1. **(a)** Along vertical direction

$u_y = 0$

$v_y^2 = u_y^2 + 2a_y g_y$

$a_y = +g$

$= (0)^2 + 2 \times 10 \times 10$

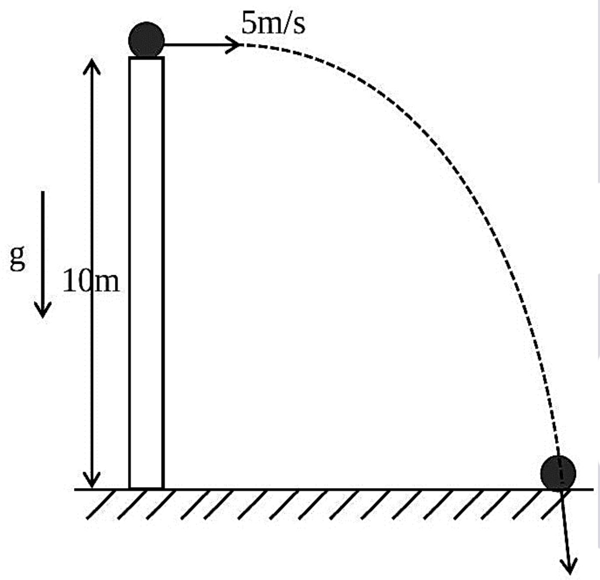
$v_y = ?$

$v_y^2 = 200$

$S_y = 10\ \text{m}$

$v_y^2 = 200$

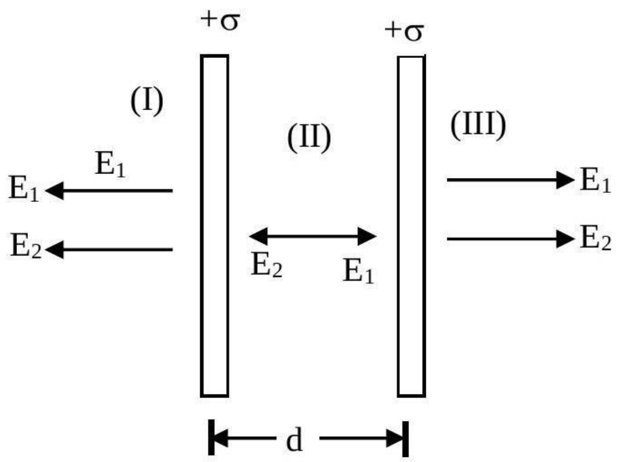
$\therefore v = \sqrt{v_x^2 + v_y^2}$



$= \sqrt{25 + 200} = \sqrt{225}$

$= 15\ \text{m/s}$

2. **(c)**



$\therefore E_I = -\frac{\sigma}{E_0}\hat{n}$

$\therefore E_{II} = 0$

$\therefore E_{III} = -\frac{\sigma}{E_0}\hat{n}$

3. (d) [Volume of bigger drop] = [volume of smaller drop] × 125

$\frac{4}{3}\pi R^3 = 125 \times \frac{4}{3}\pi r^3$

$R^3 = 125r^3$

$\therefore R = 5 \times r$

$\Rightarrow$ Gain in sinface energy = TdA

$= 0.45 \times [A_2 - A_1]$
$= 0.45 \times [125 \times 4\pi r^2 - 4\pi R^2]$
$= 0.45 \times \left[125 \times 4\pi \left(\frac{R}{5}\right)^2 - 4\pi R^2\right]$
$= 0.45 \times [20\pi R^2 - 4\pi R^2]$
$= 0.45 \times 16\pi R^2$
$= 0.45 \times 16 \times 3.14 \times (10^{-3})^2$
$= 2.26 \times 10^{-5}$ J

4. (d)

$M_E = 9M_P$
$R_E = 2R_P$

$V_c^1 = \sqrt{\frac{2GM_P}{R_P}} = \sqrt{\frac{2G\frac{M_E}{9}}{\frac{R_E}{2}}}$

$= \sqrt{\frac{2GM_E}{R_E}} \times \sqrt{\frac{2}{9}}$

$V_c^1 = \frac{V_e}{3}\sqrt{2}$

5. **(b)** Work done in the process is given by

$$W = \frac{R}{\gamma - 1}(T_1 - T_2)$$

For adiabatic process:

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$TV^{\frac{3}{2}-1} = T_2(2V)^{\frac{3}{2}-1}$$

$$TV^{\frac{1}{2}} = T_2(2V)^{\frac{1}{2}}$$

$$T^2 V = T_2^2 \times 2V$$

$$\therefore T_2 = \frac{T}{\sqrt{2}}$$

$$\therefore W = \frac{R}{\gamma - 1} \times \left(T - \frac{T}{\sqrt{2}}\right)$$

$$= 2RT\left[1 - \frac{1}{\sqrt{2}}\right]$$

$$= RT\left[2 - \frac{2}{\sqrt{2}}\right]$$

$$= RT[2 - \sqrt{2}]$$

$$W = RT[2 - \sqrt{2}]$$

6. **(a)** $[b] = [L^3]$

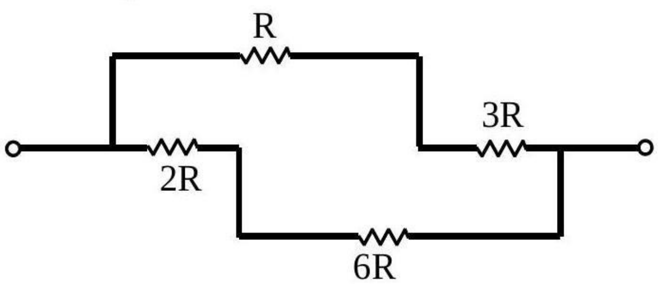
$$[a] = [PV^2]$$

$$= [ML^{-1}T^{-2}][L^6]$$

$$= [ML^5 T^{-2}]$$

$$\frac{[b^2]}{[a]} = \frac{[L^6]}{[ML^5 T^{-2}]} = [M^{-1} L^1 T^2]$$

7. **(a)** The given network is wheat-stone network



$$\therefore\ R_{eq} = \frac{4R \times 8R}{4R + 8R}$$
$$= \frac{4R \times 8R}{12R}$$
$$R_{eq} = \frac{8}{3}R$$

8. (c)

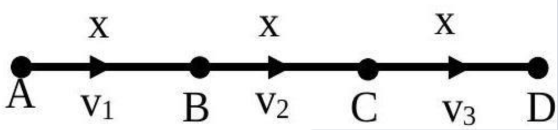
(A) A.C. generator → II. Electro-magnetic induction

(B) transformer → IV Mutual induction

(C) Resonance phenomenon to occur → (I) presence of both L and C

(D) Sharpness of resonance → (III) Quality factor

9. (c)



$$\langle v\rangle = \frac{\text{Total distance}}{\text{Total time}}$$
$$= \frac{3x}{\frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}}$$
$$= \frac{3}{\left[\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3}\right]} = \frac{3}{\left[\frac{v_2 v_3 + v_1 v_3 + v_1 v_2}{v_1 v_2 v_3}\right]}$$
$$< v >= \frac{3 v_1 v_2 v_3}{[v_3 v_2 + v_1 v_2 + v_1 v_2]}$$

