Physics
Section-A

1. **(a)** The direction of propagation of light is perpendicular to the wave front and is symmetric about x, y and z axis.

$\therefore$ Angle made by the light with x, y&z axis is same.

$\therefore \cos\alpha = \cos\beta = \cos\gamma$ ($\alpha, \beta \& \gamma$ are angle made by light with x, y&z axis respectively)

Also $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ [Sum of direction cosines]

$\therefore \alpha = \cos^{-1}\frac{1}{\sqrt{3}}$

2. **(d)**

$\left[P + \frac{a}{V^2}\right](V - b) = RT$

$\therefore [a] = [P][V^2] = ML^{-1}T^{-2}L^6 = ML^5T^{-2}$

$[b] = [V] = L^3$

$[ab^{-2}] = ML^5T^{-2}L^{-6} = ML^{-1}T^{-2}$

Dimension of energy density.

3. **(a)**

$W_F = 20 \times 1 = 20\text{ J}$

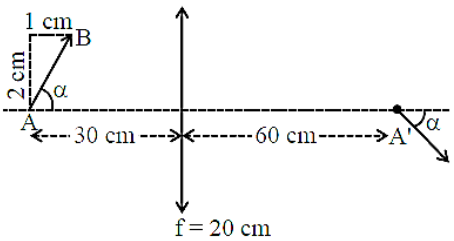
$\therefore \Delta KE = 20\text{ J} = \frac{1}{2}I\omega^2$

$I = MR^2 = 10 \times 0.1^2 = 0.1\text{ kg m}^2$

$\therefore 20 = \frac{1}{2} \times 0.1 \times \omega^2$

$\Rightarrow \omega = 20\text{rad/sec}$

4. **(b)**



Location of image of A :-

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-30} = \frac{1}{20} \Rightarrow \frac{1}{v} = \frac{1}{60} \Rightarrow v = 60\text{ cm}$

$\therefore m = 2$

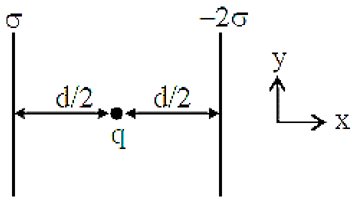
Since size of object is small wrt the location hence

$dv = m^2 du \Rightarrow dv = 4 \times 1 = 4\text{ cm}$

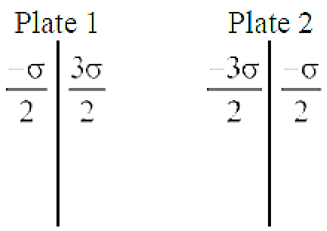
$h_i = mh_0 \Rightarrow h_i(dy) = 2 \times 2 = 4\text{ cm}$

$\therefore$ Angle made with principle axis $= -45°$

5. **(b)**

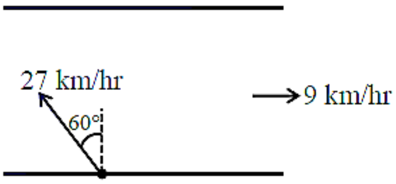


Final charge distribution will be



$\therefore F_{net} = \frac{3\sigma}{2\epsilon_0}q$

6. (b)

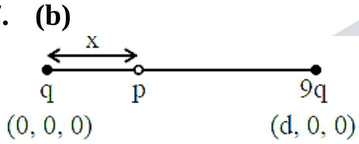


$\therefore V_{\perp} =$ river flow $= 27 \times \cos 60^{\circ} = \frac{27}{2}$ km/hr

Time taken = 30sec.

$\therefore S = Vt = \frac{27}{2} \times \frac{5}{18} \times 30 \text{ m} = 112.5 \text{ m}$

7. (b)



Let $E_P = 0$

$\therefore \frac{kq}{x^2} = \frac{k9q}{(d-x)^2}$

$\Rightarrow \frac{d-x}{x} = 3 \Rightarrow x = \frac{d}{4}$

$\therefore$ co-ordinate of P is $\left(\frac{d}{4}, 0, 0\right)$

8. (c) Given V = 4.2 volt

$\therefore$ Energy supplied by battery

$= vq = 4.2 \times 5800 \times 3600 \times 10^{-3}$ J $= 87.696$ kJ

$\therefore$ Energy stored in the battery when fully charged

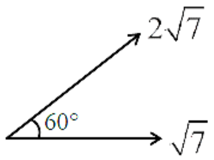
$= 87.696$ kJ ≈ 87.7 kJ

9. (a)

$x_1 = \sqrt{7}\sin 5t$

$x_2 = 2\sqrt{7}\sin\left(5t + \frac{\pi}{3}\right)$

From phasor,



$\therefore$ Amplitude of resultant SHM = 7

$$\phi = \tan^{-1}\frac{2\sqrt{7} \times \sqrt{3}/2}{\sqrt{7} + 2\sqrt{7} \times \frac{1}{2}} = \tan^{-1}\frac{\sqrt{21}}{2\sqrt{7}} = \tan^{-1}\frac{\sqrt{3}}{2}$$

$\therefore X_R = 7\sin(5t + \phi)$

$a_R = -7 \times 25\sin(5t + \phi)$

$\therefore a_{max} = 175$ cm/sec $= 175 \times 10^{-2}$ m/sec

10. (a)

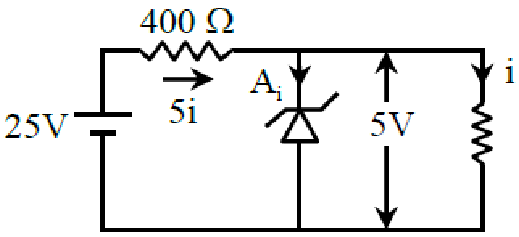
Sol. We have

$\mu_r = (1+\chi) \Rightarrow \chi = (\mu_r - 1)$

$\mu = \mu_0 \mu_r \Rightarrow \mu_r = \frac{\mu}{\mu_0}$

$\therefore \chi = \left(\frac{\mu}{\mu_0} - 1\right)$

11. (d)



