1. **(b)** $\frac{1}{2}\epsilon_0 E^2$ = Energy density

$$\therefore \left[\frac{1}{2}\epsilon_0 E^2\right] = \frac{ML^2 T^{-2}}{L^3} = ML^{-1} T^{-2}$$

$\therefore a = 1, b = -1, c = -2$

$\therefore 2a - b + c = 2 + 1 - 2 = 1$

2. **(b)** $Y = \frac{F\ell}{A\Delta\ell}$

$$\therefore \frac{\Delta Y}{Y} = \frac{\Delta F}{F} + \frac{\Delta\ell}{\ell} + 2\frac{\Delta d}{d} + \frac{\Delta(\Delta\ell)}{\Delta\ell}$$

$$\left[A = \pi\frac{d^2}{4} \Rightarrow \frac{\Delta A}{A} = \frac{2\Delta d}{d}\right]$$

Since F is known exactly $\Rightarrow \Delta F = 0$

$$\frac{\Delta Y}{Y} = \frac{0.1}{150} + \frac{2 \times 0.001}{0.08} + \frac{0.001}{0.5} = 0.0277$$

$\therefore \Delta Y = 0.0277Y$

$$Y = \frac{100 \times 1.5}{\frac{\pi}{4}\left(\frac{0.08}{100}\right)^2 \times 0.5 \times 10^{-2}} = 5.97 \times 10^{10}\ N/m^2$$

$\therefore \Delta Y = 0.0277 \times 5.97 \times 10^{10}\ N/m^2 = 1.65 \times 10^9\ N/m^2$

3. **(b)** $\vec{v} = -x\hat{\imath} + 2y\hat{\jmath} - z\hat{k}$

$$\vec{a} = \frac{d\vec{v}}{dt} = -\frac{dx}{dt}\hat{\imath} + 2\frac{dy}{dt}\hat{\jmath} - \frac{dz}{dt}\hat{k}$$

$$= -(-x)\hat{\imath} + 2(2y)\hat{\jmath} - (-z)\hat{k} = x\hat{\imath} + 4y\hat{\jmath} + z\hat{k}$$

$\therefore \vec{a}$ at $(1,2,4) = \hat{\imath} + 8\hat{\jmath} + 4\hat{k}$

$\therefore |\vec{a}| = 9\ m/s^2$

4. **(d)** Given $\vec{r} = 10t^2\hat{\imath} + 5t^3\hat{\jmath} \Rightarrow \vec{r}_{t=1} = 10\hat{\imath} + 5\hat{\jmath}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 20t\hat{\imath} + 15t^2\hat{\jmath} \Rightarrow \vec{v}_{t=1sec} = 20\hat{\imath} + 15\hat{\jmath}$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = 20\hat{\imath} + 30t\hat{\jmath} \Rightarrow \vec{a}_{t=1\ sec} = 20\hat{\imath} + 30\hat{\jmath}$$

$\therefore \vec{P} = m\vec{v} = 2\hat{\imath} + 1.5\hat{\jmath}$

$\vec{F} = m\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$

$\vec{\tau} = \vec{r} \times \vec{F} = 20\hat{k}$

$\vec{L} = \vec{r} \times \vec{P} = 5\hat{k}$

5. **(b)** From Kepler's law,

$$T^2 = \frac{4\pi^2}{GM_S} r^3$$

$$\therefore\ T_{P_1} = \frac{2\pi}{\sqrt{G(2M)}} R^{3/2}$$

$$T_{P_2} = \frac{2\pi}{\sqrt{G(4M)}} (2R)^{3/2}$$

$$\therefore \frac{T_{P_2}}{T_{P_1}} = \frac{(2)^{3/2}}{\sqrt{4}} \times \frac{\sqrt{2}}{1} = \frac{2\sqrt{2} \times \sqrt{2}}{2} = 2$$

6. **(c)** $\theta = \frac{5t^4}{40} - \frac{t^3}{3}$

$$\frac{d\theta}{dt} = \frac{5}{40}(4t^3) - \frac{3t^2}{3} = \frac{t^3}{2} - t^2$$

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{3t^2}{2} - 2t$$

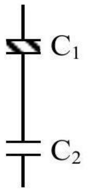
at t = 10

$\alpha = \frac{3}{2}(10)^2 - 2 \times 10 = 130\text{rad/sec}^2$

7. **(b)** Initial capacitance

$C_i = \frac{A\epsilon_0}{d} = C$

Final capacitance



$C_1 = \frac{A \in_0 \times 5}{\frac{d}{2}} = 10C$

$C_2 = \frac{A \in_0}{\frac{d}{2}} = 2C$

$C_f = \frac{C_1C_2}{C_1 + C_2} = \frac{10 \times 2}{10 + 2}C = \frac{5}{3}C$

% increase $= \frac{\frac{5}{3}C - C}{C} \times 100 = 66.67$

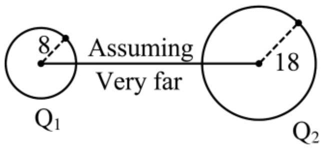
8. **(d)** $\Delta Q_p = nC_p\Delta T = \frac{7}{2}nR\Delta T$

