

Physics

Section- A

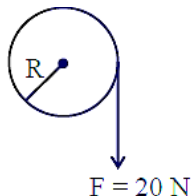
1. A light wave is propagating with plane wave fronts of the type $x + y + z = \text{constant}$. The angle made by the direction of wave propagation with the x -axis is :

- (a) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\cos^{-1}\left(\frac{2}{3}\right)$
 (c) $\cos^{-1}\left(\frac{1}{3}\right)$ (d) $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

2. The equation for real gas is given by $\left(P + \frac{a}{V^2}\right)(V - b) = RT$, where P, V, T and R are the pressure, volume, temperature and gas constant, respectively. The dimension of ab^{-2} is equivalent to that of :

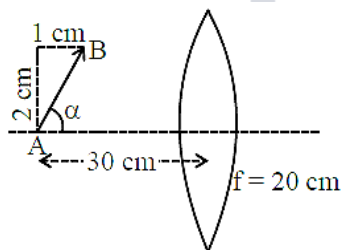
- (a) Planck's constant (b) Compressibility
 (c) Strain (d) Energy density

3. A cord of negligible mass is wound around the rim of a wheel supported by spokes with negligible mass. The mass of wheel is 10 kg and radius is 10 cm and it can freely rotate without any friction. Initially the wheel is at rest. If a steady pull of 20 N is applied on the cord, the angular velocity of the wheel, after the cord is unwound by 1 m , would be :



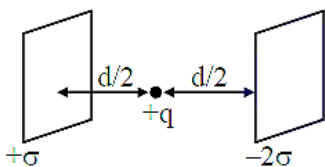
- (a) 20rad/s (b) 30rad/s
 (c) 10rad/s (d) 0rad/s

4. A slanted object AB is placed on one side of convex lens as shown in the diagram. The image is formed on the opposite side. Angle made by the image with principal axis is:



- (a) $-\frac{\alpha}{2}$ (b) -45°
 (c) $+45^\circ$ (d) $-\alpha$

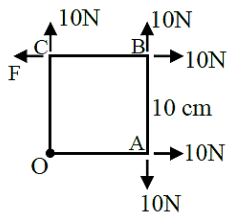
5. Consider two infinitely large plane parallel conducting plates as shown below. The plates are uniformly charged with a surface charge density $+\sigma$ and -2σ . The force experienced by a point charge $+q$ placed at the mid point between two plates will be :



- (a) $\frac{\sigma q}{4\epsilon_0}$ (b) $\frac{3\sigma q}{2\epsilon_0}$
 (c) $\frac{3\sigma q}{4\epsilon_0}$ (d) $\frac{\sigma q}{2\epsilon_0}$

6. A river is flowing from west to east direction with speed of 9 km h^{-1} . If a boat capable of moving at a maximum speed of 27 km h^{-1} in still water, crosses the river in half a minute, while moving with maximum speed at an angle of 150° to direction of river flow, then the width of the river is :

- (a) 300 m (b) 112.5 m
 (c) 75 m (d) $112.5 \times \sqrt{3} \text{ m}$

7. A point charge $+q$ is placed at the origin. A second point charge $+9q$ is placed at $(d, 0, 0)$ in Cartesian coordinate system. The point in between them where the electric field vanishes is :
- (a) $(4d/3, 0, 0)$ (b) $(d/4, 0, 0)$
 (c) $(3d/4, 0, 0)$ (d) $(d/3, 0, 0)$
8. The battery of a mobile phone is rated as 4.2 V , 5800 mAh . How much energy is stored in it when fully charged?
- (a) 43.8 kJ (b) 48.7 kJ
 (c) 87.7 kJ (d) 24.4 kJ
9. A particle is subjected two simple harmonic motions as :
 $x_1 = \sqrt{7} \sin 5t$ cm and $x_2 = 2\sqrt{7} \sin \left(5t + \frac{\pi}{3} \right)$ cm
 where x is displacement and t is time in seconds. The maximum acceleration of the particle is $x \times 10^{-2} \text{ ms}^{-2}$. The value of x is :
- (a) 175 (b) $25\sqrt{7}$
 (c) $5\sqrt{7}$ (d) 125
10. The relationship between the magnetic susceptibility (χ) and the magnetic permeability (μ) is given by :
 (μ_0 is the permeability of free space and μ_r is relative permeability)
- (a) $\chi = \frac{\mu}{\mu_0} - 1$
 (b) $\chi = \frac{\mu_r}{\mu_0} + 1$
 (c) $\chi = \mu_r + 1$
 (d) $\chi = 1 - \frac{\mu}{\mu_0}$
11. A Zener diode with 5 V Zener voltage is used to regulate an unregulated dc voltage input of 25 V . For a 400Ω resistor connected in series, the zener current is found to be 4 times load current. The load current (I_L) and load resistance (R_L) are :
- (a) $I_L = 20 \text{ mA}$; $R_L = 250\Omega$
 (b) $I_L = 10 \text{ A}$; $R_L = 0.5\Omega$
 (c) $I_L = 0.02 \text{ mA}$; $R_L = 250\Omega$
 (d) $I_L = 10 \text{ mA}$; $R_L = 500\Omega$
12. In an adiabatic process, which of the following statements is true ?
- (a) The molar heat capacity is infinite
 (b) Work done by the gas equals the increase in internal energy
 (c) The molar heat capacity is zero
 (d) The internal energy of the gas decreases as the temperature increases
13. A square Lamina $OABC$ of length 10 cm is pivoted at 'O'. Forces act at Lamina as shown in figure. If Lamina remains stationary, then the magnitude of F is :
- 
- (a) 20 N (b) 0 (zero)
 (c) 10 N (d) $10\sqrt{2}$ N
14. Let B_1 be the magnitude of magnetic field at center of a circular coil of radius R carrying current I . Let B_2 be the magnitude of magnetic field at an axial distance ' x ' from the center. For $x:R = 3:4$, $\frac{B_2}{B_1}$ is :
- (a) 4:5 (b) 16:25
 (c) 64:125 (d) 25:16
15. Considering Bohr's atomic model for hydrogen atom :
- (A) the energy of H atom in ground state is same as energy of He^+ ion in its first excited state.
 (B) the energy of H atom in ground state is same as that for Li^{++} ion in its second excited state.

(C) the energy of H atom in its ground state is same as that of He^+ ion for its ground state.

(D) the energy of He^+ ion in its first excited state is same as that for Li^{++} ion in its ground state Choose the correct answer from the options given below :

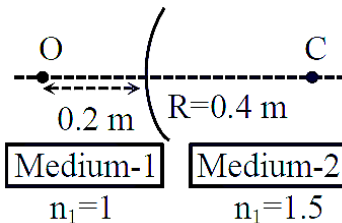
- (a) (B), (D) only (b) (A), (B) only
(c) (A), (D) only (d) (A), (C) only

16. Moment of inertia of a rod of mass 'M' and length 'L' about an axis

passing through its center and normal to its length is ' α '. Now the rod is cut into two equal parts and these parts are joined symmetrically to form a cross shape. Moment of inertia of cross about an axis passing through its center and normal to plane containing cross is :

- (a) α (b) $\alpha/4$
(c) $\alpha/8$ (d) $\alpha/2$

17. A spherical surface separates two media of refractive indices 1 and 1.5 as shown in figure Distance of the image of an object 'O', is : (C is the center of curvature of the spherical surface and R is the radius of curvature)



- (a) 0.24 m right to the spherical surface
(b) 0.4 m left to the spherical surface
(c) 0.24 m left to the spherical surface
(d) 0.4 m right to the spherical surface

18. Match List-I with List-II.

List-I

List-II

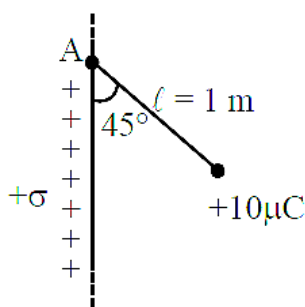
- | | |
|------------------------------|---------------------------------------|
| (A) Coefficient of viscosity | (I) $[\text{ML}^0 \text{T}^{-3}]$ |
| (B) Intensity of wave | (II) $[\text{ML}^{-2} \text{T}^{-2}]$ |
| (C) Pressure gradient | (III) $[\text{M}^{-1} \text{LT}^2]$ |
| (D) Compressibility | (IV) $[\text{ML}^{-1} \text{T}^{-1}]$ |

Choose the correct answer from the options given below :

- (a) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)
(b) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
(c) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)
(d) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

19. A small bob of mass 100 mg and charge $+10\mu\text{C}$ is connected to an

insulating string of length 1 m . It is brought near to an infinitely long nonconducting sheet of charge density ' σ ' as shown in figure. If string subtends an angle of 45° with the sheet at equilibrium the charge density of sheet will be :



(Given, $\epsilon_0 = 8.85 \times 10^{-12} \frac{\text{F}}{\text{m}}$ and acceleration due to gravity, $g = 10 \text{ m/s}^2$)

- (a) 0.885 nC/m^2 (b) 17.7 nC/m^2
(c) 885 nC/m^2 (d) 1.77 nC/m^2

20. A monochromatic light is incident on a metallic plate having work function ϕ . An electron, emitted normally to the plate from a point A with maximum kinetic energy, enters a constant magnetic field, perpendicular to the initial velocity of electron. The electron passes through a curve and hits back the plate at a point B. The distance between A and B is :

(Given : The magnitude of charge of an electron is e and mass is m , h is Planck's constant and c is velocity of light. Take the magnetic field exists throughout the path of electron)

- (a) $\sqrt{2m\left(\frac{hc}{\lambda} - \phi\right)}/eB$ (b) $\sqrt{m\left(\frac{hc}{\lambda} - \phi\right)}/eB$
 (c) $\sqrt{8m\left(\frac{hc}{\lambda} - \phi\right)}/eB$ (d) $2\sqrt{m\left(\frac{hc}{\lambda} - \phi\right)}/eB$

21. A vessel with square cross-section and height of 6 m is vertically partitioned. A small window of 100 cm^2 with hinged door is fitted at a depth of 3 m in the partition wall. One part of the vessel is filled completely with water and the other side is filled with the liquid having density $1.5 \times 10^3 \text{ kg/m}^3$. What force one needs to apply on the hinged door so that it does not get opened? (Acceleration due to gravity = 10 m/s^2)

22. A steel wire of length 2 m and Young's modulus $2.0 \times 10^{11} \text{ Nm}^{-2}$ is stretched by a force. If Poisson ratio and transverse strain for the wire are 0.2 and 10^{-3} respectively, then the elastic potential energy density of the wire is $\times 10^5$ (in SI units)

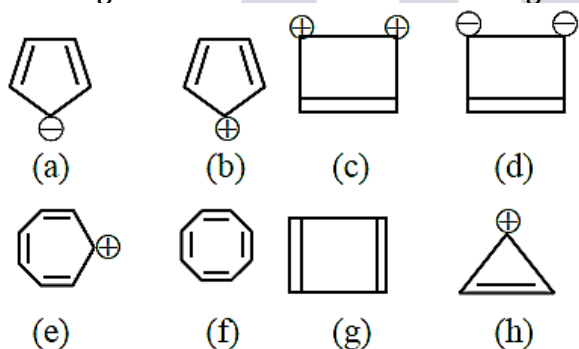
23. If the measured angular separation between the second minimum to the left of the central maximum and the third minimum to the right of the central maximum is 30° in a single slit diffraction pattern recorded using 628 nm light, then the width of the slit is $\text{ } \mu\text{m}$.

24. γ_A is the specific heat ratio of monoatomic gas A having 3 translational degrees of freedom. γ_B is the specific heat ratio of polyatomic gas B having 3 translational, 3 rotational degrees of freedom and 1 vibrational mode. If $\frac{\gamma_A}{\gamma_B} = \left(1 + \frac{1}{n}\right)$, then the value of n is .

25. A person travelling on a straight line moves with a uniform velocity v_1 for a distance x and with a uniform velocity v_2 for the next $\frac{3}{2}x$ distance. The average velocity in this motion is $\frac{50}{7} \text{ m/s}$. If v_1 is 5 m/s then $v_2 = \text{ } \text{m/s}$.

Chemistry Section-A

26. Designate whether each of the following compounds is aromatic or not aromatic.



- (a) e, g aromatic and a, b, c, d, f, h not aromatic
 (b) b, e, f, g aromatic and a, c, d, h not aromatic
 (c) a, b, c, d aromatic and e, f, g, h not aromatic
 (d) a, c, d, e, h aromatic and b, f, g not aromatic

27. An optically active alkyl halide $\text{C}_4\text{H}_9\text{Br}$ [A] reacts with hot KOH dissolved in ethanol and forms alkene [B] as major product which reacts with bromine to give dibromide [C]. The compound [C] is converted into a gas [D] upon reacting with alcoholic NaNH_2 . During hydration 18 gram of water is added to 1 mole of gas [D] on warming with mercuric sulphate and dilute acid at 333 K to form compound [E]. The IUPAC name of compound [E] is :

- (a) But-2-yne (b) Butan-2-ol
 (c) Butan-2-one (d) Butan-1-al

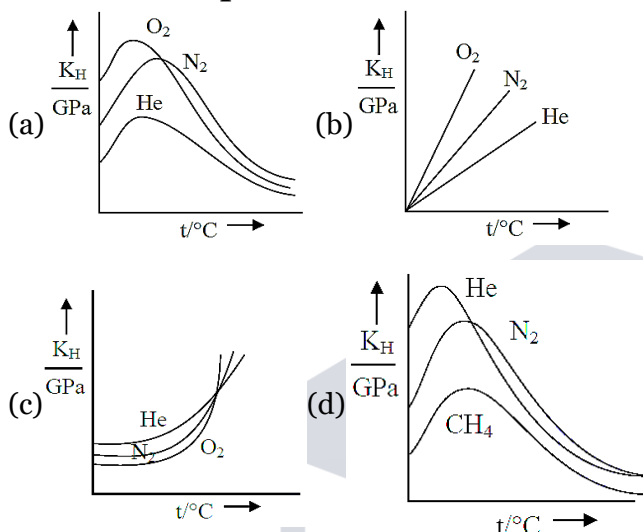
28. The property/properties that show irregularity in first four elements of group-17 is/are :

- (A) Covalent radius
- (B) Electron affinity
- (C) Ionic radius
- (D) First ionization energy

Choose the correct answer from the options given below:

- (a) B and D only
- (b) A and C only
- (c) B only
- (d) A, B, C and D

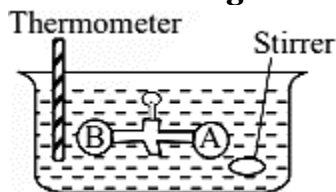
29. Which of the following graph correctly represents the plots of K_H at 1 bar gases in water versus temperature?



30. According to Bohr's model of hydrogen atom, which of the following statement is incorrect?

- (a) Radius of 3rd orbit is nine times larger than that of 1th orbit.
- (b) Radius of 8th orbit is four times larger than that of 4th orbit.
- (c) Radius of 6th orbit is three times larger than that of 4th orbit.
- (d) Radius of 4th orbit is four times larger than that of 2nd orbit.

31. Two vessels A and B are connected via stopcock. The vessel A is filled with a gas at a certain pressure. The entire assembly is immersed in water and is allowed to come to thermal equilibrium with water. After opening the stopcock the gas from vessel A expands into vessel B and no change in temperature is observed in the thermometer. Which of the following statement is true?

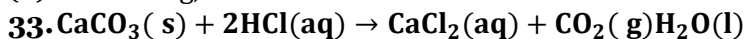


- (a) $dw \neq 0$
- (b) $dq \neq 0$
- (c) $dU \neq 0$
- (d) The pressure in the vessel B before opening the stopcock is zero.

32. A solution is made by mixing one mole of volatile liquid A with 3 moles of volatile liquid B. The vapour pressure of pure A is 200 mm Hg and that of the solution is 500 mm Hg. The vapour pressure of pure B and the least volatile component of the solution, respectively, are :

- (a) 1400 mmHg, A
- (b) 1400 mmHg, B

- (c) 600 mmHg, B
(d) 600 mmHg, A



Consider the above reaction, what mass of CaCl_2 will be formed if 250 mL of 0.76 M HCl reacts with 1000 g of CaCO_3 ?

(Given : Molar mass of Ca, C, O, H and Cl are 40, 12, 16, 1 and 35.5 g mol^{-1} , respectively)

- (a) 3.908 g
(b) 2.636 g
(c) 10.545 g
(d) 5.272 g

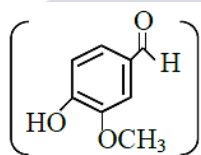
34. If equal volumes of AB_2 and XY (both are salts) aqueous solutions are mixed, which of the following combination will give a precipitate of AY_2 at 300 K ? (Given K_{sp} (at 300 K) for $\text{AY}_2 = 5.2 \times 10^{-7}$)

- (a) $3.6 \times 10^{-3} \text{MAB}_2, 5.0 \times 10^{-4} \text{MXY}$
(b) $2.0 \times 10^{-4} \text{MAB}_2, 0.8 \times 10^{-3} \text{MXY}$
(c) $2.0 \times 10^{-2} \text{MAB}_2, 2.0 \times 10^{-2} \text{MXY}$
(d) $1.5 \times 10^{-4} \text{MAB}_2, 1.5 \times 10^{-3} \text{MXY}$

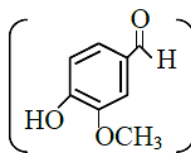
35. Among SO_2 , NF_3 , NH_3 , XeF_2 , ClF_3 and SF_4 , the hybridization of the molecule with non-zero dipole moment and highest number of lone-pairs of electrons on the central atom is.

- (a) sp^3 (b) dsp^2
(c) $\text{sp}^3 \text{d}^2$ (d) $\text{sp}^3 \text{d}$

36. Given below are two statements :



Statement (I) : Vanillin will react with NaOH and also with Tollen's reagent.



Statement (II) : Vanillin will undergo self aldol condensation very easily.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (a) Statement I is incorrect but Statement II is correct
(b) Statement I is correct but Statement II is incorrect
(c) Both Statement I and Statement II are incorrect
(d) Both Statement I and Statement II are correct

37. Identify the correct statement among the following:

- (a) All naturally occurring amino acids except glycine contain one chiral center.
(b) All naturally occurring amino acids are optically active.
(c) Glutamic acid is the only amino acid that contains a $-\text{COOH}$ group at the side chain.
(d) Amino acid, cysteine easily undergo dimerization due to the presence of free SH group.

38. The correct order of basic nature on aqueous solution for the bases

NH_3 , $\text{H}_2\text{N}-\text{NH}_2$, $\text{CH}_3\text{CH}_2\text{NH}_2$, $(\text{CH}_3\text{CH}_2)_2\text{NH}$ and $(\text{CH}_3\text{CH}_2)_3\text{N}$ is :

- (a) $\text{NH}_3 < \text{H}_2\text{N}-\text{NH}_2 < (\text{CH}_3\text{CH}_2)_3\text{N} < \text{CH}_3\text{CH}_2\text{NH}_2 < (\text{CH}_3\text{CH}_2)_2\text{NH}$
(b) $\text{NH}_3 < \text{H}_2\text{N}-\text{NH}_2 < \text{CH}_3\text{CH}_2\text{NH}_2 < (\text{CH}_3\text{CH}_2)_2\text{NH} < (\text{CH}_3\text{CH}_2)_3\text{N}$
(c) $\text{H}_2\text{N}-\text{NH}_2 < \text{NH}_3 < (\text{CH}_3\text{CH}_2)_3\text{N} < \text{CH}_3\text{CH}_2\text{NH}_2 < (\text{CH}_3\text{CH}_2)_2\text{NH}$
(d) $\text{NH}_2-\text{NH}_2 < \text{NH}_3 < \text{CH}_3\text{CH}_2\text{NH}_2 < (\text{CH}_3\text{CH}_2)_3\text{N} < (\text{CH}_3\text{CH}_2)_2\text{NH}$

39. Given below are two statements :

Statement (I) : The metallic radius of Al is less than that of Ga .

Statement (II) : The ionic radius of Al^{3+} is less than that of Ga^{3+} .

In the light of the above statements, choose the most appropriate answer from the options given below :

- (a) Both Statement I and Statement II are incorrect
- (b) Statement I is incorrect but Statement II is correct
- (c) Statement I is correct but Statement II is incorrect
- (d) Both Statement I and Statement II are correct

40. Given below are two statements :

Statement (I) : In octahedral complexes, when $\Delta_o < P$ high spin complexes are formed. When $\Delta_o > P$ low spin complexes are formed.

Statement (II) : In tetrahedral complexes because of $\Delta_t < P$, low spin complexes are rarely formed.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (a) Statement I is correct but Statement II is incorrect.
- (b) Both Statement I and Statement II are incorrect
- (c) Statement I is incorrect but Statement II is correct
- (d) Both Statement I and Statement II are correct

41. Choose the correct tests with respective observations.

- (A) CuSO_4 (acidified with acetic acid) + $\text{K}_4[\text{Fe}(\text{CN})_6] \rightarrow$ Chocolate brown precipitate.
- (B) $\text{FeCl}_3 + \text{K}_4[\text{Fe}(\text{CN})_6] \rightarrow$ Prussian blue precipitate.
- (C) $\text{ZnCl}_2 + \text{K}_4[\text{Fe}(\text{CN})_6]$, neutralised with $\text{NH}_4\text{OH} \rightarrow$ White or bluish white precipitate.
- (D) $\text{MgCl}_2 + \text{K}_4[\text{Fe}(\text{CN})_6] \rightarrow$ Blue precipitate.
- (E) $\text{BaCl}_2 + \text{K}_4[\text{Fe}(\text{CN})_6]$, neutralised with $\text{NaOH} \rightarrow$ White precipitate.

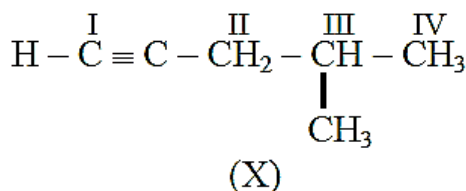
Choose the correct answer from the options given below :

- (a) A, D and E only
- (b) B, D and E only
- (c) A, B and C only
- (d) C, D and E only

42. On complete combustion 1.0 g of an organic compound (X) gave 1.46 g of CO_2 and 0.567 g of H_2O . The empirical formula mass of compound (X) is ____ g. (Given molar mass in gmol^{-1} : C: 12, H: 1, O: 16)

- (a) 30
- (b) 45
- (c) 60
- (d) 15

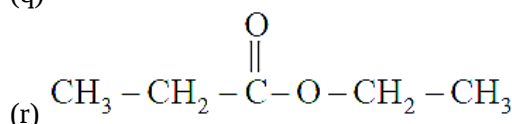
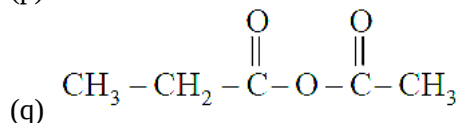
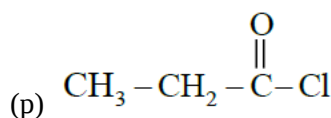
43. Consider the following compound (X)

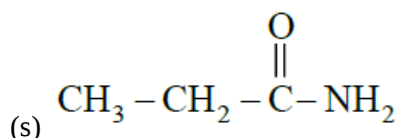


The most stable and least stable carbon radicals, respectively, produced by homolytic cleavage of corresponding C-H bond are :

- (a) II, IV
- (b) III, II
- (c) I, IV
- (d) II, I

44. Consider the following molecules :





The correct order of rate of hydrolysis is :

- (a) $r > q > p > s$ (b) $q > p > r > s$
(c) $p > r > q > s$ (d) $p > q > r > s$

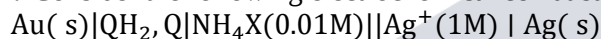
45. A molecule with the formula AX_4Y has all its elements from p-block.

Element A is rarest, monoatomic, non-radioactive from its group and has the lowest ionization enthalpy value among A, X and Y. Elements X and Y have first and second highest electronegativity values respectively among all the known elements. The shape of the molecule is :

- (a) Square pyramidal
(b) Octahedral
(c) Pentagonal planar
(d) Trigonal bipyramidal

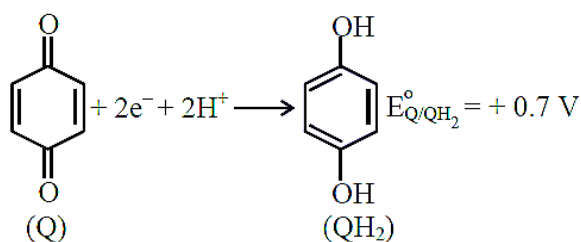
46. A transition metal (M) among Mn, Cr, Co and Fe has the highest standard electrode potential ($\text{M}^{3+}/\text{M}^{2+}$). It forms a metal complex of the type $[\text{M}(\text{CN})_6]^{4-}$. The number of electrons present in the e_g orbital of the complex is .

47. Consider the following electrochemical cell at standard condition.



$$E_{\text{cell}} = +0.4 \text{ V}$$

The couple QH_2/Q represents quinhydrone electrode, the half cell reaction is given below

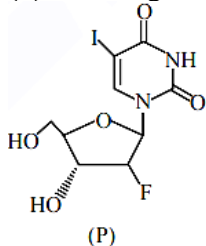


$$\left[\text{Given : } E_{\text{Ag}^+/\text{Ag}}^0 = +0.8 \text{ V and } \frac{2.303RT}{F} = 0.06 \text{ V} \right]$$

The pK_b value of the ammonium halide salt (NH_4X) used here is . (nearest integer)

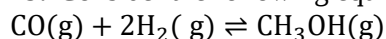
48. 0.1 mol of the following given antiviral compound

(P) will weigh $\times 10^{-1} \text{ g}$



(Given : molar mass in gmol^{-1} : H: 1, C: 12, N: 14, O: 16, F: 19, I: 127)

49. Consider the following equilibrium,



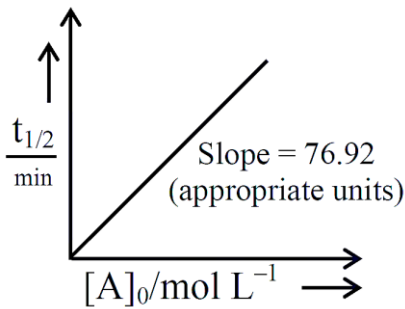
0.1 mol of CO along with a catalyst is present in a

2dm^3 flask maintained at 500 K. Hydrogen is introduced into the flask until the pressure is 5 bar and 0.04 mol of CH_3OH is formed. The K_p^0 is $\times 10^{-3}$ (nearest integer).

Given : $R = 0.08\text{dm}^3 \text{ bar K}^{-1} \text{ mol}^{-1}$

Assume only methanol is formed as the product and the system follows ideal gas behaviour.

50. For the reaction $\text{A} \rightarrow \text{products}$.



The concentration of A at 10 minutes is $\times 10^{-3} \text{ mol L}^{-1}$ (nearest integer).

The reaction was started with 2.5 mol L^{-1} of A.

Mathematics

Section-A

51. The largest $n \in \mathbb{N}$ such that 3^n divides $50!$ is:

- (a) 21 (b) 22
(c) 20 (d) 23

52. Let one focus of the hyperbola $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be at $(\sqrt{10}, 0)$ and the

corresponding directrix be $x = \frac{9}{\sqrt{10}}$. If e and l respectively are the eccentricity and the length of the latus rectum of H , then $9(e^2 + l)$ is equal to:

- (a) 14 (b) 15
(c) 16 (d) 12

53. The number of sequences of ten terms, whose terms are either 0 or 1 or 2, that contain exactly five 1s and exactly three 2s, is equal to

- (a) 360 (b) 45
(c) 2520 (d) 1820

54. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a twice differentiable function such that

$(\sin x \cos y)(f(2x + 2y) - f(2x - 2y)) = (\cos x \sin y)(f(2x + 2y) + f(2x - 2y))$, for all $x, y \in \mathbb{R}$. If $f'(0) = \frac{1}{2}$, then the value of $24f''\left(\frac{5\pi}{3}\right)$ is:

- (a) 2 (b) -3
(c) 3 (d) -2

55. Let $A = \begin{bmatrix} \alpha & -1 \\ 6 & \beta \end{bmatrix}$, $\alpha > 0$, such that $\det(A) = 0$ and $\alpha + \beta = 1$. If I denotes 2×2 identity matrix, then the matrix $(1 + A)^8$ is:

- (a) $\begin{bmatrix} 4 & -1 \\ 6 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 257 & -64 \\ 514 & -127 \end{bmatrix}$
(c) $\begin{bmatrix} 1025 & -511 \\ 2024 & -1024 \end{bmatrix}$ (d) $\begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$

56. The term independent of x in the expansion of $\left(\frac{(x+1)}{\left(\frac{2}{x^3} + 1 - x^3 \right)} - \frac{(x+1)}{\left(x - x^2 \right)} \right)^{10}$, $x >$

1 is:

- (a) 210 (b) 150
(c) 240 (d) 120

57. If $\theta \in [-2\pi, 2\pi]$, then the number of solutions of $2\sqrt{2}\cos^2 \theta + (2 - \sqrt{6})\cos \theta - \sqrt{3} = 0$, is equal to:

- (a) 12 (b) 6
(c) 8 (d) 10

58. Let $a_1, a_2, a_3 \dots$ be in an A.P. such that $\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5}a_1$, $a_1 \neq 0$. If $\sum_{k=1}^n a_k ='$

0, then n is:

- (a) 11 (b) 10
(c) 18 (d) 17

59. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its local

maximum and local minimum values at p and q , respectively, such that $p^2 = q$, then $f(3)$ is equal to:

- (a) 55 (b) 10
(c) 23 (d) 37

60. Let z be a complex number such that $|z| = 1$. If $\frac{2+k^2z}{k+\bar{z}} = kz$, $k \in \mathbb{R}$, then the maximum distance of $k + ik^2$ from the circle $|z - (1 + 2i)| = 1$ is:

- (a) $\sqrt{5} + 1$ (b) 2
(c) 3 (d) $\sqrt{3} + 1$

61. If $\vec{a}$ is nonzero vector such that its projections on the vectors $2\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} + 2\hat{j} - 2\hat{k}$ and $\hat{k}$ are equal, then a unit vector along $\vec{a}$ is:

- (a) $\frac{1}{\sqrt{155}}(-7\hat{i} + 9\hat{j} + 5\hat{k})$
(b) $\frac{1}{\sqrt{155}}(-7\hat{i} + 9\hat{j} - 5\hat{k})$
(c) $\frac{1}{\sqrt{155}}(7\hat{i} + 9\hat{j} + 5\hat{k})$
(d) $\frac{1}{\sqrt{155}}(7\hat{i} + 9\hat{j} - 5\hat{k})$

62. Let A be the set of all functions $f: \mathbb{Z} \rightarrow \mathbb{Z}$ and R be a relation on A such that $R = \{(f, g): f(0) = g(1) \text{ and } f(1) = g(0)\}$. Then R is:

- (a) Symmetric and transitive but not reflexive
(b) Symmetric but neither reflexive nor transitive
(c) Reflexive but neither symmetric nor transitive
(d) Transitive but neither reflexive nor symmetric

63. For α, β, γ, R , if $\lim_{x \rightarrow 0} \frac{x^2 \sin \alpha x + (\gamma - 1)e^{x^2}}{\sin 2x - \beta x} = 3$, then $\beta + \gamma - \alpha$ is equal to:

- (a) 7 (b) 4
(c) 6 (d) -1

64. If the system of linear equations

$$3x + y + \beta z = 3$$

$$2x + \alpha y - z = -3$$

$$x + 2y + z = 4$$

has infinitely many solutions, then the value of $22\beta - 9\alpha$ is :

- (a) 49 (b) 31
(c) 43 (d) 37

65. Let $P_n = \alpha^n + \beta^n$, $n \in \mathbb{N}$. If $P_{10} = 123$, $P_9 = 76$, $P_8 = 47$ and $P_1 = 1$, then the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is:

- (a) $x^2 - x + 1 = 0$
(b) $x^2 + x - 1 = 0$
(c) $x^2 - x - 1 = 0$
(d) $x^2 + x + 1 = 0$

66. If S' and S' are the foci of the ellipse $\frac{x^2}{18} + \frac{y^2}{9} = 1$ and P be a point on the ellipse, then $\min(SP, S'P) + \max(SP, S'P)$ is equal to :

- (a) $3(1 + \sqrt{2})$ (b) $3(6 + \sqrt{2})$
(c) 9 (d) 27

67. Let the vertices Q and R of the triangle PQR lie on the line $\frac{x+3}{5} = \frac{y-1}{2} =$

$\frac{z+4}{3}$, $QR = 5$ and the coordinates of the point P be $(0, 2, 3)$. If the area of the triangle PQR is $\frac{m}{n}$ then :

- (a) $m - 5\sqrt{21}n = 0$
(b) $2m - 5\sqrt{21}n = 0$
(c) $5m - 2\sqrt{21}n = 0$
(d) $5m - 21\sqrt{2}n = 0$

68. Let $ABCD$ be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the areas of the triangles ABC, ACD and ADB be 5, 6 and 7 square units respectively. Then the area (in square units) of the $\triangle BCD$ is equal to :

- (a) $\sqrt{340}$ (b) 12
(c) $\sqrt{110}$ (d) $7\sqrt{3}$

69. Let $a \in \mathbb{R}$ and A be a matrix of order 3×3 such that $\det(A) = -4$ and

$A + I = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}$, where I is the identity matrix of order 3×3 . If $\det((a+1)\text{adj}((a-1)A))$ is

$2^m 3^n$, $m, n \in \{0, 1, 2, \dots, 20\}$, then $m + n$ is equal to :

- (a) 14 (b) 17
(c) 15 (d) 16

70. Let the focal chord PQ of the parabola $y^2 = 4x$ make an angle of 60°

with the positive x -axis, where P lies in the first quadrant. If the circle, whose one diameter is PS , S being the focus of the parabola, touches the y -axis at the point $(0, \alpha)$, then $5\alpha^2$ is equal to :

- (a) 15 (b) 25
(c) 30 (d) 20

71. Let $[\cdot]$ denote the greatest integer function. If $\int_0^{e^3} \left[\frac{1}{e^{x-1}} \right] dx = \alpha - \log_e 2$, then α^3 is equal to _____.

72. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a thrice differentiable odd function satisfying

$f'(x) \geq 0, f'(x) = f(x), f(0) = 0, f'(0) = 3$. Then $9f(\log_e 3)$ is equal to .

73. If the area of the region $\{(x, y): |4 - x^2| \leq y \leq x^2, y \leq 4, x \geq 0\}$ is $\left(\frac{80\sqrt{2}}{\alpha} - \beta\right)$, $\alpha, \beta \in \mathbb{N}$, then $\alpha + \beta$ is equal to.

74. Three distinct numbers are selected randomly from the set $\{1, 2, 3, \dots, 40\}$. If the probability, that the selected numbers are in an increasing G.P. is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to _____.

75. The absolute difference between the squares of the radii of the two circles passing through the point $(-9, 4)$ and touching the lines $x + y = 3$ and $x - y = 3$, is equal to .