10. (d) According to Malus law:

$$I = \frac{I_0}{2}[\cos^2 45 \times \cos^2 45 \times \cos^2 45 \times \ldots (n-1) \text{ times }]$$
$$\frac{I_0}{64} = \frac{I_0}{2} \times \left(\frac{1}{2}\right)^{n-1}$$
$$\frac{1}{32} = \frac{1}{2^{n-1}} \Rightarrow \frac{1}{(2)^5} = \frac{1}{2^{n-1}}$$
$$\therefore n - 1 = 5$$
$$\therefore n = 6$$

11. (b)

(A) Micro-wave (IV) Klystron valve
(B) Gamma rays (I) Radio-active decay of nucleus

(C) Radio-waves (II) Rapid acceleration and deceleration of electrons in aerials
(D) X-rays (III) Inner shell electron

12. (c) The wavelength of matter is given by

$$\lambda = \frac{h}{p}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{p_\alpha}{p_p} = \frac{\sqrt{2k_\alpha m_\alpha}}{\sqrt{2k_p m_p}} = 1$$

$$\therefore \frac{k_\alpha}{k_p} \times \frac{m_\alpha}{m_p} = 1 \Rightarrow \frac{k_\alpha}{k_p} = \frac{m_p}{m_\alpha}$$

$$\frac{k_\alpha}{k_p} = \frac{1}{4}$$

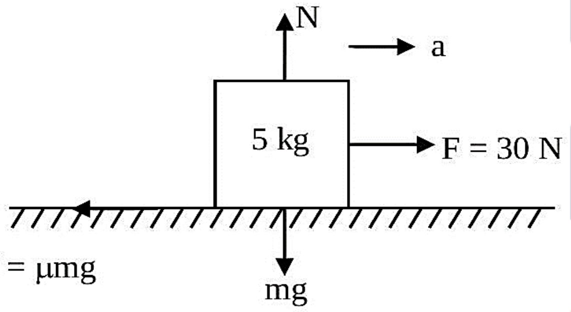
13. (d) $S = ut + \frac{1}{2}at^2$

$$50 = 0 \times t + \frac{1}{2} \times a \times (10)^2$$

$$50 = \frac{1}{2} \times a \times 100$$

$$a = \frac{100}{100} \Rightarrow a = 1\ m/s^2$$

$$\sum F_x = ma_x$$



$$30 - \mu mg = ma$$

$$30 - \mu \times 50 = 5$$

$$50\mu = 25$$

$$\mu = \frac{25}{50}$$

$$= \frac{1}{2}$$

$$\Rightarrow \mu = 0.5$$

14. (b)

Statement (I) is true But

Statement (II) is false

15. (a) The Frequencies for FM Broadcast is between 87.5MHz to 108MHz.

16. (a) $2P + 2n = {}_{2}^{4}He + E$

$\therefore B.E = [2 \times (1.0073 + 1.0087) - 4.0015] \times 931$

$= 0.0305 \times 931$

$= 28.3955 MeV$

17. (a) The velocity of Transverse wave on string is given by

$V = \sqrt{\frac{T}{\mu}}$

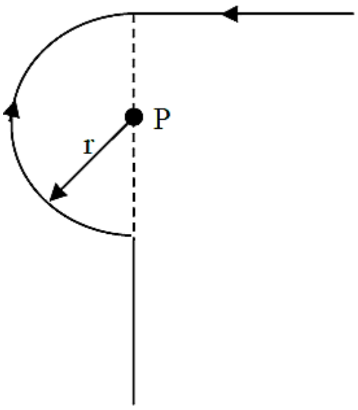
$= \sqrt{\frac{70}{7 \times 10^{-3}}} = \sqrt{\frac{70 \times 10^{3}}{7}}$

$= \sqrt{10^{4}} = 100 \ m/s$

18. (a)

(A) Intrinsic	(II) Fermi-level in the middle of valence and conduction band
(B) n-type semiconductor	(III) Fermi-level near conduction band
(C) p-type semiconductor	(I) Fermi-level near valence band
(D) Metals	(IV) Fermi-level inside the conduction band

19. (b)



$B_P = B_1 + B_2$

$= \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4r}$

$= \frac{\mu_0 i}{4r}\left[\frac{1}{\pi} + 1\right]$

$B_P = \frac{\mu_0 i}{2r}\left[\frac{1}{2\pi} + \frac{1}{2}\right]$

20. **(a)** The average kinetic energy of gas molecule is given by,

$K.E_{avg} = \frac{3}{2}KT$

$\therefore K \cdot E_{avg} \propto T$

21. **(40)** $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

$= (5\hat{i} - 2\hat{j} + \hat{k}) - (2\hat{i} + 3\hat{j} - 4\hat{k})$

$(2,3,-4)$

$(5,-2,1)$

$\overrightarrow{\Delta r} = 3\hat{i} - 5\hat{j} + 5\hat{k}$

$\therefore W = \vec{F} \cdot \overrightarrow{\Delta r}$

$= (5\hat{i} + 2\hat{j} + 7\hat{k}) \cdot (3\hat{i} - 5\hat{j} + 5\hat{k})$

$= 15 - 10 + 35$

$= 5 + 35$

$W = 40\text{ J}$

22. **(1)** Bulk Modulus $= V\frac{dP}{dV}$

$$\frac{(B)_{water}}{(B)_{liquid}} = \frac{VdP/dV}{VdP/dV} = \frac{dP/0.01}{dP/0.03}$$

$$\therefore \frac{(B)_{water}}{(B)_{liquid}} = \frac{0.03}{0.01} = \frac{3}{1}$$

$$\frac{(B)_{water}}{(B)_{liquid}} = \frac{3}{1}$$

$\therefore$ On comparing with $\frac{3}{x}$, The value of " x " will be " 1 '.

23. **(828)**

The energy of electron in ground state $= -13.6\text{eV}$

$E_n - E_1 = 12.75$

$\therefore E_n = 12.75 - 13.6$

$E_n = -0.85$

So " n " is given by

$$E_n = -\frac{13.6}{n^2}$$

$n^2 = \frac{-13.6}{-0.85}$

$n^2 = 16 \Rightarrow n = 4$

$\Rightarrow L = \frac{nh}{2\pi} = \frac{x}{\pi} \times 10^{-17}$

$\Rightarrow 4 \times \frac{h}{2\pi} = \frac{x}{\pi} \times 10^{-17}$

$4 \times \frac{4.14 \times 10^{-15}}{2\pi} = \frac{x}{\pi} \times 10^{-17} \Rightarrow \frac{2 \times 4.14 \times 10^{-15}}{10^{-17}} = x$

$x = 8.28 \times 10^2 \Rightarrow x = 828$

24. (144)

$R = \frac{mV}{qB} = \frac{p}{qB}$

$R = \frac{\sqrt{2mq\Delta V}}{qB}$

$3 \times 10^{-2} = \frac{\sqrt{2\ m \times 2 \times 10^{-6} \times 10^2}}{2 \times 10^{-6} \times 4 \times 10^{-3}}$

$3 \times 10^{-2} \times 2 \times 10^{-6} \times 4 \times 10^{-3} = \sqrt{4\ m \times 10^{-4}}$

