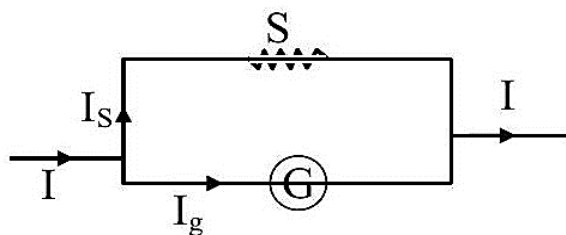


1. (a)



$$I_S S = I_g G$$

$$\frac{95}{100} I S = \frac{5I}{100} G$$

$$S = \frac{G}{19}$$

$$R_A = \frac{SG}{S+G} = \frac{\frac{G^2}{19}}{\frac{20G}{19}}$$

$$R_A = \frac{G}{20}$$

2. (c)

$$\text{For no deflection } \frac{0.8}{1} = \frac{R}{3}$$

$$\Rightarrow R = 2.4 \text{ m}\Omega$$

$$\text{Temperature fall in } 10 \text{ s} = 20^\circ\text{C}$$

$$\Delta R = R \propto \Delta t$$

$$\alpha = \frac{\Delta R}{R \Delta t} = \frac{-0.6}{3 \times 20}$$

$$= -10^{-2} \text{ C}^{-1}$$

3. (c)

$$4. \text{ (d) } \gamma = 1 + \frac{2}{f} = 1.4 \Rightarrow \frac{2}{f} = 0.4$$

$$\Rightarrow f = 5$$

$$W = nR\Delta T = 200 \text{ J}$$

$$Q = \left( \frac{f+2}{2} \right) nR\Delta T$$

$$= \frac{7}{2} \times 200 = 700 \text{ J}$$

5. (c) Only the translational kinetic energy of disc changes into gravitational potential energy. And rotational KE remains unchanged as there is no friction.

$$\frac{1}{2}mv^2 = mgh$$

$$h = \frac{v^2}{2g}$$

6. (a)

$$E_g = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{660 \times 10^{-9}} \text{ J}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{660 \times 10^{-9} \times 1.6 \times 10^{-19}} \text{ eV}$$

$$= \frac{15}{8} \text{ eV}$$

So  $x = 15$

7. (d) Let's say radius of small droplets is  $r$  and that of big drop is  $R$

$$\frac{4}{3}\pi R^3 = 1000 \frac{4}{3}\pi r^3$$

$$R = 10r$$

$$U_i = 1000(4\pi r^2 S)$$

$$U_f = 4\pi R^2 S$$

$$= 100(4\pi r^2 S)$$

$$U_f = \frac{1}{10} U_i$$

8. (c)  $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{60 \times 10^6} = 5 \text{ m}$

9. (d)

$$F_{av} = \frac{\Delta p}{\Delta t}$$

$$= \frac{0.12 \times 25}{0.1} = 30 \text{ N}$$

10. (c)  $P = nh\nu$

$$n = \frac{P}{h\nu} = \frac{2 \times 10^{-3}}{6.63 \times 10^{-34} \times 6 \times 10^{14}}$$

$$= 5 \times 10^{15}$$

11. (c) For first minima  $a \sin \theta = \lambda$

$$\sin \theta = \frac{\lambda}{a} = \frac{1}{2}$$

$$\theta = 30^\circ$$

$$\text{Angular spread} = 60^\circ$$

$$12. (a) \phi_{\text{smaller cube}} = \frac{2Q}{\epsilon_0}$$

$$\phi_{\text{bigger cube}} = \frac{5Q}{\epsilon_0}$$

$$\frac{\phi_{\text{smaller cube}}}{\phi_{\text{bigger cube}}} = \frac{2}{5}$$

13. (b)

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{M_2}{M_1}} \Rightarrow \frac{2}{V_2} = \sqrt{\frac{32}{2}}$$

$$V_2 = 0.5 \text{ km/s}$$

14. (c) B ↓ 30 m/s

$$A \uparrow 20 \text{ m/s}$$

$$V_A = 20 \text{ m/s}$$

$$V_B = -30 \text{ m/s}$$

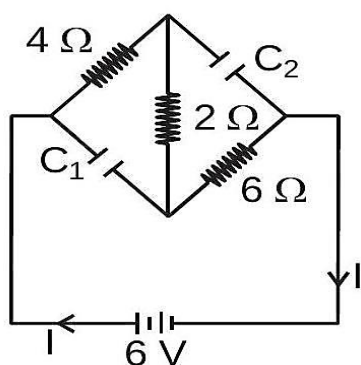
Velocity of B w.r.t. A

$$V_{B/A} = -50 \text{ m/s}$$

Velocity of ground w.r.t. B

$$V_{G/B} = 30 \text{ m/s}$$

15. (d)



In steady state

$$R_{eq} = 12\Omega$$

$$I = \frac{6}{12} = 0.5 \text{ A}$$

$$\text{P.D across } C_1 = 3 \text{ V}$$

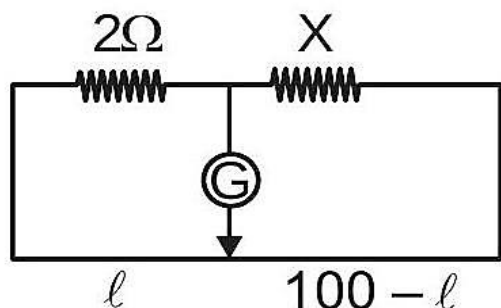
$$\text{P.D across } C_2 = 4 \text{ V}$$

$$q_1 = C_1 V_1 = 12\mu\text{C}$$

$$q_2 = C_2 V_2 = 24\mu\text{C}$$

$$\frac{q_1}{q_2} = \frac{1}{2}$$

16. (a)



$$\text{First case } \frac{2}{40} = \frac{X}{60} \Rightarrow X = 3\Omega$$

$$\text{In second case } X' = \frac{2 \times 3}{2+3} = 1.2\Omega$$

$$\frac{2}{\ell} = \frac{1.2}{100 - \ell}$$

$$200 - 2\ell = 1.2\ell$$

$$\ell = \frac{200}{3.2} = 62.5 \text{ cm}$$

Balance length changes by 22.5 cm

17. (c)

$$\text{18. (b) Efficiency} = \frac{E_S I_S}{E_P I_P}$$

$$0.8 = \frac{240 I_S}{4000}$$

$$I_S = \frac{3200}{240} = 13.33 \text{ A}$$

19. (a)

$$F = \frac{GMm}{R^{3/2}} = m\omega^2 R$$

$$\omega^2 \propto \frac{1}{R^{5/2}} \quad \therefore T = \frac{2\pi}{\omega} \text{ so}$$

$$T^2 \propto R^{5/2}$$

20. (c)

$$\text{Net force} = 8\hat{i} + 4\hat{j} + 4\hat{k}$$

$$\vec{a} = \frac{\vec{F}}{m} = 2\hat{i} + \hat{j} + \hat{k}$$

21. (3)

$$f_1 = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{k}{9m}}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{9}{1}} = \frac{3}{1}$$

