1. **(c)** Given $|\vec{a}+\vec{b}|=\sqrt{21}$ and $|\vec{a}-\vec{b}|=3$.
Squaring both equations:
$|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}=21$
$|\vec{a}|^2+|\vec{b}|^2-2\vec{a}\cdot\vec{b}=9$
Subtracting the second equation from the first gives:
$4\vec{a}\cdot\vec{b}=12\Rightarrow\vec{a}\cdot\vec{b}=3$
Adding the two equations gives:
$2\left(|\vec{a}|^2+|\vec{b}|^2\right)=30\Rightarrow|\vec{a}|^2+|\vec{b}|^2=15$
Since $\vec{a}$ and $(\vec{a}-\vec{b})$ are perpendicular, their dot product is zero:
$\vec{a}\cdot(\vec{a}-\vec{b})=0\Rightarrow|\vec{a}|^2-\vec{a}\cdot\vec{b}=0\Rightarrow|\vec{a}|^2=\vec{a}\cdot\vec{b}=3$
Substituting $|\vec{a}|^2=3$ into $|\vec{a}|^2+|\vec{b}|^2=15$ gives:
$3+|\vec{b}|^2=15\Rightarrow|\vec{b}|^2=12$
Using Lagrange's identity to find $|\vec{a}\times\vec{b}|$:
$|\vec{a}\times\vec{b}|^2=|\vec{a}|^2|\vec{b}|^2-(\vec{a}\cdot\vec{b})^2$
The position vectors of P and R are $\vec{a}$ and $\vec{a}+\vec{b}$ respectively. The area of triangle OPR is:
$\frac{1}{2}|\overrightarrow{OP}\times\overrightarrow{OR}|=\frac{1}{2}|\vec{a}\times(\vec{a}+\vec{b})|=\frac{1}{2}|\vec{a}\times\vec{a}+\vec{a}\times\vec{b}|=\frac{1}{2}|\vec{a}\times\vec{b}|$
Substituting the value of $|\vec{a}\times\vec{b}|$:
Area $=\frac{3\sqrt{3}}{2}$

2. **(c)** The equation of the parabola is $y^2=16x$. Comparing with $y^2=4ax$, we get $a=4$.
Let the parametric coordinates of the point $(64,32)$ be $(at_1^2,2at_1)$.
$2at_1=32\Rightarrow 8t_1=32\Rightarrow t_1=4$
The slope of the tangent T at t_1 is $m_1=\frac{1}{t_1}=\frac{1}{4}$.
Let the tangent L be at the point (x_1,y_1) with parameter t_2. Its slope is $m_2=\frac{1}{t_2}$.
Since T and L are perpendicular, $m_1m_2=-1$.
$\frac{1}{4}\times\frac{1}{t_2}=-1\Rightarrow t_2=-\frac{1}{4}$
The x-coordinate of the point (x_1,y_1) is $x_1=at_2^2=4\left(-\frac{1}{4}\right)^2=\frac{1}{4}$.
The distance of a point (x_1,y_1) on the parabola from the focus is given by its focal distance x_1+a.
Distance $=\frac{1}{4}+4=\frac{17}{4}$

3. **(b)** The given differential equation is
$$\frac{dy}{dx}=\frac{e^{5x}y^3+y^3}{e^x+e^xy^4}$$
Factoring the numerator and denominator, we get
$$\frac{dy}{dx}=\frac{y^3(e^{5x}+1)}{e^x(1+y^4)}$$
Separating the variables x and y, we obtain
$$\frac{1+y^4}{y^3}dy=\frac{e^{5x}+1}{e^x}dx$$
$$\left(\frac{1}{y^3}+y\right)dy=(e^{4x}+e^{-x})dx$$
Integrating both sides yields
$$\int(y^{-3}+y)dy=\int(e^{4x}+e^{-x})dx$$
$$-\frac{1}{2y^2}+\frac{y^2}{2}=\frac{e^{4x}}{4}-e^{-x}+C$$
Multiplying by 2, we get

$$y^2 - \frac{1}{y^2} = \frac{e^{4x}}{2} - 2e^{-x} + 2C$$

Using the initial condition $y(0) = \frac{1}{\sqrt{2}}$, we substitute $x = 0$ and $y = \frac{1}{\sqrt{2}}$:

$$\left(\frac{1}{\sqrt{2}}\right)^2 - \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2} = \frac{e^0}{2} - 2e^0 + 2C$$

$$\frac{1}{2} - 2 = \frac{1}{2} - 2 + 2C$$

This gives $2C = 0$, so $C = 0$. The equation becomes

$$y^2 - \frac{1}{y^2} = \frac{e^{4x}}{2} - 2e^{-x}$$

To find $y(\log_e 2)$, we substitute $x = \log_e 2$ into the equation. We have $e^{4x} = e^{4\log_e 2} = 16$ and $e^{-x} = e^{-\log_e 2} = \frac{1}{2}$.

$$y^2 - \frac{1}{y^2} = \frac{16}{2} - 2\left(\frac{1}{2}\right)$$

$$y^2 - \frac{1}{y^2} = 8 - 1 = 7$$

Let $y^2 = t$. Since $y \in (0, \infty), t > 0$.

$$t - \frac{1}{t} = 7$$

$$t^2 - 7t - 1 = 0$$

Solving for t using the quadratic formula gives

$$t = \frac{7 \pm \sqrt{49 - 4(1)(-1)}}{2} = \frac{7 \pm \sqrt{53}}{2}$$

Since $t = y^2 > 0$, we must choose the positive root:

$$y^2 = \frac{7 + \sqrt{53}}{2}$$

Taking the positive square root since $y > 0$, we get

$$y = \sqrt{\frac{7 + \sqrt{53}}{2}}$$

4. **(b)** Let $I = \int_0^2 \frac{1}{3^x + 3} dx$

Substitute $t = 3^x$, which gives $dt = 3^x \ln 3 dx \Rightarrow dx = \frac{dt}{t \ln 3}$.

When $x = 0, t = 1$.

When $x = 2, t = 9$.

The integral becomes:

$$I = \int_1^9 \frac{1}{t+3} \frac{dt}{t \ln 3} = \frac{1}{\ln 3} \int_1^9 \frac{1}{t(t+3)} dt$$

Using partial fractions:

$$\frac{1}{t(t+3)} = \frac{1}{3}\left(\frac{1}{t} - \frac{1}{t+3}\right)$$

$$I = \frac{1}{3\ln 3} \int_1^9 \left(\frac{1}{t} - \frac{1}{t+3}\right) dt$$

$$I = \frac{1}{3\ln 3} [\ln t - \ln(t+3)]_1^9$$

$$I = \frac{1}{3\ln 3} \left[\ln\left(\frac{t}{t+3}\right)\right]_1^9$$

$$I = \frac{1}{3\ln 3} \left(\ln\left(\frac{9}{12}\right) - \ln\left(\frac{1}{4}\right)\right)$$

$$I = \frac{1}{3\ln 3} \left(\ln\left(\frac{3}{4}\right) - \ln\left(\frac{1}{4}\right)\right)$$

$$I=\frac{1}{3\ln 3}\ln\left(\frac{3/4}{1/4}\right)=\frac{1}{3\ln 3}\ln 3$$
$$I=\frac{1}{3}$$

5. **(a,b,c)** Given $f(x)=\frac{d^{10}}{dx^{10}}((x^2-1)^{10})$.

Using the binomial expansion, we have:

$$(x^2-1)^{10}=\sum_{k=0}^{10}(-1)^{k10}C_k x^{20-2k}$$

Differentiating 10 times with respect to x, we get:

$$f(x)=\sum_{k=0}^{5}(-1)^{k10}C_k\frac{(20-2k)!}{(10-2k)!}x^{10-2k}$$

For the coefficient of x^8, we set $10-2k=8 \Rightarrow k=1$.

The coefficient is $(-1)^{1\,10}C_1\frac{18!}{8!}=-10\left(\frac{18!}{8!}\right)$. Thus, statement (a) is true.

The highest power of x in f(x) corresponds to $k=0$, which gives x^{10} with a non-zero coefficient of $\frac{20!}{10!}$.

Therefore, the degree of the polynomial f(x) is 10 . Thus, statement (c) is true.

For the constant term, we set $10-2k=0 \Rightarrow k=5$.

The constant term is $(-1)^{5\,10}C_5\frac{10!}{0!}=-\frac{10!}{5!5!}10!=-\left(\frac{10!}{5!}\right)^2$.

Thus, statement (d) is false.

To find f(1) and f(−1), we use the Leibniz rule for the n-th derivative of a product:

$$f(x)=\frac{d^{10}}{dx^{10}}((x-1)^{10}(x+1)^{10})=\sum_{r=0}^{10}{}^{10}C_r\frac{d^{10-r}}{dx^{10-r}}((x-1)^{10})\frac{d^r}{dx^r}((x+1)^{10})$$

Evaluating at $x=1$, all terms in the sum are zero except when $r=0$ (since $\frac{d^{10-r}}{dx^{10-r}}((x-1)^{10})$ contains a factor of $(x-1)$ for $r>0$).

$f(1)={}^{10}C_0(10!)(1+1)^{10}=10!\,2^{10}$

Evaluating at $x=-1$, all terms are zero except when $r=10$.

$f(-1)={}^{10}C_{10}(-1-1)^{10}(10!)=10!\,2^{10}$

Adding these values gives:

$f(1)+f(-1)=10!\,2^{10}+10!\,2^{10}=10!\,2^{11}$

Thus, statement (b) is true.

6. **(a,b,c)** Given a,b,c are in arithmetic progression, we have $2b=a+c$.

Let the integer roots of the equation $ax^2+bx+c=0$ be α and β.

Sum of the roots is $\alpha+\beta=-\frac{b}{a}$ and product of the roots is $\alpha\beta=\frac{c}{a}$.

Dividing the arithmetic progression condition by a, we get:

$$\frac{2b}{a}=1+\frac{c}{a}$$

Substituting the sum and product of the roots:

$-2(\alpha+\beta)=1+\alpha\beta$

$\alpha\beta+2\alpha+2\beta+1=0$

Adding 3 to both sides to factorize:

$\alpha\beta+2\alpha+2\beta+4=3$

$(\alpha+2)(\beta+2)=3$

Since α and β are integers, $(\alpha+2)$ and $(\beta+2)$ must be integer factors of 3 . The possible pairs are $(1,3),(3,1),(-1,-3)$, and $(-3,-1)$.

If $(\alpha+2,\beta+2)=(1,3)$ or $(3,1)$, the roots are -1 and 1 . The sum of the roots is 0 , which implies $b=0$. This is rejected because b is a positive integer.

If $(\alpha+2,\beta+2)=(-1,-3)$ or $(-3,-1)$, the roots are -3 and -5 . The sum of the roots is -8 , which implies $-\frac{b}{a}=-8 \Rightarrow b=8a$.

The product of the roots is 15 , which implies $\frac{c}{a}=15 \Rightarrow c=15a$.
Since a is a positive integer, $b=8a$ and $c=15a$ are also positive integers. The roots of the equation are always -3 and -5 .
Evaluating the given statements:
$c-b=15a-8a=7a$, which is an integer multiple of a.
The roots are -3 and -5 , both of which are odd integers.
If $c=15$, then $15a=15 \Rightarrow a=1$. Consequently, $b=8(1)=8$. Thus, $ab=1\times 8=8$.
If $b=8$, then $8a=8 \Rightarrow a=1$. The roots are -3 and -5 , so $x=3$ is not a root.

