

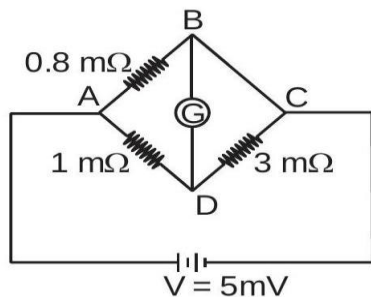
PART- I PHYSICS

SECTION- A

1. In an ammeter, 5% of the main current passes through the galvanometer. If resistance of the galvanometer is G , the resistance of ammeter will be:

- (a) $\frac{G}{20}$ (b) $\frac{G}{199}$
(c) $199G$ (d) $200G$

2. To measure the temperature coefficient of resistivity α of a semiconductor, an electrical arrangement shown in the figure is prepared. The arm BC is made up of the semiconductor. The experiment is being conducted at 25°C and resistance of the semiconductor arm is $3\text{ m}\Omega$. Arm BC is cooled at a constant rate of 2°C/s . If the galvanometer G shows no deflection after 10 s, then α is :



- (a) $-2 \times 10^{-2} ^\circ\text{C}^{-1}$ (b) $-1.5 \times 10^{-2} ^\circ\text{C}^{-1}$
(c) $-1 \times 10^{-2} ^\circ\text{C}^{-1}$ (d) $-2.5 \times 10^{-2} ^\circ\text{C}^{-1}$

3. From the statements given below:

- (A) The angular momentum of an electron in n^{th} orbit is an integral multiple of h .
(B) Nuclear forces do not obey inverse square law.
(C) Nuclear forces are spin dependent.
(D) Nuclear forces are central and charge independent.
(E) Stability of nucleus is inversely proportional to the value of packing fraction.

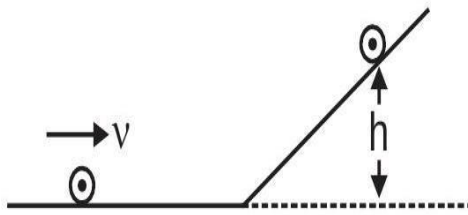
Choose the correct answer from the options given below:

- (a) (A), (B), (C), (D) only (b) (A), (C), (D), (E) only
(c) (A), (B), (C), (E) only (d) (B), (C), (D), (E) only

4. A diatomic gas ($\gamma = 1.4$) does 200 J of work when it is expanded isobarically. The heat given to the gas in the process is:

- (a) 850 J (b) 800 J
(c) 600 J (d) 700 J

5. A disc of radius R and mass M is rolling horizontally without slipping with speed v . It then moves up an inclined smooth surface as shown in figure. The maximum height that the disc can go up the incline is :



- (a) $\frac{v^2}{g}$ (b) $\frac{3v^2}{4g}$
(c) $\frac{1}{2} \frac{v^2}{g}$ (d) $\frac{2}{3} \frac{v^2}{g}$

6. Conductivity of a photodiode starts changing only if the wavelength of incident light is less than 660 nm. The band gap of photodiode is found to be $\left(\frac{X}{8}\right)$ eV. The value of X is :

(Given, $h = 6.6 \times 10^{-34}$ Js, $e = 1.6 \times 10^{-19}$ C)

- (a) 15 (b) 11
(c) 13 (d) 21

7. A big drop is formed by coalescing 1000 small droplets of water. The surface energy will become:

- (a) 100 times (b) 10 times
(c) $\frac{1}{100}$ th (d) $\frac{1}{10}$ th

8. If frequency of electromagnetic wave is 60MHz and it travels in air along z direction then the corresponding electric and magnetic field vectors will be mutually perpendicular to each other and the wavelength of the wave (in m) is :

- (a) 2.5 (b) 10
(c) 5 (d) 2

9. A cricket player catches a ball of mass 120 g moving with 25 m/s speed. If the catching process is completed in 0.1 s then the magnitude of force exerted by the ball on the hand of player will be (in SI unit):

- (a) 24 (b) 12
(c) 25 (d) 30

10. Monochromatic light of frequency 6×10^{14} Hz is produced by a laser. The power emitted is 2×10^{-3} W. How many photons per second on an average, are emitted by the source ?