$\Delta U = nC_v\Delta T = \frac{5}{2}nR\Delta T$

$W = nR\Delta T$

$\Delta Q : \Delta U : W :: 7 : 5 : 2$

9. **(d)**



$V_1 = V_2$

$\frac{kQ_1}{8} = \frac{kQ_2}{18} \left\{V_{\text{surface}} = \frac{kQ}{R}\right\}$

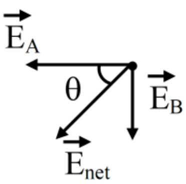
$\frac{Q_1}{Q_2} = \frac{4}{9}$

$\frac{E_{S_1}}{E_{S_2}} = \frac{kQ_1}{r_1^2}\frac{r_2^2}{kQ_2}\left\{E_{\text{surface}} = \frac{kQ}{R^2}\right\}$

$= \frac{Q_1}{Q_2}\left(\frac{r_2}{r_1}\right)^2 = \frac{4}{9} \times \left(\frac{9}{4}\right)^2 = \frac{9}{4}$

10. **(d)** $v = \frac{\text{coeff of t}}{\text{coeff of x}} = \frac{200}{\frac{1}{150}\times 100} = 300 \text{ m/s}$

11. **(b)**



$\vec{E}_A = \frac{2kP_1}{x^3}(-\hat{\imath})$\{ Axial point\}

$\vec{E}_B = \frac{kP_2}{x^3}(-\hat{j})$ {Equatorial point}

Now : $\tan\theta = \frac{|\vec{E}_B|}{|\vec{E}_A|}$

$\Rightarrow \tan 60^\circ = \frac{P_2}{2P_1}$

$\Rightarrow \frac{P_2}{P_1} = 2\sqrt{3}$

12. **(b)** For the +ve half cycle of input

Middle branch has ideal diode in forward bias and Zener diode in reverse bias Thus, we get constant 5 V output due to Zener breakdown. For this situation component between x&y should act as open circuit.

For -ve half cycle of input

middle branch has ideal diode in reverse bias, which makes it open branch without any current. for this situation component between x&y should give a constant -5 V output due to Zener breakdown.

13. **(a)** Power dissipated : $P = \frac{\varepsilon^2}{R}$

Induced emf $\varepsilon = -NA\frac{dB}{dt}$

Resistance of coil : $R = \rho\frac{\ell}{\pi r^2}$

Now : $\ell = N(2\pi R_{coil})$

Also : $A = \pi R_{coil}^2 \Rightarrow R_{coil} = \sqrt{\frac{A}{\pi}}$

Thus ; $R = \frac{\rho N(2\pi)}{\pi r^2}\sqrt{\frac{A}{\pi}}$

$P = \frac{N^2 A^2\left(\frac{dB}{dt}\right)^2}{\frac{2\rho N}{r^2}\sqrt{\frac{A}{\pi}}} \Rightarrow P \propto NA^{3/2}r^2 \quad \ldots(i)$

$P' \propto (2N)(2A)^{3/2}(3r)^2 \quad \ldots(ii)$

$\frac{(2)}{(1)} : \frac{P'}{P} = 36\sqrt{2} \Rightarrow P' = (\sqrt{2})(36)P$

Thus : $\alpha = 36$

14. **(a)** $B_1 = \frac{\mu_0 I}{4\pi r} \times 2 + \frac{\mu_0 I}{4r} = \frac{\mu_0 I}{4r}\left(\frac{2}{\pi} + 1\right)$

$B_2 = \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4r} = \frac{\mu_0 I}{4r}\left(\frac{1}{\pi} + 1\right)$

$\frac{B_1}{B_2} = \frac{2+\pi}{1+\pi}$

15. **(a)** We have for thin prism minimum deviation :

$\delta_m = (\mu - 1)A$

$\Rightarrow \delta_m = (1.5 - 1)A \Rightarrow \delta_m = \frac{A}{2}$

Also, $i = \frac{A+\delta_m}{2} \Rightarrow i = \frac{A+\frac{A}{2}}{2} \Rightarrow i = \frac{3A}{4}$

Thus $\frac{i}{\delta_m} = \frac{3}{2}$

16. **(a)** $\frac{\mu_2}{v} - \frac{\mu_1}{\mu} = \frac{\mu_2 - \mu_1}{R}$

$\frac{1.54}{v} - \frac{1}{(-40)} = \frac{1.54 - 1}{-20}$

$\frac{1.54}{v} = -\frac{1}{40} - \frac{0.54}{20}$

$\frac{1.54}{v} = \frac{-2.08}{40}$

$v = -29.61$ cm

$\frac{h_i}{h_0} = \frac{\mu_1 v}{\mu_2 u} = \frac{1(-29.61)}{1.54(-40)} = 0.48$

$h_i = 0.48 \times 2 = 0.96 \approx 1 \text{ cm}$

17. (b) $\frac{hc}{\lambda} - \phi = 3eV_0 \quad \dots(i)$

$\frac{hc}{2\lambda} - \phi = eV_0 \quad \dots(ii)$

$\frac{hc}{\lambda} - \phi = 3\left[\frac{hc}{2\lambda} - \phi\right]$

$\phi = \frac{hc}{4\lambda}$

$\frac{hc}{\lambda_0} = \frac{hc}{4\lambda}$

$\lambda_0 = 4\lambda$

$\alpha = 4$

18. (a) $E_y = 300\sin\omega\left(t - \frac{x}{v}\right)$

$F_M = qvB \quad \dots(i)$

$F_e = qE \quad \dots(ii)$

$\frac{F_e}{F_m} = \frac{qE}{qvB} = \frac{E}{vB} = \frac{c}{v} = \frac{3 \times 10^8}{1.5 \times 10^6} = 200$