7. **(a,d)** The direction ratios of line L passing through $P(1,2,-1)$ and $Q(2,3,1)$ are $(2-1,3-2,1-(-1))=(1,1,2)$.
The equation of line L is $\frac{x-1}{1}=\frac{y-2}{1}=\frac{z+1}{2}=\lambda$.
Any point on L can be written as $S(\lambda+1,\lambda+2,2\lambda-1)$.
Since S is the foot of the perpendicular from $R(4,-1,5)$ to L, the direction ratios of RS are $(\lambda-3,\lambda+3,2\lambda-6)$.
As $RS \perp L$, the dot product of their direction ratios is zero:
$1(\lambda-3)+1(\lambda+3)+2(2\lambda-6)=06\lambda-12=0 \Rightarrow \lambda=2$
Thus, the coordinates of S are (3,4,3).
The length of PS is $\sqrt{(3-1)^2+(4-2)^2+(3-(-1))^2}=\sqrt{4+4+16}=2\sqrt{6}$.
The length of the altitude RS is $\sqrt{(3-4)^2+(4-(-1))^2+(3-5)^2}=\sqrt{1+25+4}=\sqrt{30}$.
Since S divides PT internally in the ratio 1: 2, we have:
$$S=\frac{T+2P}{3} \Rightarrow T=3S-2P$$
$T=3(3,4,3)-2(1,2,-1)=(7,8,11)$
The length of the base PT is $3|PS|=6\sqrt{6}$.
The area of $\triangle$ PRT is $\frac{1}{2}\times|PT|\times|RS|=\frac{1}{2}\times 6\sqrt{6}\times\sqrt{30}=18\sqrt{5}$.
The orthocentre H lies on the altitude RS. The line RS passes through S(3,4,3) and has direction ratios proportional to $R-S=(1,-5,2)$.
Let the coordinates of H be $(3+t,4-5t,3+2t)$.
Since H is the orthocentre, $PH \perp RT$.
The direction ratios of RT are $(7-4,8-(-1),11-5)=(3,9,6)$, which is proportional to (1,3,2).
The direction ratios of PH are $(3+t-1,4-5t-2,3+2t-(-1))=(t+2,2-5t,2t+4)$.
Taking the dot product of PH and RT :
$1(t+2)+3(2-5t)+2(2t+4)=0$
$t+2+6-15t+4t+8=0$
$$16-10t=0 \Rightarrow t=\frac{8}{5}$$
Substituting $t=\frac{8}{5}$ into the coordinates of H, we get:
$$H=\left(3+\frac{8}{5},4-5\left(\frac{8}{5}\right),3+2\left(\frac{8}{5}\right)\right)=\left(\frac{23}{5},-4,\frac{31}{5}\right)$$

8. **(b,d)** The given differential equation is $x\frac{dy}{dx}=y-x^3$.
Dividing by x, we get:
$$\frac{dy}{dx}-\frac{1}{x}y=-x^2$$
This is a linear differential equation of the form $\frac{dy}{dx}+P(x)y=Q(x)$, where $P(x)=-\frac{1}{x}$ and $Q(x)=-x^2$.
$$\text{Integrating Factor (IF)}=e^{\int -\frac{1}{x}dx}=e^{-\ln x}=\frac{1}{x}$$
Multiplying the differential equation by the IF:
$$\frac{d}{dx}\left(y\cdot\frac{1}{x}\right)=-x$$
Integrating both sides with respect to x :
$$\frac{y}{x}=-\frac{x^2}{2}+C$$

$y = -\frac{x^3}{2} + Cx$

Given $y(1) = 0$:

$0 = -\frac{1}{2} + C \Rightarrow C = \frac{1}{2}$

Thus, $f(x) = \frac{x}{2} - \frac{x^3}{2}$.

To find local extrema, we find $f'(x)$:

$f'(x) = \frac{1}{2} - \frac{3x^2}{2}$

Setting $f'(x) = 0$ gives $3x^2 = 1 \Rightarrow x = \frac{1}{\sqrt{3}}$ (since $x > 0$).

Now, $f''(x) = -3x$. At $x = \frac{1}{\sqrt{3}}, f''\left(\frac{1}{\sqrt{3}}\right) = -\sqrt{3} < 0$.

So, $f(x)$ has a local maximum at $x = \frac{1}{\sqrt{3}}$.

For $x \in (1,2), x^2 > 1$, so $f'(x) = \frac{1-3x^2}{2} < 0$. Thus, $f(x)$ is decreasing in $(1,2)$.

For the intersection of $f(x)$ and $g(x)$:

$\frac{x}{2} - \frac{x^3}{2} = 4x^3 - 5x^2 + \frac{3}{2}x$

$x - x^3 = 8x^3 - 10x^2 + 3x$

$9x^3 - 10x^2 + 2x = 0$

$x(9x^2 - 10x + 2) = 0$

Since $x \in (0, \infty), x \neq 0$. We solve $9x^2 - 10x + 2 = 0$.

Discriminant $\Delta = (-10)^2 - 4(9)(2) = 100 - 72 = 28 > 0$.

Sum of roots $= \frac{10}{9} > 0$ and Product of roots $= \frac{2}{9} > 0$.

Both roots are real and positive. Therefore, there are exactly 2 elements in the set.

9. **(b,d)** We are given the matrices $S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

First, we compute the product ST :

$ST = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$

Comparing this with $ST = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, we get $a = 0, b = -1, c = 1$, and $d = 1$.

Evaluating Option (a):

$\frac{b + ia}{d + ic} = \frac{-1 + i(0)}{1 + i(1)} = \frac{-1}{1 + i} = \frac{-1(1 - i)}{(1 + i)(1 - i)} = \frac{-1 + i}{2}$

This is not equal to i, so statement (a) is FALSE.

Evaluating Option (b):

Given $\omega = \frac{-1+i\sqrt{3}}{2}$, which is a complex cube root of unity, satisfying $\omega^2 + \omega + 1 = 0$ and $\omega^3 1$.

$\frac{a\omega + b}{c\omega + d} = \frac{0 \cdot \omega - 1}{1 \cdot \omega + 1} = \frac{-1}{\omega + 1}$

Since $\omega + 1 = -\omega^2$, we have:

$\frac{-1}{-\omega^2} = \frac{1}{\omega^2} = \frac{\omega^3}{\omega^2} = \omega$

Thus, statement (b) is TRUE.

Evaluating Option (c):

The characteristic equation of ST is $\lambda^2 - \mathrm{Tr}(ST)\lambda + \det(ST) = 0$, which gives $\lambda^2 - \lambda + 1 = 0$.

By the Cayley-Hamilton theorem, $(ST)^2 - ST + I = 0$.

Multiplying by $(ST + I)$, we get $(ST)^3 + I = 0 \Rightarrow (ST)^3 = -I$.

Squaring both sides gives $(ST)^6 = I$.

We are given $(ST)^2 = (ST)^m$, which implies $(ST)^{m-2} = I$.

This means $m - 2$ must be a multiple of 6 , so $m = 6k + 2$ for some integer k.

For $k = 1, m = 8$ (which is a multiple of 8).

For $k = 2, m = 14$ (which is NOT a multiple of 8).

Thus, statement (c) is FALSE.

Evaluating Option (d):
For $z \in H$, let $z = x + iy$ where $y > 0$.
$$\frac{az+b}{cz+d} = \frac{-1}{z+1} = \frac{-1}{(x+1)+iy}$$
Multiplying the numerator and denominator by the conjugate $(x+1) - iy$:
$$\frac{-1}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy} = \frac{-(x+1)+iy}{(x+1)^2+y^2}$$
The imaginary part of this new complex number is $\frac{y}{(x+1)^2+y^2}$.
Since $y > 0$ and $(x+1)^2 + y^2 > 0$, the imaginary part is strictly positive.
Therefore, $\frac{az+b}{cz+d} \in H$, making statement (d) TRUE.

10. (1860)

The condition $g(f(x)) = 2^x$ for all $x \in A$ implies that if $f(x_1) = f(x_2)$, then $g(f(x_1)) = g(f(x_2))$, which gives $2^{x_1} = 2^{x_2} \Rightarrow x_1 = x_2$
Thus, f must be an injective (one-to-one) function. Conversely, if f is injective, such a function g can always be constructed. Therefore, T is the set of all injective functions $f: A \to B$ such that $f(2) \neq 2$ and $f(4) \neq 4$.
The total number of injective functions from A to B is ${}^7P_5 = 7 \times 6 \times 5 \times 4 \times 3 = 2520$.
Let E_2 be the set of injective functions where $f(2) = 2$. The number of such functions is ${}^6P_4 = 6 \times 5 \times 4 \times 3 = 360$.
Let E_4 be the set of injective functions where $f(4) = 4$. The number of such functions is ${}^6P_4 = 360$.
Let $E_2 \cap E_4$ be the set of injective functions where both $f(2) = 2$ and $f(4) = 4$. The number of such functions is ${}^5P_3 = 5 \times 4 \times 3 = 60$.
Using the Principle of Inclusion-Exclusion, the number of injective functions where $f(2) = 2$ or $f(4) = 4$ is:
$|E_2 \cup E_4| = |E_2| + |E_4| - |E_2 \cap E_4| = 360 + 360 - 60 = 660$
The number of elements in the set T is the number of injective functions where $f(2) \neq 2$ and $f(4) \neq 4$, which is: $|T| = 2520 - 660 = 1860$

11. (100)

Total number of ways to choose 6 books from 11 books is ${}^{11}C_6 = 462$.
Let M and P be the number of Mathematics and Physics books chosen respectively. Since 6 books are chosen, $M + P = 6$.
The random variable X is given by $X = |M - P| = |2M - 6|$.
The possible selections of (M, P), the corresponding values of X, and the number of ways are as follows:
For $M = 1, P = 5: X = |1-5| = 4$, Number of ways $= {}^6C_1 \times {}^5C_5 = 6 \times 1 = 6$
For $M = 2, P = 4: X = |2-4| = 2$, Number of ways $= {}^6C_2 \times {}^5C_4 = 15 \times 5 = 75$
For $M = 3, P = 3: X = |3-3| = 0$, Number of ways $= {}^6C_3 \times {}^5C_3 = 20 \times 10 = 200$
For $M = 4, P = 2: X = |4-2| = 2$, Number of ways $= {}^6C_4 \times {}^5C_2 = 15 \times 10 = 150$
For $M = 5, P = 1: X = |5-1| = 4$, Number of ways $= {}^6C_5 \times {}^5C_1 = 6 \times 5 = 30$
For $M = 6, P = 0: X = |6-0| = 6$, Number of ways $= {}^6C_6 \times {}^5C_0 = 1 \times 1 = 1$
The mean α of the random variable X is given by:
$$\alpha = \frac{\sum X_i f_i}{\sum f_i}$$
$$\alpha = \frac{4(6) + 2(75) + 0(200) + 2(150) + 4(30) + 6(1)}{462}$$
$$\alpha = \frac{24 + 150 + 0 + 300 + 120 + 6}{462}$$
$$\alpha = \frac{600}{462} = \frac{100}{77}$$
Therefore, the value of 77α is:
$$77\alpha = 77 \times \frac{100}{77} = 100$$

12. (44)

Given $\frac{\sum_{i=1}^{10} x_i}{10} = 5 \Rightarrow \sum_{i=1}^{10} x_i = 50$

$$\frac{\sum_{i=1}^{10} x_i^2}{10} - 5^2 = 7 \Rightarrow \sum_{i=1}^{10} x_i^2 = 320$$

For the first 8 observations:

$$\frac{\sum_{i=1}^{8} x_i}{8} = 4 \Rightarrow \sum_{i=1}^{8} x_i = 32$$

$$\frac{\sum_{i=1}^{8} x_i^2}{8} - 4^2 = 3.5 \Rightarrow \sum_{i=1}^{8} x_i^2 = 8(3.5+16) = 156$$

Subtracting the sums, we get:

$x_9 + x_{10} = 50 - 32 = 18$

$x_9^2 + x_{10}^2 = 320 - 156 = 164$

Using $(x_9 + x_{10})^2 = x_9^2 + x_{10}^2 + 2x_9x_{10}$:

$18^2 = 164 + 2x_9x_{10} \Rightarrow 2x_9x_{10} = 324 - 164 = 160 \Rightarrow x_9x_{10} = 80$

The numbers x_9 and x_{10} are roots of the equation $t^2 - 18t + 80 = 0$.

$(t-8)(t-10) = 0 \Rightarrow t = 8,10$

Since $x_9 < x_{10}$, we have $x_9 = 8$ and $x_{10} = 10$.

Therefore, $3x_9 + 2x_{10} = 3(8) + 2(10) = 24 + 20 = 44$.

13. (18)

For the ellipse E: $\frac{x^2}{18} + \frac{y^2}{12} = 1$, we have $a_E^2 = 18$ and $b_E^2 = 12$.

The eccentricity e_E is given by:

$$e_E = \sqrt{1 - \frac{b_E^2}{a_E^2}} = \sqrt{1 - \frac{12}{18}} = \frac{1}{\sqrt{3}}$$

The foci of the ellipse E are $(\pm a_E e_E, 0) = \left(\pm\sqrt{18} \times \frac{1}{\sqrt{3}}, 0\right) = (\pm\sqrt{6}, 0)$.

For the hyperbola H, the eccentricity e_H is the reciprocal of e_E, so $e_H = \sqrt{3}$.

Since H has the same foci as E, its foci are $(\pm\sqrt{6}, 0)$.

Let the equation of H be $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$.

The foci are $(\pm Ae_H, 0)$, which gives $A\sqrt{3} = \sqrt{6} \Rightarrow A = \sqrt{2} \Rightarrow A^2 = 2$.

Also, $B^2 = A^2(e_H^2 - 1) = 2(3-1) = 4$.

Thus, the equation of the hyperbola H is $\frac{x^2}{2} - \frac{y^2}{4} = 1$.

To find the points of intersection of H and the parabola $\sqrt{5}y = x^2$, we substitute $x^2 = \sqrt{5}y$ into the equation of H :

$$\frac{\sqrt{5}y}{2} - \frac{y^2}{4} = 1$$

Multiplying by 4 , we get:

$2\sqrt{5}y - y^2 = 4 \Rightarrow y^2 - 2\sqrt{5}y + 4 = 0$

Solving for y using the quadratic formula:

$$y = \frac{2\sqrt{5} \pm \sqrt{20-16}}{2} = \sqrt{5} \pm 1$$

Let $y_1 = \sqrt{5} - 1$ and $y_2 = \sqrt{5} + 1$. Both are positive.

The corresponding x^2 values are:

$x_1^2 = \sqrt{5}(\sqrt{5} - 1) = 5 - \sqrt{5}$

$x_2^2 = \sqrt{5}(\sqrt{5} + 1) = 5 + \sqrt{5}$

Since the points P and Q lie in the first quadrant, $x > 0$. Thus, $x_1 = \sqrt{5 - \sqrt{5}}$ and $x_2 = \sqrt{5 + \sqrt{5}}$.