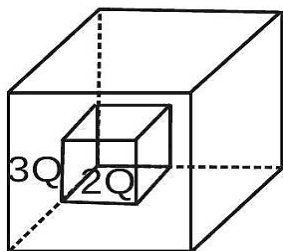
(Given $h = 6.63 \times 10^{-34}$ Js)

- (a) 9×10^{18} (b) 6×10^{15}
(c) 5×10^{15} (d) 7×10^{16}

11. A microwave of wavelength 2.0 cm falls normally on a slit of width 4.0 cm. The angular spread of the central maxima of the diffraction pattern obtained on a screen 1.5 m away from the slit, will be:

- (a) 30° (b) 15°
(c) 60° (d) 45°

12. C_1 and C_2 are two hollow concentric cubes enclosing charges $2Q$ and $3Q$ respectively as shown in figure. The ratio of electric flux passing through C_1 and C_2 is:



- (a) 2:5 (b) 5:2
(c) 2:3 (d) 3:2

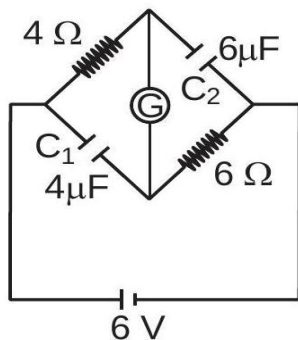
13. If the root mean square velocity of hydrogen molecule at a given temperature and pressure is 2 km/s, the root mean square velocity of oxygen at the same condition in km/s is :

- (a) 2.0 (b) 0.5
(c) 1.5 (d) 1.0

14. Train A is moving along two parallel rail tracks towards north with speed 72 km/h and train B is moving towards south with speed 108 km/h. Velocity of train B with respect to A and velocity of ground with respect to B are (in ms^{-1}) :

- (a) -30 and 50 (b) -50 and -30
(c) -50 and 30 (d) 50 and -30

15. A galvanometer (G) of 2Ω resistance is connected in the given circuit. The ratio of charge stored in C_1 and C_2 is:



- (a) $\frac{2}{3}$ (b) $\frac{3}{2}$
(c) 1 (d) $\frac{1}{2}$

16. In a metre-bridge when a resistance in the left gap is 2Ω and unknown resistance in the right gap, the balance length is found to be 40 cm. On shunting the unknown resistance with 2Ω , the balance length changes by:

- (a) 22.5 cm (b) 20 cm
(c) 62.5 cm (d) 65 cm

17. Match List - I with List - II.

List - I
(Number)

- (A) 1001
(B) 010.1
(C) 100.100
(D) 0.0010010

List - II
(Significant figure)

- (I) 3
(II) 4
(III) 5
(IV) 6

Choose the correct answer from the options given below :

- (a) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(b) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(c) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
(d) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

18. A transformer has an efficiency of 80% and works at 10 V and 4 kW. If the secondary voltage is 240 V, then the current in the secondary coil is :

- (a) 1.59 A (b) 13.33 A
(c) 1.33 A (d) 15.1 A

19. A light planet is revolving around a massive star in a circular orbit of radius R with a period of revolution T. If the force of attraction between planet and star is proportional to $R^{-3/2}$ then choose the correct option :

- (a) $T^2 \propto R^{5/2}$ (b) $T^2 \propto R^{7/2}$
(c) $T^2 \propto R^{3/2}$ (d) $T^2 \propto R^3$

20. A body of mass 4 kg experiences two forces $\vec{F}_1 = 5\hat{i} + 8\hat{j} + 7\hat{k}$ and $\vec{F}_2 = 3\hat{i} - 4\hat{j} - 3\hat{k}$. The acceleration acting on the body is:

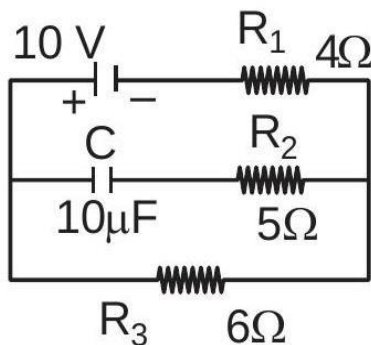
- (a) $-2\hat{i} - \hat{j} - \hat{k}$ (b) $4\hat{i} + 2\hat{j} + 2\hat{k}$
(c) $2\hat{i} + \hat{j} + \hat{k}$ (d) $2\hat{i} + 3\hat{j} + 3\hat{k}$

SECTION- B

21. A mass m is suspended from a spring of negligible mass and the system oscillates with a frequency f_1 . The frequency of oscillations if a mass $9m$ is suspended from the same spring is f_2 . The value of $\frac{f_1}{f_2}$ is _____
22. A particle initially at rest starts moving from reference point. $x = 0$ along x -axis, with velocity v that varies as $v = 4\sqrt{x}$ m/s. The acceleration of the particle is _____ ms^{-2} .
23. A moving coil galvanometer has 100 turns and each turn has an area of 2.0 cm^2 . The magnetic field produced by the magnet is 0.01 T and the deflection in the coil is 0.05 radian when a current of 10 mA is passed through it. The torsional constant of the suspension wire is $x \times 10^{-5} \text{ N-m/rad}$. The value of x is _____.
24. One end of a metal wire is fixed to a ceiling and a load of 2 kg hangs from the other end. A similar wire is attached to the bottom of the load and another load of 1 kg hangs from this lower wire. Then the ratio of longitudinal strain of upper wire to that of the lower wire will be _____.

[Area of cross section of wire = 0.005 cm^2 , $Y = 2 \times 10^{11} \text{ Nm}^{-2}$ and $g = 10 \text{ ms}^{-2}$]

25. A particular hydrogen-like ion emits the radiation of frequency $3 \times 10^{15} \text{ Hz}$ when it makes transition from $n = 2$ to $n = 1$. The frequency of radiation emitted in transition from $n = 3$ to $n = 1$ is $\frac{x}{9} \times 10^{15} \text{ Hz}$, when $x =$ _____.
26. In an electrical circuit drawn below the amount of charge stored in the capacitor is _____ μC .



27. A coil of 200 turns and area 0.20 m^2 is rotated at half a revolution per second and is placed in uniform magnetic field of 0.01 T perpendicular to axis of rotation of the coil. The maximum voltage generated in the coil is $\frac{2\pi}{\beta}$ volt. The value of β is _____
28. In Young's double slit experiment, monochromatic light of wavelength 5000 Å is used. The slits are 1.0 mm apart and screen is placed at 1.0 m away from slits. The distance from the centre of the screen where intensity becomes half of the maximum intensity for the first time is _____ $\times 10^{-6} \text{ m}$.
29. A uniform rod AB of mass 2 kg and Length 30 cm at rest on a smooth horizontal surface. An impulse of force 0.2 Ns is applied to end B. The time taken by the rod to turn through at right angles will be $\frac{\pi}{x} \text{ s}$, where $x =$ _____.
30. Suppose a uniformly charged wall provides a uniform electric field of $2 \times 10^4 \text{ N/C}$ normally. A charged particle of mass 2 g being suspended through a silk thread of length 20 cm and remain stayed at a distance

of 10 cm from the wall. Then the charge on the particle will be $\frac{1}{\sqrt{x}} \mu\text{C}$ where $x = \underline{\hspace{2cm}}$ [use $g = 10 \text{ m/s}^2$]

PART-II CHEMISTRY

SECTION- A

31. The transition metal having highest 3rd ionisation enthalpy is:

- (a) Cr (b) Mn
(c) V (d) Fe

32. Given below are two statements:

Statement (I): A π bonding MO has lower electron density above and below the inter-nuclear axis.