$24 \times 10^{-11} = \sqrt{4\ m \times 10^{-4}}$

$m = \frac{24 \times 24 \times 10^{-22}}{4 \times 10^{-4}}$

$m = 144 \times 10^{-18} \text{Kg}$

25. (2)

$K.E = P.E + \frac{25}{100} \times P.E.$

$K.E = P.E + \frac{1}{4} P.E$

$K.E = \frac{5}{4} P \cdot E$

$\frac{1}{2} K(A^2 - x^2) = \frac{5}{4} \times \frac{1}{2} Kx^2$

$4(A^2 - x^2) = 5x^2$

$4A^2 - 4x^2 = 5x^2$

$9x^2 = 4A^2$

$x^2 = \frac{4}{9} \times (3)^2$

$\therefore x = \pm 2$

26. (25)

$E_1 = K\ell_1$

$E_2 = K\ell_2$

$\therefore \frac{E_2}{E_1} = \frac{\ell_2}{\ell_1}$

$\frac{E}{1.5} = \frac{100}{60}$

$\therefore E = 1.5 \times \frac{10}{6}$

$= \frac{3}{2} \times \frac{10}{6}$

$= \frac{5}{2}$

$= \frac{2.5}{}$

$= \frac{25}{10}$

$\therefore x = 25$

27. (32)

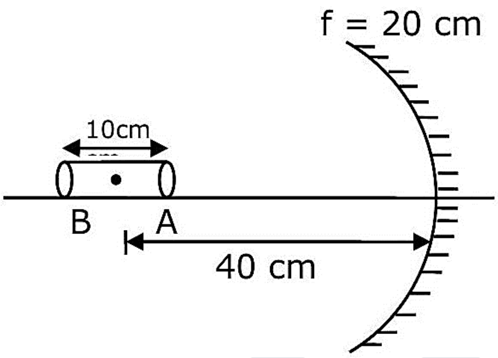


Image of end A:

$u = -35$ cm

$f = -20$ cm

$v = ?$

$v = \frac{uf}{u - f}$

$= \frac{-35 \times -20}{-35 + 20}$

$= \frac{-35 \times -20}{-15}$

$v = -\frac{140}{3}$

Image of end B:

$u = -45$ cm

$v = ?$

$f = -20$ cm

$v = \frac{uf}{u - f}$

$= \frac{-45 \times -20}{-45 + 20}$

$= \frac{-45 \times -20}{-25}$

$v = -36$

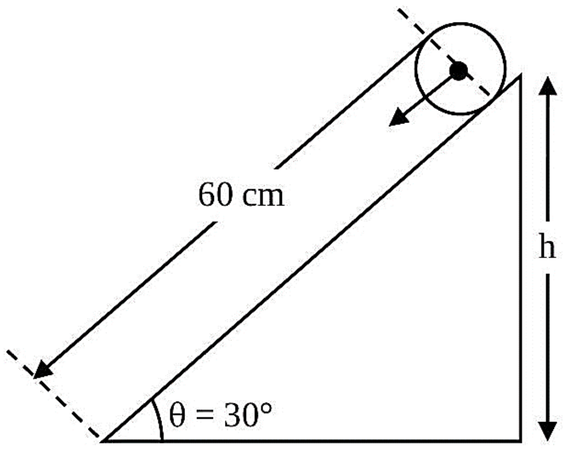
$\therefore$ length of image $= \left|-36 + \frac{140}{3}\right|$

$= \left|-\frac{108 + 140}{3}\right|$

$= \frac{32}{3}$

$\therefore$ The value of $x = 32$

28. (2)



$h = 60 \sin 30$

$\therefore h = 30$ cm

The velocity of by linder upon reaching the ground is given by

$V = \sqrt{\frac{2gh}{1 + \frac{K^2}{R^2}}}$

$\therefore V = \sqrt{\frac{2 \times 10 \times 30 \times 10^{-2}}{1 + \frac{1}{2}}}$

$= \sqrt{\frac{6 \times 2}{3}}$

$V = 2\ m/s$

29. (40) For maximum power, the LCR must be in resonance.

$\therefore X_L = X_C$

$79.6 = \frac{1}{\omega C}$

$C = \frac{1}{\omega \times 79.6}$

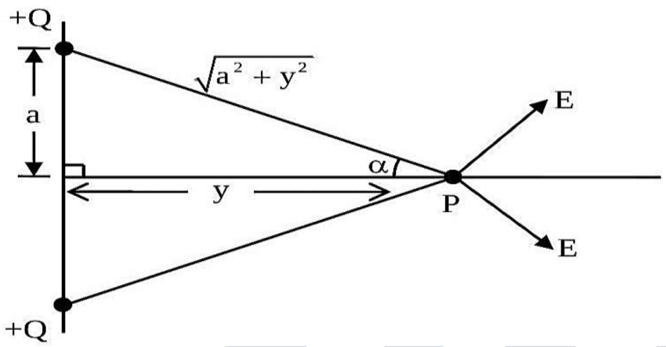
$= \frac{1}{2\pi \times 50 \times 79.6}$

$= \frac{1}{100\pi \times 79.6}$

$= 40 \times 10^{-6}$

$C = 40\mu F$

30. (2)



Electric field at point "P" due to any one change $= \frac{KQ}{a^2+y^2}$

$\therefore$ Net electric field at point "P" will be

$E_{net} = 2E\cos\alpha$

$= \frac{2KQ}{a^2+y^2} \times \frac{y}{\sqrt{a^2+y^2}}$

$E_{net} = \frac{2KQy}{(a^2+y^2)^{3/2}}$

$\Rightarrow$ Electric force $(F) = E_{net}\, q_0$

$= \frac{2KQq_0 y}{(a^2+y^2)^{3/2}}$

For $F = \max \Rightarrow \frac{dF}{dy} = 0$

By solving, we get $y = \frac{a}{\sqrt{2}}$

$\therefore$ the value of x = 2

31. (b)

$$4FeCl_3 + 3K_4[Fe(CN)_6] \longrightarrow 12KCl + \underset{\text{Pursianblue ppt}}{Fe_4[Fe(CN)_6]_3}$$

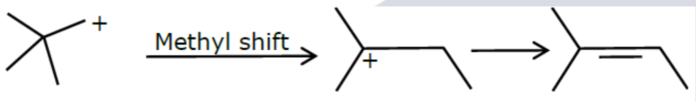
32. (d) No pollution occurs by combustion of hydrogen and very low density of hydrogen.

33. (d) Resonating structure are hypothtical and resonance hybrid is a real structure which is weighted average of all the resonating struture.