19. (c) $L = \frac{nh}{2\pi}$

$\frac{3h}{\pi} = \frac{nh}{2\pi}$

$n = 6$

$E = \frac{-13.6}{n^2} eV$

$= \frac{-13.6}{6^2} \approx -0.38eV$

20. (b) $V_1 = V_2$

$\frac{4}{3}\pi R^3 = 512\left(\frac{4}{3}\pi r^3\right)$

$R = (512)^{1/3} r$

$1 = 8r$

$r = \frac{1}{8} \text{ mm}$

$\Delta SE = SE_2 - SE_1$

$= 512[T \times 4\pi r^2] - [T \times 4\pi R^2]$

$= 4\pi T[512r^2 - R^2]$

$= 4\pi(0.08)\left[512\left(\frac{1}{8}\right)^2 - 1^2\right] \times 10^{-6} \approx 7 \times 10^{-6}$

$\alpha = 7$

21. (36) Angular width of central maxima $\theta_{cm} = \frac{2\lambda}{d}$

$= \frac{2 \times 628 \times 10^{-9}}{0.2 \times 10^{-3}}$ rad.

$= (628 \times 10^{-5}) \times \frac{180}{\pi}$ degrees

$\approx 36 \times 10^{-2}$ degrees

$\alpha = 36$

22. (120) $r = \frac{C_P}{C_V} = \frac{3}{2}$

$\because P_1 V_1^\gamma = P_2 V_2^\gamma$

$\Rightarrow V_2 = \left(\frac{P_1}{P_2}\right)^{1/\gamma} V_1$

$\Rightarrow V_2 = \left(\frac{8}{1}\right)^{2/3} \times 0.15$

$\Rightarrow V_2 = 0.6 \text{ m}^3$

$\because W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$

$= \left(\frac{8 \times 0.15 - 1 \times 0.6}{1.5 - 1}\right) \times 10^5 = 1.2 \times 10^5$

$\Rightarrow W = 120 \text{ kJ}$

23. (171) $\vec{F} = q(\vec{v} \times \vec{B})$

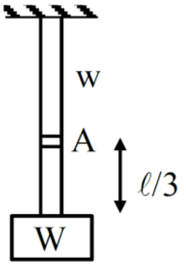
$= 1 \times 10^{-6} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 3 & -5 \end{vmatrix}$

$= 10^{-6}[\hat{i} + 11\hat{j} + 7\hat{k}]$

$\Rightarrow \vec{F} = \sqrt{171} \times 10^{-6} \text{ N}$

$\therefore \alpha = 171$

24. (6)

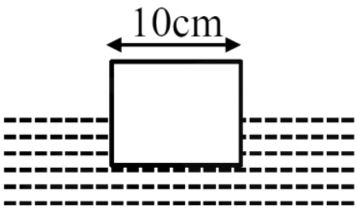


$\sigma = \frac{T}{A} = \frac{W + \frac{W}{3}}{A}$

$\Rightarrow \sigma = \frac{W}{A} + \frac{2}{6}\frac{w}{A}$

$\therefore \gamma = 6$

25. (387)



Weight of coin = Additional up thrust

$\Rightarrow m_c \, g = \rho_\ell V_{add} \, g$

$\Rightarrow m_c = 1 \times 3.87 \times (10)^2$

$\Rightarrow m_c = 387$ gram

26. (d) $3Fe(s) + 4H_2O(g) \rightarrow Fe_3O_4(s) + 4H_2(g)$

$n_{steam} = \frac{18}{18} = 1$

n_{Fe} required $= \frac{3}{4}$

mass of Fe required $= \frac{3}{4} \times 56$

$= 42$gm

27. (c) $E_{Li^{2+}} = E_H \times z^2$

$= 2.18 \times 10^{-18} \times (3)^2$

$= 1.962 \times 10^{-17} \text{ Jatom}^{-1}$

28. (c)

Species	Bond order	unpaired electrons
$O_2^{\oplus}$	2.5	1
O_2	2	2
O_2^-	1.5	1
O_2^{2-}	1	0