The square of the distance d between P and Q is:

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

First, we evaluate $(x_2 - x_1)^2$:

$(x_2 - x_1)^2 = x_2^2 + x_1^2 - 2x_1x_2$

$x_2^2 + x_1^2 = (5 + \sqrt{5}) + (5 - \sqrt{5}) = 10$

$x_1x_2 = \sqrt{(5-\sqrt{5})(5+\sqrt{5})} = \sqrt{25-5} = \sqrt{20} = 2\sqrt{5}$

So, $(x_2 - x_1)^2 = 10 - 4\sqrt{5}$.

Next, we evaluate $(y_2 - y_1)^2$:

$(y_2 - y_1)^2 = ((\sqrt{5}+1) - (\sqrt{5}-1))^2 = 2^2 = 4$

Adding these together gives d^2 :

$d^2 = 10 - 4\sqrt{5} + 4 = 14 - 4\sqrt{5}$

Comparing this with $d^2 = a + b\sqrt{5}$, we get $a = 14$ and $b = -4$.

Therefore, $a - b = 14 - (-4) = 18$.

14. (56)

For the function $f(x) = [x^3]\log_e(1+\sin^2(\pi(x-[x])))$, we can simplify the argument of the sine function. Since $[x]$ is an integer, $\sin(\pi(x-[x])) = \sin(\pi x - \pi[x]) = \pm\sin(\pi x)$. Thus, $\sin^2(\pi(x-[x])) = \sin^2(\pi x)$.

The function $f(x)$ can be rewritten as $f(x) = [x^3]\log_e(1+\sin^2(\pi x))$.

The term $\log_e(1+\sin^2(\pi x))$ is continuous everywhere. The term $[x^3]$ has jump discontinuities where x^3 is an integer. For $x \in (-3,3), x^3 \in (-27,27)$. The integer values of x^3 in this interval are $k \in \{-26,-25,\ldots,26\}$, which gives 53 points of the form $x = k^{1/3}$.

At these points, $[x^3]$ is discontinuous. For $f(x)$ to be continuous at $x = k^{1/3}$, the continuous factor must be zero: $\log_e(1+\sin^2(\pi x)) = 0 \Longrightarrow \sin^2(\pi x) = 0 \Longrightarrow x$ is an integer.

The integers in $(-3,3)$ are $-2,-1,0,1,2$. Their cubes are $-8,-1,0,1,8$, which are 5 values among the 53 points.

At these 5 points, $f(x)$ is continuous. At the remaining $53 - 5 = 48$ points, $f(x)$ is discontinuous.

Thus, $|A| = 48$.

For the function $g(x) = x^3\sin^2(\pi\log_e(1+x-[x]))$, we can write $x - [x] = \{x\}$, which is the fractional part of x.

The function $g(x) = x^3\sin^2(\pi\log_e(1+\{x\}))$ is continuous everywhere except possibly at the integers, where $\{x\}$ is discontinuous.

The integers in $(-3,3)$ are $c \in \{-2,-1,0,1,2\}$.

Let's check the continuity at $x = c$:

Right-hand limit $(x \to c^+)$: $\{x\} \to 0$, so $g(x) \to c^3\sin^2(\pi\log_e 1) = 0$.

Left-hand limit $(x \to c^-)$: $\{x\} \to 1$, so $g(x) \to c^3\sin^2(\pi\log_e 2)$.

For $g(x)$ to be continuous at $x = c$, we must have $c^3\sin^2(\pi\log_e 2) = 0$. Since $\log_e 2 = 0.693, \pi\log_e 2$ is not an integer multiple of π, meaning $\sin^2(\pi\log_e 2) \neq 0$. Therefore, we must have $c^3 = 0 \Longrightarrow c = 0$.

So, $g(x)$ is continuous at $x = 0$ and discontinuous at $x \in \{-2,-1,1,2\}$.

Thus, $B = \{-2,-1,1,2\}$ and $|B| = 4$.

Since A contains no integers and B contains only integers, $A \cap B = \emptyset$, which means $|A \cap B| = 0$.

Finally, we calculate the required value:

$|A| + 2|B| - |A \cap B| = 48 + 2(4) - 0 = 48 + 8 = 56$.

15. (11)

Equating the equations of the curves C_1 and C_2 :

$e^{-x} = e^{-x}(\sin x + \cos x)$

Since $e^{-x} \neq 0$ for all real x, dividing both sides by e^{-x} gives:

$\sin x + \cos x = 1$

Multiplying both sides by $\frac{1}{\sqrt{2}}$:

$$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$$

$$\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

The general solution is given by:

$x + \frac{\pi}{4} = 2m\pi + \frac{\pi}{4}$ or $x + \frac{\pi}{4} = 2m\pi + \frac{3\pi}{4}$ for integer m

This simplifies to:

$$x = 2m\pi \text{ or } x = 2m\pi + \frac{\pi}{2}$$

Given the interval $x \in [0,10\pi]$, the valid values for x are:

For $x = 2m\pi$: $x = 0, 2\pi, 4\pi, 6\pi, 8\pi, 10\pi$ (6 solutions)

For $x = 2m\pi + \frac{\pi}{2}$: $x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}, \frac{17\pi}{2}$ (5 solutions)

The total number of points of intersection is $n = 6 + 5 = 11$.

16. (2.5)

To find the points of intersection of the curves C_1 and C_2, we equate their equations:

$e^{-x} = e^{-x}(\sin x + \cos x)$

Since $e^{-x} \neq 0$, we have:

$\sin x + \cos x = 1$

$\Rightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right) = 1 \Rightarrow \sin\left(x+\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

The general solution is $x + \frac{\pi}{4} = 2k\pi + \frac{\pi}{4}$ or $x + \frac{\pi}{4} = 2k\pi + \frac{3\pi}{4}$ for any integer k.

Thus, $x = 2k\pi$ or $x = 2k\pi + \frac{\pi}{2}$.

For $x \in [0,10\pi]$, the first four points of intersection are:

$\alpha_1 = 0$

$\alpha_2 = \frac{\pi}{2}$

$\alpha_3 = 2\pi$

$\alpha_4 = 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$

The area β enclosed between the curves from $x = \alpha_1$ to $x = \alpha_4$ is given by:

$$\beta = \int_0^{\frac{5\pi}{2}} |e^{-x}(\sin x + \cos x - 1)|dx$$

We analyze the sign of $\sin x + \cos x - 1$ in the intervals $\left(0,\frac{\pi}{2}\right), \left(\frac{\pi}{2},2\pi\right)$, and $\left(2\pi,\frac{5\pi}{2}\right)$:

For $x \in \left(0,\frac{\pi}{2}\right)$, $\sin x + \cos x > 1$

For $x \in \left(\frac{\pi}{2},2\pi\right)$, $\sin x + \cos x < 1$

For $x \in \left(2\pi,\frac{5\pi}{2}\right)$, $\sin x + \cos x > 1$

Let $I = \int e^{-x}(\sin x + \cos x - 1)dx$. Using integration by parts or standard formulas, we get:

$$\int e^{-x}\sin x\,dx = -\frac{e^{-x}}{2}(\sin x + \cos x)$$

$$\int e^{-x}\cos x\,dx = \frac{e^{-x}}{2}(\sin x - \cos x)$$

$$\Rightarrow \int e^{-x}(\sin x + \cos x)dx = -e^{-x}\cos x$$

Thus, $I = -e^{-x}\cos x + e^{-x} = e^{-x}(1 - \cos x)$.

Now, we evaluate the area β by splitting the integral:

$$\beta = \int_0^{\frac{\pi}{2}} e^{-x}(\sin x + \cos x - 1)dx - \int_{\frac{\pi}{2}}^{2\pi} e^{-x}(\sin x + \cos x - 1)dx + \int_{2\pi}^{\frac{5\pi}{2}} e^{-x}(\sin x + \cos x - 1)dx$$

$$\beta = [e^{-x}(1-\cos x)]_0^{\frac{\pi}{2}} - [e^{-x}(1-\cos x)]_{\frac{\pi}{2}}^{2\pi} + [e^{-x}(1-\cos x)]_{2\pi}^{\frac{5\pi}{2}}$$

$$\beta = \left(e^{-\frac{\pi}{2}} - 0\right) - \left(0 - e^{-\frac{\pi}{2}}\right) + \left(e^{-\frac{5\pi}{2}} - 0\right)$$

$$\beta = 2e^{-\frac{\pi}{2}} + e^{-\frac{5\pi}{2}}$$

We are required to find the value of $-\frac{1}{\pi}\log_e\left(\beta - 2e^{-\frac{\pi}{2}}\right)$:

$$\beta - 2e^{-\frac{\pi}{2}} = e^{-\frac{5\pi}{2}}$$

$$\Rightarrow -\frac{1}{\pi}\log_e\left(e^{-\frac{5\pi}{2}}\right) = -\frac{1}{\pi}\times\left(-\frac{5\pi}{2}\right) = \frac{5}{2} = 2.5$$

17. (7.5)

The given equations of the ellipses are $x^2 + 4y^2 = 1$ and $4x^2 + y^2 = 1$.

To find the point of intersection, we equate the two expressions:

$x^2 + 4y^2 = 4x^2 + y^2$

$3x^2 = 3y^2 \Rightarrow x = y$ (since P lies in the first quadrant)
Substituting $x = y$ into the first ellipse equation:

$$x^2 + 4x^2 = 1 \Rightarrow 5x^2 = 1 \Rightarrow x = \frac{1}{\sqrt{5}}$$

Thus, the point of intersection is $P\left(\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$.
Differentiating $x^2 + 4y^2 = 1$ with respect to x gives the slope of the tangent to the first ellipse:

$$2x + 8y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{4y}$$

At $P\left(\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$, the slope is $m_1 = -\frac{1}{4}$.
Differentiating $4x^2 + y^2 = 1$ with respect to x gives the slope of the tangent to the second ellipse:

$$8x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{4x}{y}$$

At $P\left(\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$, the slope is $m_2 = -4$.
The acute angle θ between the tangents is given by:

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{-\frac{1}{4} - (-4)}{1 + \left(-\frac{1}{4}\right)(-4)}\right| = \left|\frac{\frac{15}{4}}{2}\right| = \frac{15}{8}$$

Therefore, the value of $4\tan\theta$ is:

$$4\tan\theta = 4 \times \frac{15}{8} = \frac{15}{2} = 7.5$$

18. (0.75)

The given ellipses are $E_1: x^2 + 4y^2 = 1$ and $E_2: 4x^2 + y^2 = 1$.
By symmetry, the common region is symmetric about both the coordinate axes and the lines $y = x$ and $y = -x$.
The total area α is 8 times the area of the region in the first quadrant bounded by $\theta = 0$ and $\theta = \frac{\pi}{4}$.
In polar coordinates, substituting $x = r\cos\theta$ and $y = r\sin\theta$ into the equation of E_2 (which is the inner boundary for $0 \le \theta \le \frac{\pi}{4}$), we get:

$$4r^2\cos^2\theta + r^2\sin^2\theta = 1 \Rightarrow r^2 = \frac{1}{4\cos^2\theta + \sin^2\theta}$$

The area of this sector is given by:

$$A = \frac{1}{2}\int_0^{\frac{\pi}{4}} r^2 d\theta = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{1}{4\cos^2\theta + \sin^2\theta} d\theta$$

Dividing the numerator and the denominator by $\cos^2\theta$:

$$A = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\sec^2\theta}{4 + \tan^2\theta} d\theta$$

Substituting $t = \tan\theta$, we have $dt = \sec^2\theta d\theta$. The limits change from 0 to 1 :

$$A = \frac{1}{2}\int_0^1 \frac{dt}{4 + t^2} = \frac{1}{2}\left[\frac{1}{2}\arctan\left(\frac{t}{2}\right)\right]_0^1 = \frac{1}{4}\arctan\left(\frac{1}{2}\right)$$

The total area α of the common region is:

$$\alpha = 8A = 8 \times \frac{1}{4}\arctan\left(\frac{1}{2}\right) = 2\arctan\left(\frac{1}{2}\right)$$

We need to find the value of $\cot\alpha$:

$$\cot\alpha = \cot\left(2\arctan\left(\frac{1}{2}\right)\right)$$

Let $\phi = \arctan\left(\frac{1}{2}\right)$, which implies $\tan\phi = \frac{1}{2}$.
Using the double angle formula for tangent:

$$\tan(2\phi) = \frac{2\tan\phi}{1-\tan^2\phi} = \frac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2} = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

Therefore, $\cot a = \frac{1}{\tan(2\phi)} = \frac{3}{4}$.

19. **(c)** The number density of conduction electrons n is given by the number of atoms per unit volume, since there is one conduction electron per atom:

$$n = \frac{d \times N_A}{M}$$

Substituting the given values:

$$n = \frac{6.35 \times 10^3 \times 6 \times 10^{23}}{63.5 \times 10^{-3}} = 6 \times 10^{28}\ m^{-3}$$

The resistance of the wire R is:

$$R = \frac{L}{\sigma A}$$

$$R = \frac{100}{2 \times 10^8 \times 0.5 \times 10^{-6}} = \frac{100}{10^2} = 1\Omega$$

The total resistance of the circuit is $R_{total} = R + r = 1 + 1 = 2\Omega$.