Statement (II): The π^* antibonding MO has a node between the nuclei.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (a) Both Statement I and Statement II are false
(b) Both Statement I and Statement II are true
(c) Statement I is false but Statement II is true
(d) Statement I is true but Statement II is false

33. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): In aqueous solutions Cr^{2+} is reducing while Mn^{3+} is oxidising in nature.

Reason (R): Extra stability to half-filled electronic configuration is observed than incompletely filled electronic configuration.

In the light of the above statement, choose the most appropriate answer from the options given below:

- (a) Both (A) and (R) are true and (R) is the correct explanation of (A)
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A)
(c) (A) is false but (R) is true
(d) (A) is true but (R) is false

34. Match List - I with List - II.

List-I	List-II
(Reactants)	Products
(A) Phenol, Zn/Δ	(I) Salicylaldehyde
(B) Phenol, CHCl_3 , NaOH , HCl	(II) Salicylic acid
(C) Phenol, CO_2 , NaOH , HCl	(III) Benzene
(D) Phenol, Conc. HNO_3	(IV) Picric acid

- (a) (A)-(IV), (B), (II), (C)-(I), (D)-(III)
(b) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
(c) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)
(d) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

35. Given below are two statements:

Statement (I): Both metal and non-metal exist in p and d-block elements.

Statement (II): Non-metals have higher ionisation enthalpy and higher electronegativity than the metals.

In the light of the above statements, choose the most appropriate answer from the option given below:

- (a) Both Statement I and Statement II are false
- (b) Statement I is false but Statement II is true
- (c) Statement I is true but Statement II is false
- (d) Both Statement I and Statement II are true

36. The strongest reducing agent among the following is:

- (a) NH_3 (b) SbH_3
- (c) BiH_3 (d) PH_3

37. Which of the following compounds show colour due to d-d transition?

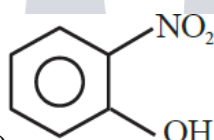
- (a) $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$ (b) $\text{K}_2\text{Cr}_2\text{O}_7$
- (c) K_2CrO_4 (d) KMnO_4

38. The set of meta directing functional groups from the following sets is:

- (a) $-\text{CN}$, $-\text{NH}_2$, $-\text{NHR}$, $-\text{OCH}_3$
- (b) $-\text{NO}_2$, $-\text{NH}_2$, $-\text{COOH}$, $-\text{COOR}$
- (c) $-\text{NO}_2$, $-\text{CHO}$, $-\text{SO}_3\text{H}$, $-\text{COR}$
- (d) $-\text{CN}$, $-\text{CHO}$, $-\text{NHCOCH}_3$, $-\text{COOR}$

39. Select the compound from the following that will show intramolecular hydrogen bonding.

- (a) H_2O (b) NH_3



- (c) $\text{C}_2\text{H}_5\text{OH}$ (d)

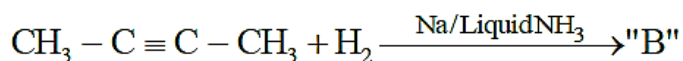
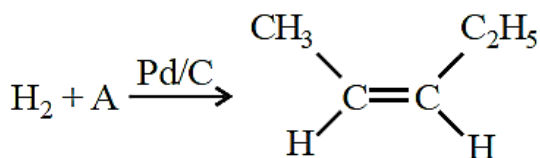
40. Lassaigne's test is used for detection of:

- (a) Nitrogen and Sulphur only
- (b) Nitrogen, Sulphur and Phosphorous Only
- (c) Phosphorous and halogens only
- (d) Nitrogen, Sulphur, phosphorous and halogens

41. Which among the following has highest boiling point?

- (a) $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_3$ (b) $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_2 - \text{OH}$
- (c) $\text{CH}_3\text{CH}_2\text{CH}_2\text{CHO}$ (d) $\text{H}_5\text{C}_2 - \text{O} - \text{C}_2\text{H}_5$

42. In the given reactions identify A and B.



- (a) A: 2-Pentyne B: trans -2 - butene
 (b) A: n-Pentane B: trans -2 - butene
 (c) A: 2 - Pentyne B: Cis -2 - butene
 (d) A: n-Pentane B: Cis -2 - butene