From the circuit diagram,

$5i = \frac{20}{400} = \frac{1}{20}$ A

$\therefore i = \frac{1}{100}$ A $= 10$ mA $=$ Load current

Also, $V_L = 5$ V

$\therefore R_L = \frac{5}{10 \times 10^{-3}} \Omega = 500\Omega$

12. (c)

Sol. For adiabatic process, $dQ = 0$

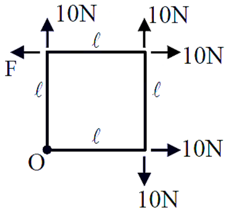
$\therefore$ Molar heat capacity $= 0$

$\because dQ = 0 \Rightarrow dU = -dW$

Also $dU = \frac{f}{2} nRdT$

13. (c)

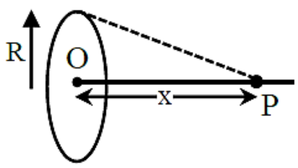
Since the lamina is equilibrium.



$\therefore F_{net} = 0 \& \tau_{net} = 0$

$T_O = 10\ell - F\ell \Rightarrow F = 10$ N

14. (c)



$B_1 = \frac{\mu_0 i}{2R}$

$B_2 = B_1 \sin^3\theta$

$\therefore \frac{B_2}{B_1} = \sin^3\theta = \left(\frac{4}{5}\right)^3 = \frac{64}{125}$

15. (b)

$$E \propto \frac{Z}{n^2}$$

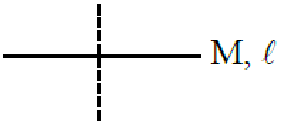
$Z_H = 1 \; Z_{He^+} = 2 \; Z_{Li^{+2}} = 3$

1^{st} excited state $\Rightarrow n = 2$

2^{nd} excited state $\Rightarrow n = 3$

From the given statements only A & B are correct.

16. (b)



$\alpha = \frac{M\ell^2}{12}$ ... (i)

$\frac{M}{2}, \frac{\ell}{2}$

$$\alpha' = 2\left[\frac{\frac{M}{2}\left(\frac{\ell}{2}\right)^2}{12}\right]$$

$$\alpha' = \frac{M\ell^2}{48} = \frac{\alpha}{4}$$

17. (b)

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{v} - \frac{1}{(-0.2)} = \frac{1.5 - 1}{0.4}$$

$$\frac{1.5}{v} = \frac{0.5}{0.4} - \frac{1}{0.2}$$

$$\frac{1.5}{v} = -\frac{1.5}{0.4}$$

$v = -0.4$ m

18. (b)

(A) Coefficient of viscosity

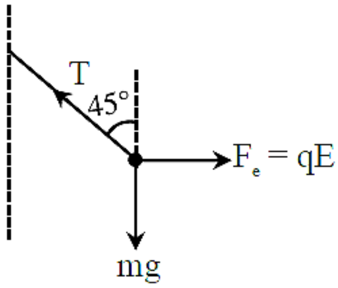
$[\eta] = [M^1 \, L^{-1} \, T^{-1}]$

(B) Intensity $[I] = [M^1 \, L^0 \, T^{-3}]$

(C) Pressure gradient $= [ML^{-2} \, T^{-2}]$

(D) Compressibility $[K] = [M^{-1} \, L^1 \, T^2]$

19. (d)



$qE = mg$

$$q\left[\frac{\sigma}{2\varepsilon_0}\right] = mg$$

$$\sigma = \frac{2\varepsilon_0 mg}{q}$$

$$\sigma = \frac{2 \times 8.85 \times 10^{-12} \times 100 \times 10^{-6} \times 10}{10 \times 10^{-6}}$$

$\sigma = 17.7 \times 10^{-10} C/m^2$

$\sigma = 1.77 nC/m^2$

20. (c)

$KE_{max} = \frac{hc}{\lambda} - \phi$

$p = \sqrt{2mK_{max}}$

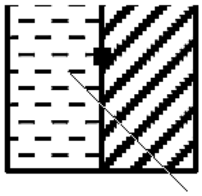
$p = \sqrt{2\,m\left(\frac{hc}{\lambda} - \phi\right)}$

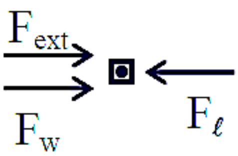
$d_{A-B} = 2R$

$= 2\left[\frac{p}{qB}\right]$

$d_{AB} = \frac{2\sqrt{2\,m\left(\frac{hc}{\lambda} - \phi\right)}}{eB} = \frac{\sqrt{8\,m\left(\frac{hc}{\lambda} - \phi\right)}}{eB}$

21. (150)





in equilibrium

$F_{ext} + F_w = F_\ell$

$\Rightarrow F_{ext} = F_\ell - F_w$

$= (P_0 + \rho_\ell gh)A - (P_0 + \rho_w gh)A$

$= (\rho_\ell - \rho_w)\,ghA$

$= (1500 - 1000) \times 10 \times 3 \times (100 \times 10^{-4})$

$= 150$ m

22. (25)

$\ell = 2$ m; $Y = 2 \times 10^{11}\,\frac{N}{m^2}$

$\mu = -\frac{\left(\frac{\Delta r}{r}\right)}{\left(\frac{\Delta \ell}{\ell}\right)} \Rightarrow \frac{\Delta \ell}{\ell} = \frac{1}{\mu} \times \left(\frac{\Delta r}{r}\right)$