The current in the circuit is:

$$I = \frac{E}{R_{total}} = \frac{2}{2} = 1\ A$$

The drift velocity v_d is given by the relation $I = neAv_d$:

$$v_d = \frac{I}{neA}$$

$$v_d = \frac{1}{6 \times 10^{28} \times 1.6 \times 10^{-19} \times 0.5 \times 10^{-6}}$$

$$v_d = \frac{1}{4.8 \times 10^3}\ m\ s^{-1}$$

Converting to mms^{-1} :

$$v_d = \frac{1000}{4800}\ mm\ s^{-1} = \frac{5}{24}\ mm\ s^{-1} = 0.208\ mm\ s^{-1}$$

20. **(a)** Let N be the number of active nuclei at time t. The rate of change of the number of nuclei is given by $\frac{dN}{dt} = \alpha - \lambda N$

Integrating with the initial condition $N = 0$ at $t = 0$:

$$\int_0^N \frac{dN}{\alpha - \lambda N} = \int_0^t dt$$

$$-\frac{1}{\lambda}\ln\left(\frac{\alpha - \lambda N}{\alpha}\right) = t$$

$$N = \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)$$

The rate of decay of the nuclide at time t is

$$A = \lambda N = \alpha\left(1 - e^{-\lambda t}\right)$$

Since each decay produces energy E_0, the rate of energy production is

$$\frac{dE}{dt} = AE_0 = \alpha E_0\left(1 - e^{-\lambda t}\right)$$

This energy is used to heat the liquid. The rate of heat absorption is

$$\frac{dQ}{dt} = ms\frac{dT}{dt}$$

Equating the rate of energy production to the rate of heat absorption:

$$ms\frac{dT}{dt} = \alpha E_0\left(1 - e^{-\lambda t}\right)$$

$$\frac{dT}{dt} = \frac{\alpha E_0}{ms}\left(1 - e^{-\lambda t}\right)$$

21. **(b)** For a thin prism, the angle of deviation is given by $D = (n-1)A$.

To find the smallest value of deviation D_m, we need to find the minimum value of the refractive index $n(\lambda)$.
Given $n(\lambda) = \alpha\lambda + \frac{\beta}{\lambda^2}$.
Differentiating $n(\lambda)$ with respect to λ and equating to zero for minimum:
$$\frac{dn}{d\lambda} = \alpha - \frac{2\beta}{\lambda^3} = 0$$
$$\lambda^3 = \frac{2\beta}{\alpha}$$
Substituting the given values $\alpha = 3\mu m^{-1}$ and $\beta = 0.096\mu m^2$:
$$\lambda^3 = \frac{2 \times 0.096}{3} = 0.064$$
$$\lambda = 0.4\mu\ m$$
Now, substituting $\lambda = 0.4\mu$ m back into the expression for $n(\lambda)$:
$$n_{min} = 3(0.4) + \frac{0.096}{(0.4)^2}$$
$$n_{min} = 1.2 + \frac{0.096}{0.16} = 1.2 + 0.6 = 1.8$$
The smallest angle of deviation is:
$$D_m = (n_{min} - 1)A$$
$$D_m = (1.8 - 1) \times 6° = 0.8 \times 6° = 4.8°$$

22. **(a)** The equation of motion for the radial distance r is given by:
$$m\frac{d^2r}{dt^2} = F(r) + \frac{\ell^2}{mr^3} = -\frac{k}{r^2} + \frac{\ell^2}{mr^3}$$
For a circular orbit of radius r_0, the radial acceleration is zero:
$$-\frac{k}{r_0^2} + \frac{\ell^2}{mr_0^3} = 0 \Rightarrow r_0 = \frac{\ell^2}{mk}$$
Let the particle be displaced by a small distance x such that $r = r_0 + x$. The restoring force is:
$$m\frac{d^2x}{dt^2} = -\frac{k}{(r_0 + x)^2} + \frac{\ell^2}{m(r_0 + x)^3}$$
Using the binomial expansion for $x \ll r_0$:
$$m\frac{d^2x}{dt^2} = -\frac{k}{r_0^2}\left(1 - \frac{2x}{r_0}\right) + \frac{\ell^2}{mr_0^3}\left(1 - \frac{3x}{r_0}\right)$$
Since $\frac{k}{r_0^2} = \frac{\ell^2}{mr_0^3}$, the constant terms cancel out:
$$m\frac{d^2x}{dt^2} = \left(\frac{2k}{r_0^3} - \frac{3\ell^2}{mr_0^4}\right)x$$
Substituting $\frac{\ell}{mr_0^4} = \frac{k}{r_0^3}$ into the equation:
$$m\frac{d^2x}{dt^2} = \left(\frac{2k}{r_0^3} - \frac{3k}{r_0^3}\right)x = -\frac{k}{r_0^3}x$$
This represents simple harmonic motion with angular frequency $\omega = \sqrt{\frac{k}{mr_0^3}}$.

The time period is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{mr_0^3}{k}}$.

Substituting $r_0 = \frac{\ell^2}{mk}$:
$$T = 2\pi\sqrt{\frac{m}{k}\left(\frac{\ell^2}{mk}\right)^3} = 2\pi\sqrt{\frac{m\ell^6}{km^3k^3}} = \frac{2\pi\ell^3}{mk^2}$$

23. **(a,d)** From the given geometry, faces a_1b_1 and a_2b_2 are parallel to each other and perpendicular to the mirror M.
Let the mirror M be horizontal, which makes faces a_1b_1 and a_2b_2 vertical.
The normal to face a_1b_1 is horizontal. The ray emerges from prism 1 at an angle e_1 with the normal, meaning it makes an angle e_1 with the horizontal.
Upon striking the horizontal mirror M, the ray reflects. By the law of reflection, the reflected ray also makes an angle e_1 with the horizontal.

The reflected ray then strikes face a_2b_2 of prism 2 . Since the normal to a_2b_2 is also horizontal, the angle of incidence i_2 is exactly the angle the ray makes with the horizontal. Thus, we have $i_2 = e_1$.

For Option (a): If both prisms are at minimum deviation, the angle of emergence equals the angle of incidence for each prism. For prism 1, $\sin e_1 = n_1 \sin\left(\frac{A_1}{2}\right)$. For prism 2, $\sin i_2 = n_2 \sin\left(\frac{A_2}{2}\right)$. Since $i_2 = e_1$, we get $n_1 \sin\left(\frac{A_1}{2}\right) = n_2 \sin\left(\frac{A_2}{2}\right)$, which gives $\frac{n_2}{n_1} = \frac{\sin(A_1/2)}{\sin(A_2/2)}$. This statement is correct.

For Option (b): If prism 2 is at minimum deviation, $\sin i_2 = n_2 \sin\left(\frac{A_2}{2}\right)$. Since $i_2 = e_1$, we have $\sin e_1 = n_2 \sin\left(\frac{A_2}{2}\right)$.

However, $i_1 = e_1$ is only true if prism 1 is also at minimum deviation. Therefore, $\sin i_1 = n_2 \sin\left(\frac{A_2}{2}\right)$ is not always true. This statement is incorrect.

For Option (c): The angle θ is the angle between the extended faces a_1c_1 and a_2c_2. From the geometry, $\theta = A_1 + A_2$. For thin prisms at minimum deviation, $\delta_m = (n-1)A$, so $A_1 = \frac{\delta_{m1}}{n_1-1}$ and $A_2 = \frac{\delta_{m2}}{n_2-1}$. Thus, $\theta = \frac{\delta_{m1}}{n_1-1} + \frac{\delta_{m2}}{n_2-1}$. The given formula has an extra factor of 2 in the denominator. This statement is incorrect.

For Option (d): If prism 1 is at minimum deviation, $\sin e_1 = n_1 \sin\left(\frac{A_1}{2}\right)$. Since $i_2 = e_1$ is always true from the reflection geometry, we have $\sin i_2 = n_1 \sin\left(\frac{A_1}{2}\right)$ regardless of the state of prism 2 . This statement is correct.

24. (a,c) For the time interval $0 \le t \le 0.2$ s, the particle is under the influence of the electric field and gravity.

The charge-to-mass ratio of the particle is $\frac{q}{m} = \frac{10^{-6}}{10^{-6}} = 1 \mathrm{Ckg}^{-1}$.

The acceleration of the particle is:

$$\vec{a} = \frac{q\vec{E}}{m} + \vec{g} = 1(\hat{\imath}) - 10\hat{\jmath} = \hat{\imath} - 10\hat{\jmath} \text{ ms}^{-2}$$

Given the initial velocity $\vec{v}_0 = \hat{\imath} + 2\hat{\jmath}$ ms^{-1} and initial vertical position $y_0 = 0$ (since it is projected from the XZ plane), we can find the velocity and position at $t = 0.2$ s.

Velocity at $t = 0.2$ s :

$\vec{v}(0.2) = \vec{v}_0 + \vec{a}t = (\hat{\imath} + 2\hat{\jmath}) + (\hat{\imath} - 10\hat{\jmath})(0.2) = 1.2\hat{\imath}$ ms^{-1}

Vertical position at $t = 0.2$ s :

$$y(0.2) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = 0 + 2(0.2) - \frac{1}{2}(10)(0.2)^2 = 0.4 - 0.2 = 0.2 \text{ m} = 20 \text{ cm}$$

For $t > 0.2$ s, the electric field is switched off and the magnetic field $\vec{B} = 6\hat{\jmath}$ T is switched on. The magnetic force $\vec{F}_m = q(\vec{v} \times \vec{B})$ acts only in the XZ plane because $\vec{B}$ is along the y-axis. The vertical motion is only affected by gravity.

Let $t' = t - 0.2$ be the time elapsed after the fields are switched. The initial vertical velocity for this phase is $v_y(0.2) = 0$. The vertical position as a function of t' is:

$$y(t') = y(0.2) + v_y(0.2)t' - \frac{1}{2}gt'^2 = 0.2 - 5t'^2$$

Evaluating the options:

At $t = 0.3$ s$(t' = 0.1$ s) :

$y(0.1) = 0.2 - 5(0.1)^2 = 0.2 - 0.05 = 0.15$ m $= 15$ cm (Option a is correct)

At $t = 0.4$ s$(t' = 0.2$ s) :

$y(0.2) = 0.2 - 5(0.2)^2 = 0.2 - 0.2 = 0$ m $= 0$ cm. (Option b is incorrect)

At $t = 0.35$ s$(t' = 0.15$ s) :

$y(0.15) = 0.2 - 5(0.15)^2 = 0.2 - 0.1125 = 0.0875$ m $\neq 0$. (Option d is incorrect)

For the radius of the trajectory for $t > 0.2$ s, the particle moves in a helical path. The radius of the circular projection in the XZ plane is determined by the velocity perpendicular to the magnetic field, $v_\perp = 1.2$ ms^{-1}.

$$R = \frac{mv_\perp}{qB} = \frac{10^{-6} \times 1.2}{10^{-6} \times 6} = 0.2 \text{ m} = 20 \text{ cm. (Option c is correct)}$$

25. (a,b,c) The potential at a point (x, y) due to charge Q_1 is $V_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{(x-a)^2+(y-b)^2}}$.

The potential at (x, y) due to charge Q_2 is $V_2 = \frac{1}{4\pi\epsilon_0} \frac{mq}{\sqrt{(x-ma)^2+(y-mb)^2}}$.

Given $|V_1| = |V_2|$, we have:

$$\frac{1}{(x-a)^2+(y-b)^2}=\frac{m^2}{(x-ma)^2+(y-mb)^2}$$

Cross-multiplying and expanding both sides:

$(x-ma)^2+(y-mb)^2=m^2[(x-a)^2+(y-b)^2]$

$x^2+m^2a^2-2max+y^2+m^2b^2-2mby=m^2(x^2+a^2-2ax+y^2+b^2-2by)$

$x^2+y^2-2m(ax+by)+m^2(a^2+b^2)=m^2(x^2+y^2)-2m^2(ax+by)+m^2(a^2+b^2)$

Canceling $m^2(a^2+b^2)$ from both sides and rearranging:

$(m^2-1)(x^2+y^2)-2m(m-1)(ax+by)=0$

Since $m\neq 1$, dividing by $m-1$ gives the general equation of the locus:

$(m+1)(x^2+y^2)-2m(ax+by)=0$

For $m=-1, 0-2(-1)(ax+by)=0\Rightarrow ax+by=0$. This represents a straight line.

For $m=2$,$3(x^2+y^2)-4(ax+by)=0\Rightarrow x^2+y^2-\frac{4}{3}ax-\frac{4}{3}by=0$.

This represents a circle with center $\left(\frac{2}{3}a,\frac{2}{3}b\right)$ and radius $\sqrt{\left(\frac{2}{3}a\right)^2+\left(\frac{2}{3}b\right)^2}=\frac{2}{3}\sqrt{a^2+b^2}$.

For $m=-2,(-1)(x^2+y^2)-2(-2)(ax+by)=0\Rightarrow x^2+y^2-4ax-4by=0$.

This represents a circle with center $(2a,2b)$ and radius $\sqrt{(2a)^2+(2b)^2}=2\sqrt{a^2+b^2}$.

For $m=-3,(-2)(x^2+y^2)-2(-3)(ax+by)=0\Rightarrow x^2+y^2-3ax-3by=0$.

This represents a circle, not a straight line.