34. (c)

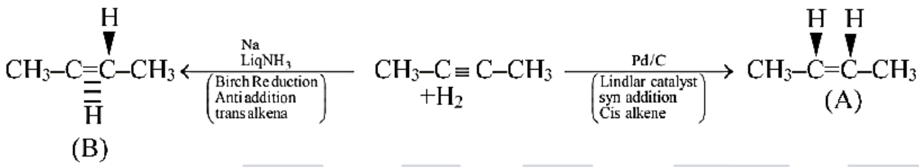
A → (iii)
B → (i)
C → (ii)
D → (iv)

35. (d)



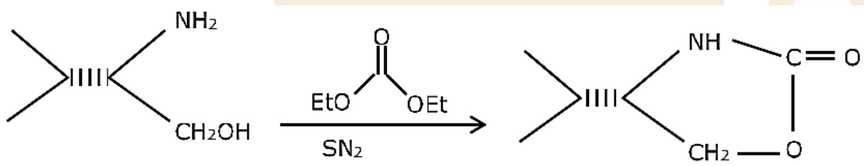
In question given option reaction is incorrect

36. (b)

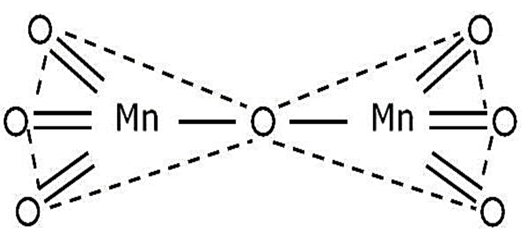


A) Cis has dipole monent, more soluble than trans (B)
B) B.P. (cis > trans), M.P. (trans > cis)
C) Dipole moment (A > B) but $\mu_A \neq 0$
D) Br add easilv to A not B

37. (c)



38. (a, c)



39. (d)

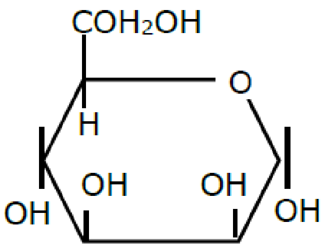
Slaked Lime → $Ca(OH)_2$

Dead burnt plaster → $CaSO_4$

Caustic Soda → NaOH

Washing Soda → $Na_2CO_3 \cdot 10H_2O$

40. (b)



Haworth structure of mannose

41. (c)

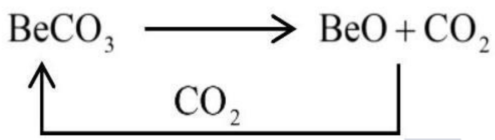
42. (a) Double salt contain's two or more types of salts.

$CuSO_4.4NH_3.H_2O$ and $Fe(CN)_2.4KCN$ are complex compounds.

43. (d) Ease of hydration α stability of carbocation b > d > c > a

44. (a) Chlorine oxides, Cl_2O, ClO_2, Cl_2O_6 and Cl_2O_7 are heighly Reactive oxidising Agents and tend to explode.

45. (a) BeO is Amphoteric



BeS04 is solube in water

Due to small size Be shows anomalous behaviour

46. (a) Some vacancy generated by this type defect.

47. (c) $2C_{(S)} + O_{2(g)} \rightarrow 2CO_{(g)}$

ΔS° is the, $\Delta G^\circ = \Delta H^e - T\Delta S$
Thus slope is Negative.
As temperature Increase ΔC becomes more Negative thus it has loner tendency to get decomposed.

48. (a) Assertion A correct but Reason is wrong.

49. (b)

A → (II) C → (III)
B → (I) D → (IV)

50. (a) By using catalytic convertors in the automobiles / industry.

51. (12)

$x(g) \rightleftharpoons y(g) + z(g)\ Kp_1 = 3$

$t = 0\ 1\ 0\ 0$

teq $1 - x\ x\ x$

Partial Pressure $\frac{(1-x)}{1+x}P_1 \quad \frac{xP_1}{1+x} \quad \frac{xP_1}{1+x}$

$A(g) \rightleftharpoons 2\,B(g)$

$t = 0\ 1\ 0$

teq $1 - x 2x$

Partial Pressure $\frac{1-x}{1+x} \times P_2 \quad \frac{2x}{1+x} \times P_2$

$$KP_1 = \frac{\left(\frac{xP}{1+x}\right)\left(\frac{xP_1}{1+x}\right)}{\left(\frac{1-x}{1+x}P_1\right)}$$

$$Kp_2 = \frac{(2x)^2 \times P_2{}^2}{\left(\frac{1-x}{1+x}\right)P_2}$$

$$\frac{KP_1}{KP_2} = \frac{3}{1} = \frac{P_1}{4P_2}$$

$$\frac{P_1}{P_2} = \frac{12}{1}$$

52. (2)

(A) $\lambda = \frac{h}{mv} = \frac{6\times10^{-34}}{9\times10^{-31}\times1000}$

$= 666.67 \times 10^{-9}$ m

(C) The cathode ray start from Cathode and move towards anode.

53. (15)

Condition $\Rightarrow [B] = 4[A]$

For A $A \underset{15\text{min}}{\overset{t_1}{\rightarrow}} \frac{A}{2}$

For B

$4A \underset{15\text{min}}{t_{1/2}} 2A \frac{t_{1/2}}{5\text{ min}} A \frac{t_{1/2}}{5\text{ min}} A/2$

54. (12)

Bromic Acid $\rightarrow HBrO_5 \rightarrow +5$

Perbromic Acid $\rightarrow HBrO_7 \rightarrow +7$

Sum of oxidation state = 5 + 7 = 12

55. (3)

$$\underset{V=25ml}{KCl} + \underset{\substack{V=20ml \\ M=1M}}{AgNO_3} \rightarrow AgCl + KNO_3$$

At equivalence point,

Mmole of KCl = mmole of $AgNO_3$ = 20 mmole

Volume of solution = 25ml

Mass of solution = 25gm

Mass of solvent = 25 − mass of solute

$= 25 - [20 \times 10^{-3} \times 74.5]$

$= 23.51\text{gm}$

Molality of KCl $= \frac{\text{mole of KCl}}{\text{mass of solvent in kg}}$

$= \frac{20 \times 10^{-3}}{23.51 \times 10^{-3}} = 0.85$

i of KCl = 2(100% ionisation)

$\Delta T_f = i \times K_f \times m$

$= 2 \times 2 \times 0.85$

$= 3.4$

$\simeq 3$

56. (499)

$2H_2O(g) \rightarrow H_2(g) + 2(g)$	$+(242 \times 2)$
$H_2(g) + O_2(g) \rightarrow 2OH$	+78
$H_2(g) \rightarrow H_2$	+436
$2H_2O \rightarrow 2H + 2OH$	+998KJ/mole

$H_2O \rightarrow H + OH$ $\qquad$ $998 \times \frac{1}{2} = +499 KJ/mole$

57. (3)

$$MnO_4^- + 84^{\oplus} + 5e^{\oplus} \rightleftharpoons Mn^{+2} + 4H_2O$$
$$E = E^\circ - \frac{0.059}{5} \log \frac{[mn^{+2}]}{[mnO_4^-][H^+]^8}$$
$$1.282 = 1.54 - \frac{0.059}{5} \log \frac{10^{-3}}{10^{-1} \times [H^+]}$$
$$\frac{0.258 \times 5}{0.059} = \log \frac{10^{-2}}{[H^+]^8}$$
$$21.86 = -2 + 8pH$$
$$pH = 2.98 = 3$$

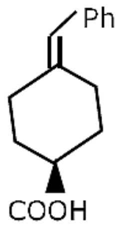
58. (**364**)

$$m = \frac{1000 \times M}{1000\, d - M \times M.wt} = \frac{1000 \times 3}{1000 \times 1 - (3 \times 58.5)} = 3.64$$
$$= 364 \times 10^{-2}$$