29. (d) $2Al(s) + 6HCl(aq) \rightarrow Al_2Cl_6(aq) + 3H_2(g)$

$\Delta_r H_1 = -1200\ kJ/mol$

$[H_2(g) + Cl_2(g) \rightarrow 2HCl(g)] \times 3$

$\Delta_r H_2 = -163 \times 3\ kJ/mol$

$[HCl(g) + aq \rightarrow HCl(aq)] \times 6$

$\Delta_r H_3 = -83 \times 6\ kJ/mol$

$Al_2Cl_6(aq) \rightarrow Al_2Cl_6(s) +$ aq. $\Delta_r H_4 = +663\ kJ/mole$

$2Al(s) + 3Cl_2 \rightarrow Al_2Cl_6(s)\ \Delta_f H$

$\Delta_f H = \Delta_r H_1 + \Delta_r H_2 + \Delta_r H_3 + \Delta_r H_4$

$= -1200 - 163 \times 3 - 83 \times 6 + 663$

$= -1527\ kJ/mole$

30. (d) $\Delta T_f = iK_f m$

no. of moles $= \frac{19.5}{78} = \frac{1}{4}$ mole

$$1 = i \times 1.86 \times \frac{1/4}{1/2}$$

$$i = \frac{2}{1.86}$$

$i = 1 + (n-1)\alpha$

$i = 1 + \alpha$

$$\alpha = \frac{2}{1.86} - 1 = 0.075$$

$$K_a = \frac{C^2}{1-\alpha}$$

$$= \frac{1}{2} \times \frac{(0.075)^2}{(1-0.075)}$$

$= 3 \times 10^{-3}$

31. (d) For $Ag_2C_2O_4$

$4\ S_1{}^3 = 32x$

$\Rightarrow S_1 = 2x^{1/3}$

For AgBr

$S_2{}^2 = 4y$

$\Rightarrow S_2 = 2y^{1/2}$

Ratio $\left(\frac{S_1}{S_2}\right) = \frac{\sqrt[3]{x}}{\sqrt{y}}$

32. (a) Overall reaction

$Fe(s) + 2OH^-(aq) + 2AgBr(s) \rightleftharpoons Fe(OH)_2(s) + 2Ag(s) + 2Br^-(aq)$

$E^o_{cell} = E^o_{R(\text{ cathode })} - E^o_{R(\text{ anode })}$

$= 0.07 - (-0.88) = 0.95\ V$

33. (b) Zero -order

$$t_{100\%} = \frac{a}{k}; t_{1/2} = \frac{a}{2k} t_{100\%} = 2t_{1/2}$$

Ist order

$t_{100\%} = \infty$

$$t_{1/2} = \frac{\ell n2}{k}$$

$t_{100\%} = (t_{1/2})^{\infty}$

34. **(d)**(I) Na < Mg < Cl < Ar → Ist ionization energy

(II) Among B, Ca, Al, Fe

Ca has maximum 3rd ionization energy among given due to inert gas configuration.

35. **(a)**(I) $F_2 > Cl_2 > Br_2 > I_2$ (Oxidizing power order)

(II) $NaBr + Cl_2\text{-water} \rightarrow Br_2 + NaCl$

$NaI + Cl_2\text{-water} \rightarrow I_2 + NaCl$

36. **(a)** Reducing agent $+ KmnO_4 \xrightarrow[\text{medium}]{\text{acidic}} Mn^{+2} + \text{Product}$

Given Reducing Agent

$Fe^{+2} \rightarrow Fe^{+3}$

$Sn^{+2} \rightarrow Sn^{+4}$

$I^- \rightarrow I_2$

$S^{-2} \rightarrow S$

37. **(d)** CFSE is proportional to strength of ligand followed by chelation.

$CN^-$is stronger ligand than en

$CN^- > en > H_2O > F^-$

38. **(d)** Glycerol is separated by distillation under reduced pressure.

39. **(a)** Rate of $SN^1 \propto$ Stability of carbocation Order of stability of carbocation :

$Ph - \overset{\oplus}{C}H_2 > CH_3 - \overset{\oplus}{C}H_2$

Hyperconjugation in $CH_3 - \overset{\oplus}{C}H_2$

Resonance in $Ph - \overset{\oplus}{C}H_2$

40. **(c)**

41. **(d)** U is predominant site towards attack of electrophile because of +M nature of nitrogen.

42. **(d)** . - Phenol gives violet colour with neutral $FeCl_3$ not ethanol.

• 1° amine gives positive test with $CHCl_3/KOH$ not 2° amine.

• Aldehyde gives positive test with ammoniacal silver nitrate not ketone.

• Cinnamic acid gives bromine water test.

43. **(a)** Rate of ESR $\propto$ nucleophilicity of benzene ring.

44. **(c)**

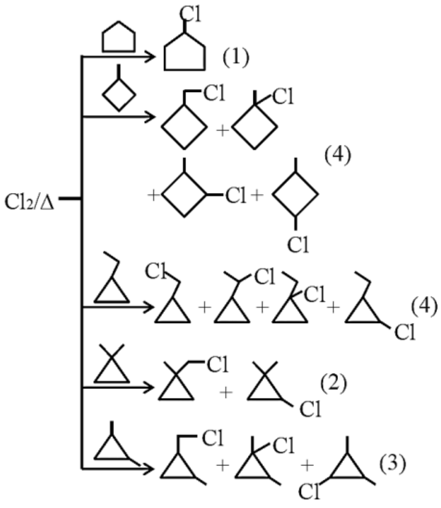
45. **(b)**(i) Ba^{+2} gives apple green colour while Ca^{+2} and Sr^{+2} gives brick red and crimson red in flame test.