26. (b,d) The net force on the dipole in a uniform electric field is zero, as the forces on the two charges are equal and opposite ($qE\hat{j}$ and $-qE\hat{j}$). Therefore, the center of mass remains at rest and is not deflected. Option (a) is incorrect.

The moment of inertia of the dipole about its center of mass is $I=m\left(\frac{d}{2}\right)^2+m\left(\frac{d}{2}\right)^2=\frac{md^2}{2}$.

The torque on the dipole is $\vec{\tau}=\vec{p}\times\vec{E}$. The angle between the dipole moment $\vec{p}$ and the electric field $\vec{E}$ is $90^\circ-\theta$.

The magnitude of the torque is $\tau=pE\sin(90^\circ-\theta)=qdE\cos\theta$.

The work done by the electric field as the dipole rotates from $\theta=0$ to $\theta=\theta_f$ is equal to the change in kinetic energy:

$$\Delta K=\int_0^{\theta_f}\tau d\theta=\int_0^{\theta_f}qdE\cos\theta d\theta=qdE\sin\theta_f$$

Equating this to the final kinetic energy $\frac{1}{2}I\omega_f^2$:

$$\frac{1}{2}\left(\frac{md^2}{2}\right)\omega_f^2=qdE\sin\theta_f$$

$$\frac{md^2}{4}\omega_f^2=qdE\sin\theta_f$$

For option (b), substituting $\omega_f=\sqrt{\frac{2qE}{md}}$:

$$\frac{md^2}{4}\left(\frac{2qE}{md}\right)=qdE\sin\theta_f$$

$$\frac{qdE}{2}=qdE\sin\theta_f\Rightarrow\sin\theta_f=\frac{1}{2}\Rightarrow\theta_f=\frac{\pi}{6}$$

Thus, option (b) is correct.

For option (c), if $\theta_f=\frac{\pi}{3}$, the change in kinetic energy is $\Delta K=qdE\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}qEd$. Thus, option (c) is incorrect.

For option (d), after $t>t_f$, the electric field is turned off. The net torque on the dipole becomes zero. By Newton's first law of rotational motion, the dipole will continue to rotate about its center of mass with a constant angular velocity. Thus, option (d) is correct.

For option (d), after $t>t_f$, the electric field is turned off. The net torque on the dipole becomes zero. By Newton's first law of rotational motion, the dipole will continue to rotate about its center of mass with a constant angular velocity. Thus, option (d) is correct.

27. (a,b,c) For state a: $n=10, T_a=27^\circ C=300\ K, P_a=1\ atm, V_a=V_0$.

For a monoatomic gas, $C_v=\frac{3}{2}R$ and $\gamma=\frac{5}{3}$.

Process $a\rightarrow b$ is a sudden compression, which is an adiabatic process.

$V_b = \frac{V_0}{3}$

Using $T_a V_a^{\gamma-1} = T_b V_b^{\gamma-1}$:

$T_b = T_a \left(\frac{V_a}{V_b}\right)^{\gamma-1} = 300 \times (3)^{5/3-1} = 300 \times 3^{2/3} = 300 \times (9)^{1/3}$

Given $9^{1/3} = 2.08$, we get $T_b = 300 \times 2.08 = 624$ K.

Using $P_a V_a^{\gamma} = P_b V_b^{\gamma}$:

$P_b = P_a \left(\frac{V_a}{V_b}\right)^{7} = 1 \times (3)^{5/3} = 3 \times 3^{2/3} = 3 \times 2.08 = 6.24$ atm.

Thus, the pressure at state b is 6.24 times the atmospheric pressure. Option (d) is incorrect.

The change in internal energy for process a → b is:

$\Delta U_{ab} = nC_v(T_b - T_a) = 10 \times \frac{3}{2} R \times (624 - 300) = 15R \times 324 = 4860R$

Option (b) is correct.

Process b → c is isochoric (stationary piston), so $V_c = \frac{V_0}{3}$.

The gas cools to the water bath temperature, $T_c = 11^\circ C = 284$ K.

Process c → f is isothermal (slow expansion in water bath), so $T_f = T_c = 284$ K and $V_f = V_0$.

The net change in internal energy for the entire process a → f depends only on the initial and final temperatures:

$\Delta U_{net} = nC_v(T_f - T_a) = 10 \times \frac{3}{2} R \times (284 - 300) = 15R \times (-16) = -240R.$

Option (c) is correct.

For the P-V diagram:

Process a → b is an adiabatic compression (curve upwards and to the left).

Process b → c is an isochoric cooling (vertical line downwards).

Process c → f is an isothermal expansion (curve downwards and to the right).

Comparing pressures at volume V_0 :

$P_a = \frac{nR(300)}{V_0}$ and $P_f = \frac{nR(284)}{V_0}$

Since $P_a > P_f$, point a lies above point f. The given schematic diagram correctly represents all these features.

Option (a) is correct.

28. (1915)

The least count (LC) of the screw gauge is given by:

$LC = \frac{\text{Pitch}}{\text{Number of divisions on CS}} = \frac{0.5}{100} = 0.005$ mm

For Wire-1, the measured reading is:

$\text{Reading}_1 = LS + CS \times LC$

$\text{Reading}_1 = 0.5 + 42 \times 0.005 = 0.5 + 0.210 = 0.710$ mm

The actual diameter of Wire-1 is 0.650 mm . The zero error of the screw gauge is:

Zero Error = Measured Reading - Actual Value $= 0.710 - 0.650 = +0.060$ mm

For Wire-2, the measured reading is:

$\text{Reading}_2 = LS + CS \times LC$

$\text{Reading}_2 = 1.5 + 95 \times 0.005 = 1.5 + 0.475 = 1.975$ mm

The actual diameter d of Wire-2 is obtained by subtracting the zero error from its measured reading:

$d = \text{Reading}_2 - \text{Zero Error} = 1.975 - 0.060 = 1.915$ mm

Converting the diameter into micrometers (μ m):

$d = 1.915 \times 1000\mu\text{ m} = 1915\mu\text{ m}$

29. (0.46)

Taking the natural logarithm on both sides, we get:

$\ln \lambda = \ln a + \ln(\sin\theta)$

Differentiating to find the maximum fractional error:

$\frac{\Delta\lambda}{\lambda} = \frac{\Delta a}{a} + \cot\theta\Delta\theta$

Since $\theta = 2°$ is very small, $\cos\theta = 1$, which gives $\cot\theta = \frac{1}{\sin\theta}$.
Given $\sin(2°) = 0.035$, we can approximate the angle in radians as $\theta = \sin\theta = 0.035\text{rad}$.
The error in the angle is $\Delta\theta = 40' = \left(\frac{40}{60}\right)^\circ = \left(\frac{2}{3}\right)^\circ$.
Converting $\Delta\theta$ into radians:
$$\Delta\theta = \frac{2}{3} \times \left(\frac{0.035}{2}\right) = \frac{0.035}{3}\text{rad}$$
The fractional error in the angular term is:
$$\cot\theta\Delta\theta = \frac{\Delta\theta}{\sin\theta} = \frac{\frac{0.035}{3}}{0.035} = \frac{1}{3}$$
The fractional error in the slit width is:
$$\frac{\Delta a}{a} = \frac{0.002}{0.016} = \frac{1}{8}$$
Substituting these values into the error equation:
$$\frac{\Delta\lambda}{\lambda} = \frac{1}{8} + \frac{1}{3} = \frac{3+8}{24} = \frac{11}{24}$$

30. (60)

Let the angle of incidence at the first interface (water to glass) be i.
By applying Snell's law at the successive parallel interfaces, we can find the angle of incidence at the interface where the reflection occurs (water to medium n_p).
For the first interface (water to glass):
$$n_w \sin i = n_g \sin r_1$$
For the second interface (glass to water):
$$n_g \sin r_1 = n_w \sin r_2$$
From the above two equations, we get:
$$n_w \sin i = n_w \sin r_2 \Longrightarrow r_2 = i$$
Thus, the angle of incidence at the third interface (water to medium n_p) is also i.
The problem states that the reflected ray is completely polarized with its electric field perpendicular to the plane of incidence (indicated by the dots on ray CD). This complete polarization upon reflection occurs when the light is incident at Brewster's angle θ_B.
According to Brewster's law for the interface between water and medium n_p :
$$\tan\theta_B = \frac{n_p}{n_w}$$
Since the angle of incidence at this interface is i, we have $i = \theta_B$. Substituting the given refractive indices:
$$\tan i = \frac{4/\sqrt{3}}{4/3}$$
$$\tan i = \frac{3}{\sqrt{3}} = \sqrt{3}$$
Therefore, the angle of incidence is:
$i = 60^0$

31. (694.44)

Let the resistance of the galvanometer be G and the series resistance be R_1.
When the key K is open, the current through the galvanometer is:
$$I_1 = \frac{V}{R_1 + G}$$
When the key K is closed, the total resistance of the circuit becomes:
$$R_{eq} = R_1 + \frac{GR_2}{G + R_2}$$
The total current from the battery is $I = \frac{V}{R_{eq}}$. The current through the galvanometer is:
$$I_2 = I \times \frac{R_2}{G + R_2} = \frac{V}{R_1 + \frac{GR_2}{G+R_2}} \times \frac{R_2}{G + R_2} = \frac{VR_2}{R_1(G + R_2) + GR_2}$$
In the half-deflection condition, $I_2 = \frac{I_1}{2}$:

$$\frac{VR_2}{R_1(G+R_2)+GR_2}=\frac{V}{2(R_1+G)}$$
Cross-multiplying and simplifying:
$2R_2(R_1+G)=R_1(G+R_2)+GR_2$
$2R_1R_2+2GR_2=R_1G+R_1R_2+GR_2$
$R_1R_2+GR_2=R_1G$
$R_1(G-R_2)=GR_2$
$$R_1=\frac{GR_2}{G-R_2}$$
Given $G=6\Omega$ and $R_2=4\Omega$, we can find R_1 :
$$R_1=\frac{6\times 4}{6-4}=\frac{24}{2}=12\Omega$$
In the half-deflection condition (key K closed), the total equivalent resistance of the circuit is:
$$R_{eq}=12+\frac{6\times 4}{6+4}=12+2.4=14.4\Omega$$
The current through the resistor R_1 is the total current from the battery:
$$I=\frac{V}{R_{eq}}=\frac{10}{14.4}=\frac{100}{144}=\frac{25}{36}\text{ A}$$
Converting the current to mA :
$$I=\frac{25}{36}\times 1000\text{ mA}=\frac{25000}{36}\text{ mA}=694.44\text{ mA}$$

32. (25)

The dimensional formula for permeability of free space μ_0 is $[MLT^{-2}A^{-2}]$.
The dimensional formula for permittivity of free space ϵ_0 is $[M^{-1}L^{-3}T^4A^2]$.
The dimensional formula for $\sqrt{\frac{\mu_0}{\epsilon_0}}$ is:
$$\left(\frac{[MLT^{-2}A^{-2}]}{[M^{-1}L^{-3}T^4A^2]}\right)^{1/2}=([M^2L^4T^{-6}A^{-4}])^{1/2}=[ML^2T^{-3}A^{-2}]$$
Let the magnitude of 1 SI unit in the new system be n. Using the principle of dimensional homogeneity:
$n_1u_1=n_2u_2$
$1\times[M_1L_1^2T_1^{-3}A_1^{-2}]=n\times[M_2L_2^2T_2^{-3}A_2^{-2}]$
Given the new units are $M_2=5$ kg, $L_2=5$ m, $T_2=5$ s, and $A_2=5$ A, while the SI units are $M_1=1$ kg, $L_1=1$ m, $T_1=1$ s, and $A_1=1$ A.
$$n=\left(\frac{M_1}{M_2}\right)\left(\frac{L_1}{L_2}\right)^2\left(\frac{T_1}{T_2}\right)^{-3}\left(\frac{A_1}{A_2}\right)^{-2}$$
$$n=\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)^2\left(\frac{1}{5}\right)^{-3}\left(\frac{1}{5}\right)^{-2}$$
$n=5^{-1}\times 5^{-2}\times 5^3\times 5^2$
$n=5^{-1-2+3+2}=5^2=25$
Thus, the magnitude of one SI unit in the new system of units is 25 .

33. (1.25)

Let A be the cross-sectional area of each chamber and a be the cross-sectional area of the hole.
From the given dimensions, the area of each chamber is $A=1\text{ m}\times 1\text{ m}=1\text{ m}^2$.
The area of the hole is $a=\sqrt{10}\text{ cm}^2=\sqrt{10}\times 10^{-4}\text{ m}^2$.
Let h_1 and h_2 be the heights of the liquid in the left and right chambers at time t, respectively.
By conservation of volume, $Ah_1+Ah_2=AH$, where $H=2$ m is the initial height of the liquid in the left chamber.
$\Rightarrow h_1+h_2=2$
The velocity of efflux through the hole is given by Torricelli's law:
$v=\sqrt{2g(h_1-h_2)}$
The rate of flow of the liquid is:
$$-A\frac{dh_1}{dt}=a\sqrt{2g(h_1-h_2)}$$
Substitute $h_2=2-h_1$ into the equation:

$-A\dfrac{dh_1}{dt} = a\sqrt{2g(2h_1 - 2)}$

Let $y = 2h_1 - 2$. Then $dy = 2dh_1$, which gives $dh_1 = \frac{dy}{2}$.