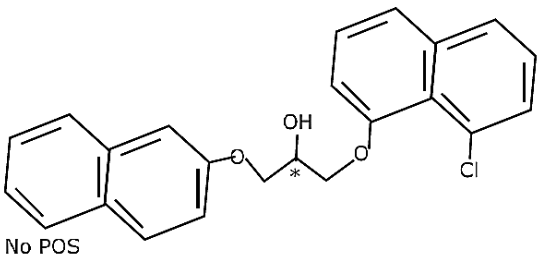
59. (2)

Do not dissolve in NaOH, So no acidic group
Do not dissolve in HCl, So no basic group, no alkene
Do not give orange PPT with 2, 4-DNP so no carbonyl group
Possible compounds - cis and trans of $Ph - CH = CH - O - CH_3$
(Also Many possible products are there)

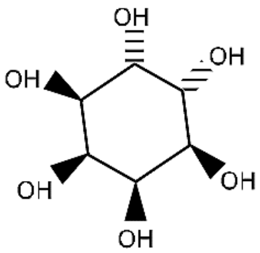
60. (2)



No POS
(1) No COS



No POS
(2) No COS

(3) 

No POS
No COS

61. (b) Given D.E. is linear D.E.

$\text{I.F.} = e^{\int \tan x dx}$

$= e^{\ell n \sec x} = \sec x$

Solution is -

$y \sec x = \int x \sec^2 x dx$

$= x \tan x - \int \tan x dx$

$\Rightarrow \; y \sec x = x \tan x - \ell n \sec x + c$

Put $y(0) = 1$

$1 = 0 - 0 + c \Rightarrow c = 1$

$$Y(x) = \frac{x \tan x}{\sec x} - \frac{\ell n \sec x}{\sec x} + \frac{1}{\sec x}$$

$$y\left(\frac{\pi}{6}\right) = \frac{\left(\frac{\pi}{6}\right)\left(\frac{1}{\sqrt{3}}\right)}{\left(\frac{2}{\sqrt{3}}\right)} - \frac{\ln\left(\frac{2}{\sqrt{3}}\right)}{\left(\frac{2}{\sqrt{3}}\right)} + \frac{\sqrt{3}}{2}$$

$$= \frac{\pi}{12} - \frac{\sqrt{3}}{2}\ln\left(\frac{2}{\sqrt{3}}\right) + \frac{\sqrt{3}}{2}\ln e$$

$$= \frac{\pi}{12} - \frac{\sqrt{3}}{2}\ln\left(\frac{2}{e\sqrt{3}}\right)$$

62. (c)

$(a, a) \in R \Rightarrow 3a - 3a + \sqrt{7}$
$\quad = \sqrt{7}$ (irrational)

$\Rightarrow$ R is reflexive

Let $a = \frac{2\sqrt{7}}{3}$ and $b = \frac{\sqrt{7}}{3}$

$(a, b) \in R \Rightarrow 2\sqrt{7} - \sqrt{7} + \sqrt{7}$

$\quad = 2\sqrt{7}$ (irration)

$(b, a) \in R \Rightarrow \sqrt{7} - 2\sqrt{7} + \sqrt{7}$
$\quad = 0$ (rational)

$\Rightarrow$ R is no symmetric

Let $a = \frac{2\sqrt{7}}{3}, b = \frac{\sqrt{7}}{3}, C = \frac{3\sqrt{7}}{3}$

(a; b) $\in$ R $\Rightarrow 2\sqrt{7}$ (irrational)

(b ; c) $\in$ R $\Rightarrow \sqrt{7}$ (irrational)

(a, c) $\in$ R $\Rightarrow 2\sqrt{7} - 3\sqrt{7} + \sqrt{7}$

$= 0$ (rational)

R is not transitive

$\Rightarrow$ R is reflexive but neither symmetric nor transitive

63. (d)

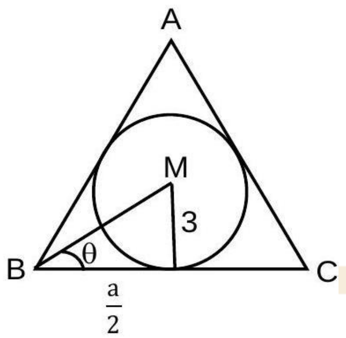
$$\begin{aligned} \text{Let} \quad P &= \cos 2A + \cos 2B + \cos 2C \\ &= 2\cos(A+B)\cos(A-B) + 2\cos^2 C - 1 \\ &= 2\cos(\pi - C)\cos(A-B) + 2\cos^2 C - 1 \\ &= -2\cos C[\cos(A-B) + \cos(A+B)] - 1 \\ &= -1 - 4\cos A\cos B\cos C \end{aligned}$$

for P to be minimum

$\cos A\cos B\cos C$ must be maximum

$\Rightarrow \triangle$ ABC is equilateral triangle.

Let side length of triangle is a



$\tan\theta = \frac{3}{a/2}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{6}{a} \Rightarrow a = 6\sqrt{3}$

area of triangle $= \frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4}(108) = 27\sqrt{3}$

64. (d)

65. **(c)** Given system of equation is inconsistent

$\Rightarrow \Delta = 0$

$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - 3\lambda + 2 = 0$

$\Rightarrow (\lambda - 1)^2(\lambda + 2) = 0$

$\Rightarrow \lambda = 1, -2$

But for $\lambda = 1$ all planes are same

Then $\lambda = -2$

$\sum_{\lambda \in s}(|\lambda|^2 + |\lambda|) = 4 + 2 = 6$

66. (**a**) Given

$np + npq = 5$

$\Rightarrow np(1 + q) = 5$

and (np) (npq) = 6

$\Rightarrow n^2p^2q = 6$

(i) $^2 \div$ (ii)

$$\frac{(1+9)^2}{9} = \frac{25}{6}$$

$\Rightarrow 6q^2 - 13q + 6 = 0$

$\Rightarrow q = \frac{2}{3}, \frac{3}{2}$ (reject)

$$P = 1 - \frac{2}{3} = \frac{1}{3}$$

$$\frac{n}{3}\left(1 + \frac{2}{3}\right) = 5$$

$\Rightarrow n = 9$

$6(n + p - q) = 52$

67. **(a)** For pair of st. liens in form

$ax^2 + b^2 + 2hxy + 2gx + 2fy + c = 0$

equation of angle bisector is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

for $2x^2 + xy - 3y^2 = 0$

$a = 2, b = -3, h = \frac{1}{2}$

equation of angle bisector is

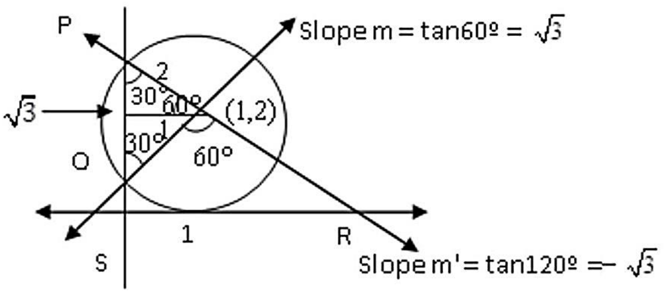
$\frac{x^2 - y^2}{5} = \frac{xy}{1/2}$

$\Rightarrow x^2 - y^2 - 10xy = 0$

68. (b)