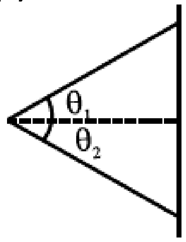
$= \frac{1}{0.2} \times (10^{-3})$

$\Rightarrow \frac{\Delta \ell}{\ell} = 5 \times 10^{-3}$

$u = \frac{1}{2} y \varepsilon_\ell^2 = \frac{1}{2} \times 2 \times 10^{11} \times [5 \times 10^{-3}]^2$

$= 25$

23. (6)



$\theta_1 = \sin^{-1}\left(\frac{2\lambda}{a}\right)$

$\theta_2 = \sin^{-1}\left(\frac{3\lambda}{a}\right)$

$\because\ \theta_1 + \theta_2 = 30°$

$\Rightarrow \sin^{-1}\left(\frac{2\lambda}{a}\right) + \sin^{-1}\left(\frac{3\lambda}{a}\right) = \frac{\pi}{6}$

$$\Rightarrow \frac{2\lambda}{a}\sqrt{1-\left(\frac{3\lambda}{a}\right)^{2}}+\frac{3\lambda}{a}\sqrt{1+\left(\frac{2\lambda}{a}\right)^{2}}=\sin\frac{\pi}{6}$$

Here $\lambda = 628$ nm

After solving
$A = 6.07\mu$ m

24.(3)

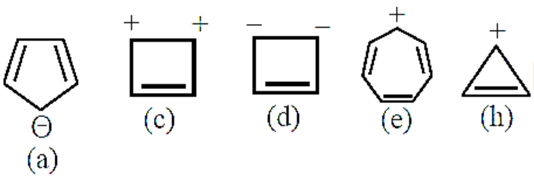
$$\frac{\gamma_A}{\gamma_B}=\frac{f_A+2}{f_A}\times\frac{f_B}{f_B+2}$$
$$=\frac{3+2}{3}\times\frac{(6+2)}{(6+2)+2}$$
$$=\frac{5}{3}\times\frac{8}{10}=\frac{40}{30}$$
$$\because \frac{40}{30}=1+\frac{1}{n}$$
$$\Rightarrow \frac{40}{30}-1=\frac{1}{n}$$
$$\Rightarrow n=3$$

25.(10)

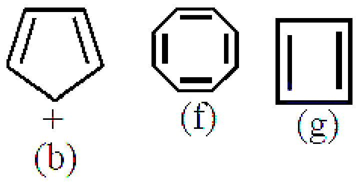
$$v_{avg}=\frac{x_1+x_2}{t_1+t_2}$$
$$\Rightarrow \frac{50}{7}=\frac{x+\frac{3x}{2}}{\frac{x}{5}+\frac{3x}{2v_2}}$$
$$\Rightarrow \frac{50}{7}=\frac{5/2}{\frac{1}{5}+\frac{3}{2v_2}}$$
$$\Rightarrow \frac{1}{5}+\frac{3}{2v_2}=\frac{7}{20}$$
$$\Rightarrow \frac{3}{2v_2}=\frac{7}{20}-\frac{1}{5}=\frac{7-4}{20}$$
$$\Rightarrow \frac{3}{2v_2}=\frac{3}{20}$$
$$\Rightarrow v_2=10\ \text{m/s}$$

26. (d)

Aromatic compounds



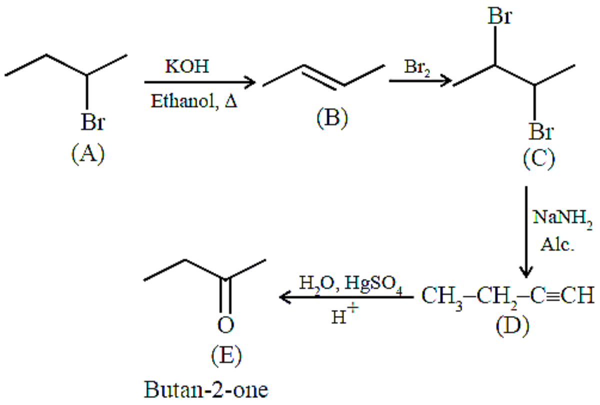
a, c, d, e, h follow Huckel's rule



(Not Aromatic)

b, f, g, are not aromatic, these compounds do not follow Huckel's rule

27. (c)



28.(c)

Sol. The order of first four elements of group-17 are as follows.

F < Cl < Br < I (Covalent radius)

Cl > F > Br > I (Electron affinity)

$F^- < Cl^- < Br^- < I^-$(Ionic radius)

F > Cl > Br > I (I^{st} ionization energy)

Electron affinity order is irregular.

29. (d)

As temperature increases solubility first decrease then increase hence K_H first increase than decrease also at moderate temperature K_H value He > N_2 > CH_4.

30. (c)

$r \propto n^2$

(a) $\frac{r_3}{r_1} = \frac{9}{1}$

(b) $\frac{r_8}{r_4} = \frac{64}{16} = 4$

(c) $\frac{r_6}{r_4} = \left(\frac{6}{4}\right)^2 = \frac{9}{4}$

(d) $\frac{r_4}{r_2} = \left(\frac{4}{2}\right)^2 = 4$

31. (d)

It is free expansion of gas $\Rightarrow P_{ext} = 0$ Where

$w = 0, q = 0$ and $\Delta U = 0$

32.(d)

$P_S = P_A^o \cdot X_A + P_B^o \cdot X_B$

$500 = 200 \times \frac{1}{4} + P_B^o \cdot \frac{3}{4}$

$P_B^o = 600$ mmHg

As $P_A^\circ < P_B^\circ \Rightarrow$ A is least volatile.