The differential equation becomes:

$-\dfrac{A}{2}\dfrac{dy}{dt} = a\sqrt{2gy}$

$\Rightarrow \dfrac{dy}{\sqrt{y}} = -\dfrac{2a\sqrt{2g}}{A}d$

Integrating both sides from $t = 0$ to $t = 500$ s. At $t = 0, h_1 = 2$ m, so $y(0) = 2(2) - 2 = 2$ m.

$\displaystyle\int_2^y y^{-1/2}dy = -\frac{2a\sqrt{2g}}{A}\int_0^{500} dt$

$[2\sqrt{y}]_2^y = -\dfrac{2 \times (\sqrt{10} \times 10^{-4}) \times \sqrt{20}}{1} \times 500$

$2\sqrt{y} - 2\sqrt{2} = -1000 \times 10^{-4} \times \sqrt{200}$

$2\sqrt{y} - 2\sqrt{2} = -0.1 \times 10\sqrt{2}$

$2\sqrt{y} - 2\sqrt{2} = -\sqrt{2}$

$2\sqrt{y} = \sqrt{2}$

$\sqrt{y} = \dfrac{\sqrt{2}}{2}$

Squaring both sides, we get:

$y = 0.5$

Since $y = 2h_1 - 2$, we have:

$2h_1 - 2 = 0.5$

$2h_1 = 2.5$

$h_1 = 1.25$ m

34. (1.97)

Let A be the area of each chamber. $A = 1 \times 1 = 1\ m^2$. Distance between plates is $d = 2$ m.

At $t = 0$, the left chamber is completely filled with liquid ($\epsilon_r = 15$) and right chamber is empty ($\epsilon_r = 1$).

Initial capacitance $C(0) = C_1 + C_2 = \frac{15\epsilon_0 A}{d} + \frac{\epsilon_0 A}{d} = \frac{15\epsilon_0(1)}{2} + \frac{\epsilon_0(1)}{2} = 8\epsilon_0$.

Let y_1 and y_2 be the liquid levels in the left and right chambers at time t. By conservation of volume, $y_1 + y_2 = 2$ m.

Let $h = y_1 - y_2$. The velocity of efflux from the left chamber is $v = \sqrt{2gh}$.

Rate of flow $Q = -A\frac{dy_1}{dt} = a\sqrt{2gh}$, where $a = \sqrt{10} \times 10^{-4}\ m^2$ is the hole area.

Since $y_1 + y_2 = 2$, we have $dy_2 = -dy_1$, so $dh = 2dy_1 \Rightarrow \frac{dy_1}{dt} = \frac{1}{2}\frac{dh}{dt}$.

$-\dfrac{1}{2}\dfrac{dh}{dt} = a\sqrt{2gh} \Rightarrow \dfrac{dh}{\sqrt{h}} = -2a\sqrt{2g}dt$

Integrating from $t = 0(h = 2)$ to $t = 500\ s(h)$:

$\displaystyle\int_2^h h^{-1/2}dh = -2a\sqrt{2g}\int_0^{500} dt$

$2\sqrt{h} - 2\sqrt{2} = -2(\sqrt{10} \times 10^{-4})(\sqrt{20})(500) = -\sqrt{200} \times 10^{-4} \times 1000 = -10\sqrt{2} \times 0.1$

$= -\sqrt{2}$

$2\sqrt{h} = \sqrt{2} \Rightarrow \sqrt{h} = \dfrac{1}{\sqrt{2}} \Rightarrow h = 0.5$ m

From $y_1 + y_2 = 2$ and $y_1 - y_2 = 0.5$, we get $y_1 = 1.25$ m and $y_2 = 0.75$ m.

At $t = 500$ s, each chamber acts as two capacitors in series (liquid and air).

$C_{left} = \dfrac{1}{\dfrac{y_1}{15\epsilon_0 A} + \dfrac{2 - y_1}{\epsilon_0 A}} = \dfrac{15\epsilon_0}{30 - 14y_1} = \dfrac{15\epsilon_0}{30 - 14(1.25)} = \dfrac{15\epsilon_0}{12.5} = \dfrac{6}{5}\epsilon_0 = 1.2\epsilon_0$.

$$C_{right} = \frac{1}{\frac{y_2}{15\epsilon_0 A} + \frac{2 - y_2}{\epsilon_0 A}} = \frac{15\epsilon_0}{30 - 14y_2} = \frac{15\epsilon_0}{30 - 14(0.75)} = \frac{15\epsilon_0}{19.5} = \frac{150}{195}\epsilon_0 = \frac{10}{13}\epsilon_0.$$

$$C(500) = C_{left} + C_{right} = \frac{6}{5}\epsilon_0 + \frac{10}{13}\epsilon_0 = \frac{128}{65}\epsilon_0 = 1.97\epsilon_0.$$

Difference in capacitance $= C(0) - C(500) = 8\epsilon_0 - \frac{128}{65}\epsilon_0 = \left(8 - \frac{128}{65}\right)\epsilon_0.$

Comparing this with the given expression $(8 - n)\epsilon_0$, we get $n = \frac{128}{65} = 1.97$

35. (0.15)

Moment of inertia of the disk about the pivot C is $I_C = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$.

Substituting $M = 1$ kg and $R = 0.2$ m, we get $I_C = \frac{3}{2}(1)(0.2)^2 = 0.06\ \text{kg m}^2$.

The initial angular momentum of the particle about C is $L_i = muy_\perp$, where $y_\perp$ is the perpendicular distance from C to the initial line of motion.

$$L_i = mu(R + R\cos 45^\circ) = (0.02)(100)(0.2)\left(1 + \frac{1}{\sqrt{2}}\right) = 0.4 + 0.2\sqrt{2}\ \text{kg m}^2/\text{s}.$$

This initial angular momentum is in the clockwise direction.

The final angular momentum of the particle about C is $L_f = mvx_\perp$, where $x_\perp$ is the perpendicular distance from C to the final line of motion.

$$L_f = mv(R\sin 45^\circ) = (0.02)(90)(0.2)\left(\frac{1}{\sqrt{2}}\right) = 0.18\sqrt{2}\ \text{kg m}^2/\text{s}.$$

This final angular momentum is also in the clockwise direction.

By conservation of angular momentum about C during the collision, $L_i = L_f + I_C\omega$, where ω is the angular velocity of the disk just after the collision.

$$0.4 + 0.2\sqrt{2} = 0.18\sqrt{2} + 0.06\omega$$

$$0.06\omega = 0.4 + 0.02\sqrt{2}$$

$$\omega = \frac{40 + 2\sqrt{2}}{6} = \frac{20 + \sqrt{2}}{3}\ \text{rad/s}.$$

Applying conservation of mechanical energy for the disk to find the maximum change in height h of its center O :

$$\frac{1}{2}I_C\omega^2 = Mgh$$

$$h = \frac{I_C\omega^2}{2Mg} = \frac{0.06}{2(1)(10)}\left(\frac{20 + \sqrt{2}}{3}\right)^2$$

$$h = 0.003\left(\frac{400 + 40\sqrt{2} + 2}{9}\right)$$

$$h = \frac{3}{1000}\left(\frac{402 + 40\sqrt{2}}{9}\right)$$

$$h = \frac{402 + 40\sqrt{2}}{3000} = \frac{201 + 20\sqrt{2}}{1500} = 0.15$$

36. (17.47)

Let the center of the uniform circular disk be the origin $O(0,0)$. The disk is pivoted at its top point $C(0, R)$.

Radius of disk $R = 0.2$ m, Mass $M = 1$ kg.

Mass of particle $m = 20\ \text{g} = 0.02$ kg.

Initial velocity of particle $\vec{v}_i = -100\hat{i}\ \text{ms}^{-1}$.

Final velocity of particle $\vec{v}_f = -90\hat{j}\ \text{ms}^{-1}$.

The particle hits the disk at point P. From the figure, the line OP makes an angle of 45° with the negative y-axis.

Coordinates of P with respect to O are $(R\sin 45^\circ, -R\cos 45^\circ)$.

Position vector of P with respect to the pivot C is:

$$\vec{r}_{P/C} = (R\sin 45^\circ - 0)\hat{i} + (-R\cos 45^\circ - R)\hat{j} = \frac{R}{\sqrt{2}}\hat{i} - R\left(1 + \frac{1}{\sqrt{2}}\right)\hat{j}$$

Angular momentum is conserved about the pivot C during the collision.

Initial angular momentum of the system about C is only due to the particle:

$\vec{L}_i = \vec{r}_{P/C} \times (m\vec{v}_i) = \left[\frac{R}{\sqrt{2}}\hat{\imath} - R\left(1+\frac{1}{\sqrt{2}}\right)\hat{\jmath}\right] \times (-100m\hat{\imath})$

$\vec{L}_i = -100mR\left(1+\frac{1}{\sqrt{2}}\right)(\hat{\jmath} \times \hat{\imath}) = -100mR\left(1+\frac{1}{\sqrt{2}}\right)\hat{k}$

Substituting $m = 0.02$ kg and $R = 0.2$ m :

$\vec{L}_i = -100(0.02)(0.2)\left(1+\frac{1}{\sqrt{2}}\right)\hat{k} = -0.4\left(1+\frac{1}{\sqrt{2}}\right)\hat{k} = -(0.4+0.2\sqrt{2})\hat{k}\ \text{kg m}^2\ \text{s}^{-1}$

Final angular momentum of the particle about C :

$\vec{L}_{f,p} = \vec{r}_{P/C} \times (m\vec{v}_f) = \left[\frac{R}{\sqrt{2}}\hat{\imath} - R\left(1+\frac{1}{\sqrt{2}}\right)\hat{\jmath}\right] \times (-90m\hat{\jmath})$

$\vec{L}_{f,p} = -90m\frac{R}{\sqrt{2}}(\hat{\imath} \times \hat{\jmath}) = -\frac{90mR}{\sqrt{2}}\hat{k}$

$\vec{L}_{f,p} = -\frac{90(0.02)(0.2)}{\sqrt{2}}\hat{k} = -\frac{0.36}{\sqrt{2}}\hat{k} = -0.18\sqrt{2}\hat{k}\ \text{kg m}^2\ \text{s}^{-1}$

Let ω be the angular velocity of the disk after collision. Its moment of inertia about C is:

$I_C = I_{cm} + MR^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2 = \frac{3}{2}(1)(0.2)^2 = 0.06\ \text{kg m}^2$

Final angular momentum of the disk is $\vec{L}_{f,d} = I_C\omega\hat{k} = 0.06\omega\hat{k}$

By conservation of angular momentum about C :

$\vec{L}_i = \vec{L}_{f,p} + \vec{L}_{f,d}$

$-(0.4+0.2\sqrt{2}) = -0.18\sqrt{2} + 0.06\omega$

$0.06\omega = -0.4 - 0.02\sqrt{2} \Rightarrow \omega = -\frac{40+2\sqrt{2}}{6} = -\frac{20+\sqrt{2}}{3}\ \text{rad s}^{-1}$

Initial kinetic energy of the system:

$E_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(0.02)(100)^2 = 100\ \text{J}$

Final kinetic energy of the system:

$E_f = \frac{1}{2}mv_f^2 + \frac{1}{2}I_C\omega^2 = \frac{1}{2}(0.02)(90)^2 + \frac{1}{2}(0.06)\left(\frac{20+\sqrt{2}}{3}\right)^2$

$E_f = 81 + 0.03\left(\frac{400+2+40\sqrt{2}}{9}\right) = 81 + \frac{402+40\sqrt{2}}{300} = 81 + \frac{201+20\sqrt{2}}{150}$

Amount of energy loss:

$\Delta E = E_i - E_f = 100 - \left(81 + \frac{201+20\sqrt{2}}{150}\right) = 19 - \frac{201+20\sqrt{2}}{150}$

$\Delta E = \frac{2850-201-20\sqrt{2}}{150} = \frac{2649-20\sqrt{2}}{150}$

Using $\sqrt{2} = 1.414$,

$\Delta E = \frac{2649-28.28}{150} = \frac{2620.72}{150} = 17.47\ \text{J}$

37. (c) Using the Debye-Hückel-Onsager equation, $\Lambda_m = \Lambda_m^\circ - b\sqrt{C}$, we can find the limiting molar conductivity (Λ_m°) for each salt.

For $NaNO_3$:

$\Lambda_m^\circ - b\sqrt{0.01} = 111 \Rightarrow \Lambda_m^\circ - 0.1b = 111$

$\Lambda_m^\circ - b\sqrt{0.04} = 101 \Rightarrow \Lambda_m^\circ - 0.2b = 101$

Subtracting the two equations gives $0.1b = 10 \Rightarrow b = 100$.

$\Lambda_m^\circ(NaNO_3) = 111 + 100(0.1) = 121\ \text{S cm}^2\ \text{mol}^{-1}$

For NaCl :

$\Lambda_m^\circ - 0.1b = 117$

$\Lambda_m^\circ - 0.2b = 107$

Subtracting gives $0.1b = 10 \Rightarrow b = 100$.