43. The number of radial node/s for 3p orbital is:

- (a) 1 (b) 4
 (c) 2 (d) 3

44. Match List - I with List - II.

List - I

List - II

Compound

Use

(A) Carbon tetrachloride

(I) Paint remover

(B) Methylene chloride

(II) Refrigerators and air conditioners

(C) DDT

(III) Fire extinguisher

(D) Freons

(IV) Non- Biodegradable insecticide

Choose the correct answer from the options given below :

- (a) (A)-(I), (B), (II), (C)-(III), (D)-(IV)
 (b) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)
 (c) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)
 (d) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

45. The functional group that shows negative resonance effect is:

- (a) $-\text{NH}_2$ (b) $-\text{OH}$
 (c) $-\text{COOH}$ (d) $-\text{OR}$

46. $[\text{Co}(\text{NH}_3)_6]^{3+}$ and $[\text{CoF}_6]^{3-}$ are respectively known as:

- (a) Spin free Complex, Spin paired Complex
 (b) Spin paired Complex, Spin free Complex
 (c) Outer orbital Complex, Inner orbital Complex
 (d) Inner orbital Complex, Spin paired Complex

47. Given below are two statements:

Statement (I): SiO_2 and GeO_2 are acidic while SnO and PbO are amphoteric in nature.

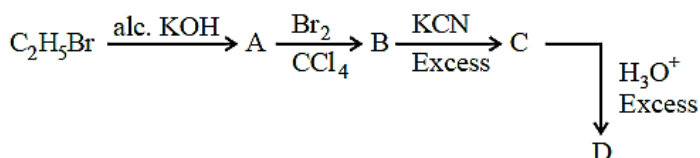
Statement (II): Allotropic forms of carbon are due to property of catenation and $p\pi - d\pi$ bond formation.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (a) Both Statement I and Statement II are false

- (b) Both Statement I and Statement II are true
 (c) Statement I is true but Statement II is false
 (d) Statement I is false but Statement II is true

48.



Acid D formed in above reaction is:

- (a) Gluconic acid (b) Succinic acid
 (c) Oxalic acid (d) Malonic acid

49. Solubility of calcium phosphate (molecular mass, M) in water is W_g per 100 mL at 25°C . Its solubility product at 25°C will be approximately.

- (a) $10^7 \left(\frac{W}{M}\right)^3$ (b) $10^7 \left(\frac{W}{M}\right)^5$
 (c) $10^3 \left(\frac{W}{M}\right)^5$ (d) $10^5 \left(\frac{W}{M}\right)^5$

50. Given below are two statements:

Statement (I): Dimethyl glyoxime forms a sixmembered covalent chelate when treated with NiCl_2 solution in presence of NH_4OH .

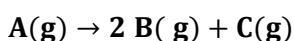
Statement (II): Prussian blue precipitate contains iron both in (+2) and (+3) oxidation states. In the light of the above statements, choose the most appropriate answer from the options given below:

- (a) Statement I is false but Statement II is true
 (b) Both Statement I and Statement II are true
 (c) Both Statement I and Statement II are false
 (d) Statement I is true but Statement II is false