33.(c)

$CaCO_3 + 2HCl \rightarrow CaCl_2 + CO_2 + H_2O$

Moles of $CaCO_3 = \frac{1000}{100} = 10$

Moles of HCl $= 0.76 \times \frac{250}{1000} = 0.19$ (L.R.)

Moles of $CaCl_2$ formed $= \frac{0.19}{2}$

Mass of $CaCl_2 = \frac{0.19}{2} \times 111 = 10.545$gm

34.(c)

When equal volumes are mixed molarity reduce to half.

For precipitation $Q_{SP} = [A^{+2}][Y^-]^2 > K_{SP}$

(1) $Q_{SP} = (1.8 \times 10^{-3})\left(\frac{5}{2} \times 10^{-4}\right)^2 < K_{sp}$

(2) $Q_{SP} = (10^{-4})(0.4 \times 10^{-3})^2 < K_{SP}$

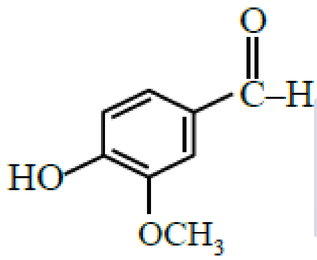
(3) $Q_{sp} = (10^{-2})(10^{-2})^2 > K_{sp}$

(4) $Q_{SP} = \left(\frac{1.5}{2} \times 10^{-4}\right)\left(\frac{1.5}{2} \times 10^{-3}\right)^2 < K_{SP}$

35.(d)

Molecule	Hybridisation	Dipole Moment	Lone pair on the central atom
SO_2	sp^3	Nonzero	1
NF_3	sp^3	Nonzero	1
NH_3	sp^3	Nonzero	1
XeF_2	sp^3d	zero	3
ClF_3	sp^3d	Nonzero	2
SF_4	sp^3d	Nonzero	1

36.(b)



Phenolic group soluble in NaOH
Benzaldehyde derivative react with Tollen's reagent.
Vanillin does not give self-aldol reaction due to lack of acidic H for condensation.

37.(d)

Isoleucine has 2 chiral center
Glycine is optically inactive
Aspartic acid also contain COOH group at the side chain.
Cysteine easily dimmerise due to free SH group

38.(d)

Basic strength of amine depends on hydrogen bonding and electronic inductive effect.

$NH(Et)_2 > N(Et)_3 > NH_2Et > \ddot{N}H_3 > NH_2 - NH_2$

39.(b)

⇒ The metallic radius order of Al&Ga is B< $\underset{\downarrow}{\overbrace{}^{Ga < Al}}$ <In<Tl

(due to poor shielding of d-subshell electrons) ⇒ The ionic radius order of Al^{+3}&Ga^{+3} is $B^{+3} < Al^{+3} < Ga^{+3} < In^{+3} < T\ell^{+3}$

40. (d)

In octahedral complex (CN = 6) If $\Delta_0 <$ P.E. , then high spin complexes are formed If $\Delta_0 >$ P.E. , then low spin complexes are formed But in tetrahedral complex (CN = 4)

$\Delta_t <$ P.E. , then mainly high spin complexes are formed and rarely low spin complexes are formed.

41. (c)

$2CuSO_4 + K_4[Fe(CN)_6] \xrightarrow{CH_3COOH}$
$Cu_2[Fe(CN)_6] + 2\,K_2SO_4$
(Chocolate brown ppt.)

$4FeCl_3 + 3\,K_4[Fe(CN)_6] \longrightarrow$
$Fe_4[Fe(CN)_6]_3 + 12KCl$
(Prussian Blue ppt.)

$3ZnCl_4 + 2\,K_4[Fe(CN)_6] \xrightarrow{NH_4OH} \underset{\text{(White or bluish white ppt.)}}{K_2Zn_3[Fe(CN)_6]_2} + 6KCl$

42.(a)

Moles of ' C ' $= n_{CO_2} = \dfrac{1.46}{44} = 0.033$

Moles of ' C ' $= W_c = 0.033 \times 12$

Moles of ' H ' $= 2 \times n_{H_2O} = 2 \times \dfrac{0.567}{18} = 0.063$

Mass of ' H ' $= 0.0063$

Mass of Oxygen(O) $= 1 - (W_c + W_H)$

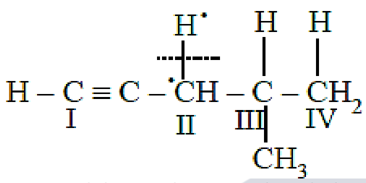
$= 1 - (0.033 \times 12 + 0.063 \times 1) = 0.541\text{gm}$

Moles of ' O ' $= \dfrac{0.541}{16} = 0.033$

Empirical formula $= CH_2O$

Empirical formula mass $= 30$.

43.(d)

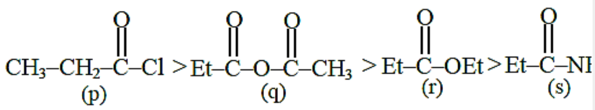


II most stable carbon radical due to resonance stablise

I least stable carbon radical due to no stabilising factor.

44. (d)

Rate of hydrolysis ∝ Leaving group ability.



45.(a)

Given A is rarest, monoatomic, non-radioactive p-block element and form AX_4Y type of molecule.

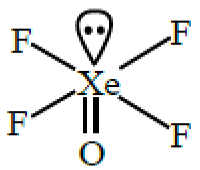
∴ It is concluded that it is Xe

It is given the electronegativity of A is less than X & Y

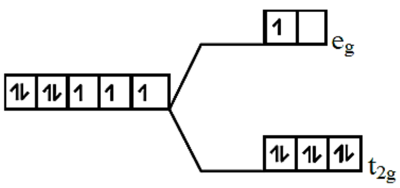
It is given the electronegativity of X&Y is highest and second highest respectively among all element.

∴ X&Y are F & O

∴ Compound is consider as $XeOF_4$ with square pyramidal shape.



46.(a)



Co has highest standard electrode potential (M^{+3}/M^{+2}) among Mn, Cr, Co, Fe

∴ Complex is $[Co(CN)_6]^{4-}$ and its splitting is as follows.

∴ electron in e_g orbital is one.

47. (6)

$QH_2 + 2Ag^+ \rightarrow 2Ag + Q + 2H^+$

$E = E^\circ - \frac{0.06}{2}\log[H^+]^2$

$E = E^\circ - 0.06 \times \log[H^+]$

$pH = -\log(H^+) = \frac{E - E^\circ}{0.06} = \frac{0.4 - 0.1}{0.06}$

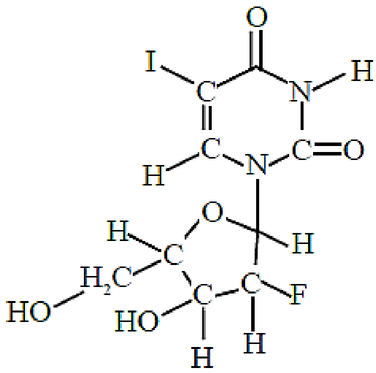
$= \frac{0.3}{0.06} = 5$

$pH + NH_4X = 7 - \frac{1}{2}pK_b - \frac{1}{2}\log C$

$5 = 7 - \frac{1}{2} \times pK_b - \frac{1}{2}\log(10^{-2})$

$.pK_b = 6$

48. (372)



Molar mass $= 372$gm

$\therefore$ 0.1 mole has $= 372 \times 10^{-1}$gm

49. (74)

$CO(g) + 2H_2(g) \rightleftharpoons CH_3OH(g)$

$t = 0$ 0.1 mol a mol

t_{eq} $0.1 - x$ $a - 2x_X = 0.04$

$= 0.06 = a - 0.08$

$= 0.23 - 0.08$

$= 0.15$ mole

$V = 2$ L

$T = 500$ K

$P_{total} = 5$bar

$n_{Total} = 0.25 = \frac{1}{4}$ mol.

$P_{total} = n_{total} \times \frac{RT}{V}$

$\Rightarrow 5 = (0.06 + a - 0.08 + 0.04) \times \frac{0.08 \times 500}{2}$

$\Rightarrow 10 = (0.02 + a) \times 0.08 \times 500$

$\Rightarrow a = 0.25 - 0.02 = 0.23$ mol

$K_P = \frac{X_{CH_3OH}}{X_{CO} \times X_{H_2}^2} \times \frac{1}{(P_T)^2} = \frac{0.04}{0.06 \times (0.15)^2} \times \left[\frac{1/4}{5}\right]^2$

$= \frac{4}{6 \times (0.15)^2 \times 16} \times \frac{1}{25}$

$= \frac{100 \times 100}{24 \times 225 \times 25} = \frac{100 \times 100}{135000} = 0.074 = 74 \times 10^{-3}$

50. (2435)

$t_{1/2} \propto [A]_0 \Rightarrow$ Order $=$ zero

$t_{1/2} = \frac{A_0}{2K} \Rightarrow$ Slope $= \frac{1}{2K} = 76.92$

$K = \frac{1}{2 \times 76.92}$

$[A]_{10} = -Kt + A_0 = -\frac{1}{2 \times 76.92} \times 10 + 2.5 = 2.435$

$= 2435 \times 10^{-3}$ mol/L

51. (b)

Sol. $2^{\alpha} \cdot 3^{\beta} \cdot 5^{\gamma}$

$B = \left[\frac{50}{3}\right] + \left[\frac{50}{3^2}\right] + \left[\frac{50}{3^3}\right] + \left[\frac{50}{3^4}\right]$

$= 16 + 5 + 1$

$= 2$

Maximum value of n is 22

52. (c)

$ae = \sqrt{10}$ and $\frac{a}{e} = \frac{9}{10}$

$\Rightarrow a^2 = 9$ and $e = \frac{\sqrt{10}}{3}$

Now $(ae)^2 = a^2 + b^2$

$10 = 9 + b^2 \Rightarrow b^2 = 1$

$\ell = \frac{2\,b^2}{a} = \frac{2(1)}{3}$

$\Rightarrow 9(e^2 + \ell)$

$= 9\left(\frac{10}{9} + \frac{2}{3}\right)$

$= 10 + 6$

$= 16$

53. (c)

1111122200

No. of sequences $= \frac{10!}{5!3!2!} = 2520$

Note : Sequence can start with 0 .

54. (b)

$(\sin x \cos y)(f(2x+2y) - f(2x-2y)) = (\cos x \sin y)$

$(f(2x+2y) + f(2x-2y))$

$f(2x+2y)(\sin(x-y)) = f(2x-2y)\sin(x+y)$

$\frac{f(2x+2y)}{\sin(x+y)} = \frac{f(2x-2y)}{\sin(x-y)}$

Put $2x + 2y = m, 2x - 2y = n$

$\frac{f(m)}{\sin\left(\frac{m}{2}\right)} = \frac{f(n)}{\sin\left(\frac{n}{2}\right)} = K$

$\Rightarrow f(m) = K\sin\left(\frac{m}{2}\right)$

$\therefore f(x) = K\sin\left(\frac{x}{2}\right)$

$f'(x) = \frac{K}{2}\cos\left(\frac{x}{2}\right)$

Put $x = 0; \frac{1}{2} = \frac{K}{2} \Rightarrow K = 1$

$f'(x) = \frac{1}{2}\cos\frac{x}{2}$

$f''(x) = -\frac{1}{4}\sin\frac{x}{2}$

$4f''\left(\frac{5\pi}{3}\right) = \left(-\frac{1}{4}\sin\left(\frac{5\pi}{6}\right)\right)24$

$= \frac{-24}{8} = -3$

55. (d)

$|A| = 0$

$\alpha\beta + 6 = 0$

$\alpha\beta = -6$

$\alpha + \beta = 1$

$\Rightarrow \alpha = 3, \beta = -2$

$A = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}$

$\therefore A^2 = A$

$A = A^2 = A^3 = A^4 = A^5$

$(I + A)^8$

$= I + {}^8C_1\, A^7 + {}^8C_2\, A^6 + \cdots . + {}^8C_8\, A^8$

$= I + A({}^8C_1 + {}^8C_2 + \cdots .. {}^8C_8)$

$= I + A(2^8 - 1)$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 765 & -255 \\ 1530 & -510 \end{bmatrix}$

$= \begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$

56. (a)

$$\left(\frac{(x+1)}{\left(x^{\frac{2}{3}} + 1 - x^{\frac{1}{3}}\right)} - \frac{(x-1)}{\left(x - x^{\frac{1}{2}}\right)}\right)^{10}$$

$$= \left(\left(x^{\frac{1}{3}} + 1\right) - \left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$$

$$= \left(x^{\frac{1}{3}} - \frac{1}{\sqrt{x}}\right)^{10}$$

$T_{r+1} = {}^{10}C_r (x)^{\frac{10-r}{3}}(-1)^r (x)^{-\frac{r}{2}}$

$\frac{10-r}{3} - \frac{r}{2} = 0$

$(20 - 2r) - 3r = 0$

$\Rightarrow {}^{10}C_4(-1)^4 = 210$

57. (c)

$2\sqrt{2}\cos^2\theta + 2\cos\theta - \sqrt{6}\cos\theta - \sqrt{3} = 0$

$(2\cos\theta - \sqrt{3})(\sqrt{2}\cos\theta + 1) = 0$

$\cos\theta = \frac{\sqrt{3}}{2}, \frac{-1}{\sqrt{2}}$

Number of solution $= 8$

58. (a)

Let $a_1 = a$, common difference $= d$

$a_1 + a_3 + a_5 + \cdots ... + a_{23} = -\frac{72}{5}a$

$\frac{12}{2}[2a + 11 \times 2d] = -\frac{72}{5}a$

$12a + 132d = -\frac{72}{5}a$
$132a + 132 \times 5d = 0$
$a = -5d$
$\frac{n}{2}(2a + (n-1)d) = 0 \Rightarrow -10d + nd - d = 0$

59. (d)
$f'(x) = 6x^2 - 18ax + 12a^2$
$f'(x) = 6(x^2 - 3ax + 2a^2)$
roots are a, 2a
$p^2 = q \Rightarrow a^2 = 2a$
$a = 2$
$f(x) = 2x^3 - 18x^2 + 48x + 1$
$f(3) = 37$

60. (a)
$\frac{2 + k^2 z}{k + \bar{z}} = kz$
$|z|^2 k = 2$
$k = 2$
point p(2,4); center (1,2)
distance from circle
$(x-1)^2 + (y-2)^2 = 1$ is max
if $(OP + r) = \sqrt{1+4} + 1 = \sqrt{5} + 1$

61. (c)
Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$
$a_1^2 + a_2^2 + a_3^2 = 1$
Let $\bar{b} = 2\vec{\imath} - \hat{\jmath} + 2\hat{k}, \bar{c} = \vec{\imath} - 2\hat{\jmath} - 2\hat{k}$
$\bar{d} = \hat{k}$
$\frac{\bar{a} \cdot \bar{b}}{|\bar{b}|} = \frac{\bar{a} \cdot \bar{c}}{|c|} = \frac{\bar{a} \cdot \bar{d}}{|\bar{d}|}$
$\frac{2a_1 - a_2 + 2a_3}{3} = \frac{a_1 + 2a_2 - 2a_3}{3} = a_3$
By solving
$a_1 = \frac{7}{\sqrt{155}}, a_2 = \frac{9}{\sqrt{155}}, a_3 = \frac{5}{\sqrt{155}}$

62.(b)
$R = \{(f, g): f(0) = g(1) \text{ and } f(1) = g(0)\}$
Reflexive: $(f, f) \in R$
$= f(0) = f(1)$ and $f(1) = f(0) \rightarrow$ must hold
$\Rightarrow$ but this is not true for all function
so not reflexive
Symmetric: If $(f, g) \in R \Rightarrow (g, f) \in R$
Now, $g(0) = f(1)$ and $g(1) = f(0) \rightarrow$ true
$\therefore$ symmetric
Transitive : If $(f, g) \in R$ and $(g, h) \in R$
$\Rightarrow (f, h) \in R$
Now $(f, g) \in R \Rightarrow f(0) = g(1)$ and $f(1) = g(0)$ $(g, h) \in R \Rightarrow g(0) = h(1)$ and $g(1) = h(0)$
For $(f, h) \in R$ we need $f(0) = h(1)$ and $f(1) = h(0)$
Now $f(0) = g(1) = h(0)$ and $f(1) = g(0) = h(1)$
Hence not transitive

63. (a)
$$\lim_{x \to 10} \frac{x^2(\alpha x) + (\gamma - 1)\left(1 + \frac{x^2}{1}\right)}{2x - \frac{8x^3}{6} - \beta x} = 3$$

$$\lim_{x\to 0}\frac{(\gamma-1)+(\gamma-1)x^2+\alpha x^3}{(2-\beta)x-\frac{4}{3}x^3}=3$$

$\gamma-1, \beta=2, \frac{-3\alpha}{4}=+3 \Rightarrow \alpha=-4$

$\beta+\gamma-\alpha=7$

64.(b)

$$\Delta=\begin{vmatrix}3 & 1 & \beta\\ 2 & \alpha & -1\\ 1 & 2 & 1\end{vmatrix}=0$$

$3\alpha+4\beta-\alpha\beta+3=0$

$$\Delta_3=\begin{vmatrix}3 & 1 & 3\\ 2 & \alpha & -3\\ 1 & 2 & 4\end{vmatrix}=0$$

$9\alpha+19=0$

$\alpha=\frac{-19}{9}, \beta=\frac{6}{11}$

$\Rightarrow 22\beta-9\alpha=31$

65. (b)

$\alpha^{10}+\beta^{10}=123$

$\alpha+\beta=1$

$\alpha^9+\beta^9=76$

$\alpha^8+\beta^8=47$

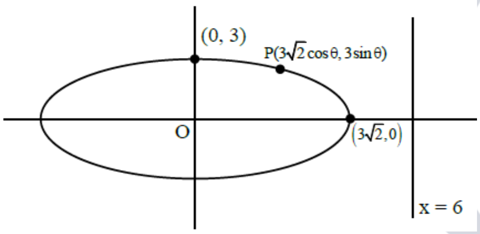
$P_{10}=P_9+P_8$

$x^2=x+1 \Rightarrow x^2-x-1=0$

$\alpha+\beta=1, \alpha\beta=-1$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{1}{-1}=-1, \frac{1}{\alpha\beta}=-1$

66. (d)



$PS+PS'=2\times 3\sqrt{2}$

$b^2=a^2(1-e^2) \Rightarrow 9=18(1-e^2)$

$\Rightarrow e=\frac{1}{\sqrt{2}}$

Directrix $x=\frac{a}{e}=\frac{3\sqrt{2}}{\frac{1}{\sqrt{2}}}=6$

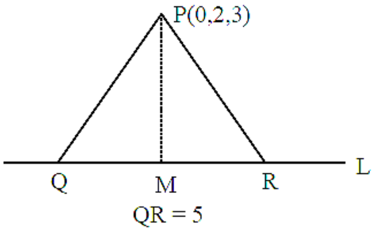
$$PS \cdot PS'=\left|\frac{1}{\sqrt{2}}(3\sqrt{2}\cos\theta-6)\frac{1}{\sqrt{2}}(3\sqrt{2}\cos\theta+6)\right|$$

$=\frac{1}{2}|18\cos^2\theta-36|$

$(PS\cdot PS')_{max}=18; (PS\cdot PS')_{min}=9$

sum $=27$

67. (b)



$M(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$
Drs of PM $\Rightarrow 5\lambda - 3, 2\lambda - 1, 3\lambda - 7$
Drs of line $L \Rightarrow 5,2,3$
$PM \perp L$
$\Rightarrow (5\lambda - 3)5 + (2\lambda - 1)2 + (3\lambda - 7)3 = 0$

$\Rightarrow \lambda = 1$

$\therefore M(2,3,-1)$

$PM = \sqrt{4+1+16} = \sqrt{21}$

Area $= \frac{1}{2} \times 5 \times \sqrt{21} = \frac{m}{n}$

$2m - 5\sqrt{21}n = 0$

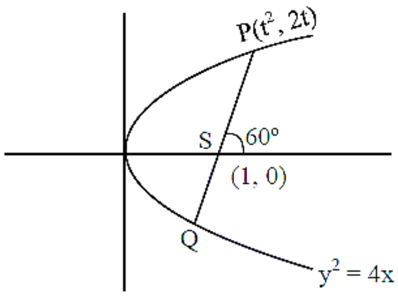
68. (c)

$Ar(\triangle BCD)$
$= \sqrt{(Ar(\triangle ABC))^2 + (Ar(ACD))^2 + (Ar(\triangle ADB))^2}$
$= \sqrt{5^2 + 6^2 + 7^2}$
$= \sqrt{110}$

69. (d)

$A = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix} - I = \begin{bmatrix} 0 & a & 1 \\ 2 & 0 & 0 \\ a & 1 & 1 \end{bmatrix}$
$|A| = -4 \Rightarrow 2 - 2a = -4 \Rightarrow a = 3$
$|(a+1)adj(a-1)A| = |4adj3\,A|$
$= 4^3|adj3\,A|$
$= 4^3 \times |3\,A|^{3-1} = 64|3\,A|^2$
$= 64 \times (3^3)^2|\,A|^2$
$= 2^6 \times 3^6 \times 16$
$2^m \times 3^n = 2^{10} \times 3^6$
$\therefore\ m = 10, n = 6$
$\Rightarrow\ m + n = 16$

70. (a)



$\tan 60^\circ = \dfrac{2t - 0}{t^2 - 1} = \sqrt{3} \Rightarrow t = \sqrt{3}$

$\therefore P(3, 2\sqrt{3})$
Circle :
$(x-1)(x-3) + (y-0)(y-2\sqrt{3}) = 0$

at $x = 0$

$\Rightarrow 3 + y^2 - 2\sqrt{3}y = 0$

$\Rightarrow y = \sqrt{3} = \alpha$

$5\alpha^2 = 15$

71. (8)

$f(x) = \dfrac{1}{e^{x-1}} = e^{1-x}$

$f(x) = 2\, f(x) = 1$

$\dfrac{1}{e^{x-1}} = 2$

$x = 1 - \ell n2$

$f(0) = e^1 = 2.71$

$f(e^3) = e^{1-e^3} \in (0,1)$

$I = \int_0^{1-n2} 2\, dx + \int_{1-\ln 2}^{1} 1dx + \int_1^{e^3} 0dx$

$= 2(1 - \ln 2 - 0) + 1(1 - 1 + \ln 2) + 0$

$\alpha - \ln 2 = 2 - \ln 2$

$\alpha = 2$

$\alpha^3 = 8$

72. (36)

$f''(x) = f(x)$

$\Rightarrow f'(x) \cdot f''(x) = f'(x) \cdot f(x)$

$\Rightarrow \dfrac{(f'(x))^2}{2} = \dfrac{(f(x))^2}{2} + C$

$\Rightarrow (f'(x))^2 = (f(x))^2 + C'$

$f(0) = 0, f'(0) = 3 \;\Rightarrow C' = 9$

$\therefore (f'(x))^2 = (f(x))^2 + 9$

$f'(x) = \sqrt{(f(x))^2 + 9}$

$\int \dfrac{dy}{\sqrt{y^2 + 9}} = \int dx \Rightarrow \ln\left|y + \sqrt{y^2 + 9}\right| = x + C$

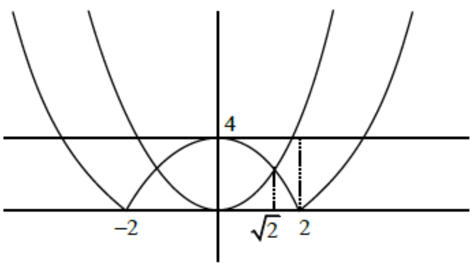
$\Rightarrow f(0) = 0 \Rightarrow C = \ell n3$

$\Rightarrow y + \sqrt{y^2 + 9} = 3e^x$

at $x = \ell n3; y = 4$

$\therefore 9f(\ell n3) = 36$

73. (22)



$A = \int_0^4 \sqrt{4 + y}dy - \int_0^2 \sqrt{4 - y}dy - \int_2^4 \sqrt{y}dy$

$= \left(\dfrac{(4 + y)^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4 + \left(\dfrac{(4 - y)^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^2 - \left(\dfrac{y^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4$

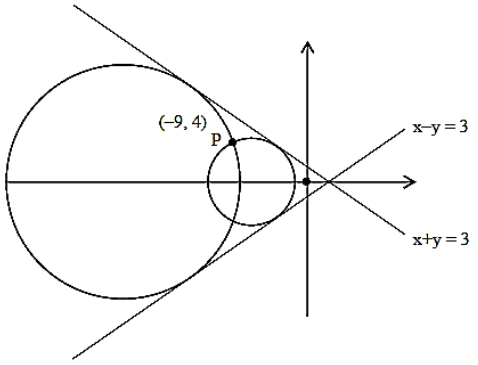
$\dfrac{80\sqrt{2}}{3} - 16 = \dfrac{40\sqrt{2}}{3} - 16$

$\alpha = 6, \beta = 16$
$\alpha + \beta = 22$

74. (2477)

$1 \leq a < ar < ar^2 \leq 40$
(If $r \in N$)
If $r = 2$
$1 \leq a < 2a < 4a \leq 40$
$a \in \{1, \dots \dots, 10\}$
If $r = 3$
$1 \leq a < 3a < 9a \leq 40$
$a \in \{1,2,3,4\}$
If $r = 4$
$1 \leq a < 4a < 16a \leq 40$
$a \in \{1,2\}$
If $r = 5$
$1 \leq a < 5a < 25a \leq 40$
$a \in \{1\}$
If $r = 6$
$1 \leq a < 6a < 36a \leq 40$
$a \in \{1\}$
$\left(P = \frac{18}{9880} = \frac{9}{4940}\right)$ as per NTA for $r \in N$
$m + n = 4949$
If $r \notin N$ (also possible)
$r = \frac{3}{2}$
$ar^2 = \frac{9a}{4}; a = 4k$
$$\left.\begin{array}{l}(4,6,9)\\(8,12,18)\\(12,18,27)\\(16,24,36)\end{array}\right\} 4GP$$
$r = \frac{5}{2}\ ar^2 = \frac{25a}{4}; a = 4k$
(4,10,25)
(1) GP
$r = \frac{4}{3}\ ar^2 = \frac{16a}{9} \rightarrow a = 9k$
(9,12,16), (18,24,32)
(2) GP
$r = \frac{5}{3}\ ar^2 = \frac{25a}{9}; a = 9k$
(9,15,25)
(1) GP
$r = \frac{5}{4}\ ar^2 = \frac{25a}{16}; a = 16k$
(16,20,25)
(1) GP
$r = \frac{6}{5}\ ar^2 = \frac{36a}{25}; a = 25k$
(25,30,36)
(1) GP
Total $= 18 + 10 = 28$
$P = \frac{28}{{}^{40}C_3} = \frac{28}{9880} = \frac{7}{2470}\ m + n = 2477$

75. (768)



Centre (a, 0)

$$r = \left|\frac{a-0-3}{\sqrt{2}}\right|$$

$$\text{circle}(x-a)^2 + y^2 = \left(\frac{a-3}{\sqrt{2}}\right)^2$$

passes through $(-9,4)$

$2(a^2 + 18a + 81 + 16) = (a^2 - 6a + 9)$

$a^2 + 42a + 185 = 0$

$(a+37)(a+5) = 0$

$\Rightarrow a = -37, -5$

$r_1 = \left|\frac{-37-3}{\sqrt{2}}\right| = 20\sqrt{2}$

$r_2 = \left|\frac{-5-3}{\sqrt{2}}\right| = 4\sqrt{2}$

$\left|r_1^2 - r_2^2\right| = |800 - 32| = 768$