$\Lambda_m^\circ(NaCl) = 117 + 100(0.1) = 127\ \text{S cm}^2\ \text{mol}^{-1}$

For $AgNO_3$:

$\Lambda_m^\circ - 0.1b = 125$
$\Lambda_m^\circ - 0.2b = 116$
Subtracting gives $0.1b = 9 \Rightarrow b = 90$.
$\Lambda_m^\circ(AgNO_3) = 125 + 90(0.1) = 134\ S\ cm^2\ mol^{-1}$
According to Kohlrausch's law of independent migration of ions:
$\Lambda_m^\circ(AgCl) = \Lambda_m^\circ(AgNO_3) + \Lambda_m^\circ(NaCl) - \Lambda_m^\circ(NaNO_3)$
$\Lambda_m^\circ(AgCl) = 134 + 127 - 121 = 140\ S\ cm^2\ mol^{-1}$
The molar conductivity of the saturated AgCl solution is equal to its limiting molar conductivity, so $\Lambda_m(AgCl) = 140\ S\ cm^2\ mol^{-1}$.
The relationship between molar conductivity, conductivity (κ), and solubility (X in $molL^{-1}$) is:
$$\Lambda_m = \frac{\kappa \times 1000}{X}$$
Substituting the given values:
$$140 = \frac{1.40 \times 10^{-6} \times 1000}{X}$$
$$140 = \frac{1.40 \times 10^{-3}}{X}$$
$$X = \frac{1.40 \times 10^{-3}}{140} = 10^{-5}\ mol\ L^{-1}$$
We need to find $\log_{10}(X^{-1})$:
$$X^{-1} = \frac{1}{10^{-5}} = 10^5$$
$$\log_{10}(X^{-1}) = \log_{10}(10^5) = 5$$

38. **(b)** For NO_2^+, the central nitrogen atom is sp hybridized with no lone pairs, resulting in a linear geometry and a bond angle of 180°.
For NO_2, the central nitrogen atom is sp^2 hybridized with one unpaired electron. The repulsion from a single electron is less than that from a bond pair, so the bond angle opens up to be greater than 120° (approximately 134°).
For NO_3^-, the central nitrogen atom is sp^2 hybridized with no lone pairs, resulting in a trigonal planar geometry and a bond angle of exactly 120°.
For NO_2^-, the central nitrogen atom is sp^2 hybridized with one lone pair. The lone pair-bond pair repulsion is greater than bond pair-bond pair repulsion, compressing the bond angle to less than 120° (approximately 115°).
Therefore, the correct order of O-N-O bond angles is $NO_2^- < NO_3^- < NO_2 < NO_2^+$.

39. **(a)** Natural rubber is cis-1,4-polyisoprene, with the repeating unit $[-CH_2 - C(CH_3) = CH - CH_2 -]_n$.
Complete reductive ozonolysis ($O_3/Zn - H_2O$) of natural rubber cleaves the double bonds to give 4-oxopentanal (levulinaldehyde) as compound X
$$[-CH_2 - C(CH_3) = CH - CH_2 -]_n \xrightarrow{O_3/Zn-H_2O} nCH_3 - C(=O) - CH_2 - CH_2 - CHO$$
Compound $X(CH_3 - CO - CH_2 - CH_2 - CHO)$ has a methyl ketone group, which gives a positive iodoform test, and an aldehyde group, which gives a positive Tollen's test.
When X is heated with aqueous NaOH , it undergoes an intramolecular aldol condensation. The base abstracts an α-proton from the terminal methyl group to form an enolate, which then attacks the aldehyde carbonyl carbon to form a stable five-membered ring.
The intermediate formed is 3-hydroxycyclopentanone, which upon heating undergoes dehydration to yield cyclopent-2-en-1one as the major product Y .

40. **(a)**

41. **(c,d)** For a first-order reaction R → P, the rate law is given by:
$$-\frac{d[R]}{dt} = \frac{dP]}{dt} = k[R]$$
Integrating this, the concentration of the reactant at time t is $[R] = [R]_0 e^{-kt}$.
The concentration of the product at time t is $[P] = [R]_a - [R] = [R]_a\left(1 - e^{-kt}\right)$.
The plot of [P] versus t follows the equation $[P] = [R]_0\left(1 - e^{-kt}\right)$. This represents a curve that starts from the origin and is concave down, approaching a constant value $[R]_0$. The graph in (a) is concave up, making it incorrect.

The plot of $\frac{d[R]}{dt}$ versus [R] follows the equation $\frac{d[R]}{dt}=-k[R]$. This is a straight line passing through the origin with a negative slope ($-k$). The graph in (b) has a positive slope, making it incorrect.
The plot of $\frac{dP]}{dt}$ versus t follows the equation $\frac{dP]}{dt}=k[R]_a e^{-kt}$. This represents an exponentially decreasing curve with respect to time. The graph in (c) matches this behavior, making it correct.
The rate constant k depends only on the temperature and is independent of time. Therefore, the plot of k versus t is a horizontal straight line. The graph in (d) matches this, making it correct.

42. **(a,c,d)** From the given reaction conditions:

$Xe(g)+2F_2(g)\xrightarrow[1.5\,ratio]{873\text{ K},7\text{bar}}XeF_4$

Thus, P is XeF_4.

$XeF_4+O_2F_2\xrightarrow{143\text{ K}}XeF_6+O_2$

Thus, Q is XeF_6.

$XeF_6+3H_2O\xrightarrow{\text{complete hydrolysia}}XeO_3+6HF$

Thus, R is XeO_3.

Evaluating the given options:
(a)P(XeF_4) has 8 valence electrons on the central Xe atom. It forms 4 single bonds with F , leaving 4 electrons which form 2 lone pairs. This statement is correct.
(b)Q(XeF_6) has 6 bond pairs and 1 lone pair on the central Xe atom. Its geometry is distorted octahedral, not perfect octahedral. This statement is incorrect.
(c)Q(XeF_6) is a strong fluorinating agent. This statement is correct.
(d)R(XeO_3) has 3 double bonds and 1 lone pair on the central Xe atom. Its molecular structure is trigonal pyramidal. This statement is correct.

43. **(a,b)** For (a): The electronic configuration of $B^+$is $1s^2 2s^2$ and that of $C^+$is $1s^2 2s^2 2p^1$. Removing an electron from the fully filled, more penetrating 2s orbital of B^+requires more energy than removing a 2p electron from C^+. Thus, the second ionization enthalpy of carbon is less than that of boron. Statement (a) is correct.
For (b): Al^{3+}, Mg^{2+}, and $Na^+$are isoelectronic species with 10 electrons each. The nuclear charge increases in the order $Na^+(11)<Mg^{2+}(12)<Al^{3+}(13)$. Higher effective nuclear charge leads to a stronger pull on the electrons, resulting in a smaller ionic radius. Thus, the order of ionic radii is $Al^{3+}<Mg^{2+}<Na^+$. Statement (b) is correct.
For (c): The density of potassium ($0.86\ g/cm^3$) is less than that of sodium ($0.97\ g/cm^3$) due to an abnormal increase in atomic volume caused by the presence of empty 3d orbitals in potassium. Statement (c) is incorrect.
For (d): The bond dissociation enthalpy of $H-H(436\ kJ/mol)$ is much higher than that of $F-F(159\ kJ/mol)$ due to strong interelectronic repulsions between the lone pairs on the small fluorine atoms. Statement (d) is incorrect.

44. **(b,d)** In the first reaction sequence:
Step 1:Aniline reacts with acetic anhydride in the presence of pyridine to undergo acetylation, forming acetanilide.
Step 2:Acetanilide is treated with a nitrating mixture (conc. HNO_3 and conc. H_2SO_4) at 288 K . The $-NHCOCH_3$ group is moderately activating and ortho/para directing. Due to steric hindrance, the major product P is p-nitroacetanilide.
Step 3:P is hydrolyzed with $H_3O^+$to yield p-nitroaniline.
Step 4:p-nitroaniline is diazotized using $NaNO_2/HCl$ at $273-278$ K to form the diazonium salt Q, which is p nitrobenzenediazonium chloride.
In the second reaction sequence:
Step 1:Cumene (isopropylbenzene) is oxidized by O_2 followed by acid hydrolysis (H_3O^+)to produce phenol and acetone. The major aromatic product is phenol.
Step 2:Phenol undergoes the Kolbe-Schmitt reaction with NaOH and CO_2 to form sodium salicylate.
Step 3:Acidification with H_3O^+yields salicylic acid as the major product S.
In the final reaction:
Salicylic acid (S) reacts with p-nitrobenzenediazonium chloride (Q) in an aqueous NaOH medium. This is an electrophilic aromatic substitution (azo coupling). The phenoxide ion formed is highly activating, and coupling occurs at the para position with respect to the -OH group (position 5 of salicylic acid). The resulting product T is an azo dye, which is a highly conjugated and coloured compound.

Evaluating the options:
(a)Treatment of Q (p-nitrobenzenediazonium chloride) with ethanol reduces the diazonium group, yielding nitrobenzene, not an aromatic aldehyde. (Incorrect)
(b)The phthalein dye test is a characteristic test for phenols. Salicylic acid (S) contains a phenolic $-OH$ group with a free para position (position 5) and gives a positive phthalein dye test when heated with phthalic anhydride and conc. H_2SO_4. (Correct)
(c)P is p-nitroacetanilide, which is a mononitro compound, not a dinitro compound. (Incorrect)
(d)T is an azo dye, which is a coloured compound. (Correct)

45. (b,c) Statement(a)is incorrect: Nitric acid (HNO_3) is a strong oxidizing agent that oxidizes both glucose and gluconic acid to saccharic acid.
Statement(b)is correct: Fehling's reagent provides an alkaline medium. Under these conditions, fructose (a ketose) undergoes the Lobry de Bruyn-van Ekenstein rearrangement to form an equilibrium mixture of fructose, glucose, and mannose. Glucose and mannose are aldohexoses that readily reduce Fehling's solution, giving a positive test.
Statement(c)is correct: Hydrolysis of sucrose (a disaccharide) yields an equimolar mixture of D-(+)-glucose and D-(-)fructose. This mixture is known as invert sugar because the sign of specific rotation changes from positive (for sucrose) to negative (for the mixture).
Statement(d)is incorrect: The specific rotation of L-(-)-glucose is -52.5°, so for D-(+)-glucose it is $+52.5^{\circ}$. The specific rotation of L-(+)-fructose is $+92.5^{\circ}$, so for D-(-)-fructose it is -92.5°.
The specific rotation of invert sugar (an equimolar mixture of D-glucose and D-fructose, which have the same molar mass) is the average of their specific rotations:
$$[\alpha]_{\text{mix}} = \frac{+52.5^{\circ} + (-92.5^{\circ})}{2} = -20^{\circ}$$

46. (3)
For a hydrogen-like species, the wavelength of the absorbed light during a transition from n_1 to n_2 is given by the Rydberg formula:
$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$
For X^{a+}, the transition is from $n = 1$ to $n = 2$:
$$\frac{1}{\lambda} = RZ_X^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}RZ_X^2 \Rightarrow \lambda = \frac{4}{3RZ_X^2}$$
For Y^{b+}, the transition is from $n = 2$ to $n = 4$:
$$\frac{1}{9\lambda} = RZ_Y^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = RZ_Y^2\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3}{16}RZ_Y^2 \Rightarrow 9\lambda = \frac{16}{3RZ_Y^2}$$
Substituting the expression for λ from the first equation into the second:
$$9\left(\frac{4}{3RZ_X^2}\right) = \frac{16}{3RZ_Y^2}$$
$$\frac{12}{Z_X^2} = \frac{16}{3Z_Y^2}$$
$$36Z_Y^2 = 16Z_X^2$$
$$9Z_Y^2 = 4Z_X^2 \Rightarrow 3Z_Y = 2Z_X$$
$$\frac{Z_X}{Z_Y} = \frac{3}{2}$$
For the lowest possible integer values of atomic numbers, we get $Z_X = 3$ and $Z_Y = 2$.
Since X^{a+} and Y^{b+} are hydrogen-like species, they must contain exactly 1 electron. Therefore, the charge on the ion is $(Z - 1)$.
For $X^{a+}(Z_X = 3)$: $a = 3 - 1 = 2$
For $Y^{b+}(Z_Y = 2)$: $b = 2 - 1 = 1$
The lowest possible value of $(a + b)$ is $2 + 1 = 3$.

47. (1.5)
Given the pH of the resulting solution is 3.0 , the concentration of H^+ions is:
$[H^+] = 10^{-3}M$
For the dissociation of acetic acid ($CH_3COOH \rightleftharpoons CH_3COO^- + H^+$), the acid dissociation constant is given by:

$K_a = \frac{[CH_3COO^-][H^+]}{[CH_3COOH]}$

Assuming $[CH_3COO^-] = [H^+]$and the equilibrium concentration of acetic acid is C , we can use the approximation:

$K_a = \frac{[H^+]^2}{C}$

$1.0 \times 10^{-5} = \frac{(10^{-3})^2}{C}$

$C = \frac{10^{-6}}{1.0 \times 10^{-5}} = 0.1M$

The number of moles of acetic acid remaining in the 50 mL(0.05 L) solution at equilibrium is:

$n_{eq} = C \times V = 0.1 \text{ mol L}^{-1} \times 0.05 \text{ L} = 0.005 \text{ mol}$

The mass of acetic acid remaining in the solution is:

$W_{eq} = n_{eq} \times \text{ Molar mass } = 0.005 \text{ mol} \times 60 \text{ g mol}^{-1} = 0.30 \text{ g}$

The initial mass of acetic acid was 0.45 g . The mass of acetic acid adsorbed (x) by the charcoal is:

$x = 0.45 \text{ g} - 0.30 \text{ g} = 0.15 \text{ g}$

The Freundlich adsorption isotherm is given by:

$\frac{x}{m} = kC^{1/n}$

Taking the logarithm on both sides:

$\log_{10}\left(\frac{x}{m}\right) = \log_{10}k + \frac{1}{n}\log_{10}C$

The plot of $\log_{10}(x/m)$ against $\log_{10}C$ is a straight line with a slope of 1 . Therefore:

$\frac{1}{n} = 1 \Rightarrow n = 1$

Substitute the values into the isotherm equation ($m = 1.0$ g):

$\frac{0.15}{1.0} = k(0.1)^1$

$k = \frac{0.15}{0.1} = 1.5 \text{ L mol}^{-1}$

48. (33.33)

The elevation in boiling point is given by $\Delta T_b = iK_b m$.