SECTION- B

51. Total number of isomeric compounds (including stereoisomers) formed by monochlorination of 2-methylbutane is _____.

52. The following data were obtained during the first order thermal decomposition of a gas A at constant volume:

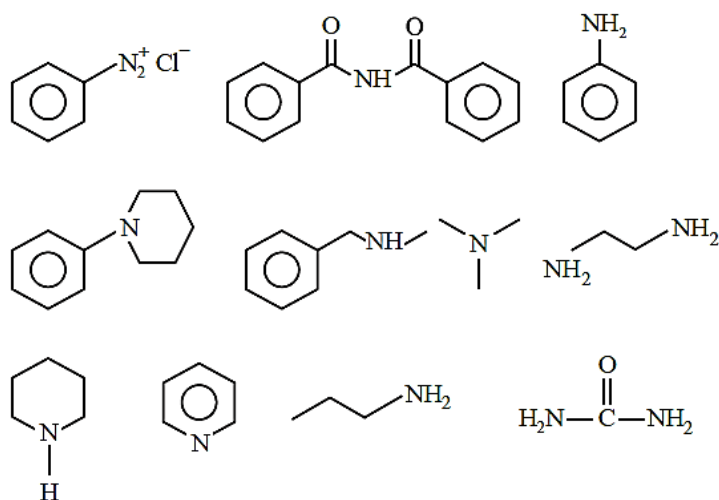


S. No	Time/s	Total pressure/(atm)
1.	0	0.1
2.	115	0.28

The rate constant of the reaction is _____ $\times 10^{-2} \text{ s}^{-1}$ (nearest integer)

53. The number of tripeptides formed by three different amino acids using each amino acid once is _____.

54. Number of compounds which give reaction with Hinsberg's reagent is _____.



55. Mass of ethylene glycol (antifreeze) to be added to 18.6 kg of water to protect the freezing point at -24°C is _____ kg (Molar mass in gmol^{-1} for ethylene glycol 62, K_f of water = $1.86 \text{ K kg mol}^{-1}$)
56. Following Kjeldahl's method, 1 g of organic compound released ammonia, that neutralised 10 mL of $2\text{M H}_2\text{SO}_4$. The percentage of nitrogen in the compound is _____ %.
57. The amount of electricity in Coulomb required for the oxidation of 1 mol of H_2O to O_2 is _____ $\times 10^5 \text{ C}$.
58. For a certain reaction at 300 K, $K = 10$, then ΔG° for the same reaction is - _____ $\times 10^{-1} \text{ kJ mol}^{-1}$.
(Given $R = 8.314 \text{ JK}^{-1} \text{ mol}^{-1}$)

59. Consider the following redox reaction:



The standard reduction potentials are given as below (E_{red}°)

$$E_{\text{MnO}_4^-/\text{Mn}^{2+}}^{\circ} = +1.51 \text{ V}$$

$$E_{\text{CO}_2/\text{H}_2\text{C}_2\text{O}_4}^{\circ} = -0.49 \text{ V}$$

If the equilibrium constant of the above reaction is given as $K_{\text{eq}} = 10^x$, then the value of $x =$ _____ (nearest integer)

60. 10 mL of gaseous hydrocarbon on combustion gives 40 mL of CO_2 (g) and 50 mL of water vapour. Total number of carbon and hydrogen atoms in the hydrocarbon is _____.

PART- III MATHEMATICS

SECTION- A

61. Let $f(x) = |2x^2 + 5|x| - 3|$, $x \in \mathbb{R}$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then $m + n$ is equal to:
(a) 5 (b) 2
(c) 0 (d) 3
62. Let α and β be the roots of the equation $px^2 + qx - r = 0$, where $p \neq 0$. If p, q and r be the consecutive terms of a non-constant G.P and $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$, then the value of $(\alpha - \beta)^2$ is :
(a) $\frac{80}{9}$ (b) 9
(c) $\frac{20}{3}$ (d) 8
63. The number of solutions of the equation $4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0$; $x \in [-2\pi, 2\pi]$ is:
(a) 1 (b) 3

(c) 2 (d) 0

64. The value of $\int_0^1 (2x^3 - 3x^2 - x + 1)^{\frac{1}{3}} dx$ is equal to:

(a) 0 (b) 1

(c) 2 (d) -1

65. Let P be a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Let the line passing through P and parallel to y-axis meet the circle $x^2 + y^2 = 9$ at point Q such that P and Q are on the same side of the x-axis. Then, the eccentricity of the locus of the point R on PQ such that PR: RQ = 4: 3 as P moves on the ellipse, is :

(a) $\frac{11}{19}$ (b) $\frac{13}{21}$
(c) $\frac{\sqrt{139}}{23}$ (d) $\frac{\sqrt{13}}{7}$