(ii) $Ba^{+2} + CrO_4^{2-} \rightarrow \underset{\text{Yellow ppt}}{BaCrO_4}$

(iii) $PO_4^{3-} + \text{Conc } HNO_3 + (NH_4)_2MoO_4 \rightarrow \underset{\text{Canery yellow ppt}}{(NH_4)_3[P(Mo_3O_{10})_4]}$

46. **(33)** $[Cr(H_2O)_6]Cl_3 + \underset{\text{excess}}{AgNO_3} \rightarrow \underset{\text{ppt}}{3AgCl(s)} \dfrac{\text{no. of moles of complex reacted}}{\text{no. of moles of AgCl precipitated}} \times 100 = \frac{1}{3} \times 100 = 33.3$

47. (14)



$1 + 4 + 4 + 2 + 3 = 14$

48. (17) $C_nH_{2n+2} + \left(\frac{3n+1}{2}\right)O_2 \rightarrow nCO_2 + (n+1)H_2O$

$1\left(\frac{3n+1}{2}\right)$

$\frac{3n+1}{2} = 8 \Rightarrow 3n + 1 = 16$

$\Rightarrow 3n = 15$

$n = 5$

So, alkane is $C_5H_{12} \Rightarrow 17$

49. (41) $\ell n\left(\frac{K_2}{K_1}\right) = \frac{E_a}{R}\left[\frac{T_2 - T_1}{T_1\,T_2}\right]$

$\ln\left(\frac{4.5 \times 10^3}{1.5 \times 10^3}\right) = \frac{60 \times 10^3}{8.3}\left[\frac{T_2 - 300}{300.\,T_2}\right]$

$\ln 3 = \frac{60 \times 10^3}{8.3}\left[\frac{T_2 - 300}{300.\,T_2}\right]$

$\Rightarrow T_2 = 314.4\ K$

$\Rightarrow t = 41.4°C$

50. (727) $\Delta G° = 0$ (eqm. at standard pressure)

$105 - 35\log\ T = 0$

$\log T = 3$

$T = 1000\ K$

$T = 727°C$

51. (c) $nx^2 + x(2 + 4 + 6 + \cdots .. + 2n) + (1.3 + (n+1)) = 4n$

$nx^2 + n(n+1)x + \frac{n(n-1)(2n+5)}{6} = 4n$

$x^2 + (n+1)x + \frac{(n-1)(2n+5)}{6} = 4$

D must be a perfect square

$D = \frac{122 - 2n^2}{6} = 20 - \left(\frac{n^2 - 1}{3}\right)$

If make perfect square $= \frac{n^2-1}{3} = 16 \Rightarrow n = 7$

So Equation is

$\Rightarrow x^2 + 8x + \frac{8 \times 15}{6} - 5 = 0$

$x^2 + 8x + 15 = 0$

$x = -3, -5$

$\alpha = -5, \alpha + 2 = -3$

$\alpha + n = 7 - 5 = 2$

52. (d) $50\left(\frac{2x(1-3i)}{10} - \frac{y(1+2i)}{5}\right) = 31 + 17i$

$10x - (30x)i - 10y - 20yi = 31 + 17i$

$\left.\begin{aligned} 10(x-y) &= 31 \\ -30x - 20y &= 17 \end{aligned}\right\}$

$30x - 30y = 93$

$-10(3x + 2y) = 17$

$x = 0.9, y = -2.2$

$10(x - 3y) = 75$

53. (b) $x + 2y + z = 5$

$2x + y + \alpha z = 5$

$8x + 4y + \beta z = 18$

for no solution

$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & \alpha \\ 8 & 4 & \beta \end{vmatrix} = 0$

$= \beta - 4\alpha - 4\beta + 16\alpha = 0$

$\Rightarrow 4\alpha = \beta$

54. (b) $A = \begin{bmatrix} 1 & 2 \\ 1 & \alpha \end{bmatrix}$

$A^2 - 4A + I = 0$

trace of matrix

$\alpha + 1 = 4$

$\alpha = 3$

$B = \begin{bmatrix} 3 & 3 \\ \beta & 2 \end{bmatrix}$

$B^2 - 5B - 6I = 0$

$\det.(B) = 6 - 3\beta = -6$

$3\beta = 12$

$\beta = 4$

$B - A = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}; B + A = \begin{bmatrix} 4 & 5 \\ 5 & 5 \end{bmatrix}$

$(B - A)(B + A) = \begin{bmatrix} 13 & 15 \\ 7 & 10 \end{bmatrix}$

$\operatorname{adj}(A) = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}, \operatorname{adj}B = \begin{bmatrix} 2 & -3 \\ -4 & 3 \end{bmatrix}$

$\operatorname{adj}(A + B) = \begin{bmatrix} 5 & -5 \\ -5 & 4 \end{bmatrix}$

55. (b) AP_1 Set A = {1,6,11,16 101 terms)