$$\frac{dy}{dx} + \frac{x + \alpha}{y - 2} = 0, y(a) = 0$$
$$\frac{dy}{dx} = \frac{-(x + \alpha)}{y - 2}$$
$$\int (y - 2)dy = -\int (x + \alpha)dx$$
$$\frac{y^2}{2} - 2y = -\left[\frac{x^2}{2} + \alpha x\right] + \lambda$$

$$y(a) = 0$$
$$x = 1 \Rightarrow y = 0$$
$$0 - 0 = -\left[\frac{1}{2} + \alpha\right] + \lambda$$
$$\frac{y^2}{2} - 2y = -\left[\frac{x^2}{2} + \alpha x\right] + \frac{1}{2} + \alpha$$
$$\frac{x^2 + y^2}{2} = 2y - \alpha x + \frac{1}{2} + \alpha$$
$$x^2 + y^2 + 2\alpha x - 4y - 1 - 2\alpha = 0$$
$$\text{Area } = 4\pi$$
$$\pi r^2 = 4\pi$$
$$r^2 = 4$$
$$\alpha^2 + 4 + 1 + 2\alpha = 4$$
$$\alpha^2 + 2\alpha + 1 = 0$$
$$(\alpha + 1)^2 = 0 \Rightarrow [\alpha = -1]$$
$$x^2 + y^2 - 2x - 4y + 1 = 0$$



$y - 2 = \sqrt{3}(x - 1)$ $\quad$ $y - 2 = -\sqrt{3}(x - 1)$

$y = 0$ $\quad$ $y = 0$

$\frac{-2}{\sqrt{3}} = x - 1$ $\quad\quad$ $1 + \frac{2}{\sqrt{3}} = x$

$1 - \frac{2}{\sqrt{3}} = x$ $\quad\quad$ $R\left(1 + \frac{2}{\sqrt{3}}, 0\right)$

$S\left(1 - \frac{2}{\sqrt{3}}, 0\right)$

$$RS = \left(1 + \frac{2}{\sqrt{3}}\right) - \left(1 - \frac{2}{\sqrt{3}}\right) = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

69. (a)

$S = \frac{1}{1!\,50!} + \frac{1}{3!\,48!} + \frac{1}{5!\,46!} + \cdots \ldots + \frac{1}{49!\,2!} + \frac{1}{51!\,1!}$

$= \frac{1}{51!\,1!}\left(\frac{51!}{1!\,50!} + \frac{51!}{3!\,48!} + \frac{51!}{5!\,46!} + \cdots \ldots + \frac{51!}{49!\,2!} + \frac{51!}{51!\,0!}\right)$

$= \frac{1}{51!\,1!}\left({}^{51}C_{50} + {}^{51}C_{48} + {}^{51}C_{46} + \cdots \ldots\ldots + {}^{51}C_2 + {}^{51}C_0\right)$

$\because\ {}^nC_0 + {}^nC_2 + {}^nC_4 + \cdots \ldots = 2^{n-1}$

$S = \frac{2^{50}}{51!}$

70. (b) Let remaining two observations are a and b

$5 = \frac{1 + 3 + 5 + a + b}{5}$

$\Rightarrow a + b = 16$

$8 = \frac{1^2 + 3^2 + 5^2 + a^2 + b^2}{5} - 25$

$\Rightarrow a^2 + b^2 = 130$

$a^2 + b^2 = 130$

$(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow 256 = 130 + 2ab$

$ab = 63$

$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$= (16)^3 - 3(63)(16)$

$= 4096 - 3024$

$\Rightarrow a^3 + b^3 = 1072$

71. (a)

$$T_n = \frac{n}{1+n^2+n^4}$$

$$= \frac{n}{(n^2-n+1)(n^2+n+1)}$$

$$= \frac{1}{2}\left[\frac{(n^2+n+1)-(n^2-n+1)}{(n^2-n+1)(n^2+n+1)}\right]$$

$$\Rightarrow T_n = \frac{1}{2}\left[\frac{1}{(n^2-n+1)} - \frac{1}{(n^2+n+1)}\right]$$

$$S_n = \sum_{n=1}^{10} T_n$$

$$= \frac{1}{2}\Sigma\left(\frac{1}{n^2-n+1} - \frac{1}{n^2+n+1}\right)$$

$$= \frac{1}{2}\left[\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{13}\right)\right.$$

$$\left.\ldots\ldots\ldots\ldots\ldots\left(\frac{1}{91}-\frac{1}{111}\right)\right]$$

$$= \frac{1}{2}\left[1-\frac{1}{111}\right] = \frac{55}{111}$$

72. (c) $L_1: \frac{x-5}{1} = \frac{y-2}{2} = \frac{z-4}{-3}$

$$\vec{a_1} = 5\hat{i} + 2\hat{j} + 4\hat{k}$$

$$\vec{r_1} = \hat{i} + 2\hat{j} - 3\hat{k}$$

$$L_2: \frac{x+3}{1} = \frac{y+5}{4} = \frac{z-1}{-5}$$

$$\vec{a_2} = -3\hat{i} - 5\hat{j} + \hat{k}$$

$$\vec{r_2} = \hat{i} + 4\hat{j} - 5\hat{k}$$

$$\vec{r_1} \times \vec{r_2} = \begin{vmatrix} \hat{i} & j & k \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}$$

$$= 2\hat{i} + 2\hat{j} + 2\hat{k}$$

Shortest distance (d) $= \left|\frac{(\vec{r_1}\times\vec{r_2})\cdot(\vec{a_1}-\vec{a_2})}{|\vec{r_1}\times\vec{r_2}|}\right|$

$$= \frac{36}{2\sqrt{3}} = 6\sqrt{3}$$

73. (a) $\lim_{n\to\infty}\left[\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+\cdots\ldots+\frac{1}{2n}\right]$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{r+n}$$

$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\left(\frac{1}{\frac{r}{n}+1}\right)$

$= \int_0^1 \frac{dx}{x+1}$

$= \log_e(1+x)|_0^1$

$= \log_e^2$

74. (c) let $Q(\alpha,\beta,\gamma)$ is image of $P(2,-1,3)$ in the plane $x+2y-z=0$

$\frac{\alpha-2}{1} = \frac{\beta+1}{2} = \frac{\gamma-3}{-1} = \frac{-2(2-2-3)}{1^2+2^2+(-1)^2} = 1$

$\alpha = 3, \beta = 1, \gamma = 2$

Distance of $Q(3,1,2)$ from

$3x+2y+z+29=0$

$D = \frac{|3(c)+2(a)+2+29|}{\sqrt{3^2+2^2+1^2}}$

$= \frac{42}{\sqrt{14}} = 3\sqrt{14}$

75. (b) $f'(x) = 2 + \frac{1}{1+x^2} > 0$ for $x \in [0,3]$

$f(x) \uparrow$ for $x \in [0,3]$

$f(0) = 0, f(c) = 6 + \tan^{-1}(c)$

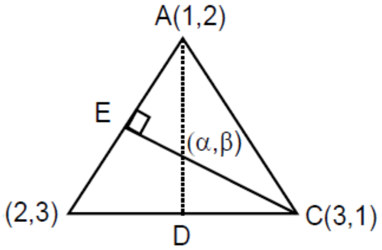
$g'(x) = \frac{\frac{x}{\sqrt{x^2+1}}+1}{x+\sqrt{1+x^2}} = \frac{1}{\sqrt{1+x^2}} > 0$ for $x \in [0,3]$