22. (8)

$$V = 4\sqrt{x}$$

$$a = V \frac{dv}{dx}$$

$$= 4\sqrt{x} \times 4 \times \frac{1}{2} x^{-1/2} = 8 \text{ m/s}^2$$

23. (4)

$$\tau = BINA \sin \phi$$

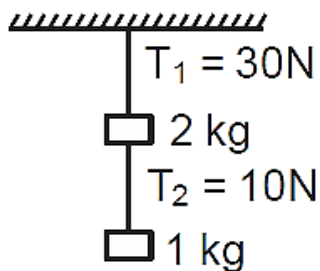
$$C\theta = BINA \sin 90^\circ$$

$$C = \frac{BINA}{\theta} = \frac{0.01 \times 10 \times 10^{-3} \times 100 \times 2 \times 10^{-4}}{0.05}$$

$$= 4 \times 10^{-5} \text{ N-m/rad.}$$

$$x = 4$$

24. (3)



$$\Delta L = \frac{FL}{AY}$$

$$\frac{\Delta L}{L} = \frac{F}{AY}$$

$$\frac{\frac{\Delta L_1}{L_1}}{\frac{\Delta L_2}{L_2}} = \frac{F_1}{F_2} = \frac{30}{10} = 3$$

25. (32)

$$E = -13.6z^2 \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$$

$$E = C \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$h\nu = C \left[ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right]$$

$$\frac{v_1}{v_2} = \frac{\left[ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right]_{2-1}}{\left[ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right]_{3-1}}$$

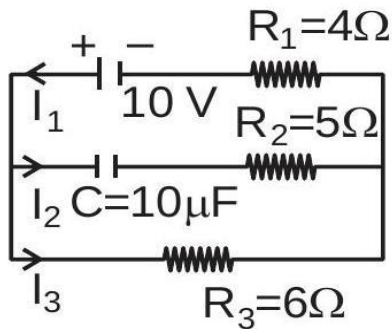
$$= \frac{\left[ \frac{1}{1} - \frac{1}{4} \right]}{\left[ \frac{1}{1} - \frac{1}{9} \right]} = \frac{3/4}{8/9}$$

$$= \frac{3}{4} \times \frac{9}{8}$$

$$\frac{v_1}{v_2} = \frac{27}{32}$$

$$v_2 = \frac{32}{27} v_1 = \frac{32}{27} \times 3 \times 10^{15} \text{ Hz} = \frac{32}{9} \times 10^{15} \text{ Hz}$$

26. (60)



In steady state there will be no current in branch of capacitor, so no voltage drop across  $R_2 = 5\Omega$

$$I_2 = 0$$

$$I_1 = I_3 = \frac{10}{4 + 6} = 1 \text{ A}$$

$$V_{R_3} = V_c + V_{R_2} \quad V_{R_2} = 0$$

$$I_3 R_3 = V_c$$

$$V_c = 1 \times 6 = 6 \text{ volt}$$

$$q_c = CV_c = 10 \times 6 = 60\mu\text{C}$$

27. (5)

$$\phi = NAB\cos(\omega t)$$

$$\varepsilon = -\frac{d\phi}{dt} = NAB\omega\sin(\omega t)$$

$$\varepsilon_{\max} = NAB\omega$$

$$= 200 \times 0.2 \times 0.01 \times \pi$$

$$= \frac{4\pi}{10} = \frac{2\pi}{5} \text{ volt}$$

28. (125) Let intensity of light on screen due to each slit is  $I_0$

So intensity at centre of screen is  $4I_0$

Intensity at distance  $y$  from centre-

$$I = I_0 + I_0 + 2\sqrt{I_0 I_0} \cos \phi$$

$$I_{\max} = 4I_0$$

$$\frac{I_{\max}}{2} = 2I_0 = 2I_0 + 2I_0 \cos \phi$$

$$\cos \phi = 0$$

$$\phi = \frac{\pi}{2}$$

$$K\Delta x = \frac{\pi}{2}$$

$$\frac{2\pi}{\lambda} d \sin \theta = \frac{\pi}{2}$$

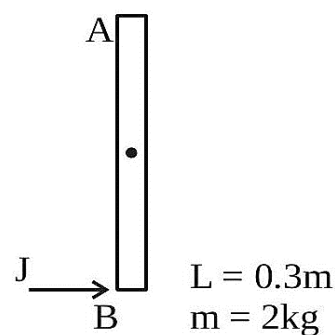
$$\frac{2}{\lambda} d \times \frac{y}{D} = \frac{1}{2}$$

$$y = \frac{\lambda D}{4d} = \frac{5 \times 10^{-7} \times 1}{4 \times 10^{-3}}$$

$$= 125 \times 10^{-6}$$

$$= 125$$

29. (4)



$$\text{Impulse } J = 0.2 \text{ N} \cdot \text{s}$$

$$J = \int F dt = 0.2 \text{ N} \cdot \text{s}$$

Angular impulse ( $\vec{M}$ )

$$\vec{M}_c = \int \tau dt$$

$$= \int F \frac{L}{2} dt$$

$$= \frac{L}{2} \int F dt = \frac{L}{2} \times J$$

$$= \frac{0.3}{2} \times 0.2$$

$$= 0.03$$

$$I_{\text{cm}} = \frac{ML^2}{12} = \frac{2 \times (0.3)^2}{12} = \frac{0.09}{6}$$

$$M = I_{\text{cm}}(\omega_f - \omega_i)$$

$$0.03 = \frac{0.09}{6}(\omega_f)$$

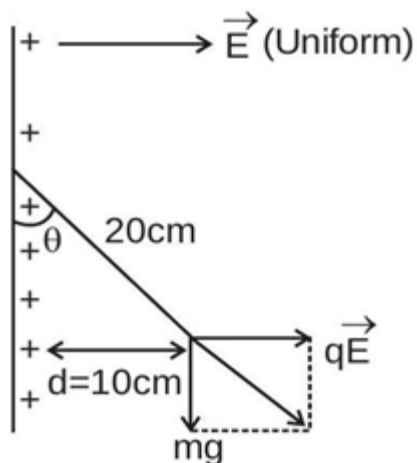
$$\omega_f = 2\text{rad/s}$$

$$\theta = \omega t$$

$$t = \frac{\theta}{\omega} = \frac{\pi}{2 \times 2} = \frac{\pi}{4} \text{ sec.}$$

$$X = 4$$

30. (3)



$$\sin \theta = \frac{10}{20} = \frac{1}{2}$$

$$\theta = 30^\circ$$

$$\tan \theta = \frac{qE}{mg}$$

$$\tan 30^\circ = \frac{q \times 2 \times 10^4}{1 \times 10^{-3} \times 10}$$

$$\frac{1}{\sqrt{3}} = q \times 10^6$$

$$q = \frac{1}{\sqrt{3}} \times 10^{-6} \text{ C}$$

$$x = 3$$

31. (b) 3rd Ionisation energy: [NCERT Data]

V: 2833KJ/mol

Cr: 2990KJ/mol

Mn: 3260KJ/mol

Fe: 2962KJ/mol

alternative

Mn: 3 d<sup>5</sup> 4 s<sup>2</sup>

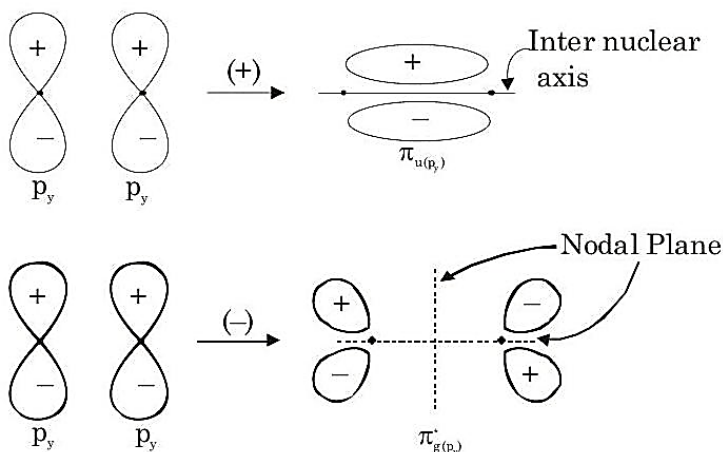
Fe: 3 d<sup>6</sup> 4 s<sup>2</sup>

Cr:  $3d^5 4s^1$

V:  $3d^3 4s^2$

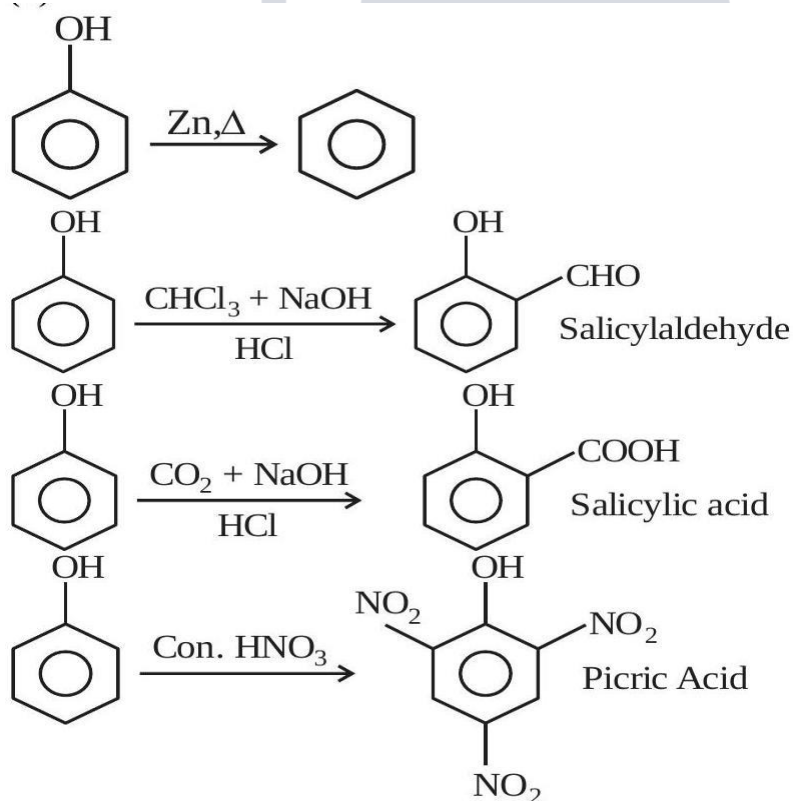
So Mn has highest 3rd IE among all the given elements due to  $d^5$  configuration.

32. (c) A  $\pi$  bonding molecular orbital has higher electron density above and below inter nuclear axis



33. (a)  $Cr^{2+}$  is reducing as its configuration changes from  $d^4$  to  $d^3$  due to formation of  $Cr^{3+}$ , which has half filled  $t_{2g}$  level, on other hand, the change  $Mn^{3+}$  to  $Mn^{2+}$  results in half filled  $d^5$  configuration which has extra stability.

34. (c)



35. (b) I. In p-Block both metals and non-metals are present but in d-Block only metals are present.

II. EN and IE of non-metals are greater than that of metals

I - False, II-True

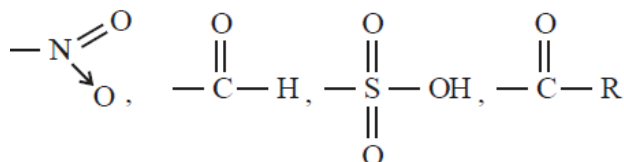
36. (c) Strongest reducing agent :  $\text{BiH}_3$  explained by its low bond dissociation energy.

37. (a)  $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$

$\text{Cu}^{2+}$ :  $3d^9 4s^0$

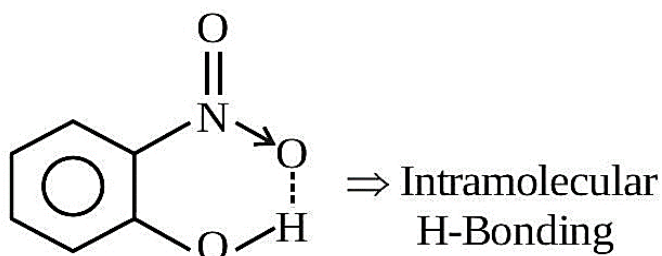
unpaired electron present so it show colour due to d-d transition.

38. (c)



All are  $-\text{M}$ , Hence meta directing groups.

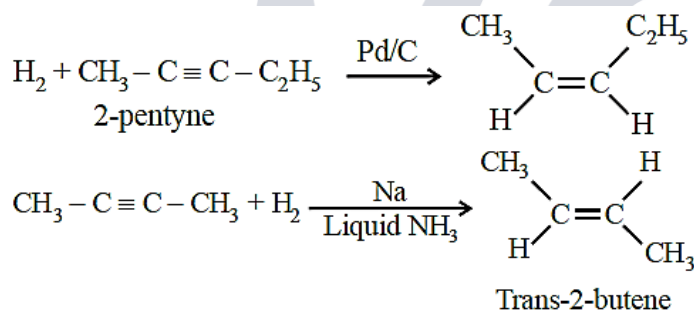
39. (d)  $\text{H}_2\text{O}, \text{NH}_3, \text{C}_2\text{H}_5\text{OH} \Rightarrow$  Intermolecular H-Bonding



40. (d) Lassaigne's test is used for detection of all element N, S, P, X.

41. (b) Due to H-bonding boiling point of alcohol is High.

42. (a)



43. (a) For  $3p$ :  $n = 3, \ell = 1$

Number of radial node =  $n - \ell - 1$

$$= 3 - 1 - 1 = 1$$

44. (b)  $\text{CCl}_4$  used in fire extinguisher.  $\text{CH}_2\text{Cl}_2$  used as paint remover. Freons used in refrigerator and AC. DDT used as non-Biodegradable insecticide.



45. (c)  $\text{—C—OH}$  shows  $-\text{R}$  effect, while rest 3 groups shows  $+\text{R}$  effect via lone pair.

46. (b)  $[\text{Co}(\text{NH}_3)_6]^{3+}$

$\text{Co}^{3+}$  (strong field ligand)  $\Rightarrow 3d^6(t_{2g}^6, e_g^0)$ ,

Hybridization:  $d^2sp^3$

Inner orbital complex (spin paired complex)

Pairing will take place.

$[CoF_6]^{3-}$

$Co^{3+}$  (weak field ligand)  $\Rightarrow 3 d^6(t_{2g}^4, e_g^2)$

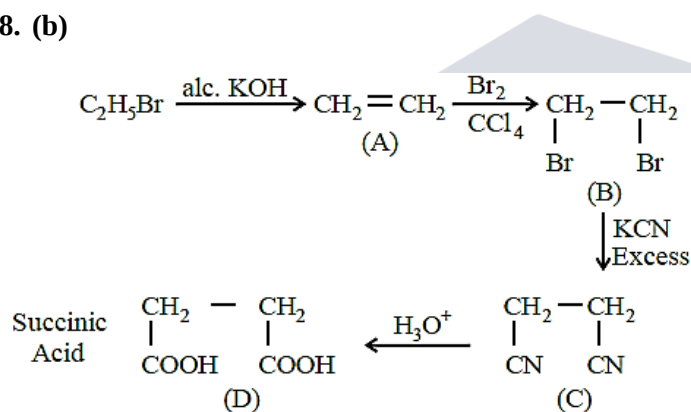
Hybridisation:  $sp^3 d^2$

Outer orbital complex (spin free complex) no pairing will take place

47. (c)  $SiO_2$  and  $GeO_2$  are acidic and  $SnO, PbO$  are amphoteric.

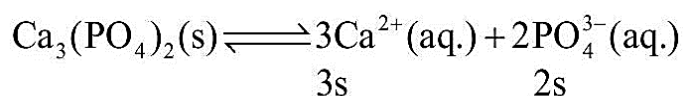
Carbon does not have d-orbitals so can not form  $p\pi - d\pi$  Bond with itself. Due to properties of catenation and  $p\pi - p\pi$  bond formation. carbon is able to show allotropic forms.

48. (b)



49. (b)

$$S = \frac{W \times 10}{M}$$



$$S = \frac{W \times 1000}{M \times 100} = \frac{W \times 10}{M}$$

$$K_{sp} = (3s)^3(2s)^2$$

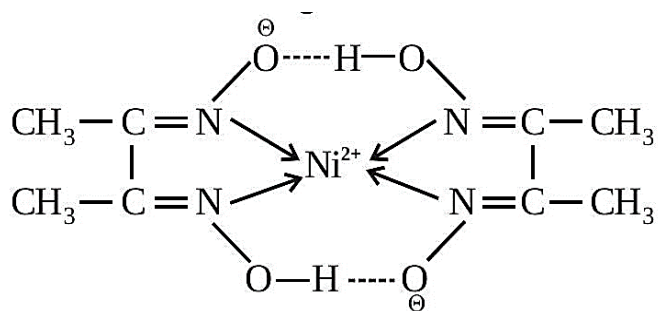
$$= 108 s^5$$

$$= 108 \times 10^5 \times \left(\frac{W}{M}\right)^5$$

$$= 1.08 \times 10^7 \left(\frac{W}{M}\right)^5$$

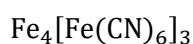
50. (a)





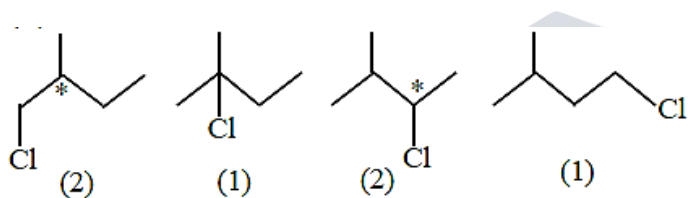
2 Five-member ring

III II

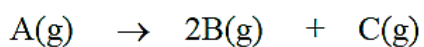


Prussian Blue

51. (6)



52. (2)



$t = 0$

0.1

$t = 115 \text{ sec.}$

$0.1 - x$

$2x$

$x$

$$0.1 + 2x = 0.28$$

$$2x = 0.18$$

$$x = 0.09$$

$$K = \frac{1}{115} \ln \frac{0.1}{0.1 - 0.09}$$

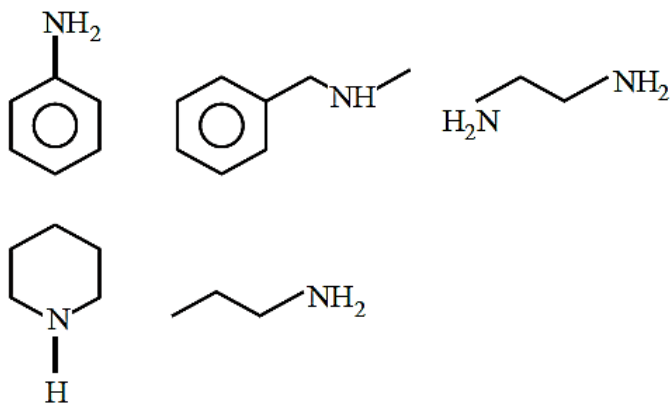
$$= 0.0200 \text{ sec}^{-1}$$

$$= 2 \times 10^{-2} \text{ sec}^{-1}$$

53. (6) Let 3 different amino acid are A, B, C then following combination of tripeptides can be formed-

ABC, ACB, BAC, BCA, CAB, CBA

54. (5)



55. (15)

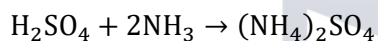
$$\Delta T_f = i K_f \times \text{molality}$$

$$24 = (a) \times 1.86 \times \frac{W}{62 \times 18.6}$$

$$W = 14880 \text{ gm}$$

$$= 14.880 \text{ kg}$$

56. (56)



$$\text{Millimole of H}_2\text{SO}_4 \rightarrow 10 \times 2$$

$$\text{So Millimole of NH}_3 = 20 \times 2 = 40$$

$$\text{Organic} \rightarrow \text{NH}_3$$

$$\text{Compound} \quad 40 \text{ Millimole}$$

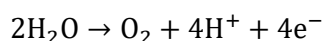
$$\therefore \text{Mole of N} = \frac{40}{1000}$$

$$\text{wt. of N} = \frac{40}{1000} \times 14$$

$$\% \text{ composition of N in organic compound} = \frac{40 \times 14}{1000 \times 1} \times 100$$

$$= 56\%$$

57. (2)



$$\frac{W}{E} = \frac{Q}{96500}$$

$$\text{mole} \times \text{n-factor} = \frac{Q}{96500}$$

$$1 \times 2 = \frac{Q}{96500}$$

$$Q = 2 \times 96500C$$

$$= 1.93 \times 10^5 C$$

58. (57)

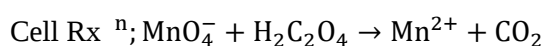
$$\Delta G^\circ = -RT \ln K$$

$$= -8.314 \times 300 \ln(10)$$

$$= 5744.14 \text{ J/mole}$$

$$= 57.44 \times 10^{-1} \text{ kJ/mole}$$

59. (338 or 339)



$$E_{\text{cell}}^\circ = E_{\text{op}}^\circ \text{ of anode} + E_{\text{RP}}^\circ \text{ of cathode}$$

$$= 0.49 + 1.51 = 2.00 \text{ V}$$

At equilibrium

$$E_{\text{cell}} = 0$$

$$E_{\text{cell}}^\circ = \frac{0.059}{n} \log K$$

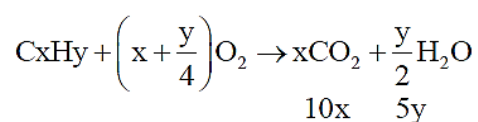
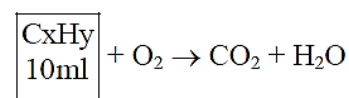
(As per NCERT  $\frac{RT}{F} = 0.059$  But  $\frac{RT}{F} = 0.0591$

can also be taken.)

$$2 = \frac{0.059}{10} \log K$$

$$\log K = 338.98$$

60. (14)



$$10x = 40$$

$$x = 4$$

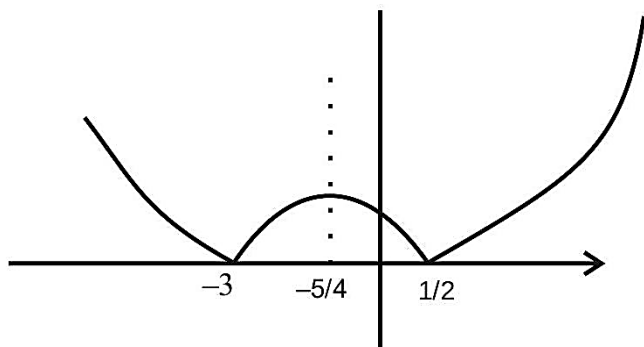
$$5y = 50$$

$$y = 10$$

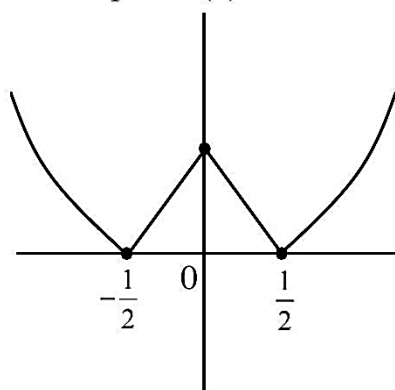


61. (d)  $f(x) = |2x^2 + 5|x| - 3|$

Graph of  $y = |2x^2 + 5x - 3|$



Graph of  $f(x)$



Number of points of discontinuity = 0 = m

Number of points of non-differentiability = 3 = n

62. (a)

$$px^2 + qx - r = 0 \quad \begin{matrix} \alpha \\ \beta \end{matrix}$$

$$p = A, q = AR, r = AR^2$$

$$Ax^2 + ARx - AR^2 = 0$$

$$x^2 + Rx - R^2 = 0 \quad \begin{matrix} \alpha \\ \beta \end{matrix}$$

$$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$$

$$\therefore \frac{\alpha + \beta}{\alpha\beta} = \frac{3}{4} \Rightarrow \frac{-R}{-R^2} = \frac{3}{4} \Rightarrow R = \frac{4}{3}$$

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = R^2 - 4(-R^2) = 5\left(\frac{16}{9}\right)$$

$$= 80/9$$

63. (d)  $4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0; x \in [-2\pi, 2\pi]$

$$4 - 4\cos^2 x - 4\cos^3 x + 9 - 4\cos x = 0$$

$$4\cos^3 x + 4\cos^2 x + 4\cos x - 13 = 0$$

$$4\cos^3 x + 4\cos^2 x + 4\cos x = 13$$

L.H.S.  $\leq 12$  can't be equal to 13.

64. (a)

$$I = \int_0^1 (2x^3 - 3x^2 - x + 1)^{\frac{1}{3}} dx$$

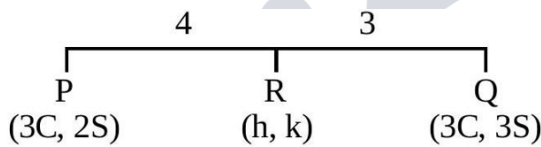
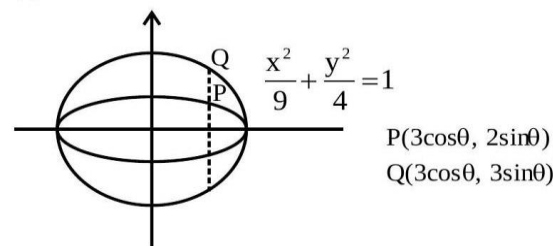
Using  $\int_0^{2a} f(x) dx$  where  $f(2a - x) = -f(x)$

Here

$$f(1 - x) = f(x)$$

$$\therefore I = 0$$

65. (d)



$$h = 3\cos\theta$$

$$k = \frac{18}{7}\sin\theta$$

$$\therefore \text{locus} = \frac{x^2}{9} + \frac{49y^2}{324} = 1$$

$$e = \sqrt{1 - \frac{324}{49 \times 9}} = \frac{\sqrt{117}}{21} = \frac{\sqrt{13}}{7}$$

66. (d)

$$\left( \frac{1}{x^3} + \frac{-2}{x^3} \right)^{18}$$

$$t_7 = {}^{18}C_6 \left( \frac{1}{x^3} \right)^{12} \left( \frac{-2}{x^3} \right)^6 = {}^{18}C_6 \frac{1}{(3)^{12}} \cdot \frac{1}{2^6}$$

$$t_{13} = {}^{18}C_{12} \left( \frac{x^{\frac{1}{3}}}{3} \right)^6 \left( \frac{x^{-\frac{2}{3}}}{2} \right)^{12} = {}^{18}C_{12} \frac{1}{(3)^6} \cdot \frac{1}{2^{12}} \cdot x^{-6}$$

$$m = {}^{18}C_6 \cdot 3^{-12} \cdot 2^{-6}; n = {}^{18}C_{12} \cdot 2^{-12} \cdot 3^{-6}$$

$$\left( \frac{n}{m} \right)^{\frac{1}{3}} = \left( \frac{2^{-12} \cdot 3^{-6}}{3^{-12} \cdot 2^{-6}} \right)^{\frac{1}{3}} = \left( \frac{3}{2} \right)^2 = \frac{9}{4}$$

67. (c)

$$f(0) = 2, \lim_{x \rightarrow -\infty} f(x) = 1$$

$$f'(x) - x \cdot f(x) = 3$$

$$I.F = e^{-\alpha x}$$

$$y(e^{-\alpha x}) = \int 3 \cdot e^{-\alpha x} dx$$

$$f(x) \cdot (e^{-\alpha x}) = \frac{3e^{-\alpha x}}{-\alpha} + c$$

$$x = 0 \Rightarrow 2 = \frac{-3}{\alpha} + c \Rightarrow \frac{3}{\alpha} = c - 2$$

$$f(x) = \frac{-3}{\alpha} + c^{\alpha x}$$

$$x \rightarrow -\infty \Rightarrow 1 = \frac{-3}{\alpha} + c(0)$$

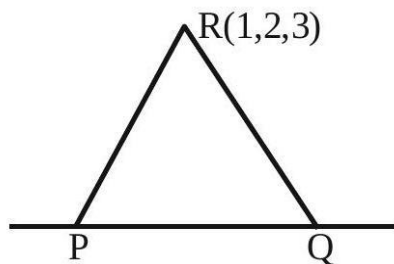
$$\alpha = -3 \therefore c = 1$$

$$f(-\ln 2) = \frac{-3}{\alpha} + c^{\alpha x}$$

$$= 1 + e^{3 \ln 2} = 9$$

(But  $\alpha$  should be greater than 0 for finite value of  $c$ )

68. (c)



$$P(8\lambda - 3, 2\lambda + 4, 2\lambda - 1)$$

$$PR = 6$$

$$(8\lambda - 4)^2 + (2\lambda + 2)^2 + (2\lambda - 4)^2 = 36$$

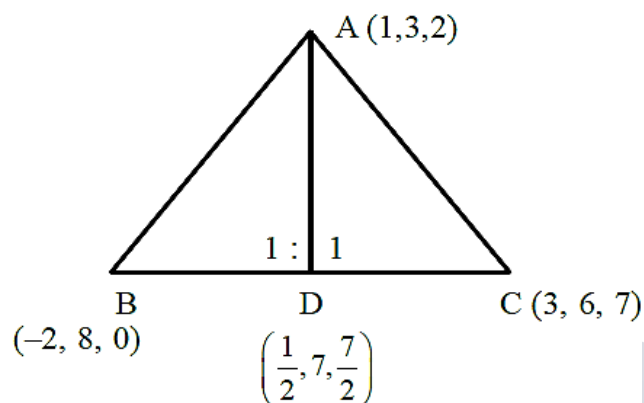
$$\lambda = 0,1$$

$$\text{Hence } P(-3,4,-1) \& Q(5,6,1)$$

$$\text{Centroid of } \triangle PQR = (1,4,1) \equiv (\alpha, \beta, \gamma)$$

$$\alpha^2 + \beta^2 + \gamma^2 = 18$$

69. (a)



$$A(1,3,2); B(-2,8,0); C(3,6,7);$$

$$\overrightarrow{AC} = 2\hat{i} + 3\hat{j} + 5\hat{k}$$

$$AB = \sqrt{9 + 25 + 4} = \sqrt{38}$$

$$AC = \sqrt{4 + 9 + 25} = \sqrt{38}$$

$$\overrightarrow{AD} = \frac{1}{2}\hat{i} - 4\hat{j} - \frac{3}{2}\hat{k} = \frac{1}{2}(\hat{i} - 8\hat{j} - 3\hat{k})$$

Length of projection of  $\overrightarrow{AD}$  on  $\overrightarrow{AC}$

$$= \frac{|\overrightarrow{AD} \cdot \overrightarrow{AC}|}{|\overrightarrow{AC}|} = \frac{37}{2\sqrt{38}}$$

70. (c)  $S_{10} = 390$

$$\frac{10}{2}[2a + (10 - 1)d] = 390$$

$$\Rightarrow 2a + 9d = 78$$

$$\frac{t_{10}}{t_5} = \frac{15}{7} \Rightarrow \frac{a + 9d}{a + 4d} = \frac{15}{7} \Rightarrow 8a = 3d$$

From (a) & (b)  $a = 3$  &  $d = 8$

$$S_{15} - S_5 = \frac{15}{2}(6 + 14 \times 8) - \frac{5}{2}(6 + 4 \times 8)$$

$$= \frac{15 \times 118 - 5 \times 38}{2} = 790$$

71. (a)

$$\int_0^{\pi/3} \cos^4 x dx$$

$$= \int_0^{\pi/3} \left( \frac{1 + \cos 2x}{2} \right)^2 dx$$

$$= \frac{1}{4} \int_0^{\pi/3} (1 + 2\cos 2x + \cos^2 2x) dx$$

$$= \frac{1}{4} \left[ \int_0^{\pi/3} dx + 2 \int_0^{\pi/3} \cos 2x dx + \int_0^{\pi/3} \frac{1 + \cos 4x}{2} dx \right]$$

$$= \frac{1}{4} \left[ \frac{\pi}{3} + (\sin 2x)_0^{\pi/3} + \frac{1}{2} \left( \frac{\pi}{3} \right) + \frac{1}{8} (\sin 4x)_0^{\pi/3} \right]$$

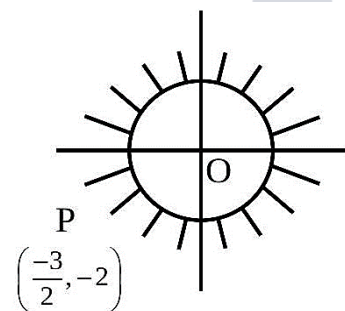
$$= \frac{1}{4} \left[ \frac{\pi}{3} + (\sin 2x)_0^{\pi/3} + \frac{1}{2} \left( \frac{\pi}{3} \right) + \frac{1}{8} (\sin 4x)_0^{\pi/3} \right]$$

$$= \frac{1}{4} \left[ \frac{\pi}{2} + \frac{\sqrt{3}}{2} + \frac{1}{8} \times \left( -\frac{\sqrt{3}}{2} \right) \right]$$

$$= \frac{\pi}{2} + \frac{7\sqrt{3}}{64}$$

$$\therefore a = \frac{1}{8}; b = \frac{7}{64}$$

$$\therefore 9a + 8b = \frac{9}{8} + \frac{7}{8} = 2$$

72. (c)  $|z| \geq 1$ 

Min. value of  $\left| z + \frac{3}{2} + 2i \right|$  is actually zero.

73. (c)

$$f(x) = \frac{\sqrt{x^2 - 25}}{4 - x^2} + \log_{10}(x^2 + 2x - 15)$$

$$\text{Domain: } x^2 - 25 \geq 0 \Rightarrow x \in (-\infty, -5] \cup [5, \infty)$$

$$4 - x^2 \neq 0 \Rightarrow x \neq \{-2, 2\}$$

$$x^2 + 2x - 15 > 0 \Rightarrow (x + 5)(x - 3) > 0$$

$$\Rightarrow x \in (-\infty, -5) \cup (3, \infty)$$

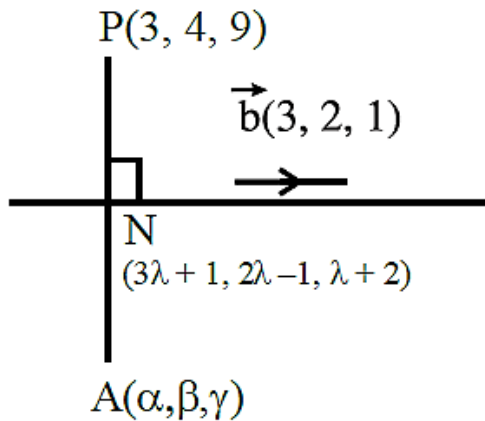
$$\therefore x \in (-\infty, -5) \cup [5, \infty)$$

$$\alpha = -5; \beta = 5$$

$$\therefore \alpha^2 + \beta^3 = 150$$

74. (b)  $R_1: a \sim b \Leftrightarrow a^2 + b^2 = 1; a, b \in \mathbb{R}$   $R_2: (a, b) \sim (c, d) \Leftrightarrow a + d = b + c; (a, b), (c, d) \in \mathbb{N}$  for  $R_1$  : Not reflexive symmetric not transitive for  $R_2$ :  $R_2$  is reflexive, symmetric and transitive Hence only  $R_2$  is equivalence relation.

75. (c)



$$\overrightarrow{PN} \cdot \vec{b} = 0 ?$$

$$3(3\lambda - 2) + 2(2\lambda - 5) + (\lambda - 7) = 0$$

$$14\lambda = 23 \Rightarrow \lambda = \frac{23}{14}$$

$$N\left(\frac{83}{14}, \frac{32}{14}, \frac{51}{14}\right)$$

$$\therefore \frac{\alpha + 3}{2} = \frac{83}{14} \Rightarrow \alpha = \frac{62}{7}$$

$$\frac{\beta + 4}{2} = \frac{32}{14} \Rightarrow \beta = \frac{4}{7}$$

$$\frac{\gamma + 9}{2} = \frac{51}{14} \Rightarrow \gamma = \frac{-12}{7}$$

$$\text{Ans. } 14(\alpha + \beta + \gamma) = 108$$

76. (b)

$$f(x) = \begin{cases} x - 1; & x = \text{even} \\ 2x; & x = \text{odd} \end{cases}$$

$$f(f(f(a))) = 21$$

C - 1: If  $a = \text{even}$

$$f(a) = a - 1 = \text{odd}$$

$$f(f(a)) = 2(a - 1) = \text{even}$$

$$f(f(f(a))) = 2a - 3 = 21 \Rightarrow a = 12$$

C-2: If  $a = \text{odd}$

$$f(a) = 2a = \text{even}$$

$$f(f(a)) = 2a - 1 = \text{odd}$$

$$f(f(f(a))) = 4a - 2 = 21 \text{ (Not possible)}$$

Hence  $a = 12$

Now

$$\lim_{x \rightarrow 12^-} \left( \frac{|x|^3}{2} - \left[ \frac{x}{12} \right] \right)$$