AP_2 Set B = {9,16 71 terms}

D = L.C.M $\{d_1, d_2\} = 35$

1st Common term is 16

$16 + (n - 1)35 \leq 499$

$n \leq 14.8$

$\Rightarrow n = 14$

$A \cap B = \{16,51,86,121,156,191,226,261,296,331,366,401,436,471\}$

terms divisible by ' 3 '

$= \{51,156,261,366,471\}$

56. (c) Exactly one digit repeated → ${}^5C_1 \times \frac{7!}{3!} = 4200$

Exactly two digits repeated → ${}^5C_2 \times \frac{7!}{2!2!} = 12600$

Total numbers = 16800

57. (b) $\frac{36}{r+1} \cdot {}^{35}C_r = 6 \cdot \frac{{}^{35}C_r}{k^2 - 3}$

$k^2 - 3 = \frac{r+1}{6}$

$k^2 = \frac{r+19}{6}$

$0 \le r \le 35$

$19 \le r + 19 \le 54$

$3.1 \le \frac{r+19}{6} \le 9$

$\therefore k^2 = 4, 9$

$k = \pm 2, \pm 3$

$\therefore (r, k) \Rightarrow (5, \pm 2) \Rightarrow (35, \pm 3)$

$\therefore 4$

58. (c)

Class	Freq.
5 – 10	2
10 – 15	k
15 – 20	28
20 – 25	54
25 – 30	k + 1
30 – 35	5

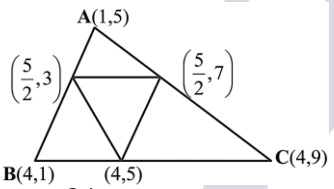
Mean

$21 = \frac{15 + 12.5k + 490 + 1215 + 27.5 + 27.5k + 162.5}{90 + 2k}$

$1890 + 42k = 1910 + 40k$

$k = 10$

59. (c)



$I = \left(\frac{24}{9}, 5\right)$

$\therefore 3h + k = 8 + 5 = 13$

60. (d) $e^2 = 1 - \frac{a^2}{b^2} \Rightarrow \frac{a^2}{b^2} = 1 - e^2 = 1 - \frac{5}{9} = \frac{4}{9}$

$\Rightarrow \frac{a^2}{b^2} = \frac{4}{9}$ ……(1)

Passes through $(4,3) \Rightarrow \frac{16}{a^2} + \frac{9}{b^2} = 1$ ……(2)

from (1) and (2)

$a^2 = 20$ and $b^2 = 45$

$LR = \frac{2a^2}{b} = \frac{2(20)}{3\sqrt{5}} = \frac{8\sqrt{5}}{3}$

61. (a) $K = \sin 10° \sin 50° \sin 70°$

$= \frac{1}{4}\sin 30° = \frac{1}{8}$

$\sin 10\, K\frac{\pi}{3} = \sin\left(10 \times \frac{1}{8} \cdot \frac{\pi}{3}\right) = \sin\frac{5\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

62. (d) $\sin x(\sin x + \cos x) \in \left[\frac{1-\sqrt{2}}{2}, \frac{1+\sqrt{2}}{2}\right]$

2 integer will be there $\Rightarrow a = 0, 1$

If $a = 0$ $\sin x(\sin x + \cos x) = 0$

$\Rightarrow \sin x = 0$ or $\sin x + \cos x = 0$

$x = -\pi, 0, \pi$

$= \tan x = -1$
3 solutions
$x = -\frac{\pi}{4}, \frac{3\pi}{4}$ 2 solutions
If $a = 1 \sin x(\sin x + \cos x) = 1$
$\sin^2 x + \sin x \cos x = 1$
$2\sin^2 x + 2\sin x \cos x = 2$
$1 - \cos 2x + \sin 2x = 2$
$\text{Sin}2x - \cos 2x = 1 \rightarrow$ square
$\text{Sin}4x = 0$
$4x = -4\pi, -3\pi, \ldots\ldots\ldots, 3\pi, 4\pi$
$$x = -\pi, -\frac{3\pi}{4}, -\frac{\pi}{2}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$$
$$\Rightarrow x = -\frac{3\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{2} \text{ 4 solutions}$$
Total 9 solution

63. (c) Line (L_1)
$$\frac{x+1}{3} = \frac{y+a}{5} = \frac{z+b+1}{7} = r_1$$
$\Rightarrow$ General point P on L_1 is
$(3r_1 - 1, 5r_1 - a, 7r_1 - b - 1)$
Line (L_2)
$$\frac{x-2}{1} = \frac{y-b}{4} = \frac{z-2a}{7} = r_2$$
$\Rightarrow$ General point Q on L_2 is
$(r_2 + 2, 4r_2 + b, 7r_2 + 2a)$
For point of intersection
$3r_1 - 1 = r_2 + 2 \Rightarrow r_2 = 3r_1 - 3$ .....(1)
$5r_1 - a = 4r_2 + b$ ......(2)
$7r_1 - b - 1 = 7r_2 + 2a$ ......(3)
Since point lies on XY plane
$\Rightarrow z-$ coordinate $= 0$
From $L_1: 7r_1 - b - 1 = 0 \Rightarrow 7r_1 = b + 1$
From $L_2: 7r_2 + 2a = 0 \Rightarrow 2a = -7r_2$
Substitute $r_2 = 3r_1 - 3$
$2a = -7(3r_1 - 3)$
$$\Rightarrow a = \frac{-21r_1 + 21}{2}$$
Put in equation (2)
$5r_1 - a = 4r_2 + b$
$$5r_1 - \left(\frac{-21r_1 + 21}{2}\right) = 4(3r_1 - 3) + (7r_1 - 1)$$
$$\Rightarrow r_1 = \frac{5}{7}$$
$\Rightarrow b = 7r_1 - 1 = 4$
$\Rightarrow a = 3 \therefore a + b = 7$

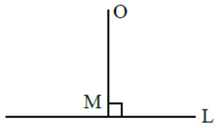
64. (c) $E = 3\sqrt{9a^2 + 4b^2 + 12a \cdot \vec{b}} + 4\sqrt{9a^2 + 4b^2 - 12\vec{a} \cdot \vec{b}}$
$= 3\sqrt{36 + 36 + 12 \times 6\cos\theta} + 4\sqrt{36 + 36 - 72\cos\theta}$
$= 3\sqrt{72 + 72\cos\theta} + 4\sqrt{72 - 72\cos\theta}$
$= 18\sqrt{2}\sqrt{1 + \cos\theta} + 24\sqrt{2}\sqrt{1 - \cos\theta}$
$$= 18\sqrt{2}\sqrt{2\cos\frac{\theta}{2} + 24\sqrt{2}\left(\sqrt{2\sin\frac{\theta}{2}}\right)}$$

$= 36\left(\cos\frac{\theta}{2}\right) + 48\sin\left(\frac{\theta}{2}\right)$

$E_{max} = \sqrt{(36)^2 + (48)^2} = \sqrt{6^4 + 6^2 \times 8^2} = 60$

65. (c) Direction vector of line is

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -2\hat{i} + 3\hat{j} - 2\hat{k}$



$(2\lambda + 1, -3\lambda + 1, 2\lambda + 1)$

$\frac{x-1}{2} = \frac{y-1}{-3} = \frac{z-1}{2}$

So, equation of line is

$\frac{x-1}{2} = \frac{y-1}{-3} = \frac{z-1}{2} = \lambda$ let

let ($2\lambda + 1, -3\lambda + 1, 2\lambda + 1$) be the foot of perpendicular from $(0,0,0)$ upon given line

$\Rightarrow \overrightarrow{OM} \cdot (2\hat{i} - 3j + 2\hat{k}) = 0$

$\Rightarrow 2(2\lambda + 1) - 3(-3\lambda + 1) + 2(2\lambda + 1) = 0$

$\Rightarrow \lambda = -\frac{1}{17}$

So $(a, b, c) \equiv \left(\frac{-2}{17} + 1, \frac{3}{17} + 1, \frac{-2}{17} + 1\right)$

$\Rightarrow 34(a + b + c) = 100$

66. (b) Denominator $= 0$ at $x = 2$

$\Rightarrow$ Numerator $= 0$ at $x = 2$

$\Rightarrow 2^3 - 5(2)^2 + a(2) + b = 0$

$\Rightarrow 2a + b = 12 \ldots (1)$

$$\therefore m = \mathrm{Lim}_{x\to 2} \frac{\frac{\sin(x^3 - 5x^2 + ax + b)}{(x^3 - 5x^2 + ax + b)} \cdot (x^3 - 5x^2 + ax + b)}{\left(\frac{x - 1 - 1}{\sqrt{x-1} + 1}\right) \cdot \frac{\log_a(1 + (x-2))}{(x-2)} \cdot (x-2)}$$

$\Rightarrow m = \mathrm{Lim}_{x\to 2} 2\left(\frac{x^3 - 5x^2 + ax + b}{(x-2)^2}\right)$

$\Rightarrow m = \mathrm{Lim}_{x\to 2} 2\left(\frac{3x^2 - 10x + a}{2(x-2)}\right)$

$\because$ Denominator $= 0$ at $x = 2$

$\Rightarrow$ Numerator $= 0$ at $x = 2$

$\Rightarrow 3(2)^2 - 10(2) + a = 0 \Rightarrow a = 8$

put $a = 8$ in (1), we get $b = -4$

$\therefore m = \mathrm{Lim}_{x\to 2} 2\left(\frac{6x - 10}{2}\right) \Rightarrow m = 2$