$g(x) \uparrow$ for $x \in [0,3]$

$g(0) = 0, g(c) = \log_e(\sqrt{10}+3)$

$\max f(x) > \max g(x)$

76. (a)



equation of AD: $x-2y+3=0$

equation of CE: $x+y-4=0$

orthocenter (α, β) is $\left(\frac{5}{3}, \frac{7}{3}\right)$

$\alpha + 4\beta = 11$ and $4\alpha + \beta = 9$

Quadratic equation is

$x^2 - (11+9)x + (11 \times 9) = 0$

$\Rightarrow x^2 - 20x + 99 = 0$

77. (a)

$$(\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4} = 10$$
$$\Rightarrow (\sqrt{3}+\sqrt{2})^{x^2-4} + \frac{1}{(\sqrt{3}+\sqrt{2})^{x^2-4}} = 10$$

Let $(\sqrt{3}+\sqrt{2})^{x^2-4} = t$

$t + \frac{1}{t} = 10$

$\Rightarrow t^2 - 10t + 1 = 0$

$t = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$

If $t = 5 + 2\sqrt{6}$
$(\sqrt{3}+\sqrt{2})^{x^2-4} = (\sqrt{3}+\sqrt{2})^2$
$\Rightarrow x^2 - 4 = 2$
$\Rightarrow x = \pm\sqrt{6}$

If $t = 5 - 2\sqrt{6}$
$(\sqrt{3}+\sqrt{2})^{x^2-4} = (\sqrt{3}+\sqrt{2})^{-2}$
$\Rightarrow x^2 - 4 = -2$
$\Rightarrow x = \pm\sqrt{2}$

$S = \{\sqrt{6}, -\sqrt{6}, \sqrt{2}, -\sqrt{2}\}$
$n(s) = 4$

78. (b) Put $z = x + iy$

$$\frac{|(x-2)+iy|}{|(x-3)+iy|} = 2$$

$\Rightarrow (x-2)^2 + y^2 = 4((x-3)^2 + y^2)$

$\Rightarrow x^2 - 4x + 4 + y^2 = 4x^2 - 24x + 36 + 4y^2$

$\Rightarrow 3x^2 + 3y^2 - 20x + 32 = 0$

$\Rightarrow x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0$

Center $(\alpha, \beta) = \left(\frac{10}{3}, 0\right)$

$$\text{Radius}(\gamma) = \sqrt{\left(-\frac{10}{3}\right)^2 - \frac{32}{3}} = \frac{2}{3}$$

$$3\left(\frac{10}{3} + 0 + \frac{2}{3}\right) = 12$$

79. (b)

$$f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin x \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$= (1+\sin^2 x) - \cos^2 x(-1) + \sin 2x$

$f(x) = 2 + \sin 2x$

$2x \in \left[\frac{\pi}{3}, \frac{2\pi}{3}\right] \Rightarrow \frac{\sqrt{3}}{2} \le \sin 2x \le 1$

$\alpha = 2 + 1 = 3$

$\beta = 2 + \frac{\sqrt{3}}{2}$

$\beta^2 - 2\sqrt{\alpha} = \left(2 + \frac{\sqrt{3}}{2}\right)^2 - 2\sqrt{3}$

$= 4 + \frac{3}{4} + 2\sqrt{3} - 2\sqrt{3}$

$= \frac{19}{4}$

80. (d)

$\sim (q \vee (\sim q) \wedge p)$
$= \sim q \wedge (q \vee \sim p)$
$= (\sim q \wedge q) \vee (\sim q \wedge \sim p)$
$= F \vee (\sim q \wedge \sim p) = (\sim q) \wedge (\sim p)$

81. (3501)

$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -3 \\ 2\alpha & 1 & -1 \end{vmatrix}$$

$= \hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k}$

$[uvw] = \vec{u}.(\vec{v} \times \vec{w})$

$= |\vec{u}||\vec{v} \times \vec{w}|\cos\theta$

since $[uvw]$ is Least $\Rightarrow \cos\theta = -1$

$[uvw] = \left(|\vec{u}|\sqrt{1+25\alpha^2+9\alpha^2}\right)(-1)$

$\Rightarrow -\alpha\sqrt{1+34\alpha^2} = -\alpha\sqrt{3401}$

$\Rightarrow \alpha^2 = 100$

$\Rightarrow \alpha = 10 \{\because \alpha > 0\}$

$\vec{u}$ is parallel to $\vec{v}\times\vec{w}$

$\vec{u} = \lambda(\vec{v}\times\vec{w})$

$\vec{u} = \lambda(\hat{\imath} - 50\hat{\jmath} - 30\hat{k})$

$|\vec{u}| = 10$

$|\lambda|\sqrt{3401} = 10$

$|\lambda| = \frac{10}{\sqrt{3401}}\ \vec{u} = \pm\frac{10}{\sqrt{3401}}(\hat{\imath} - 50\hat{\jmath} - 30\hat{k})$

$|\vec{u}.\hat{\imath}|^2 = \frac{100}{3401} = \frac{m}{n}$

$m + n = 100 + 3401 = 3501$

82. (50400)

$A-3, I-2, S-4, N-2, O-1, T-1$

As vowels are together

Total words formed $= \left(\frac{8!}{4!2!}\right)\left(\frac{6!}{3!2!}\right)$

$= \left(\frac{8\times7\times6\times5}{2}\right)\left(\frac{6\times5\times4}{2}\right) = 50400$

83. (29)

$19^{200} + 23^{200}\ a^{n+}b^{n}$

$19^3 = 6859 = 140\times49 - 1$

$= 49\lambda - 1$

$(19^3)^{66} = (49\lambda-1)^{66}$

So, Remainder of 19^{198} divided by 49

is $(-1)^{66} = 1$
$19^2 = 361$ gives remainder 18

So, 19^{200} gives remainder 18

23^2 gives remainder 39

$(23)^3$ gives remainder 15

$(23)^4$ gives remainder 2

$((23)^4)^6$ gives remainder $(2)^6 = 64$

&64 gives remainder 15

$(23)^{24} \longrightarrow 15$

$(23)^{25} \longrightarrow 2$

$((23)^{25})^8 \longrightarrow (2)^8 = 256 \longrightarrow 11$

So, Total remainder $= 18 + 11 = 29$

84. (514)

3-digit numbers divisible by either 2 or 3

$\mathrm{P} = \mathrm{n}(\text{ divisible by } 2) + \mathrm{n}(\text{ divisible by } 3) - \mathrm{n}(\text{ divisible by } 6)$

$\mathrm{P} = 450 + 300 - 150$

$\mathrm{P} = 600$

$\mathrm{Q} = \mathrm{n}(\text{ divisible by } 14) + \mathrm{n}(\text{ divisible by } 21) - \mathrm{n}(\text{ divisible by } 42)$

$= 64 + 43 - 21 = 86$

3-digit number divisible by either 2 or 3

But not divisible by -1 so $\mathrm{P} - \mathrm{Q} = 600 - 86 = 514$

85. (1)