$$= \lim_{x \rightarrow 12^-} \frac{|x|^3}{2} - \lim_{x \rightarrow 12^-} \left[ \frac{x}{12} \right]$$

$$= 144 - 0 = 144.$$

77. (b)

$$x + 2y + 3z = 5$$

$$2x + 3y + z = 9$$

$$4x + 3y + \lambda z = \mu$$

for infinite following  $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = -13$$

$$\Delta_1 = \begin{vmatrix} 5 & 2 & 3 \\ 9 & 3 & 1 \\ \mu & 3 & -13 \end{vmatrix} = 0 \Rightarrow \mu = 15$$

$$\Delta_2 = \begin{vmatrix} 1 & 5 & 3 \\ 2 & 9 & 1 \\ 4 & 15 & -13 \end{vmatrix} = 0$$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 9 \\ 4 & 3 & 15 \end{vmatrix} = 0$$

for  $\lambda = -13, \mu = 15$  system of equation has infinite solution hence  $\lambda + 2\mu = 17$

78. (a)

$$x_1, x_2 \dots \dots x_{10}$$

$$\sum_{i=1}^{10} (x_i - \alpha) = 2 \Rightarrow \sum_{i=1}^{10} x_i - 10\alpha = 2$$

$$\text{Mean } \mu = \frac{6}{5} = \frac{\sum x_i}{10}$$

$$\therefore \sum x_i = 12$$

$$10\alpha + 2 = 12 \therefore \alpha = 1$$

$$\text{Now } \sum_{i=1}^{10} (x_i - \beta)^2 = 40 \text{ Let } y_i = x_i - \beta$$

$$\therefore \sigma_y^2 = \frac{1}{10} \sum y_i^2 - (\bar{y})^2$$

$$\sigma_x^2 = \frac{1}{10} \sum (x_i - \beta)^2 - \left( \frac{\sum_{i=1}^{10} (x_i - \beta)}{10} \right)^2$$

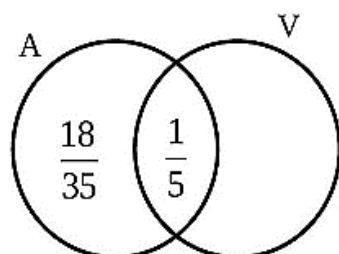
$$\frac{84}{25} = 4 - \left( \frac{12 - 10\beta}{10} \right)^2$$

$$\therefore \left( \frac{6 - 5\beta}{5} \right)^2 = 4 - \frac{84}{25} = \frac{16}{25}$$

$$6 - 5\beta = \pm 4 \Rightarrow \beta = \frac{2}{5} \text{ (not possible) or } \beta = 2$$

$$\text{Hence } \frac{\beta}{\alpha} = 2$$

79. (b)



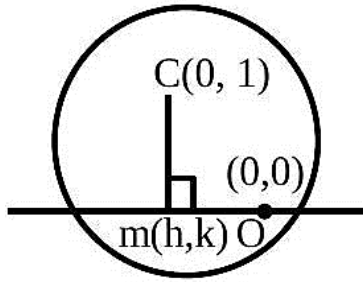
$$P(\bar{A}) = \frac{2}{7} = p$$

$$P(A \cap V) = \frac{1}{5} = q$$

$$P(A) = \frac{5}{7}$$

$$\text{Ans. } P(A \cap \bar{V}) = \frac{18}{35}$$

80. (a)

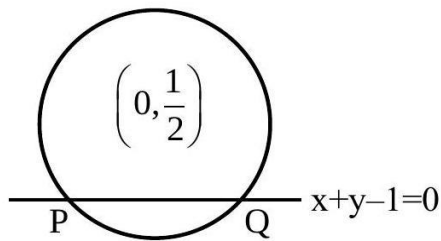


$$m_{OM} \cdot m_{CM} = -1$$

$$\frac{k}{h} \cdot \frac{k-1}{h} = -1$$

$$\therefore \text{locus is } x^2 + y(y-1) = 0$$

$$x^2 + y^2 - y = 0$$



$$p = \left| \frac{1/2}{\sqrt{2}} \right| \quad p = \frac{1}{2\sqrt{2}}$$

$$PQ = 2\sqrt{r^2 - p^2}$$

$$= 2\sqrt{\frac{1}{4} - \frac{1}{8}} = \frac{1}{\sqrt{2}}$$

81. (1)

a, ar, ar<sup>2</sup> → G.P.

Sum of any two sides > third side

$$a + ar > ar^2, a + ar^2 > ar, ar + ar^2 > a$$

$$r^2 - r - 1 < 0$$

$$r \in \left( \frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2} \right)$$

$$r^2 - r + 1 > 0$$

always true  $r^2 + r - 1 > 0$

$$r \in \left( -\infty, \frac{-1-\sqrt{5}}{2} \right) \cup \left( \frac{-1+\sqrt{5}}{2}, \infty \right)$$

Taking intersection of (a) , (b)

$$r \in \left( \frac{-1 + \sqrt{5}}{2}, \frac{1 + \sqrt{5}}{2} \right)$$

As  $r > 1$

$$r \in \left( 1, \frac{1 + \sqrt{5}}{2} \right)$$

$$[r] = 1 \quad [-r] = -2$$

$$3[r] + [-r] = 1$$

**82. (2)**

$$A = I_2 - 2MM^T$$

$$A^2 = (I_2 - 2MM^T)(I_2 - 2MM^T)$$

$$= I_2 - 2MM^T - 2MM^T + 4MM^TMM^T$$

$$= I_2 - 4MM^T + 4MM^T$$

$$= I_2$$

$$AX = \lambda X$$

$$A^2X = \lambda AX$$

$$X = \lambda(\lambda X)$$

$$X = \lambda^2 X$$

$$X(\lambda^2 - 1) = 0$$

$$\lambda^2 = 1$$

$$\lambda = \pm 1$$

Sum of square of all possible values = 2

**83. (219)**

$$F(x) = \int_0^x t \cdot f(t) dt$$

$$F^1(x) = xf(x)$$

$$F(x^2) = x^4 + x^5, \text{ let } x^2 = t$$

$$F(t) = t^2 + t^{5/2}$$

$$F'(t) = 2t + 5/2 t^{3/2}$$

$$t \cdot f(t) = 2t + 5/2 t^{3/2}$$

$$f(t) = 2 + 5/2 t^{1/2}$$

$$\sum_{r=1}^{12} f(r^2) = \sum_{r=1}^{12} 2 + \frac{5}{2} r$$

$$= 24 + 5/2 \left[ \frac{12(13)}{2} \right]$$

$$= 219$$

84. (105)

$$y = \frac{(\sqrt{x} + 1)(x^2 - \sqrt{x})}{x\sqrt{x} + x + \sqrt{x}} + \frac{1}{15}(3\cos^2 x - 5)\cos^3 x$$

$$y = \frac{(\sqrt{x} + 1)(\sqrt{x})((\sqrt{x})^3 - 1)}{(\sqrt{x})((\sqrt{x})^2 + (\sqrt{x}) + 1)} + \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x$$

$$y = (\sqrt{x} + 1)(\sqrt{x} - 1) + \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x$$

$$y' = 1 - \cos^4 x \cdot (\sin x) + \cos^2 x(\sin x)$$

$$y' \left( \frac{\pi}{6} \right) = 1 - \frac{9}{16} \times \frac{1}{2} + \frac{3}{4} \times \frac{1}{2}$$

$$= \frac{32 - 9 + 12}{32} = \frac{35}{32}$$

$$= 96y' \left( \frac{\pi}{6} \right) = 105$$

85. (38)  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$$\vec{b} = \hat{i} + 8\hat{j} + 2\hat{k}$$

$$\vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$$

$$\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$$

$$(\vec{b} - \vec{c}) \times \vec{a} = 0$$

$$\vec{b} - \vec{c} = \lambda \vec{a}$$

$$\vec{b} = \vec{c} + \lambda \vec{a}$$

$$-\hat{i} - 8\hat{j} + 2\hat{k} = (4\hat{i} + c_2\hat{j} + c_3\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$$

$$\lambda + 4 = -1 \Rightarrow \lambda = -5$$

$$\lambda + c_2 = -8 \Rightarrow c_2 = -3$$

$$\lambda + c_3 = 2 \Rightarrow c_3 = 7$$

$$\vec{c} = 4\hat{i} - 3\hat{j} + 7\hat{k}$$

$$\cos \theta = \frac{12 - 12 + 7}{\sqrt{26} \cdot \sqrt{74}} = \frac{7}{\sqrt{26} \cdot \sqrt{74}} = \frac{7}{2\sqrt{481}}$$

$$\tan^2 \theta = \frac{625 \times 3}{49}$$

$$[\tan^2 \theta] = 38$$

86. (101)

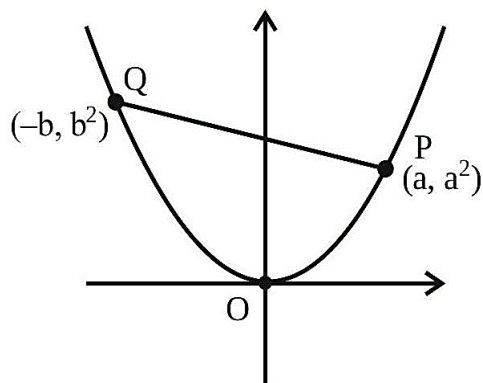
$L_1, L_3, L_5, \dots, L_{19}$  are Parallel

$L_2, L_4, L_6, \dots, L_{20}$  are Concurrent

$$\text{Total points of intersection} = {}^{20}C_2 - {}^{10}C_2 - {}^{10}C_2 + 1$$

$$= 101$$

87. (7)



$$S_2 = \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ a & a^2 & 1 \\ -b & b^2 & 1 \end{vmatrix} = \frac{1}{2}(ab^2 + a^2b)$$

$$\text{PQ: } -y - a^2 = \frac{a^2 - b^2}{a + b}(x - a)$$

$$y - a^2 = (a - b)x - (a - b)a$$

$$y = (a - b)x + ab$$

$$S_1 = \int_{-b}^a ((a - b)x + ab - x^2) dx$$

$$= (a - b) \frac{x^2}{2} + (ab)x - \frac{x^3}{3} \Big|_{-b}^a$$

$$= \frac{(a - b)^2(a + b)}{2} + ab(a + b) - \frac{(a^3 + b^3)}{3}$$

$$\frac{S_1}{S_2} = \frac{\frac{(a - b)^2}{2} + ab - \frac{(a^2 + b^2 - ab)}{3}}{\frac{ab}{2}}$$

$$= \frac{3(a-b)^2 + 6ab - 2(a^2 + b^2 - ab)}{3ab}$$

$$= \frac{1}{3} \left[ \frac{a}{b} + \frac{b}{a} + 2 \right]$$

$$= \frac{4}{3} = \frac{m}{n} \quad m + n = 7$$

**88. (8)**

$$ky^2 = 2(y-x) \quad 2y^2 = kx$$

Point of intersection  $\rightarrow$

$$ky^2 = \left( \frac{y - \frac{2y^2}{k}}{k} \right)$$

$$y = 0 \quad ky = 2 \left( \frac{1 - \frac{2y}{k}}{k} \right)$$

$$ky + \frac{4y}{k} = 2$$

$$k + \frac{2}{k} = \frac{2k}{k^2 + 4}$$

$$\int_0^{k^2+4} \left( \left( y - \frac{ky}{2} \right) - \left( \frac{2y^2}{k} \right) \right) \cdot dy$$

$$= \frac{y^2}{2} - \left( \frac{k}{2} + \frac{2}{k} \right) \cdot \frac{y^3}{3} \Big|_0^{\frac{2k}{k^2+4}}$$

$$= \left( \frac{2k}{k^2+4} \right)^2 \left[ \frac{1}{2} - \frac{k^2+4}{2k} \times \frac{1}{3} \times \frac{2k}{k^2+4} \right]$$

$$= \frac{1}{6} \times 4 \times \left( \frac{1}{k + \frac{4}{k}} \right)^2$$

$$A \cdot M \geq G \cdot M \quad \frac{\left( k + \frac{4}{k} \right)}{2} \geq 2$$

$$k + \frac{4}{k} \geq 4$$

Area is maximum when  $k = \frac{4}{k}$

$$k = 2, -2$$

**89. (5)**

$$\frac{dx}{dy} - \frac{x}{y} = \frac{1-y^2}{y}$$

$$\text{Integrating factor} = e^{\int -\frac{1}{y} dy} = \frac{1}{y}$$

$$x \cdot \frac{1}{y} = \int \frac{1-y^2}{y^2} dy$$

$$\frac{x}{y} = \frac{-1}{y} - y + c$$

$$x = -1 - y^2 + cy$$

$$x(a) = 1$$

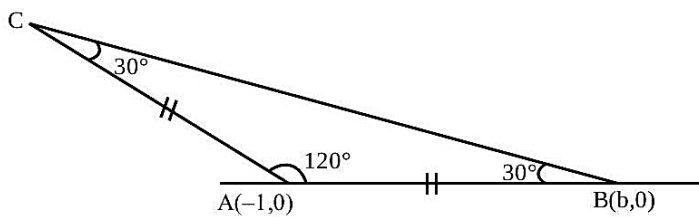
$$1 = -1 - 1 + c \Rightarrow c = 3$$

$$x = -1 - y^2 + 3y$$

$$5x(b) = 5(-1 - 4 + 6)$$

$$= 5$$

90. (36)



$$\frac{c}{\sin 30^\circ} = \frac{4\sqrt{3}}{\sin 120^\circ} \text{ [By sine rule]}$$

$$2c = 8 \Rightarrow c = 4$$

$$AB = |(b+1)| = 4$$

$$b = 3, m_{AB} = 0$$

$$m_{BC} = \frac{-1}{\sqrt{3}}$$

$$BC: -y = \frac{-1}{\sqrt{3}}(x-3)$$

$$\sqrt{3}y + x = 3$$

$$\text{Point of intersection: } y = x + 3, \sqrt{3}y + x = 3$$

$$(\sqrt{3} + 1)y = 6$$

$$y = \frac{6}{\sqrt{3} + 1}$$

$$x = \frac{6}{\sqrt{3} + 1} - 3$$

$$= \frac{6 - 3\sqrt{3} - 3}{\sqrt{3} + 1}$$

$$= 3 \frac{(1 - \sqrt{3})}{(1 + \sqrt{3})} = \frac{-6}{(1 + \sqrt{3})^2}$$

$$\frac{\beta^4}{\alpha^2} = 36$$