The ebullioscopic constant K_b is calculated as:

$K_b = \frac{R(T_b^{\dagger})^2 M_S}{1000\Delta H_{vap}}$

Substituting the given values ($T_b^{\circ} = 400$ K, $\Delta H_{vap} = 10\text{RJmol}^{-1}$):

$K_b = \frac{R(400)^2 M_S}{1000(10R)} = 16M_S$

For a dilute solution with 0.25% (mass/mass) concentration, the mass of solute $w_B = 0.25$ g and the mass of solvent $w_S = 100$ g.

The molality m is:

$m = \frac{w_B \times 1000}{M_B \times w_S} = \frac{0.25 \times 1000}{10M_S \times 100} = \frac{0.25}{M_S}$

Given $\Delta T_b = 408 - 400 = 8$ K, we substitute into the boiling point elevation formula:

$8 = i \times (16M_S) \times \left(\frac{0.25}{M_S}\right)$

$8 = 4i \Longrightarrow i = 2$

The dissociation reaction is $B \rightleftharpoons 2C + 2D$.

The van't Hoff factor i is related to the degree of dissociation α by:

$i = 1 + (n - 1)\alpha$

Here, $n = 4$ (since 1 molecule of B yields 4 particles).

$2 = 1 + (4 - 1)\alpha$

$3\alpha = 1 \Longrightarrow \alpha = \frac{1}{3}$

The mole percent of B that has been dissociated is $\alpha \times 100 = 33.33\%$.

49. (6)

In an octahedral complex, the six coordinating atoms are located at the vertices of an octahedron, which has 8 triangular faces. Each face is formed by 3 vertices, and any two adjacent (cis) vertices form an edge that is shared by exactly two faces.

For the cis- $[Co(NH_3)_4Cl_2]^+$complex:

The two Cl atoms are at adjacent (cis) positions, forming exactly one $Cl - Cl$ edge. This edge is shared by two triangular faces. Since the remaining four vertices are occupied by N atoms, the third vertex of each of these two faces must be an N atom. Therefore, there are 2 faces with exactly two Cl atoms and one N atom.

For the mer- $[Co(NH_3)_3Cl_3]$ complex:

The three Cl atoms are in a meridional arrangement, meaning two Cl atoms are trans to each other and the third is cis to both. This arrangement forms exactly two $Cl - Cl$ edges. Since the three Cl atoms do not occupy the vertices of the same face (unlike the fac isomer), no face contains three Cl atoms. Each of the two $Cl - Cl$ edges is shared by two faces, yielding $2 \times 2 = 4$ distinct faces that contain exactly two Cl atoms. The third vertex in all these 4 faces is an N atom. Therefore, there are 4 faces with exactly two Cl atoms and one N atom.

The sum of the total number of such faces in both complexes is $2 + 4 = 6$.

50. (85018)

Step 1: Identification of monomer X

The given reaction sequence for X is:

$CH_3I \xrightarrow{KCN} CH_3CN$

$CH_3CN \xrightarrow{H_3O^+,\Delta} CH_3COOH$

$CH_3COOH \xrightarrow{\text{Red P,}Br_2} BrCH_2COOH$ (Hell-Volhard-Zelinsky reaction)

$BrCH_2COOH \xrightarrow{NH_3\text{ (excess)}} H_2N - CH_2 - COOH$

Thus, X is Glycine.

Molar mass of $X(C_2H_5NO_2) = 2(12) + 5(1) + 14 + 2(16) = 75$ g/mol.

Step 2: Identification of monomer Y

Caprolactam on acidic hydrolysis gives ϵ-aminocaproic acid:

Caprolactam $\xrightarrow{H_3O^+,\Delta} H_2N - (CH_2)_5 - COOH$

Thus, Y is ϵ-aminocaproic acid.

Molar mass of $Y(C_6H_{13}NO_2) = 6(12) + 13(1) + 14 + 2(16) = 131$ g/mol.

Step 3: Calculation of the mass of copolymer Z

The copolymer Z is Nylon-2-nylon-6, which is a biodegradable polyamide formed by the condensation polymerization of Glycine and ϵ-aminocaproic acid.

The problem states that 500 mol of X and 500 mol of Y completely react to form 1 mol of a single acyclic copolymer Z.

This means each molecule of polymer Z contains 500 units of X and 500 units of Y, making a total of 1000 monomer units.

In an acyclic chain of 1000 monomers, the number of peptide (amide) bonds formed is $1000 - 1 = 999$.

For each peptide bond formed, one molecule of water (H_2O) is eliminated. Since 1 mol of polymer Z is formed, 999 mol of water are eliminated.

Applying the law of conservation of mass:

Mass of Z = (Mass of 500 mol of X) + (Mass of 500 mol of Y) -(Mass of 999 mol of H_2O)

Mass of $Z = (500 \times 75) + (500 \times 131) - (999 \times 18)$

Mass of $Z = 37500 + 65500 - 17982$

Mass of $Z = 103000 - 17982 = 85018$g

51. (2000)

For a 5 molal solution of B in A , there are 5 moles of B in 1000 g of A .

Moles of A $= \frac{1000}{50} = 20$ mol.

Mole fraction of B, $x_B = \frac{5}{20+5} = 0.2$

Mole fraction of A, $x_A = 1 - 0.2 = 0.8$

Using Raoult's law for the total vapour pressure:

$P_T = P_A^\circ x_A + P_B^\circ x_B$

$100 = 105 \times 0.8 + P_B^\circ \times 0.2$
$100 = 84 + 0.2P_B^\circ$
$0.2P_B^\circ = 16 \Rightarrow P_B^\circ = 80$ mmHg
The molar volume of pure B in the liquid phase $(V_{m,l})$ is:
$$V_{m,l} = \frac{\text{Molar mass of B}}{\text{Density of liquid B}} = \frac{57}{0.5} = 114 \text{ mL/mol} = 0.114 \text{ L/mol}$$
Assuming pure B behaves as an ideal gas in the vapour phase, its molar volume ($V_{m,v}$) at its vapour pressure P_B° is:
$$V_{m,v} = \frac{RT}{P_B^\circ} = \frac{0.08 \times 300}{\left(\frac{80}{760}\right)} = \frac{24 \times 760}{80} = 228 \text{ L/mol}$$
The ratio of the molar volume of pure B in the vapour phase to its molar volume in the liquid phase is:
$$\text{Ratio } = \frac{V_{m,v}}{V_{m,l}} = \frac{228}{0.114} = 2000$$

52. (0.16)

Molality of B in A is 5 m , which means 5 moles of B are dissolved in 1 kg(1000 g) of solvent A.
Number of moles of A, $n_A = \frac{1000}{50} = 20$ mol
Mole fraction of A in the liquid phase, $x_A = \frac{n_A}{n_A+n_B} = \frac{20}{20+5} = 0.8$
Mole fraction of B in the liquid phase, $x_B = 1 - 0.8 = 0.2$
According to Raoult's law, the partial vapour pressure of A is:
$P_A = P_A^0 x_A = 105 \times 0.8 = 84$ mmHg
Given total vapour pressure, $P_T = 100$ mmHg
Partial vapour pressure of B, $P_B = P_T - P_A = 100 - 84 = 16$mmHg
Mole fraction of B in the vapour phase, $y_B = \frac{P_B}{P_T} = \frac{16}{100} = 0.16$

53. (10)

The reaction sequence proceeds as follows:
(1) Friedel-Crafts Acylation: Reaction of m-xylene (1,3-dimethylbenzene) with chloroacetyl chloride ($ClCH_2COCl$) in the presence of anhydrous $AlCl_3$ occurs at the less sterically hindered activated position (position 4) to give 1-(2,4-dimethylphenyl)-2-chloroethan-1-one.
(2) Finkelstein Reaction: Heating with NaI replaces the chloride with iodide, yielding 1-(2,4-dimethylphenyl)-2-iodoethan-1one.
(3) Williamson Ether Synthesis: Reaction with sodium 3-nitrophenoxide gives compound J, which is 1-(2,4-dimethylphenyl)-2-(3-nitrophenoxy)ethan-1-one.
(4) Reduction and Bromination: $NaBH_4$ reduces the ketone group in J to a secondary alcohol without affecting the nitro group. Subsequent reaction with PBr_3 converts the alcohol to a bromide, yielding compound K, 1-(1-bromo-2-(3-nitrophenoxy)ethyl)-2,4-dimethylbenzene.
The molecular formula of K is $C_{16}H_{16}BrNO_3$.
Molar mass of K $= (16 \times 12) + (16 \times 1) + 80 + 14 + (3 \times 16) = 192 + 16 + 80 + 14 + 48 = 350$ g/mol, which matches the given data.
5.Amination: Reaction of K with excess NH_3 results in nucleophilic substitution of the bromide to form a primary amine, compound L, 1-(2,4-dimethylphenyl)-2-(3-nitrophenoxy)ethanamine.
The molecular formula of L is $C_{16}H_{18}N_2O_3$.
Molar mass of L $= (16 \times 12) + (18 \times 1) + (2 \times 14) + (3 \times 16) = 192 + 18 + 28 + 48 = 286$ g/mol.
Moles of L taken $= \frac{5.72 \text{ g}}{286 \text{ g/mol}} = 0.02$ mol.
In Kjeldahl's method, nitrogen present in nitro ($-NO_2$) groups is not converted to ammonium sulfate and thus is not estimated. Only the nitrogen from the primary amine ($-NH_2$) group will evolve as ammonia (NH_3).
Therefore, 1 mole of L yields 1 mole of NH_3.
Moles of NH_3 evolved $= 0.02$ mol.
The neutralization reaction with sulfuric acid is:
$2NH_3 + H_2SO_4 \rightarrow (NH_4)_2SO_4$
Moles of H_2SO_4 required $= \frac{0.02}{2} = 0.01$ mol.
Given the molarity of H_2SO_4 is 1 M :

Volume of $H_2SO_4 = \frac{\text{Moles}}{\text{Molarity}} = \frac{0.01 \text{ mol}}{1 \text{ mol/L}} = 0.01 \text{ L} = 10 \text{ mL}.$

54. (2.33)

Step 1:The starting material is m-xylene (1,3-dimethylbenzene). It undergoes Friedel-Crafts acylation with chloroacetyl chloride ($ClCH_2COCl$) in the presence of anhydrous $AlCl_3$ to form 2-chloro-1-(2,4-dimethylphenyl)ethanone.

Step 2:The chloro ketone reacts with NaI (Finkelstein reaction) to form 2-iodo-1-(2,4-dimethylphenyl)ethanone.

Step 3:This iodo compound undergoes Williamson ether synthesis with sodium 3-nitrophenoxide to form compound J. The molecular formula of J is $C_{16}H_{15}NO_4$.

Step 4:Compound J is reduced by $NaBH_4$, which converts the ketone group to a secondary alcohol. Subsequent reaction with PBr_3 replaces the hydroxyl group with a bromine atom, yielding compound K.

The molecular formula of K is $C_{16}H_{16}BrNO_3$.

Molar mass of $K = (16 \times 12) + (16 \times 1) + 80 + 14 + (3 \times 16) = 350$ g/mol, which matches the given data.

Step 5:Compound K reacts with sodium thiophenoxide (PhSNa) in an S_N2 reaction, replacing the bromide with a phenylthio group (-SPh) to form compound M.

The molecular formula of M is $C_{16}H_{16}(SC_6H_5)NO_3 \Rightarrow C_{22}H_{21}NO_3S$.

Molar mass of $M = 350 - 80$ (for Br) $+ 109$ (for SPh) $= 379$ g/mol.

Step 6:Calculate the moles of compound M used in the Carius method.

Moles of $M = \frac{3.79 \text{ g}}{379 \text{ g/mol}} = 0.01$ mol.

Step 7:Since each molecule of M contains exactly one sulphur atom, the moles of sulphur present is 0.01 mol . In the Carius method, all sulphur is quantitatively converted to barium sulphate ($BaSO_4$).Moles of $BaSO_4$ formed = 0.01 mol.

Step 8:Calculate the mass of $BaSO_4$ formed.

Molar mass of $BaSO_4 = 137(\text{Ba}) + 32(\text{S}) + 4 \times 16(\text{O}) = 233$ g/mol.

Mass of $BaSO_4 = 0.01 \text{ mol} \times 233 \text{ g/mol} = 2.33$ g.