Let $\int_0^2 f(t)dt = \lambda$

$f'(x) + f(x) = \lambda$

is linear Differential equation

$\text{I.f.} = e^{\int dx} = e^x$

$f(x) \cdot e^x = \int e^x \lambda dx$

$\Rightarrow f(x) \cdot e^x = \lambda e^x + C$

$\Rightarrow f(x) = \lambda + Ce^{-x}$

put $f(0) = e^{-2}$

$e^{-2} = \lambda + C \Rightarrow C = e^{-2} - \lambda$

$f(x) = \lambda + (e^{-2} - \lambda)e^{-x}$

$\lambda = \int_0^2 f(t)dt$

$= \int_0^2 (\lambda + (e^{-2} - \lambda)e^{-t})dt$

$\Rightarrow \lambda = \lambda + \lambda e^{-2} - e^{-4} + e^{-2}$

$\Rightarrow \lambda = e^{-2} - 1$

$f(x) = e^{-2} - 1 + e^{-x}$
$f(0) = e^{-2}$

$f(b) = 2e^{-2} - 1$

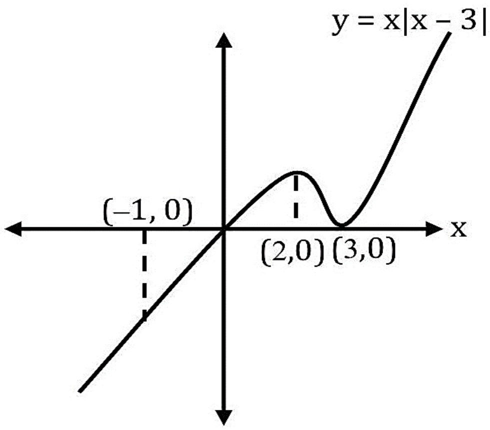
$2f(0) - f(b) = 1$

86. (14)

let $g'(1) = A$
$g''(2) = B$
$f(x) = x^2 + Ax + B$
$f(1) = A + B + 1$
$f'(x) = 2x + A$
$f''(x) = 2$
$g(x) = (A + B + 1)x^2 + x(2x + A) + 2$
$\Rightarrow g(x) = x^2(A + B + 2) + Ax + 2$
$g'(x) = 2x(A + B + 2) + A$
$g'(1) = A$
$\Rightarrow 2(A + B + 2) + A = A$
$A + B = -2$
$g''(x) = 2(A + B + 2)$
$g''(2) = B$
$\Rightarrow 2(A + B + 2) = B$
$\Rightarrow 2A + B = -4$
From (i) and (ii)
$A = -2$ and $B = 0$
$f(x) = x^2 - 2x$
$f(4) = 16 - 8 = 8$
$g(x) = -2x + 2$
$g(4) = -8 + 2 = -6$
$f(4) - g(4) = 8 - (-6) = 14$

87. (62)

$y = x|x - 3| = \begin{cases} x(x-3); x \geq 3 \\ x(3-x); x < 3 \end{cases}$



$A = -\int_{-1}^{0} x(3-x)dx + \int_{0}^{2} x(3-x)dx$

$= \int_{-1}^{0} (x^2 - 3x)dx + \int_{0}^{2} (3x - x^2)dx$

$= \left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_{-1}^{0} + \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_{0}^{2}$

$A = 0 - \left(\frac{-1}{3} - \frac{3}{2}\right) + 6 - \frac{8}{3} = \frac{31}{6}$

$A = 12\left(\frac{31}{6}\right) = 62$

88. (63)

$I = \int_{0}^{1} (x^{21} + x^{14} + x^7)(2x^{14} + 3x^7 + 6)^{\frac{1}{7}}dx$

$= \int_{0}^{1} (x^{20} + x^{13} + x^6)(2x^{21} + 3x^{14} + 6x^7)^{\frac{1}{7}}dx$

Put $2x^{21} + 3x^{14} + 6x^7 = t$

$\Rightarrow 42(x^{20} + x^{13} + x^6)dx = dt$

$\Rightarrow (x^{20} + x^{13} + x^6)dx = \frac{dt}{42}$

$I = \int_{0}^{11} \frac{t^{\frac{1}{7}}}{42} dt$

$= \frac{1}{42}\left[\frac{t^{\frac{8}{7}}}{\frac{8}{7}}\right]_{0}^{11}$

$= \left(\frac{7}{8}\right)\left(\frac{1}{42}\right)(11)^{8/7}$

$= \frac{1}{48}(11)^{8/7} = \frac{1}{\ell}(11)^{m/n}$

$\ell + m + n = 48 + 8 + 7 = 63$

89. (754)

$a_1 = 8$

$d =$ common difference

$\frac{4}{2}[16 + 3d] = 50$

$\Rightarrow d = 3$

$$\frac{4}{2}[2a_n + 3(-d)] = 170$$
$$\Rightarrow 2(a_1 + (n-1)d) - 3d = 85$$
$$\Rightarrow 16 + 6(n-1) - 9 = 85$$
$$n - 1 = 13$$
$$n = 14$$

Product of middle two terms $= T_7 \times T_8$

$= (a_1 + 6d)(a_1 + 7d)$

$= (8 + 18)(8 + 21)$

$= (26)(29) = 754$

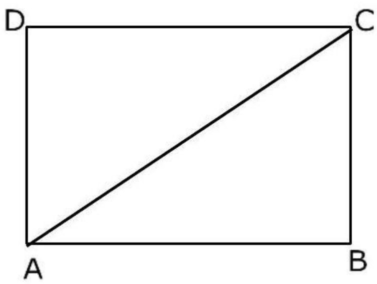
90. (11)

$\overrightarrow{AD} = 2\hat{i} - \hat{j} - 2\hat{k}$

$\overrightarrow{AC} = -3j - 3\hat{k}$

$\overrightarrow{AD} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -2 \\ 0 & -3 & -3 \end{vmatrix}$

$= -3\hat{i} + 6\hat{j} - 6\hat{k}$



Area $(\triangle ADC) = \frac{1}{2}|\overrightarrow{AD} \times \overrightarrow{AC}|$

$= \frac{1}{2}\sqrt{9 + 36 + 36} = \frac{9}{2}$

$\overrightarrow{AB} = -6\hat{i} - 6\hat{j} + (\lambda - 2)\hat{k}$

$\overrightarrow{AC} = -3\hat{j} - 3\hat{k}$

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & -6 & \lambda - 2 \\ 0 & -3 & -3 \end{vmatrix}$

$= (12 + 3\lambda)\hat{i} - 18\hat{j} + 18\hat{k}$

area $(\triangle \mathrm{ABC}) = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$= \frac{3}{2}\sqrt{(4+\lambda)^2 + 36 + 36}$

$\text{Area}(\triangle \mathrm{ABCD}) = \text{ar}(\triangle \mathrm{ADC}) + \text{ar}(\triangle \mathrm{ABC})$

$\Rightarrow 18 = \frac{9}{2} + \frac{3}{2}\sqrt{(4+\lambda)^2 + 72}$

$\Rightarrow (4+\lambda)^2 = 9$

$4 + \lambda = 3$ or $4 + \lambda = -3$

$\Rightarrow \lambda = -1$ or $\lambda = -7$ (reject)

$5 - 6\lambda = 5 + 6 = 11$