66. Let m and n be the coefficients of seventh and thirteenth terms respectively in the expansion of

$\left(\frac{1}{3}x^{\frac{1}{3}} + \frac{1}{2x^{\frac{2}{3}}}\right)^{18}$. Then $\left(\frac{n}{m}\right)^{\frac{1}{3}}$ is:

(a) $\frac{4}{9}$ (b) $\frac{1}{9}$
(c) $\frac{1}{4}$ (d) $\frac{9}{4}$

67. Let α be a non-zero real number. Suppose $f: \mathbb{R} \rightarrow \mathbb{R}$ is a differentiable function such that $f(0) = 2$ and $\lim_{x \rightarrow -\infty} f(x) = 1$. If $f'(x) = \alpha f(x) + 3$, for all $x \in \mathbb{R}$, then $f(-\log_e 2)$ is equal to ____.

(a) 3 (b) 5
(c) 9 (d) 7

68. Let P and Q be the points on the line $\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2}$ which are at a distance of 6 units from the point R(1, 2, 3). If the centroid of the triangle PQR is (α, β, γ) , then $\alpha^2 + \beta^2 + \gamma^2$ is:

(a) 26 (b) 36

(c) 18 (d) 24

69. Consider a $\triangle ABC$ where A(1, 2, 3), B(-2, 8, 0) and C(3, 6, 7). If the angle bisector of $\angle BAC$ meets the line BC at D, then the length of the projection of the vector $\vec{AD}$ on the vector $\vec{AC}$ is:

(a) $\frac{37}{2\sqrt{38}}$ (b) $\frac{\sqrt{38}}{2}$

(c) $\frac{39}{2\sqrt{38}}$ (d) $\sqrt{19}$

70. Let S_n denote the sum of the first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is 15: 7, then $S_{15} - S_5$ is equal to:

(a) 800 (b) 890

(c) 790 (d) 690

71. If $\int_0^{\frac{\pi}{3}} \cos^4 x dx = a\pi + b\sqrt{3}$, where a and b are rational numbers, then $9a + 8b$ is equal to :

(a) 2 (b) 1
(c) 3 (d) $\frac{3}{2}$

72. If z is a complex number such that $|z| \geq 1$, then the minimum value of $\left|z + \frac{1}{2}(3 + 4i)\right|$ is:

(a) $\frac{5}{2}$ (b) 2
(c) 0 (d) $\frac{3}{2}$

73. If the domain of the function $f(x) = \frac{\sqrt{x^2-25}}{(4-x^2)} + \log_{10}(x^2 + 2x - 15)$ is $(-\infty, \alpha) \cup [\beta, \infty)$, then $\alpha^2 + \beta^3$ is equal

to :

- (a) 140 (b) 175
(c) 150 (d) 125

74. Consider the relations R_1 and R_2 defined as aR_1b

$$\Leftrightarrow a^2 + b^2 = 1 \text{ for all } a, b, \in \mathbb{R} \text{ and } (a, b)R_2(c, d)$$

$$\Leftrightarrow a + d = b + c \text{ for all } (a, b), (c, d) \in \mathbb{N} \times \mathbb{N}. \text{ Then}$$

- (a) Only R_1 is an equivalence relation
(b) Only R_2 is an equivalence relation
(c) R_1 and R_2 both are equivalence relations
(d) Neither R_1 nor R_2 is an equivalence relation

75. If the mirror image of the point $P(3, 4, 9)$ in the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$ is (α, β, γ) , then $14(\alpha + \beta + \gamma)$ is :

- (a) 102 (b) 138
(c) 108 (d) 132

76.

$$\text{Let } f(x) = \begin{cases} x-1, & x \text{ is even,} \\ 2x, & x \text{ is odd,} \end{cases} \quad x \in \mathbb{N}. \text{ If for some}$$

$$a \in \mathbb{N}, f(f(f(a))) = 21, \text{ then } \lim_{x \rightarrow a^-} \left\{ \frac{|x|^3}{a} - \left\lfloor \frac{x}{a} \right\rfloor \right\},$$

where $[t]$ denotes the greatest integer less than or equal to t , is equal to:

- (a) 121 (b) 144
(c) 169 (d) 225

77. Let the system of equations $x + 2y + 3z = 5, 2x + 3y + z = 9, 4x + 3y + \lambda z = \mu$ have infinite number of solutions. Then $\lambda + 2\mu$ is equal to:

- (a) 28 (b) 17
(c) 22 (d) 15

78. Consider 10 observation $x_1, x_2, \dots, x_{10}$ such that $\sum_{i=1}^{10} (x_i - \alpha) = 2$ and $\sum_{i=1}^{10} (x_i - \beta)^2 = 40$, where α, β are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$ respectively. The $\frac{\beta}{\alpha}$ is equal to:

- (a) 2 (b) $\frac{3}{2}$
(c) $\frac{5}{2}$ (d) 1

79. Let Ajay will not appear in JEE exam with probability $p = \frac{2}{7}$, while both Ajay and Vijay will appear in the exam with probability $q = \frac{1}{5}$. Then the probability, that Ajay will appear in the exam and Vijay will not appear is:

- (a) $\frac{9}{35}$ (b) $\frac{18}{35}$
(c) $\frac{24}{35}$ (d) $\frac{3}{35}$

80. Let the locus of the mid points of the chords of circle $x^2 + (y - 1)^2 = 1$ drawn from the origin intersect the line $x + y = 1$ at P and Q. Then, the length of PQ is:

- (a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$
(c) $\frac{1}{2}$ (d) 1

SECTION- B

81. If three successive terms of a G.P. with common ratio $r (r > 1)$ are the lengths of the sides of a triangle and $[r]$ denotes the greatest integer less than or equal to r , then $3[r] + [-r]$ is equal to:

82. Let $A = I_2 - MM^T$, where M is real matrix of order 2×1 such that the relation $M^T M = I_1$ holds. If λ is a real number such that the relation $AX = \lambda X$ holds for some non-zero real matrix X of order 2×1 , then the sum of squares of all possible values of λ is equal to:

83. Let $f: (0, \infty) \rightarrow \mathbb{R}$ and $F(x) = \int_0^x t f(t) dt$. If $F(x^2) = x^4 + x^5$, then $\sum_{r=1}^{12} f(r^2)$ is equal to:

84. If $y = \frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}} + \frac{1}{15}(3\cos^2 x - 5)\cos^3 x$, then $96y'(\frac{\pi}{6})$ is equal to :

85. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = -\hat{i} - 8\hat{j} + 2\hat{k}$ and $\vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three vectors such that $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$. If the angle between the vector $\vec{c}$ and the vector $3\hat{i} + 4\hat{j} + \hat{k}$ is θ , then the greatest integer less than or equal to $\tan^2 \theta$ is:

86. The lines $L_1, L_2, \dots, L_{20}$ are distinct. For $n = 1, 2, 3, \dots, 10$ all the lines L_{2n-1} are parallel to each other and all the lines L_{2n} pass through a given point P. The maximum number of points of intersection of pairs of lines from the set $\{L_1, L_2, \dots, L_{20}\}$ is equal to:

87. Three points $O(0, 0)$, $P(a, a^2)$, $Q(-b, b^2)$, $a > 0$, $b > 0$, are on the parabola $y = x^2$. Let S_1 be the area of the region bounded by the line PQ and the parabola, and S_2 be the area of the triangle OPQ. If the minimum value of $\frac{S_1}{S_2}$ is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to:

88. The sum of squares of all possible values of k, for which area of the region bounded by the parabolas $2y^2 = kx$ and $ky^2 = 2(y - x)$ is maximum, is equal to:

89. If $\frac{dx}{dy} = \frac{1+x-y^2}{y}$, $x(a) = 1$, then $5x(b)$ is equal to:

90. Let ABC be an isosceles triangle in which A is at $(-1, 0)$, $\angle A = \frac{2\pi}{3}$, $AB = AC$ and B is on the positive x-axis.

If $BC = 4\sqrt{3}$ and the line BC intersects the line $y = x + 3$ at (α, β) , then $\frac{\beta^4}{\alpha^2}$ is: