1. (b)

2. (b)

$$g' = \frac{GMe}{(3R)^2} = \frac{1}{9}\ g$$

$$T = 2\pi\sqrt{\frac{\ell}{g'}}$$

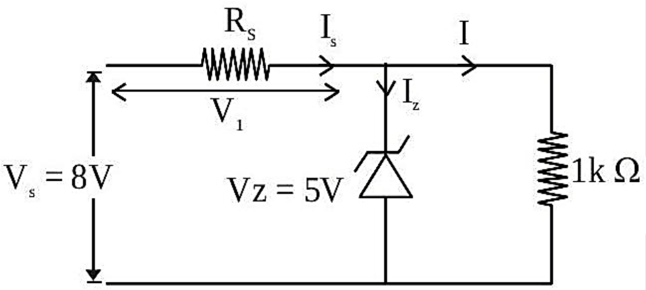
Since the time period of second pendulum is 2sec.

$T = 2\text{sec}$

$$2 = 2\pi\sqrt{\frac{\ell}{g}9}$$

$$\ell = \frac{1}{9}\ \text{m}$$

3. (d)



Pd across R_S

$V_1 = 8 - 5 = 3\ \text{V}$

Current through the load resistor

$$I = \frac{5}{1 \times 10^3} = 5\ \text{mA}$$

Maximum current through Zener diode

$$I_{z\ max.} = \frac{10}{5} = 2\ \text{mA}$$

And minimum current through Zener diode

$I_{z\ min.} = 0$

$\therefore I_{s\ max.} = 5 + 2 = 7\ \text{mA}$

And $R_{s\ min} = \frac{V_1}{I_{s\ max}} = \frac{3}{7}\text{k}\Omega$

Similarly

$I_{s\ min.} = 5\ \text{mA}$

And $R_{s\ max.} = \frac{V_1}{I_{s\ min.}} = \frac{3}{5} k\Omega$

$\therefore \frac{3}{7} k\Omega < R_s < \frac{3}{5} k\Omega$

4. (c)

$$i = \frac{E_{eq}}{r_{eq}} = \frac{8 \times 5}{8 \times 0.2}$$

$I = 25A$

$V = E - ir$

$= 5 - 0.2 \times 25$

$= 0$

5. (a)

$$V_C = \frac{q_{net}}{C_{net}} = \frac{CV + 2CV}{2C}$$

$$V_C = \frac{3\ V}{2}$$

Loss of energy

$$= \frac{1}{2} CV^2 + \frac{1}{2} C(2\ V)^2 - \frac{1}{2} 2C \left(\frac{3\ V}{2}\right)^2$$

$$= \left(\frac{CV^2}{4}\right)$$

6. (a)

$$C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$

$$= \frac{2 \times \frac{3}{2} R + 6 \times \frac{5}{2} R}{2 + 6}$$

$$= \frac{9}{4} R$$

7. (b) $T = m\omega^2 \ell$

$400 = 0.5\omega^2 \times 0.5$

$\omega = 40 \mathrm{rad/s}.$

8. (d) $I = \frac{V}{X_C} = 230 \times 300 \times 200 \times 10^{-12} = 13.8\mu A$

9. (a, b)

For $PV^x =$ constant

If work done by gas is asked then

$$W = \frac{nR\Delta T}{1-x}$$

Here $x = \frac{3}{2}$

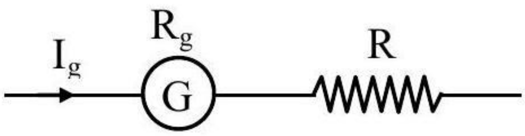
$$\therefore W = \frac{P_2 V_2 - P_1 V_1}{-\frac{1}{2}}$$

$$= 2(P_1 V_1 - P_2 V_2) \text{ ..... Option (a) is correct}$$

If work done by external is asked then

$W = -2(P_1 V_1 - P_2 V_2)$ .... Option (b) is correct

10. (c)



$$R = \frac{V}{I_g} - R_g = \frac{100}{5 \times 10^{-3}} - 50$$

$$= 20000 - 50$$

$$= 19950\Omega$$

11. (d)

$$\lambda = \frac{h}{p} = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda}$$

$$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} \times \frac{\lambda_\alpha}{\lambda_p}$$

$$= 4 \times 2 = 8$$

12. (d) $10\text{MSD} = 11\text{VSD}$

$$1\text{VSD} = \frac{10}{11}\text{MSD}$$

$$\text{LC} = 1\text{MSD} - 1\text{VSD}$$

$$= 1\text{MSD} - \frac{10}{11}\text{MSD}$$

$$= \frac{1\text{MSD}}{11}$$

$$= \frac{5}{11} \text{ units}$$

13. (c)

$\omega' = \omega$

$$\frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{LC}}$$

$\therefore L'C' = LC$

$L'(4C) = LC$

$L' = \frac{L}{4}$

$\because$ Inductance must be decreased by $\frac{3L}{4}$

14. (b)

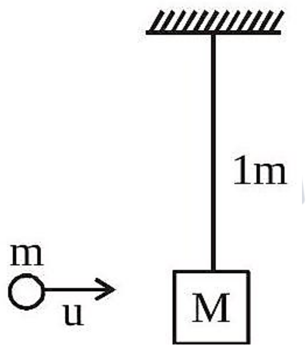
$\rho = R\frac{\rho}{\ell}$

$$\begin{aligned}\frac{\Delta\rho}{\rho} &= \frac{\Delta R}{R} + 2\frac{\Delta r}{r} + \frac{\Delta\ell}{\ell} \\ &= \frac{10}{100} + 2\times\frac{0.05}{0.35} + \frac{0.2}{15} \\ &= \frac{1}{10} + \frac{2}{7} + \frac{1}{75} \\ \frac{\Delta\rho}{\rho} &= 39.9\%\end{aligned}$$

15. (d) Angular impulse = change in angular momentum.

[Angular impulse] = [Angular momentum] = [mvr] = $[ML^2\ T^{-1}]$

16. (b)



$$\begin{aligned}&mu = (M+m)V \\ &10^{-2}\times 2\times 10^2 \cong 1\times V \\ &V \cong 2\ m/s \\ &h = \frac{V^2}{2\ g} = 0.2\ m\end{aligned}$$

17. (a)

$\frac{2\pi R}{T} = V$

Maximum height $H = \frac{v^2 \sin^2\theta}{2\ g}$

$4R = \frac{4\pi^2R^2}{T^2 2g}\sin^2\theta$

$$\sin\theta = \sqrt{\frac{2gT^2}{\pi^2 R}}$$

$$\theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2 R}\right)^{\frac{1}{2}}$$

18. (c) $f_k = \mu N = 0.04 \times 20\ g = 8$ Newton

$$a = \frac{60 - 8}{26} = 2\ m/s^2$$

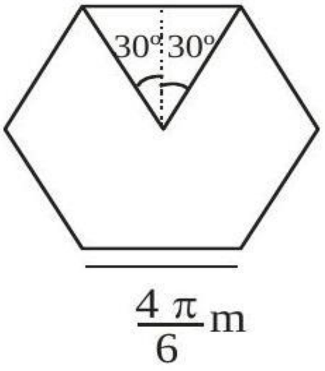
19. (d) Transition from $n = 1$ to $n = 3$

$\Delta E = 12.1 eV$

20. (b) Linear width

$$W = \frac{2\lambda d}{a} = \frac{2 \times 6 \times 10^{-7} \times 0.2}{1 \times 10^{-5}}$$
$$= 2.4 \times 10^{-2} = 24\ mm$$

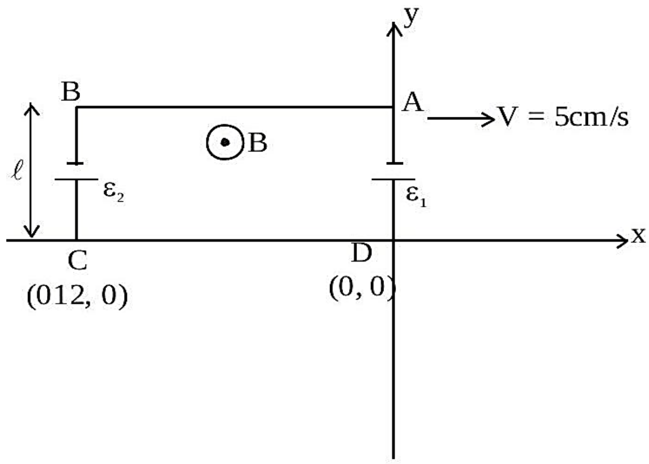
21. (72)



$$B = 6\left(\frac{\mu_0 I}{4\pi r}\right)(\sin 30° + \sin 30°)$$

$$= 6\frac{10^{-7} \times 4\pi\sqrt{3}}{\left(\frac{\sqrt{3} \times 4\pi}{2 \times 6}\right)}$$
$$= 72 \times 10^{-7}\ T$$

22. (216)



B_0 is the magnetic field at origin

$$\frac{dB}{dx} = -\frac{10^{-3}}{10^{-2}}$$

$$\int_{B_0}^{B} dB = -\int_0^x 10^{-1} dx$$

$$B - B_0 = -10^{-1}x$$

$$B = \left(B_0 - \frac{x}{10}\right)$$

Motional emf in AB $= 0$

Motional emf in CD $= 0$

Motional emf in AD $= \varepsilon_1 = B_0 \ell v$

Magnetic field on rod BC B

$$= \left(B_0 - \frac{(-12 \times 10^{-2})}{10}\right)$$

Motional emf in BC $= \varepsilon_2 = \left(B_0 + \frac{12\times10^{-2}}{10}\right)\ell \times V$

$\varepsilon_{eq} = \varepsilon_2 - \varepsilon_1 = 300 \times 10^{-7}$ V

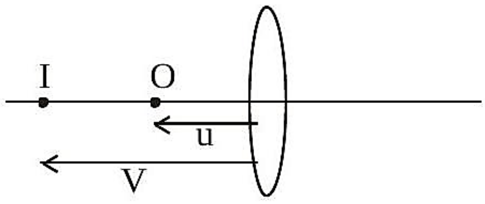
For time variation

$$(\varepsilon_{eq})' = A\frac{dB}{dt} = 60 \times 10^{-7} \text{ V}$$

$$(\varepsilon_{eq})_{net} = \varepsilon_{eq} + (\varepsilon_{eq})' = 360 \times 10^{-7} \text{ V}$$

$$\text{Power} = \frac{(\varepsilon_{eq})^2_{net}}{R} = 216 \times 10^{-9} \text{ W}$$

23. (15)



$v = 3u$

$v - u = 20 \text{ cm}$

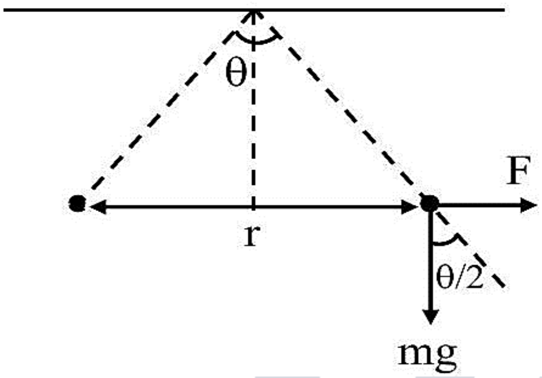
$2u = 20 \text{ cm}$

$u = 10 \text{ cm}$

$$\frac{1}{(-30)} - \frac{1}{(-10)} = \frac{1}{f}$$

$f = 15 \text{ cm}$

24. (3)



In air $\tan\frac{\theta}{2} = \frac{F}{mg} = \frac{q^2}{4\pi\varepsilon_0 r^2 mg}$

In water $\tan\frac{\theta}{2} = \frac{F'}{mg'} = \frac{q^2}{4\pi\varepsilon_0 \varepsilon_r r^2 mg_{eff}}$

Equate both equations

$$\varepsilon_0\, g = \varepsilon_0 \varepsilon_r g\left[1 - \frac{1}{1.5}\right]$$

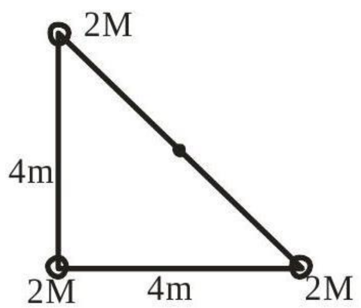
$$\varepsilon_r = 3$$

25. (27)

$R = R_0\, A^{1/3}$

$$R^3 \propto A$$
$$\left(\frac{4.8}{4}\right)^3 = \frac{64}{A}$$
$$= \frac{64}{A} = (1.2)^3$$
$$\frac{64}{A} = 1.44 \times 1.2$$
$$A = \frac{64}{1.44 \times 1.2} = \frac{1000}{x}$$
$$x = \frac{144 \times 12}{64} = 27$$

26. (3)



Position vector $\vec{r}_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$

$$\vec{r}_{COM} = \frac{2M \times 0 + 2M \times 4\hat{i} + 2M \times 4\hat{j}}{6M}$$
$$\vec{r} = \frac{4}{3}\hat{i} + \frac{4}{3}\hat{j}$$
$$|\vec{r}| = \frac{4\sqrt{2}}{3}$$
$$x = 3$$

27. (6)

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

$$f_1 = \frac{1}{2}\sqrt{\frac{6}{\mu}} \quad f_2 = \frac{1}{2}\sqrt{\frac{54}{\mu}}$$
$$\frac{f_1}{f_2} = \frac{1}{3} \quad f_2 - f_1 = 12$$
$$f_1 = 6HZ$$

28. (9600)

$A = 80\ m^2$

Using Bernonlli equation

$$A(P_2 - P_1) = \frac{1}{2}\rho(V_1^2 - V_2^2)A$$

$\mathrm{mg} = \frac{1}{2} \times 1(70^2 - 50^2) \times 80$

$\mathrm{mg} = 40 \times 2400$

$\mathrm{m} = 9600\ \mathrm{kg}$

29. (22)

$q = \int_1^2 i dt = \int_1^2 (3t^2 + 4t^3)dt$

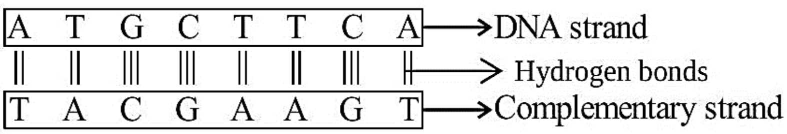
$q = (t^3 + t^4)|_1^2$
$q = 22C$

30. (52)

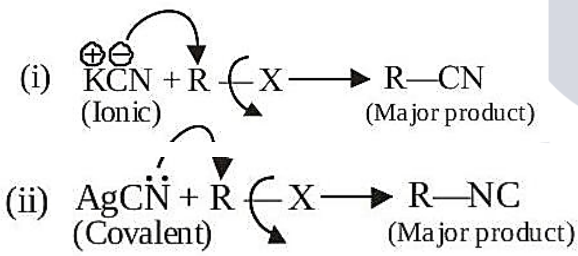
$x = 3t^3 + 18t^2 + 16t$

$\mathbf{v} = -9t^2 + 36 + 16$
$a = -18t + 36$
$a = 0$ at $t = 2$ s
$v = -9(2)^2 + 36 \times 2 + 16$
$v = 52\ \mathrm{m/s}$

31. (b) Adenine base pairs with thymine with 2 hydrogen bonds and cytosine base pairs with guanine with 3 hydrogen bonds.



32. (a)



AgCN is mainly covalent in nature and nitrogen is available for attack, so alkyl isocyanide is formed as main product.

33. (d) The balanced reaction is,

$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$

$X = 14$

$Y = 6$

$A = 7$

34. (a) When a particular oxidation state becomes less stable relative to other oxidation state, one lower, one higher, it is said to undergo disproportionation. $Cu^+ \rightarrow Cu^{2+} + Cu$

$3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

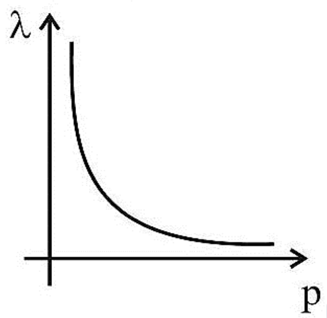
35. **(d)** In F^-, Ne, Na^+ all have $1s^2, 2s^2, 2p^6$ configuration. They have different size due to the difference in nuclear charge.

36. **(a)**

$$\lambda = \frac{h}{p}\left[\lambda \propto \frac{1}{p}\right]$$

$\Rightarrow \lambda p = h$ (constant)

So, the plot is a rectangular hyperbola.

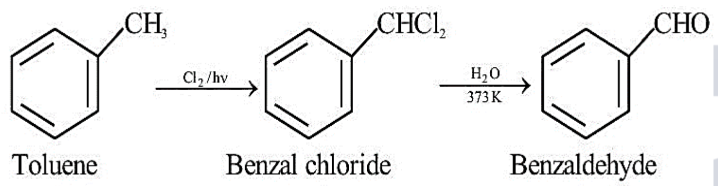


37. **(b)** $[Ni(H_2O)_6]^{+2} \rightarrow$ Green colour solution due to d-d transition

$[Ni(CN)_4]^{-2} \rightarrow$ is diamagnetic and it is colourless.

38. **(d)** Unlike NH_3, PH_3 molecules are not associated through hydrogen bonding in liquid state. That is why the boiling point of PH_3 is lower than NH_3.

39. **(b)**



40. **(b)** Aniline is also known as amino benzene.

41. **(a)** In Homoleptic complex all the ligand attached with the central atom should be the same. Hence $[Ni(CN)_4]^{2-}$ is a homoleptic complex.

42. **(d)** Higher the electron density in the benzene ring more easily it will be attacked by an electrophile. Phenol has the highest electron density amongst all the given compound.

43. **(c)** Heterolytic cleavage of Bond lead to formation of ions.

44. **(c)** Increasing order of ionic character

$N_2 < SO_2 < ClF_3 < K_2O < LiF$

Ionic character depends upon difference of electronegativity (bond polarity).

45. **(a)**

Salt	Values of i (for different conc. of a Salt)

	0.1M	0.01M	0.001M
NaCl	1.87	1.94	1.94

i approach 2 as the solution become very dilute.

46. (b) Kjeldahl's method is used for estimation of Nitrogen where $CuSO_4$ acts as a catalyst.

47. (a) Statement (I) : Potassium hydrogen phthalate is a primary standard for standardisation of sodium hydroxide solution as it is economical and its concentration does not changes with time.

Phenophthalin can acts as indicator in acid base titration as it shows colour in pH range 8.3 to 10.1

48. (b)

$$CH_3(CH_2)_5COOC_2H_5 \xrightarrow{\text{DIBAL-H, } H_2O} CH_3(CH_2)_5CHO$$

$$C_6H_5COC_6H_5 \xrightarrow{\text{Zn(Hg) \& conc. HCl}} C_6H_5CH_2C_6H_5$$

$$C_6H_5CHO \xrightarrow[H_2O]{CH_3MgBr} C_6H_5CH(OH)CH_3$$

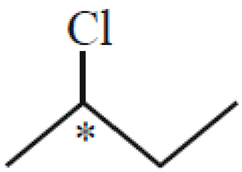
$$CH_3COCH_2COOC_2H_5 \xrightarrow{NaBH_4, H^+} CH_3CH(OH)CH_2COOC_2H_5$$

49. (d) During free expansion of an ideal gas under adiabatic condition $q = 0, \Delta T = 0, w = 0$.

50. (a) The NH_2 group in Aniline is ortho and para directing and a powerful activating group as NH_2 has strong +M effect.

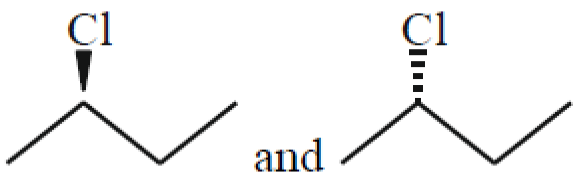
Aniline does not undergo Friedel-Craft's reaction (alkylation and acylation) as Aniline will form complex with $AlCl_3$ which will deactivate the benzene ring.

51. (2)



There is one chiral centre present in given compound.

So, Total optical isomers = 2



52. (a)

$$E = E^{o}_{H^+/H_2} - \frac{0.06}{2}\log\frac{P_{H_2}}{[H^+]^2}$$

$$E = 0.00 - \frac{0.06}{2}\log\frac{2}{[1]^2}$$

$$E = -0.03 \times 0.3 = -0.9 \times 10^{-2}\ V$$

53. (5)

$SrSO_4$ – white

$Mg(NH_4)PO_4$ – white

$BaCrO_4$ – yellow

$Mn(OH)_2$ - white

$PbSO_4$ – white

$PbCrO_4$ - yellow

AgBr - pale yellow

PbI_2 – yellow

CaC_2O_4 - white

$[Fe(OH)_2(CH_3COO)]$ – Brown Red

54. (17190)

$$\lambda t = \ln\frac{({}^{4}C/{}^{12}C)_{\text{atmosphhere}}}{({}^{14}C/{}^{12}C)_{\text{wood sample}}}$$

As per the question,

$$\frac{({}^{14}C/{}^{12}C)_{\text{wood}}}{({}^{14}C/{}^{12}C)_{\text{atmosphere}}} = \frac{1}{8}$$

So, $\lambda t = \ln 8$

$$\frac{\ln 2}{t_{1/2}} t = \ln 8$$

$t = 3 \times t_{1/2} = 17190$ years

55. (3)

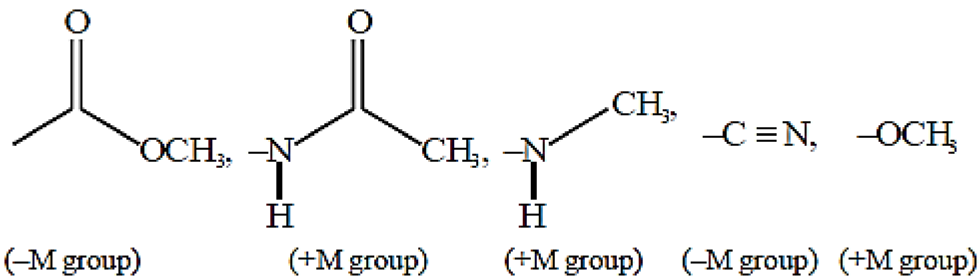
$PF_5, PCl_5, Fe(CO)_5$; Trigonal bipyramidal

BrF_5; square pyramidal

$[PtCl_4]^{-2}$; square planar

BF_3; Trigonal planar

56. (2)



57. (6) $A_2B \rightarrow 2A^+ + B^{-2}$, B^{-2} has complete octet in its dianionic form, thus in its atomic state it has 6 electrons in its valence shell. As it has negative charge, it has acquired two electrons to complete its octet.

58. (c)

Acidic oxide: Cl_2O_7, SiO_2, N_2O_5

Neutral oxide: CO, NO, N_2O

Amphoteric oxide: Al_2O_3, SnO_2, PbO_2

59. (24)

Limiting Reagent is $PbCl_2$

mmol of $Pb_3(PO_4)_2$ formed

$= \dfrac{\text{mmol of } PbCl_2 \text{ reacted}}{3}$

$= 24\text{mmol}$

60. (7)

$\mathrm{pH} = \dfrac{pK_w + pK_a - pK_b}{2}$

$pK_a = pK_b$

$\Rightarrow \mathrm{pH} = \dfrac{pK_w}{2} = 7$

61. (b)

$P(4\,W4\,B/2\,W2\,B) = P(4W4B) \times P(2W2B/4W4B)$

$\overline{P(2W6B) \times P(2W2B/2W6B) + P(3W5B) \times P(2W2B/3W5B)}$

$+ \cdots \ldots \ldots . . + P(6W2B) \times P(2W2B/6W2B)$

$$= \frac{\frac{1}{5} \times \frac{{}^4C_2 \times {}^4C_2}{{}^8C_4}}{\frac{1}{5} \times \frac{{}^2C_2 \times {}^6C_2}{{}^8C_4} + \frac{1}{5} \times \frac{{}^3C_2 \times {}^5C_2}{{}^8C_4} + \cdots + \frac{1}{5} \times \frac{{}^6C_2 \times {}^2C_2}{{}^8C_4}}$$

$= \dfrac{2}{7}$

62. (c) $\int_0^{\frac{\pi}{4}} \frac{xdx}{\sin^4(2x)+\cos^4(2x)}$

Let $2x = t$ then $dx = \frac{1}{2}dt$

$$I = \frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{tdt}{\sin^4 t + \cos^4 t}$$

$$I = \frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - t\right) dt}{\sin^4\left(\frac{\pi}{2} - t\right) + \cos^4\left(\frac{\pi}{2} - t\right)}$$

$$I = \frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2} dt}{\sin^4 t + \cos^4 t} - I$$

$$2I = \frac{\pi}{8}\int_0^{\frac{\pi}{2}} \frac{dt}{\sin^4 t + \cos^4 t}$$

$$2I = \frac{\pi}{8}\int_0^{\frac{\pi}{2}} \frac{\sec^4 tdt}{\tan^4 t + 1}$$

Let $\tan t = y$ then $\sec^2 tdt = dy$

$$2I = \frac{\pi}{8}\int_0^{\infty} \frac{(1+y^2)dy}{1+y^4}$$

$$= \frac{\pi}{16}\int_0^{\infty} \frac{1+\frac{1}{y^2}}{y^2+\frac{1}{y^2}}dy$$

Put $y - \frac{1}{y} = p$

$$I = \frac{\pi}{16}\int_{-\infty}^{\infty} \frac{dp}{p^2+(\sqrt{2})^2}$$

$$= \frac{\pi}{16\sqrt{2}}\left[\tan^{-1}\left(\frac{p}{\sqrt{2}}\right)\right]_{-\infty}^{\infty}$$

$$I = \frac{\pi^2}{16\sqrt{2}}$$

63. (b) $A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix} \Rightarrow \det(A) = 3$

$B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow \det(B) = 1$

Now $C = ABA^T \Rightarrow \det(C) = (\text{dct}(A))^2 \text{x} \det(B)$

$|C| = 9$

Now $|X| = |A^T C^2 A|$

$= |A^T||C|^2|\,A|$

$= |A|^2|C|^2$

$= 9 \times 81$

$= 729$

64. (a) Finding $\tan(A+B)$ we get

$\Rightarrow \tan(A+B) =$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{1}{\sqrt{x(x^2+x+1)}} + \frac{\sqrt{x}}{\sqrt{x^2+x+1}}}{1 - \frac{1}{x^2+x+1}}$$

$$\Rightarrow \tan(A+B) = \frac{(1+x)\left(\sqrt{x^2+x+1}\right)}{(x^2+x)(\sqrt{x})}$$

$$\frac{(1+x)\left(\sqrt{x^2+x+1}\right)}{(x^2+x)(\sqrt{x})}$$

$$\tan(A+B) = \frac{\sqrt{x^2+x+1}}{x\sqrt{x}} = \tan C$$

$A + B = C$

65. (c) Total ways to partition 5 into 4 parts are:

$5,0,0,0 \Rightarrow 1$ way

$4,1,0,0 \Rightarrow \frac{5!}{4!} = 5$ ways

$3,2,0,0, \Rightarrow \frac{5!}{3!2!} = 10$ ways

$2,2,0,1 \Rightarrow \frac{5!}{2!2!2!} = 15$ ways

$2,1,1,1 \Rightarrow \frac{5!}{2!(1!)^3 3!} = 10$ ways

$3,1,1,0 \Rightarrow \frac{5!}{3!2!} = 10$ ways

Total $\Rightarrow 1 + 5 + 10 + 15 + 10 + 10 = 51$ ways

66. (d) Let $Z = x + iy$

Then $(x-1)^2 + y^2 = 1 \ \rightarrow (a)$

$\&(\sqrt{2}-1)(2x) - i(2iy) = 2\sqrt{2} \Rightarrow (\sqrt{2}-1)x + y = \sqrt{2} \rightarrow (b)$

Solving (a) & (b) we get

Either $x = 1$ or $x = \frac{1}{2-\sqrt{2}} \rightarrow$ (c)

On solving (c) with (b) we get

For $x = 1 \Rightarrow y = 1 \Rightarrow Z_2 = 1 + i$

& for

$$x = \frac{1}{2-\sqrt{2}} \Rightarrow y = \sqrt{2} - \frac{1}{\sqrt{2}} \Rightarrow Z_1 = \left(1 + \frac{1}{\sqrt{2}}\right) + \frac{i}{\sqrt{2}}$$

Now

$$\left|\sqrt{2}z_1 - z_2\right|^2$$

$$= \left|\left(\frac{1}{\sqrt{2}} + 1\right)\sqrt{2} + i - (1+i)\right|^2$$

$$= (\sqrt{2})^2$$

$$= 2$$

67. (c)

Median $= 170 \Rightarrow 125, a, b, 170, 190, 210, 230$

Mean deviation about

Median =

$$\frac{0 + 45 + 60 + 20 + 40 + 170 - a + 170 - b}{7} = \frac{205}{7}$$

$$\Rightarrow a + b = 300$$

Mean $= \frac{170+125+230+190+210+a+b}{7} = 175$

Mean deviation

About mean =

$$\frac{50 + 175 - a + 175 - b + 5 + 15 + 35 + 55}{7} = 30$$

68. (a) $\vec{a} = -5\hat{i} + j - 3\hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$

$(\vec{a} \times \vec{b}) \times \hat{i} = (\vec{a} \cdot \hat{i})\vec{b} - (\vec{b} \cdot \hat{i})\vec{a}$

$= -5\vec{b} - \vec{a}$

$= (((-5\vec{b} - \vec{a}) \times \hat{i}) \times \hat{i})$

$= ((-11\hat{j} + 23\hat{k}) \times \hat{i}) \times \hat{i}$

$\Rightarrow (11\hat{k} + 23\hat{j}) \times \hat{i}$

$\Rightarrow (11\hat{j} - 23\hat{k})$

$\vec{c} \cdot (-\hat{i} + \hat{j} + \hat{k}) = 11 - 23 = -12$

69. (c)

$\sqrt{3} + \sqrt{2})^x + (\sqrt{3} - \sqrt{2})^x = 10$

Let $(\sqrt{3} + \sqrt{2})^x = t$

$t + \frac{1}{t} = 10$

$t^2 - 10t + 1 = 0$

$t = \frac{10 \pm \sqrt{100 - 4}}{2} = 5 \pm 2\sqrt{6}$

$(\sqrt{3} + \sqrt{2})^x = (\sqrt{3} \pm \sqrt{2})^2$

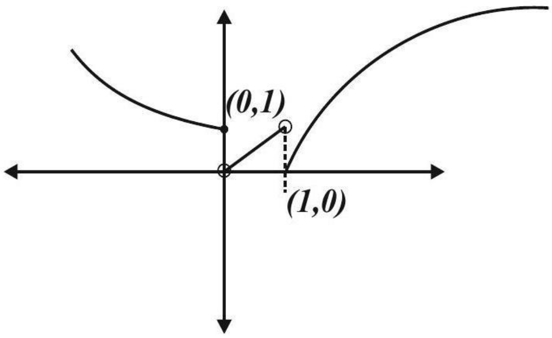
$x = 2$ or $x = -2$

Number of solutions $= 2$

70. (c)
71. (b)

$g(f(x)) = \begin{cases} f(x), f(x) \geq 0 \\ e^{f(x)}, f(x) < 0 \end{cases}$

$g(f(x)) = \begin{cases} e^{-x}, (-\infty, 0] \\ e^{\ln x}, (0,1) \\ \ln x, [1, \infty) \end{cases}$



Graph of $g(f(x))$

$g(f(x)) \Rightarrow$ Many one into

72. (b)

Using family of planes

$2x + 3y - z - 5 = k_1(x + \alpha y + 3z + 4) + k_2(3x - y$

$+\beta z - 7)$

$2 = k_1 + 3k_2, 3 = k_1\alpha - k_2, -1 = 3k_1 + \beta k_2, -5 = 4k_1 - 7k_2$

On solving we get

$$k_2 = \frac{13}{19}, k_1 = \frac{-1}{19}, \alpha = -70, \beta = \frac{-16}{13}$$

$$13\alpha\beta = 13(-70)\left(\frac{-16}{13}\right)$$

$= 1120$

73. (c)

$$e_h = \sqrt{1 + \sin^2\theta}$$

$$e_c = \sqrt{1 - \sin^2\theta}$$

$$e_h = \sqrt{7}e_c$$

$$1 + \sin^2\theta = 7(1 - \sin^2\theta)$$

$$\sin^2\theta = \frac{6}{8} = \frac{3}{4}$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3}$$

74. (d) $\frac{dy}{dx} = 2x(x + y)^3 - x(x + y) - 1$

$x + y = t$

$$\frac{dt}{dx} - 1 = 2xt^3 - xt - 1$$

$$\frac{dt}{2t^3 - t} = xdx$$

$$\frac{tdt}{2t^4 - t^2} = xdx$$

Let $t^2 = z$

$$\int \frac{dz}{2(2z^2 - z)} = \int xdx$$

$$\int \frac{dz}{4z\left(z-\frac{1}{2}\right)} = \int xdx$$

$$\ln\left|\frac{z-\frac{1}{2}}{z}\right| = x^2 + k$$

$$z = \frac{1}{2-\sqrt{e}}$$

75. (d) At $x = 1, f(x)$ is continuous therefore,

$f(1^-) = f(a) = f(1^+)$

$f(a) = 3 + c$……….. (1)

$f(1^+) = \lim_{h\to 0} 2(1+h) + 1$

$f(1^+) = \lim_{h\to 0} 3 + 2h = 3$………. (2)

from (1) & (2)

$c = 0$

at $x = 0, f(x)$ is continuous therefore,

$f(0^-) = f(0) = f(0^+)$………. (3)

$f(0) = f(0^+) = 2$………… (4)

$f(0^-)$has to be equal to 2

$$\lim_{h\to 0} \frac{a - b\cos(2h)}{h^2}$$

$$\lim_{h\to 0} \frac{a - b\left\{1 - \frac{4h^2}{2!} + \frac{16h^4}{4!} + \cdots\right\}}{h^2}$$

$$\lim_{h\to 0} \frac{a - b + b\left\{2h^2 - \frac{2}{3}h^4 \ldots\right\}}{h^2}$$

for limit to exist $a - b = 0$ and limit is $2b$……. (5)

from (3), (4) & (5)

$a = b = 1$

checking differentiability at $x = 0$

LHD : $\lim_{h\to 0} \frac{\frac{1-\cos 2h}{h^2} - 2}{-h}$

$$\lim_{h\to 0}\frac{1-\left(1-\frac{4h^2}{2!}+\frac{16h^4}{4!}\cdots\right)-2h^2}{-h^3}=0$$

RHD : $\lim_{h\to 0}\frac{(0+h)^2+2-2}{h}=0$

Function is differentiable at every point in its domain

$\therefore m=0$

$m+a+b+c=0+1+1+0=2$

76. (c)

$$e=\frac{1}{\sqrt{2}}=\sqrt{1-\frac{b^2}{a^2}}\Rightarrow\frac{1}{2}=1-\frac{b^2}{a^2}$$

$$\frac{2b^2}{a}=14$$

$$e_H=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{1}{2}}=\sqrt{\frac{3}{2}}$$

$$(e_H)^2=\frac{3}{2}$$

77. (d)

$3,a,b,c\rightarrow$ A.P $\;\Rightarrow 3,3+d,3+2\,d,3+3\,d$

$3,a-1,b+1,c+9\rightarrow$ G.P $\Rightarrow 3,2+d,4+2\,d,12+3\,d$

$a=3+d(2+d)^2=3(4+2d)$

$b=3+2\,dd=4,-2$

$c=3+3\,d$

If $d=4$ G.P $\Rightarrow 3,6,12,24$

$a=7$

$b=11$

$c=15$

$$\frac{a+b+c}{3}=11$$

78. (d)

$x^2+y^2=4$

$C(0,0)\ r_1=2$

$C'(2\lambda, 0)\ r_2 = \sqrt{4\lambda^2 - 9}$

$|r_1 - r_2| < C' < |r_1 + r_2|$

$\left|2 - \sqrt{4\lambda^2 - 9}\right| < |2\lambda| < 2 + \sqrt{4\lambda^2 - 9}$

$4 + 4\lambda^2 - 9 - 4\sqrt{4\lambda^2 - 9} < 4\lambda^2$

True $\lambda \in R$

$4\lambda^2 < 4 + 4\lambda^2 - 9 + 4\sqrt{4\lambda^2 - 9}$

$5 < 4\sqrt{4\lambda^2 - 9}$ and $\lambda^2 \geq \frac{9}{4}$

$\frac{25}{16} < 4\lambda^2 - 9\ \lambda \in \left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{3}{2}, \infty\right)$

$\frac{169}{64} < \lambda^2$

$\lambda \in \left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right)$

from (a) and (b) $\lambda \in$

$\lambda \in \left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right) \Rightarrow R - \left[-\frac{13}{8}, \frac{13}{8}\right]$

as per question $a = -\frac{13}{8}$ and $b = \frac{13}{8}$

$\therefore$ required point is $(-1,6)$ with satisfies option (d)

79. (b) $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 2, \forall x \neq 0$

Substitute $x \to \frac{1}{x}$

$5f\left(\frac{1}{x}\right) + 4f(x) = \frac{1}{x^2} - 2$

On solving (a) and (b)

$f(x) = \frac{5x^4 - 2x^2 - 4}{9x^2}$

$y = 9x^2 f(x)$

$y = 5x^4 - 2x^2 - 4$

$\frac{dy}{dx} = 20x^3 - 4x$

for strictly increasing

$\frac{dy}{dx} > 0$

$4x(5x^2 - 1) > 0$

$x \in \left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

80. (b) Passing points of lines L_1 & L_2 are

$(\lambda, 2, 1) \& (\sqrt{3}, 1, 2)$

$$\text{S.D} = \frac{\begin{vmatrix} \sqrt{3}-\lambda & -1 & 1 \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}}$$

$$1 = \left|\frac{\sqrt{3}-\lambda}{\sqrt{3}}\right|$$

$\lambda = 0, \lambda = 2\sqrt{3}$

81. (14)

$(t+1)dx = (2x + (t+1)^4)dt$

$\frac{dx}{dt} = \frac{2x + (t+1)^4}{t+1}$

$\frac{dx}{dt} - \frac{2x}{t+1} = (t+1)^3$

$I \cdot F = e^{-\int \frac{2}{t+1}dt} = e^{-2\ln(t+1)} = \frac{1}{(t+1)^2}$

$\frac{x}{(t+1)^2} = \int \frac{1}{(t+1)^2}(t+1)^3 dt + c$

$\frac{x}{(t+1)^2} = \frac{(t+1)^2}{2} + c$

$\Rightarrow c = \frac{3}{2}$

$x = \frac{(t+1)^4}{2} + \frac{3}{2}(t+1)^2$

put, $t = 1$

$x = 2^3 + 6 = 14$

82. (169)

$x + 2y + 3z = 42,\ x, y, z \geq 0$

$z = 0 \; x + 2y = 42 \Rightarrow 22$

$z = 1 \; x + 2y = 39 \Rightarrow 20$

$z = 2 \; x + 2y = 36 \Rightarrow 19$

$z = 3 \; x + 2y = 33 \Rightarrow 17$

$z = 4 \; x + 2y = 30 \Rightarrow 16$

$z = 5 \; x + 2y = 27 \Rightarrow 14$

$z = 6 \; x + 2y = 24 \Rightarrow 13$

$z = 7 \; x + 2y = 21 \Rightarrow 11$

$z = 8 \; x + 2y = 18 \Rightarrow 10$

$z = 9 \; x + 2y = 15 \Rightarrow 8$

$z = 10 \; x + 2y = 12 \Rightarrow 7$

$z = 11 \; x + 2y = 9 \Rightarrow 5$

$z = 12 \; x + 2y = 6 \Rightarrow 4$

$z = 13 \; x + 2y = 3 \Rightarrow 2$

$z = 14 \; x + 2y = 0 \Rightarrow 1$

Total: 169

83. (678)

coeff of x^{30} in $\frac{(x+1)^6(1+x^2)^7(1-x^3)^8}{x^6}$

coeff. of x^{36} in $(1+x)^6(1+x^2)^7(1-x^3)^8$

General term

$$^6C^7_{r_1}C^8_{r_2}C_{r_3}(-1)^{r_3}x^{r_1+2r_2+3r_3}$$
$$r_1 + 2r_2 + 3r_3 = 36$$

Case-I:

r_1	r_2	r_3
0	6	8
2	5	8
4	4	8
6	3	8

$r_1 + 2r_2 = 12$ (Taking $r_3 = 8$)

Case-II : $r_1 + 2r_2 = 15$ (Taking $r_3 = 7$)

r_1	r_2	r_3
1	7	7
3	6	7
5	5	7

Case-III : $r_1 + 2r_2 = 18$ (Taking $r_3 = 6$)

r_1	r_2	r_3
4	7	6
6	6	6

Coeff. $= 7 + (15 \times 21) + (15 \times 35) + (35)$
$-(6 \times 8) - (20 \times 7 \times 8) - (6 \times 21 \times 8) + (15 \times 28)$
$+(7 \times 28) = -678 = \alpha$
$|\alpha| = 678$

84. (6699)

. 3,7,11,15, 403

2,5,8,11, ….,404

LCM$(4,3) = 12$

11,23,35, …. let (403)

$403 = 11 + (n-1) \times 12 \; \frac{392}{12} = n - 1$

$33 \cdot 66 = n$

$n = 33$

Sum $\frac{33}{2}(22 + 32 \times 12)$

$= 6699$

85. (18)

Finding right hand limit

$$\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$$
$$= \lim_{h\to 0} f(h)$$
$$= \lim_{h\to 0} \frac{\cos^{-1}(1-h^2)\sin^{-1}(1-h)}{h(1-h^2)}$$
$$= \lim_{h\to 0} \frac{\cos^{-1}(1-h^2)}{h}\left(\frac{\sin^{-1}1}{1}\right)$$

Let $\cos^{-1}(1-h^2) = \theta \Rightarrow \cos\theta = 1-h^2$

$$= \frac{\pi}{2}\lim_{\theta\to 0} \frac{\theta}{\sqrt{1-\cos\theta}}$$

$$= \frac{\pi}{2}\lim_{\theta\to 0} \frac{1}{\sqrt{\frac{1-\cos\theta}{\theta^2}}}$$

$$= \frac{\pi}{2}\frac{1}{\sqrt{1/2}}$$

$$R = \frac{\pi}{\sqrt{2}}$$

Now finding left hand limit

$$L = \lim_{x\to 0^-} f(x)$$
$$= \lim_{h\to 0} f(-h)$$
$$= \lim_{h\to 0} \frac{\cos^{-1}(1-\{-h\}^2)\sin^{-1}(1-\{-h\})}{\{-h\}-\{-h\}^3}$$
$$= \lim_{h\to 0} \frac{\cos^{-1}(1-(-h+1)^2)\sin^{-1}(1-(-h+1))}{(-h+1)-(-h+1)^3}$$
$$= \lim_{h\to 0} \frac{\cos^{-1}(-h^2+2h)\sin^{-1}h}{(1-h)(1-(1-h)^2)}$$
$$= \lim_{h\to 0}\left(\frac{\pi}{2}\right)\frac{\sin^{-1}h}{(1-(1-h)^2)}$$
$$= \frac{\pi}{2}\lim_{h\to 0}\left(\frac{\sin^{-1}h}{-h^2+2h}\right)$$
$$= \frac{\pi}{2}\lim_{h\to 0}\left(\frac{\sin^{-1}h}{h}\right)\left(\frac{1}{-h+2}\right)$$
$$= \frac{32}{\pi^2}(L^2+R^2) = \frac{32}{\pi^2}\left(\frac{\pi^2}{2}+\frac{\pi^2}{16}\right)$$
$$= 16+2$$
$$= 18$$
$$=$$

86. (72)

$x^2+y^2=3$ and $x^2=2y$

$y^2 + 2y - 3 = 0 \Rightarrow (y + 3)(y - 1) = 0$

$y = -3$ or $y = 1$

$y = 1x = \sqrt{2} \Rightarrow P(\sqrt{2}, 1)$

p lies on the line

$\sqrt{2}x + y = \alpha$

$\sqrt{2}(\sqrt{2}) + 1 = \alpha$

$\alpha = 3$

For circle C_1

Q_1 lies on y axis

Let $Q_1(0, \alpha)$ coordinates

$R_1 = 2\sqrt{3}$ (Given

Line L act as tangent

Apply $P = r$ (condition of tangency)

$\Rightarrow \left|\frac{\alpha - 3}{\sqrt{3}}\right| = 2\sqrt{3}$

$\Rightarrow |\alpha - 3| = 6$

$\alpha - 3 = 6$ or $\alpha - 3 = -6$

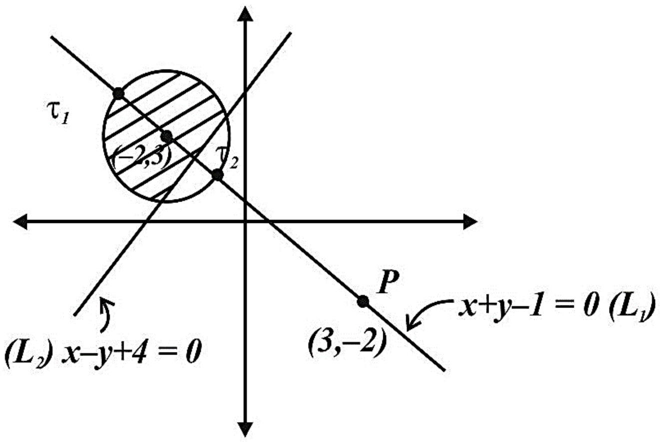
$\Rightarrow \alpha = 9\ \alpha = -3$

$$\Delta PQ_1Q_2 = \frac{1}{2}\begin{vmatrix} \sqrt{2} & 1 & 1 \\ 0 & 9 & 1 \\ 0 & -3 & 1 \end{vmatrix}$$

$= \frac{1}{2}(\sqrt{2}(12)) = 6\sqrt{2}$

$(\triangle PQ_1Q_2)^2 = 72$

87. (36)



Clearly for the shaded region z_1 is the intersection of the circle and the line passing through $P(L_1)$ and z_2 is intersection of line L_1 & L_2

Circle : $(x+2)^2+(y-3)^2=1$

$L_1: x+y-1=0$

$L_2: x-y+4=0$

On solving circle & L_1 we get

$z_1: \left(-2-\frac{1}{\sqrt{2}}, 3+\frac{1}{\sqrt{2}}\right)$

On solving L_1 and z_2 is intersection of line L_1 & L_2 we get $z_2: \left(\frac{-3}{2}, \frac{5}{2}\right)$

$$|z_1|^2+2|z_2|^2=14+5\sqrt{2}+17$$
$$=31+5\sqrt{2}$$

So $\alpha=31$

$\beta=5$

$\alpha+\beta=36$

88. (8)

$$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{(1+e^{\sin x})(1+\sin^4 x)} dx$$

Apply king

$$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x\left(e^{\sin x}\right)}{(1+e^{\sin x})(1+\sin^4 x)} dx$$

adding (a)&(b)

$$2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{1+\sin^4 x} dx$$

$I=\int_0^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{1+\sin^4 x}dx$

$\sin x = t$

$I=\int_0^1 \frac{8\sqrt{2}}{1+t^4}dx$

$I=4\sqrt{2}\int_0^1\left(\frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}-\frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}}\right)dt$

$I=4\sqrt{2}\int_0^1 \frac{\left(1+\frac{1}{t^2}\right)}{\left(t-\frac{1}{t}\right)^2+2}-\frac{\left(1-\frac{1}{t^2}\right)}{\left(t+\frac{1}{t}\right)^2-2}dt$

Let $t-\frac{1}{t}=z \& t+\frac{1}{t}=k$

$=4\sqrt{2}\left[\int_{-\infty}^{0}\frac{dz}{z^2+2}-\int_{\infty}^{2}\frac{dk}{k^2-2}\right]$

$=4\sqrt{2}\left[\frac{1}{\sqrt{2}}\tan^{-1}\frac{z}{\sqrt{2}}\right]_{-\infty}^{0}-\left[\frac{1}{2\sqrt{2}}\ln\left(\frac{k-\sqrt{2}}{k+\sqrt{2}}\right)\right]_{\infty}^{2}$

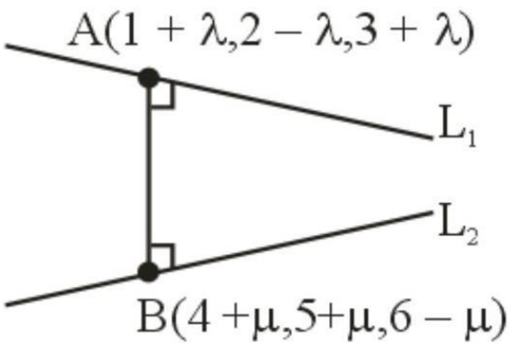
$=4\sqrt{2}\left[\frac{\pi}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\left[\ln\frac{2-\sqrt{2}}{2+\sqrt{2}}\right]\right]$

$=2\pi+2\ln(3+2\sqrt{2})$

$\alpha=2$

$\beta=2$

89. (21)



$\vec{b}=\hat{i}-\hat{j}+\hat{k}$(DR 's of L_1)

$\vec{d}=\hat{i}+\hat{j}-\hat{k}$(DR′ of L_2) $\vec{b}\times\vec{d}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 1\\ 1 & 1 & -1\end{vmatrix}$

$=0\hat{i}+2\hat{j}+2\hat{k}$ (DR's of Line perpendicular to L_1 and L_2)

DR of AB line

$= (0,2,2) = (3+\mu-\lambda, 3+\mu+\lambda, 3-\mu-\lambda)$

$\frac{3+\mu-\lambda}{0} = \frac{3+\mu+\lambda}{2} = \frac{3-\mu-\lambda}{2}$

Solving above equation we get $\mu = -\frac{3}{2}$ and $\lambda = \frac{3}{2}$

point $A = \left(\frac{5}{2}, \frac{1}{2}, \frac{9}{2}\right)$

$B = \left(\frac{5}{2}, \frac{7}{2}, \frac{15}{2}\right)$

Point of $AB = \left(\frac{5}{2}, 2, 6\right) = (\alpha, \beta, \gamma)$

$2(\alpha+\beta+\gamma) = 5+4+12 = 21$

90. (46)

$n(R_1) = 20+10+6+5+4+3+2+2+2$

$+2+\underbrace{1+\cdots+1}_{\text{10 times}}$

$n(R_1) = 66$

$R_1 \cap R_2 = \{(1,1), (2,2), \ldots (20,20)\}$

$n(R_1 \cap R_2) = 20$

$n(R_1 - R_2) = n(R_1) - n(R_1 \cap R_2)$

$= n(R_1) - 20$

$= 66 - 20$

$R_1 - R_2 = 46$ Pair