q_1) + (Q_1 - Q_2) + \cdots + (Q_4 - Q_5) = Q_0 - Q_5$

The net efficiency of the system is $\eta_{net} = \frac{W}{Q_0} = \frac{Q_0 - Q_s}{Q_0} = 1 - \frac{Q_s}{Q_0}$

Substituting the value of Q_s, we get $\eta_{met} = 1 - (1-\eta)^s$

Given $\eta_{net} = \frac{211}{243}$

$1 - (1-\eta)^5 = \frac{211}{243}$

$(1-\eta)^5 = 1 - \frac{211}{243} = \frac{32}{243}$

$(1-\eta)^5 = \left(\frac{2}{3}\right)^5$

$1 - \eta = \frac{2}{3}$

$\eta = \frac{1}{3}$

27. (0.66)

Let T_1 and T_2 be the temperatures of the gases in sections S_1 and S_2 respectively. Assume $T_1 > T_2$, so heat flows from S_1 to S_2.

The rate of heat transfer through the partition P_1 is given by Fourier's law of heat conduction:

$\frac{dQ}{dt} = \frac{KA}{x}(T_1 - T_2)$

For section S_1, the volume is constant (since P_1 is immovable). Thus, the process is isochoric. The heat lost by S_1 is:

$dQ = -n_1 C_V dT_1$

Since the gas is monoatomic and $n_1 = 1$ mole, $C_V = \frac{3}{2}R$.

$\Rightarrow dT_1 = -\frac{2dQ}{3R}$

For section S_2, the pressure is constant (since P_2 is freely movable and exposed to atmospheric pressure). Thus, the process is isobaric. The heat gained by S_2 is:

$dQ = n_2 C_P dT_2$

Since the gas is monoatomic and $n_2 = 1$ mole, $C_P = \frac{5}{2}R$.

$\Rightarrow dT_2 = \frac{2dQ}{5R}$

The change in the temperature difference $d(\Delta T) = d(T_1 - T_2)$ is:

$d(T_1 - T_2) = dT_1 - dT_2 = -\frac{2dQ}{3R} - \frac{2dQ}{5R} = -\frac{16dQ}{15R}$

Substituting the rate of heat transfer $\frac{dQ}{dt}$ into the equation:

$\frac{d(T_1 - T_2)}{dt} = -\frac{16}{15R}\left(\frac{KA}{x}(T_1 - T_2)\right)$

Let $\Delta T = T_1 - T_2$. The differential equation becomes:

$\frac{d(\Delta T)}{\Delta T} = -\frac{16KA}{15Rx}dt$

Integrating both sides from $t = 0$ to t, where the temperature difference goes from ΔT_0 to $\frac{\Delta T_0}{2}$:

$$\int_{\Delta T_0}^{\Delta T_0/2} \frac{d(\Delta T)}{\Delta T} = -\frac{16KA}{15Rx}\int_0^t dt$$

$$\ln\left(\frac{1}{2}\right) = -\frac{16KA}{15Rx}t$$

$$-\ln 2 = -\frac{16KA}{15Rx}t$$

$$t = \frac{15\,Rx\ln 2}{16KA}$$

Given that $\ln 2 = 0.7$:

$$t = \frac{15 \times 0.7}{16}\frac{xR}{KA} = \frac{10.5}{16}\frac{xR}{KA} = \frac{21}{32}\frac{xR}{KA} = 0.65625\frac{xR}{KA}$$

Comparing this with the given expression $t = n\frac{xR}{KA}$, we get:

$$n = 0.65625$$

28. (0.5)

For a point on the axis at a distance $z \gg R$ and $z \gg h$, the rotating cone can be considered as a magnetic dipole. The magnetic field on the axis of a dipole is given by:

$$B = \frac{\mu_0}{4\pi}\frac{2M}{z^3}$$

where M is the magnetic dipole moment of the cone.

To find M, consider an elemental ring on the cone at a vertical distance y from the origin, with vertical thickness dy. The radius of this ring is $r = \frac{R}{h}y$ and its slant length is $dl = \frac{\sqrt{R^2+h^2}}{h}dy$.

The surface charge density of the cone is $\sigma = \frac{Q}{\pi R\sqrt{R^2+h^2}}$.

The charge on the elemental ring is:

$$dq = \sigma(2\pi r dl) = \frac{Q}{\pi R\sqrt{R^2+h^2}}\left(2\pi\frac{R}{h}y\frac{\sqrt{R^2+h^2}}{h}dy\right) = \frac{2Q}{h^2}ydy$$

The current due to the rotation of this ring is:

$$dI = \frac{dq}{T} = \frac{dq\omega}{2\pi} = \frac{Q\omega}{\pi h^2}ydy$$

The magnetic moment of this elemental ring is:

$$dM = dI(\pi r^2) = \left(\frac{Q\omega}{\pi h^2}ydy\right)\pi\left(\frac{R}{h}y\right)^2 = \frac{Q\omega R^2}{h^4}y^3dy$$

Integrating from $y = 0$ to $y = h$, the total magnetic moment is:

$$M = \int_0^h \frac{Q\omega R^2}{h^4}y^3dy = \frac{Q\omega R^2}{h^4}\left[\frac{y^4}{4}\right]_0^h = \frac{Q\omega R^2}{4}$$

Alternatively, using the gyromagnetic ratio for a uniform charge distribution, $\frac{M}{L_{ang}} = \frac{Q}{2m}$. The moment of inertia of a hollow cone is $I = \frac{1}{2}mR^2$, so the angular momentum is $L_{ang} = \frac{1}{2}mR^2\omega$. Thus, $M = \frac{Q}{2m}\left(\frac{1}{2}mR^2\omega\right) = \frac{1}{4}QR^2\omega$.

Substituting M into the magnetic field formula:

$$B = \frac{\mu_0}{4\pi}\frac{2}{z^3}\left(\frac{1}{4}QR^2\omega\right) = \frac{\mu_0}{4\pi}\frac{QR^2\omega}{2z^3}$$

Comparing this with the given expression $B = \frac{n\mu_0}{4\pi}\frac{QR^2\omega}{z^3}$, we get:

$$n = \frac{1}{2} = 0.5$$

29. (d) For maximum amplitude at the detector D, the path difference Δx between the two paths must be an integer multiple of the wavelength λ. For the smallest value of l, we take $\Delta x = \lambda = 0.29$ m.

For configuration (P):

The sound travels through a straight tube of length l and a semi-circular tube of diameter l.

Path difference $\Delta x = \frac{\pi l}{2} - l = l\left(\frac{\pi}{2} - 1\right)$.

Equating to λ: $l\left(\frac{3.1416}{2}-1\right)=0.29 \Rightarrow l(0.5708)=0.29 \Rightarrow l=0.51$ m.

Thus, P → 3.

For configuration (Q):

The sound travels through a straight tube of length l and a rectangular path of length $0.5l+l+0.5l=2l$.

Path difference $\Delta x = 2l-l=l$.

Equating to λ: $l=0.29$ m.

Thus, Q → 4.

For configuration (R) :

The sound travels through a straight horizontal tube of length l and a path consisting of a vertical tube of length l followed by a semi-circular tube.

The diameter of the semi-circle is the hypotenuse of the right triangle formed by the tubes, which is $\sqrt{l^2+l^2}=l\sqrt{2}$.

The length of the semi-circular part is $\frac{\pi(l\sqrt{2})}{2}=\frac{\pi l}{\sqrt{2}}$.

Path difference $\Delta x=\left(l+\frac{\pi l}{\sqrt{2}}\right)-l=\frac{\pi l}{\sqrt{2}}$.

Equating to λ: $\frac{\pi l}{\sqrt{2}}=0.29 \Rightarrow l=\frac{0.29\sqrt{2}}{\pi}=\frac{0.29\times1.414}{3.1416}=0.13$ m.

Thus, R → 5.

For configuration (S):

The sound travels through a straight tube of length l and a triangular path.

The angles of the triangle are $45°, 105°$, and $180°-(45°+105°)=30°$.

Using the sine rule, the lengths of the other two sides are $l\frac{\sin30°}{\sin105°}$ and $l\frac{\sin45°}{\sin105°}$.

Path difference $\Delta x = l\frac{\sin30°+\sin45°}{\sin105°}-l$.

Given $\sin105°=\cos15°=0.97$.

$$\Delta x = l\left(\frac{0.5+1/\sqrt{2}}{0.97}-1\right)=l\left(\frac{0.5+0.707}{0.97}-1\right)=l\left(\frac{1.207}{0.97}-1\right)=0.244l.$$

Equating to λ: $0.244l=0.29 \Rightarrow l=1.19$ m.

Thus, S → 2.

Matching the results, we get P → 3, Q → 4, R → 5, S → 2.

30. **(a)** The colorful sky in the north polar region, known as Aurora Borealis, is caused by the emission of radiation from oxygen and nitrogen atoms in the upper atmosphere when they are excited by charged particles from the solar wind. Thus, P → 5.

Sunlight scattered by molecules in the atmosphere becomes partially polarized in a direction perpendicular to the incident light. Thus, Q → 4.

A rainbow is formed due to the dispersion of sunlight and its internal reflection by water droplets in the atmosphere. Thus, R → 1.

Dark and bright finges are the characteristic patterns formed due to the superposition of light waves, which occurs in phenomena like interference and diffraction. Thus, S → 3.

The correct matching is P → 5, Q → 4, R → 1, S → 3.

31. **(c)** Let us analyze the induced current $i(t)$ for each loop as it rotates clockwise with time period T. The magnetic field B is uniform in the +z direction for $x>0$. The angular velocity is $\omega=\frac{2\pi}{T}$. The induced EMF is $\varepsilon=-\frac{d\Phi}{dt}=-B\frac{dA}{dt}$, where A(t) is the area of the loop in the region $x>0$. The current $i(t)$ is proportional to $\frac{dA}{dt}$.

Loop (P):

This is a semicircle in the region $x<0$ at $t=0$. As it rotates clockwise, the top half of the straight edge enters the region $x>0$ The area in$x>0$ is a sector of angle $\theta=\omega t$.

For $0\le t\le T/2$, the area $A(t)=\frac{1}{2}R^2\omega t$ increases linearly, so $\frac{dA}{dt}$ is a positive constant. Thus, i(t) is a positive constant.

For $T/2\le t\le T$, the loop exits the region $x>0$, and the area decreases linearly, so $\frac{dA}{dt}$ is a negative constant. Thus, i(t) is a negative constant.

This matches graph (3). So, P → 3.

Loop (R):
This is a single sector of angle $60° = \frac{\pi}{3}$.
For $0 \le t \le T/6$, the sector enters $x > 0$. The area increases linearly, so $i(t)$ is a positive constant.
For $T/6 \le t \le T/2$, the sector is fully inside $x > 0$. The area is constant, so $i(t) = 0$.
For $T/2 \le t \le 2T/3$, the sector exits $x > 0$. The area decreases linearly, so $i(t)$ is a negative constant.
For $2T/3 \le t \le T$, the sector is fully outside $x > 0$. The area is zero, so $i(t) = 0$.
This matches graph (1). So, $R \to 1$.
Loop (S):
This loop consists of two identical sectors arranged symmetrically but connected such that they form a figure-8 loop (crossing at the origin). The two lobes of a figure- 8 loop have opposite area vectors. As the loop rotates, the rate of change of flux in the top lobe is exactly canceled by the rate of change of flux in the bottom lobe due to their symmetric entry and exit combined with the opposite sense of traversal. Therefore, the net induced EMF and current are zero at all times.
This matches graph (4). So, $S \to 4$.
Loop (Q):
By elimination and matching the remaining options, Q corresponds to graph (2). The loop has multiple sectors that enter and exit the magnetic field at different intervals, creating alternating positive and negative pulses of current.
So, $Q \to 2$.
The correct matching is $P \to 3, Q \to 2, R \to 1, S \to 4$.

32. (a) The moment of inertia of a uniform rod of mass m and length l about an axis passing through one end and making an angle θ with the rod is given by $I = \frac{ml^2}{3}\sin^2\theta$.

For structure (P):
The axis OO' makes an angle of $45°$ with both rods CA and CB.

$$I_P = I_{CA} + I_{CB} = \frac{ml^2}{3}\sin^2(45°) + \frac{ml^2}{3}\sin^2(45°) = 2 \times \frac{ml^2}{3} \times \frac{1}{2} = \frac{ml^2}{3}.$$

This matches with (5).
For structure (Q):
The axis OO' makes an angle of $60°$ with rods CA and CB. Rod AB is parallel to the axis at a distance $d = l\sin(60°) = \frac{\sqrt{3}}{2}l$.

$$I_Q = I_{CA} + I_{CB} + I_{AB} = 2 \times \left(\frac{ml^2}{3}\sin^2(60°)\right) + md^2$$

$$I_Q = 2 \times \frac{ml^2}{3} \times \frac{3}{4} + m\left(\frac{\sqrt{3}}{2}l\right)^2 = \frac{ml^2}{2} + \frac{3ml^2}{4} = \frac{5ml^2}{4}$$

This matches with (1).
For structure (R) :
The axis OO' is the diagonal of the square, making an angle of $45°$ with all four rods.

$$I_R = 4 \times \left(\frac{ml^2}{3}\sin^2(45°)\right) = 4 \times \frac{ml^2}{3} \times \frac{1}{2} = \frac{2ml^2}{3}.$$

This matches with (4).
For structure (S):
The axis OO' makes an angle of $30°$ with both rods CA and CB.

$$I_S = 2 \times \left(\frac{ml^2}{3}\sin^2(30°)\right) = 2 \times \frac{ml^2}{3} \times \frac{1}{4} = \frac{ml^2}{6}.$$

This matches with (2).
Therefore, the correct matching is $P \to 5, Q \to 1, R \to 4, S \to 2$.

33. (b) Let the initial, intermediate, and final volumes of the gas be V_1, V_2, and V_3 respectively.
Using the ideal gas equation $V = \frac{nRT}{P}$, we have:

$$V_1 = \frac{nRT}{2}$$

$$V_2 = \frac{nRT}{P}$$

$V_3 = \frac{nRT}{8}$

The work done on the gas during the first step against a constant external pressure P is:

$W_1 = -P_{ext,1}(V_2 - V_1) = -P\left(\frac{nRT}{P} - \frac{nRT}{2}\right) = nRT\left(\frac{P}{2} - 1\right)$

The work done on the gas during the second step against a constant external pressure of 8 bar is:

$W_2 = -P_{ext,2}(V_3 - V_2) = -8\left(\frac{nRT}{8} - \frac{nRT}{P}\right) = nRT\left(\frac{8}{P} - 1\right)$

The total work done on the gas is:

$W = W_1 + W_2 = nRT\left(\frac{P}{2} - 1\right) + nRT\left(\frac{8}{P} - 1\right) = nRT\left(\frac{P}{2} + \frac{8}{P} - 2\right)$

To find the minimum value of W, we need to minimize the expression $\left(\frac{P}{2} + \frac{8}{P}\right)$.

Using the AM-GM inequality:

$\frac{P}{2} + \frac{8}{P} \geq 2\sqrt{\frac{P}{2} \times \frac{8}{P}} = 2\sqrt{4} = 4$

The minimum value is 4 , which occurs when $\frac{P}{2} = \frac{8}{P} \Rightarrow P^2 = 16 \Rightarrow P = 4$ bar. This value of P lies within the given range $(2 < P < 8)$.

Substituting this minimum value back into the work equation:

$W_{min} = nRT(4 - 2) = 2nRT$

Given $n = 0.5$ mol and $T = 600$ K :

$W_{min} = 2 \times 0.5 \times R \times 600 = 600R$

34. (c) For the reversible first-order reaction $R \rightleftharpoons P$, at equilibrium, the rate of the forward reaction equals the rate of the backward reaction.

$k_f[R]_{eq} = k_b[P]_{eq}$

Given $k_b = 4k_f$, we can substitute this into the equilibrium condition:

$k_f[R]_{eq} = 4k_f[P]_{eq}$

$[R]_{eq} = 4[P]_{eq}$

From the conservation of mass, the total concentration remains constant at any time t :

$[R]_{eq} + [P]_{eq} = [R]_0$

Substituting $[R]_{eq} = 4[P]_{eq}$ into the mass balance equation gives:

$4[P]_{eq} + [P]_{eq} = [R]_0$

$5[P]_{eq} = [R]_0$

$[P]_{eq} = 0.2[R]_0$

Using this to find $[R]_{eq}$:

$[R]_{eq} = 4 \times 0.2[R]_0 = 0.8[R]_0$

Therefore, as $t \to \infty$, the concentration ratio $\frac{[P]}{[R]_0}$ approaches 0.2 and $\frac{[R]}{[R]_0}$ approaches 0.8 . This is correctly represented in the third graph.

35. (a) The dipole moment of a molecule depends on its geometry and the vector sum of its individual bond dipoles.

BF_3 has a symmetrical trigonal planar geometry. The three B-F bond dipoles cancel each other out completely, resulting in a net dipole moment of zero ($\mu = 0$).

NH_4^+has a symmetrical tetrahedral geometry. The four $N - H$ bond dipoles cancel each other out, resulting in a net dipole moment of zero ($\mu = 0$).

Thus, $\mu(BF_3) = \mu(NH_4^+) = 0$.

Both NH_3 and NF_3 have a trigonal pyramidal geometry with one lone pair on the central nitrogen atom.

In NH_3, nitrogen is more electronegative than hydrogen. The $N - H$ bond dipoles point towards the nitrogen atom, which is in the same direction as the orbital dipole of the lone pair. These dipoles reinforce each other, giving a higher net dipole moment.

In NF_3, fluorine is more electronegative than nitrogen. The N-F bond dipoles point away from the nitrogen atom, which is in the opposite direction to the orbital dipole of the lone pair. These dipoles partially cancel each other out, resulting in a lower net dipole moment.

Therefore, $\mu(NF_3) < \mu(NH_3)$.
The correct order of dipole moments is $BF_3 = NH_4^+ < NF_3 < NH_3$.

36. (c) From the given statement, $LiBH_4$ selectively reduces the ester group ($-CO_2Et$) to an alcohol ($-CH_2OH$) and leaves the carboxylic acid ($-CO_2H$) unchanged.
Borane (BH_3), on the other hand, is a well-known reagent for the chemoselective reduction of carboxylic acids to alcohols in the presence of esters.
Let us analyze the first set of reactions yielding P and Q :
The starting material is a 1,2-disubstituted cyclobutane derivative. At Cl , the ester group ($-CO_2Et$) is on a wedge (up) and the acid group ($-CO_2H$) is on a dash (down). The adjacent C 2 has a methyl group.
In product P (using $LiBH_4$), the wedge $-CO_2Et$ is reduced. Thus, the new $-CH_2OH$ group is on the wedge.
In product Q (using BH_3), the dash $-CO_2H$ is reduced. Thus, the new $-CH_2OH$ group is on the dash.
Comparing P and Q , the stereocenter at Cl is inverted relative to the unchanged stereocenter at C 2 . Since their relative stereochemistry (cis vs trans relationship between $-CH_2OH$ and $-CH_3$) is different, P and Q are diastereomers.
Now let us analyze the second set of reactions yielding R and S :
The starting material is a 1,3-disubstituted cyclobutane derivative. At Cl, $-CO_2Et$ is on a wedge and $-CO_2H$ is on a dash. At C3, there is a methyl group.
In product R (using $LiBH_4$), the ester on the wedge is reduced. The resulting $-CH_2OH$ group is on the wedge (cis to the methyl group if we assume methyl is up).
In product S (using BH_3), the acid on the dash is reduced. The resulting $-CH_2OH$ group is on the dash (trans to the methyl group).
Since R and S represent the cis and trans isomers of a 1,3-disubstituted cycloalkane, they are stereoisomers that are not mirror images. Therefore, R and S are also diastereomers.
Thus, P & Q are diastereomers, and R & S are diastereomers.

37. (a,b,d) For hydrogen atom (a single-electron system), the energy of an orbital depends only on the principal quantum number n. Thus, the 2 s and 2 p orbitals are degenerate.
$E_{2s}(H) = E_{2p}(H)$
For lithium atom (a multi-electron system), the energy depends on both n and l. Due to the penetration effect, the 2 s electron penetrates closer to the nucleus and experiences a higher effective nuclear charge than the 2 p electron. Thus, the 2 s orbital has lower energy than the 2 p orbital.
$E_{2s}(Li) < E_{2p}(Li)$
The energy of an orbital is given by $E \propto -\frac{Z_{eff}^2}{n^2}$.
For hydrogen, $Z_{eff} = 1$. For the 2 s electron in lithium, the nuclear charge is $Z = 3$, and it is shielded by the $1s^2$ electrons. The effective nuclear charge Z_{eff} for the 2 s electron in Li is greater than 1 (approximately 1.3).
Since $Z_{eff}(Li, 2s) > Z_{eff}(H, 2s)$, the energy of the 2 s orbital in Li is more negative than that in H .
$E_{2s}(Li) < E_{2s}(H) \Rightarrow E_{2s}(H) > E_{2s}(Li)$
Since $E_{2p}(H) = E_{2s}(H)$, we also have $E_{2p}(H) > E_{2s}(Li)$.

38. (a,c) From the given reactions, we can identify the compounds X, Y, and Z as follows:
Reaction 1: $MnO_2 + 4HCl$ (conc.) $\rightarrow MnCl_2 + Cl_2 + 2H_2O$ The greenish-yellow gas X is Cl_2.
Reaction 2: $NH_3 + 3Cl_2$ (excess) $\rightarrow NCl_3 + 3HCl$ Compound Y is NCl_3.
Reaction 3: $Cl_2 + 3\ F_2$ (excess) $\xrightarrow{573\ K} 2ClF_3$ Compound Z is ClF_3.
Evaluating the given statements:
(a)Cl_2 is widely used as a disinfectant for sterilizing drinking water. This statement is correct.
(b)In NCl_3, the nitrogen atom is sp^3 hybridized and possesses one lone pair of electrons. This gives it a trigonal pyramidal structure, not a planar one. This statement is incorrect.
(c)ClF_3 is used in the enrichment of ^{235}U because it reacts with uranium to produce the volatile compound $UF_6 (U + 3ClF_3 \rightarrow UF_6 + 3ClF)$. This statement is correct.
(d)NCl_3 is a weaker Lewis base than NH_3. The highly electronegative chlorine atoms withdraw electron density from the nitrogen atom via the -I effect, making the lone pair less available for donation compared to ammonia. This statement is incorrect.

39. (b,c) The reaction between O_2 and PtF_6 is:
$O_2 + PtF_6 \rightarrow O_2^+[PtF_6]^-$

Here, $X^+$is $O_2^+$and $Y^-$is $PtF_6]^-$.
For O_2^+, the total number of electrons is 15 . The molecular orbital configuration is $\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\sigma_{2p_z}^2\pi_{2p_x}^2\pi_{2p_y}^2\pi_{2p_x}^{*1}$. The bond order is $\frac{10-5}{2}=2.5$. Thus, the bond order is not 1.5 .
In the complex ion $[PtF_6]^-$, the oxidation state of Pt is +5 . The ground state electronic configuration of Pt is $[Xe]4f^{14}5d^96s^1$. The configuration of Pt^{5+} is $[Xe]4f^{14}5d^5$. Therefore, the valence d-orbitals of the metal ion contain 5 electrons.
During the reaction, O_2 loses an electron to form $O_2^+$and PtF_6 gains an electron to form $[PtF_6]^-$. Since PtF_6 accepts an electron, it undergoes reduction and acts as an oxidant.
Since no fluorine atoms are transferred to O_2, PtF_6 does not act as a fluorinating agent.

40. (a,b,c) Kolbe's electrolysis of sodium butyrate ($CH_3CH_2CH_2COONa$) yields n-hexane.
Aromatization of n -hexane using V_2O_5 at $500°C$ produces benzene (Q).
Benzene (Q) reacts with phthalic anhydride in the presence of anhydrous $AlCl_3$ to form o-benzoylbenzoic acid (R).
Treatment of R with PCl_5 gives o-benzoylbenzoyl chloride, which upon Rosenmund reduction ($H_2-Pd/BaSO_4$) yields obenzoylbenzaldehyde (S).
Condensation of S with hydrazine (NH_2NH_2) forms 1-phenylphthalazine (T).
Evaluating the options:
(a)S contains an aldehyde group and gives a positive Tollens' test (silver mirror).
(b)Q (benzene) reacts with excess Cl_2 under UV light to form benzene hexachloride (gammaxane).
(c)T (1-phenylphthalazine) is a heterocyclic compound.
(d)Acid-catalyzed cyclization of R yields anthraquinone, which on Clemmensen reduction ($Zn-Hg/HCl$) gives anthracene, not 9,10-dihydroxyanthracene.

41. (9.8)
Let the molar mass of He be M. Then the molar mass of Ar is 10M.
In the first cylinder, the total number of moles is given by:
$$n_1=\frac{m_1}{M}+\frac{m_2}{10M}=\frac{10m_1+m_2}{10M}$$
In the second cylinder, the total number of moles is given by:
$$n_2=\frac{m_2}{M}+\frac{m_1}{10M}=\frac{10m_2+m_1}{10M}$$
Using the ideal gas equation $PV=nRT$, since the volume and temperature are the same for both cylinders, the pressure is directly proportional to the number of moles. Thus, $\frac{P_1}{P_2}=\frac{n_1}{n_2}$.
Given that $P_1=5P_2$, we have:
$$\frac{n_1}{n_2}=5$$
Substituting the expressions for n_1 and n_2 :
$$\frac{10m_1+m_2}{10m_2+m_1}=5$$
$$10m_1+m_2=50m_2+5m_1$$
$$5m_1=49m_2$$
$$\frac{m_1}{m_2}=\frac{49}{5}=9.8$$

42. (8)
The given complex is $K[M(NCS)(NO_2)(gly)]$. The complex anion is $[M(NCS)(NO_2)(gly)]^-$.
The ligands present in the complex are:
(1)gly $^-$(glycinate): An unsymmetrical bidentate ligand coordinating through nitrogen(N) and oxygen (O). Let this be represented as (AB).
(2)NCS^-: An ambidentate monodentate ligand that can coordinate through nitrogen(-NCS) or sulfur (-SCN).
(3)NO_2^-: An ambidentate monodentate ligand that can coordinate through nitrogen(- NO_2) or oxygen (- ONO).
First, we find the number of linkage isomers due to the ambidentate ligands:
NCS^-has 2 linkage modes (N or S).
NO_2^-has 2 linkage modes (N or O).
Number of linkage combinations $=2\times2=4$.

For each linkage combination, the complex is of the type [M(a)(b)(AB)], where a and b are monodentate ligands and AB is an unsymmetrical bidentate ligand.
In a square planar geometry, the bidentate ligand (AB) must occupy adjacent (cis) positions. The monodentate ligands a and b will occupy the remaining two positions.
This gives rise to 2 geometrical isomers for each linkage combination:
(1)Ligand a is trans to A (and b is trans to B).
(2)Ligand a is trans to B (and b is trans to A).
Since square planar complexes of this type possess a molecular plane of symmetry, they do not exhibit optical isomerism.
Total number of isomers = (Number of linkage combinations) × (Number of geometrical isomers per combination) Total isomers = $4 \times 2 = 8$.

43. (4)

Reaction (1):
The reactant is 2,4-dimethyl-3-oxopentanedioic acid, which is a β-keto acid (both $-CO_2H$ groups are at the β-position with respect to the central ketone group). β-keto acids readily undergo decarboxylation upon heating. Since there are two such groups, it will undergo double decarboxylation.

$HO_2C - CH(CH_3) - C(=O) - CH(CH_3) - CO_2H \xrightarrow{\Delta} CH_3 - CH_2 - C(=O) - CH_2 - CH_3 + 2CO_2$

The major product X is 3-pentanone.
Number of carbonyl groups ($> C = O$) in X = 1.
Reaction (2):
The reactant is *cis*-3,4-dicarboxycyclopentanone. The two carboxylic acid groups are on adjacent carbons (a 1,2-dicarboxylic acid system). Heating a 1,2-dicarboxylic acid leads to dehydration, forming a stable 5-membered cyclic anhydride.
The major product **Y** is the cyclic anhydride of the reactant, which is a fused bicyclic system containing the original cyclopentanone ring and a newly formed succinic anhydride ring.
The structure of Y contains one ketone carbonyl group and two anhydride carbonyl groups.
Number of carbonyl groups ($> C = O$) in Y = $1 + 2 = 3$.
Sum of total number of carbonyl groups in X and Y = $1 + 3 = 4$.

44. (6)

Buta-1,3-diyne ($HC \equiv C - C \equiv CH$) reacts with 2 equivalents of $NaNH_2$ to undergo double deprotonation, forming the acetylide dianion:

$HC \equiv C - C \equiv CH + 2NaNH_2 \rightarrow {}^{\ominus}C \equiv C - C \equiv C^{\ominus} + 2NH_3$

This dianion acts as a strong nucleophile and undergoes an S_N2 reaction with the primary allylic halide, trans-$CH_3 - CH = CH - CH_2 - Br$ (excess ensures both sides react):

${}^{\ominus}C \equiv C - C \equiv C^{\ominus} + 2CH_3 - CH = CH - CH_2 - Br \rightarrow CH_3 - CH = CH - CH_2 - C \equiv C - C \equiv C - CH_2 - CH = CH - CH_3$

In the major product X, the central diyne unit consists of four sp hybridized carbon atoms. The geometry around sp hybridized carbons is linear (bond angle 180°).
The two sp^3 hybridized $-CH_2-$ carbons attached directly to the sp carbons also lie on this same linear axis.
Thus, the sequence of collinear carbon atoms is:
$-CH_2 - C \equiv C - C \equiv C - CH_2 -$
The carbon atoms beyond the $-CH_2-$ groups are attached at a tetrahedral angle ($\sim 109.5°$) and are not collinear with the central axis.
Therefore, the maximum number of collinear carbon atoms in X is 6 .

45. (c) (P) Physisorption is an exothermic process because attractive forces are formed between the adsorbate and the adsorbent, so $\Delta H < 0$. The gas molecules are restricted on the solid surface, decreasing randomness, so $\Delta S < 0$. This matches (2).
(Q) Diamond ⟶ Graphite is an exothermic process because graphite is the thermodynamically more stable allotrope of carbon at standard conditions ($\Delta H < 0$). Graphite has a layered structure and is less dense, so it has higher entropy than the rigid 3D network of diamond ($\Delta S > 0$). This matches (5).
(R) Denaturation of protein involves the breaking of hydrogen bonds and the unfolding of its specific 3D structure into a more random coil. Breaking these interactions requires energy ($\Delta H > 0$) and the unfolding increases

randomness ($\Delta S > 0$). This matches (1).

(S) Propene $\longrightarrow$ Cyclopropane is an endothermic process because cyclopropane has significant ring strain compared to the open-chain propene, making it less stable ($\Delta H > 0$). The formation of a rigid ring restricts the degrees of freedom (such as rotation around single bonds), decreasing entropy ($\Delta S < 0$). This matches (4).

Thus, $P \rightarrow 2, Q \rightarrow 5, R \rightarrow 1, S \rightarrow 4$.

46. (a) Let us determine the hybridization and shape of each given species by finding the number of bond pairs (bp) and lone pairs (lp) on the central atom:

$SOCl_2$: Central atom S has 6 valence electrons. It forms 3σ bonds (one with O , two with Cl) and has 1 lone pair. Steric number $= 4(sp^3)$. Shape is Trigonal Pyramidal.

$XeOF_4$: Central atom Xe has 8 valence electrons. It forms 5σ bonds (one with O , four with F) and has 1 lone pair. Steric number $= 6(sp^3d^2)$. Shape is Square Pyramidal.

ClF_3 : Central atom Cl has 7 valence electrons. It forms 3σ bonds with F and has 2 lone pairs. Steric number $= 5(sp^3d)$. Shape is T-shaped.

ClF_5 : Central atom Cl has 7 valence electrons. It forms 5σ bonds with F and has 1 lone pair. Steric number $= 6(sp^3d^2)$. Shape is Square Pyramidal.

XeF_5^+: Central atom Xe has 8 valence electrons. The positive charge leaves 7 valence electrons. It forms 5σ bonds with F and has 1 lone pair. Steric number $= 6(sp^3d^2)$. Shape is Square Pyramidal.

SO_3^{2-} : Central atom S has 6 valence electrons. The -2 charge gives 8 valence electrons. It forms 3σ bonds with O and has 1 lone pair. Steric number $= 4(sp^3)$. Shape is Trigonal Pyramidal.

XeF_3^+: Central atom Xe has 8 valence electrons. The positive charge leaves 7 valence electrons. It forms 3σ bonds with F and has 2 lone pairs. Steric number $= 5(sp^3d)$. Shape is T-shaped.

SF_4 : Central atom S has 6 valence electrons. It forms 4σ bonds with F and has 1 lone pair. Steric number $= 5(sp^3d)$. Shape is See-saw.

(P)See-saw: $1(SF_4)$
(Q)T-Shaped: $2(ClF_3, XeF_3^+)$
(R)Trigonal Planar: 0 (None)
(S)Square Pyramidal: $3(XeOF_4, ClF_5, XeF_5^+)$
Matching with List-(II):
$P \rightarrow 1$
$Q \rightarrow 2$
$R \rightarrow 5$
$S \rightarrow 3$

47. (c) The given reaction sequence involves ozonolysis of the bicyclic alkenes followed by an intramolecular aldol condensation.

For compound (P), the structure is a 6,6 -fused system (an octalin derivative) with methyl groups on opposite sides of the double bond. Ozonolysis cleaves the central double bond to form a 10 -membered ring 1,6 -diketone with methyl groups at the α positions (C2 and C7). The base-catalyzed aldol condensation proceeds via the more stable trisubstituted enolate at C2, which attacks the C6 carbonyl group. This cyclization forms a new 5-membered ring fused to a 7-membered ring (a 5,7-fused system). The fusion carbons are C2 (bearing a methyl group) and C6 (bearing a hydroxy1 group). The unreacted ketone is at C1, and the other methyl group is at C7. Both C1 and C7 are in the 7-membered ring. This matches the structure of product (2). Thus, $P \rightarrow 2$.

For compound (Q), the structure is a 7,5 -fused system with methyl groups on different rings. Ozonolysis yields a 10 -membered ring 1,7-diketone with methyl groups at C2 and C10. The more stable enolate at C2 attacks the C7 carbonyl group. This forms a 6 -membered ring fused to another 6 -membered ring (a 6,6 -fused system). The fusion carbons are C 2 (with a methyl group) and C7 (with a hydroxyl group). The unreacted ketone at C1 and the remaining methyl at C10 are both located in the same 6membered ring. This corresponds to the structure of product (1). Thus, $Q \rightarrow 1$.

For compound (R), the structure is a 7,5 -fused system with both methyl groups on the 7 -membered ring. Ozonolysis produces a 10 -membered ring 1,7-diketone with methyl groups at C2 and C6. The enolate at C2 attacks the C7 carbonyl, forming a 6,6fused system. The fusion carbons are C2 (methyl) and C7 (hydroxyl). The

unreacted ketone is at C1 (in one ring) and the remaining methyl is at C 6 (in the other ring). This corresponds to product (5). Thus, R → 5.

For compound (S), the structure is a 6,6-fused system with both methyl groups on the same side. Ozonolysis forms a 10membered ring 1,6-diketone with methyl groups at C2 and C5. The enolate at C2 attacks the C6 carbonyl, yielding a 5,7-fused system. The fusion carbons are C2 (methyl) and C6 (hydroxyl). The unreacted ketone is at C1 (in the 7-membered ring) and the remaining methyl is at C 5 (in the 5 -membered ring). This corresponds to product (3). Thus, S → 3.

48. (b) For the reaction in (P), 2-bromo-5-nitrobenzaldehyde oxime reacts with aqueous NaOH . The strong base deprotonates the oxime group. The resulting oximate anion undergoes intramolecular nucleophilic aromatic substitution (S_NAr), attacking the ortho-carbon and displacing the bromide ion to form a 1,2-benzisoxazole intermediate. Since it is derived from an aldoxime, this intermediate has a hydrogen atom at the C 3 position. In the presence of NaOH , it undergoes base-catalyzed Kemp elimination (ring opening) to form 2-hydroxy-5-nitrobenzonitrile, which corresponds to structure (1).Thus, P → 1.

In reaction (Q), the aldoxime is treated with acetic anhydride, which acetylates the oxime group to form an O-acetate. Subsequent treatment with the mild base Na_2CO_3 causes the elimination of acetic acid, converting the aldoxime acetate into a nitrile. The bromide is not displaced under these mild conditions. The product is 2-bromo-5-nitrobenzonitrile, which corresponds to structure (2).Thus, Q → 2.

For the reaction in (R), 2-bromo-5-nitroacetophenone oxime reacts with aqueous NaOH . Similar to (P), the oximate anion is formed and displaces the ortho-bromide via S_NAr to form a 1,2-benzisoxazole ring. However, because it is derived from a ketoxime, there is a methyl group at the C3 position instead of a hydrogen. Therefore, Kemp elimination cannot occur, and the reaction stops at the stable 3-methyl-5-nitro-1,2-benzisoxazole, which corresponds to structure (4).Thus, R → 4.

In reaction (S), the O-acetate of the ketoxime is treated with aqueous Na_2CO_3. Since it is a ketoxime derivative, it cannot undergo elimination to form a nitrile. The mild base simply hydrolyzes the acetate group back to the oxime. The base is not strong enough to promote the subsequent S_NAr ring closure. The product is 2-bromo-5-nitroacetophenone oxime, which corresponds to structure (5).Thus, S → 5.

Combining these results gives the sequence P → 1, Q → 2, R → 4, S → 5.