$\therefore a + b + m = 6$

67. (c) $\frac{dy}{dx} = y(2 + \ell nx)$

$\int \frac{dy}{y} = \int (2 + \ell nx)dx$

$\ell ny = 2x + x\ln x - x + C$

$\ell ny = x + x\ell nx + C$

Since it passes through $(1, e)$

$1 = 1 + 0 + C \Rightarrow C = 0$

$\ell ny = x + x\ell nx$

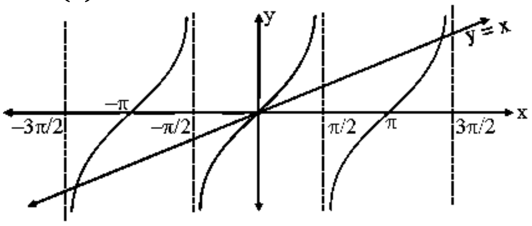
$f(x) = y = e^{x + x\ln x}$

$\Rightarrow f(e) = e^{ete} = e^{2e}$

68. (c) $\lim\limits_{x \to 0} \left|\frac{\sin x}{x}\right| = 1 = f(0) \to f(x)$ is continuous

Now $\frac{d}{dx}\left(\frac{\sin x}{x}\right) = \frac{x\cos x - \sin x}{x^2}$

$\Rightarrow f'(x) = 0 \Rightarrow \tan x = x$



3 solution in $(-2\pi, 2\pi)$ also $f'(0)$ does not exist at $x = -\pi, \pi$ total 5 points

69. (b) $\int_0^1 \frac{e^x+e^{-x}}{1} dx + \int_1^2 \frac{e^x+e^{-x}}{1} dx + \int_2^3 \frac{e^x+e^{-x}}{2} dx$

$\int_0^2 e^x dx + \int_0^2 e^{-x} dx + \frac{1}{2}\int_2^3 (e^x + e^{-x}) dx$

$(e^2 - 1) + \frac{(e^{-2} - 1)}{-1} + \frac{1}{2}\left((e^3 - e^2) - (e^{-3} - e^{-2})\right)$

$\frac{e^3 + e^2}{2} + 1 - 1 - \frac{e^{-2}}{2} - \frac{e^{-3}}{2}$

$\frac{e^3 + e^2 - e^{-2} - e^{-3}}{2}$

70. (d) $\int \frac{dy}{y+1} = -\int \frac{\cos x}{1+\sin x} dx$

$\Rightarrow \ln(y + 1) = -\ln(1 + \sin x) + C$

$\because y(0) = 0 \Rightarrow C = 0$

$\Rightarrow y + 1 = \frac{1}{1 + \sin x}$

passes through $\left(\alpha, -\frac{1}{2}\right)$

$\Rightarrow \sin x = 1$

$\Rightarrow x = \frac{\pi}{2}$ (as per options)

71. (4) For domain $\log_{0.6}\left|\frac{2x-5}{x^2-4}\right| \geq 0$

$\left|\frac{2x-5}{x^2-4}\right| \leq 1 \;\&\; x \neq \frac{5}{2}$ ...(1)

$-1 \leq \frac{2x - 5}{x^2 - 4} \leq 1$

$\frac{2x - 5}{x^2 - 4} + 1 \geq 0$

$\frac{x^2 + 2x - 9}{x^2 - 4} \geq 0$

$\frac{(x + 1)^2 - 10}{(x - 2)(x + 2)} \geq 0$

$$\begin{array}{ccccccccc} + & & - & & + & & - & & + \\ & -1-\sqrt{10} & & -2 & & 2 & & -1+\sqrt{10} & \end{array}$$

$x \in (-\infty, -1 - \sqrt{10}] \cup (-2, 2) \cup [-1 + \sqrt{10}, \infty)$ ...(2)

$\frac{2x - 5}{x^2 - 4} - 1 \leq 0$

$\frac{2x - 5 - x^2 + 4}{x^2 - 4} \leq 0$

$\frac{x^2 - 2x + 1}{x^2 - 4} \geq 0$

$$\frac{(x-1)^2}{(x-2)(x+2)} \geq 0$$

$x \in (-\infty, -2) \cup (2, \infty) \cup \{1\}$ ...(3)

$(1) \cap (2) \cap (3)$

$$x \in (-\infty, -1-\sqrt{10}] \cup \{1\} \cup \left[-1+\sqrt{10}, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$$

$a + b + c + d + e = -2 + 1 + 5 = 4$

72. (91) $a_1 + a_2 + \cdots \ldots + a_n = 6n^3$.

$a_1 + a_2 + \cdots \ldots + a_n + a_{n+1} = 6(n+1)^3$

$a_{n+1} = 6(n+1)^3 - 6n^3$

$= 6((n+1) - n)((n+1)^2 + n^2 + n(n+1))$

$a_{n+1} = 6(1)(3n^2 + 3n + 1)$

$a_n = 6(3(n-1)^2 + 3(n-1) + 1)$

$= 6(3(n^2 - 2n + 1 + n - 1) + 1)$

$= 6(3(n^2 - n) + 1)$

$a_n = 6(3n^2 - 3n + 1)$

$$\sum_{k=1}^{6} \left(\frac{6(3k^2 + 3k + 1) - 6(3k^2 - 3k + 1)}{36}\right)^2$$

$$\sum_{k=1}^{6} \left(\frac{36k}{36}\right)^2 = \sum_{k=1}^{6} k^2 = \frac{6 \times 7 \times 13}{6} = 91$$

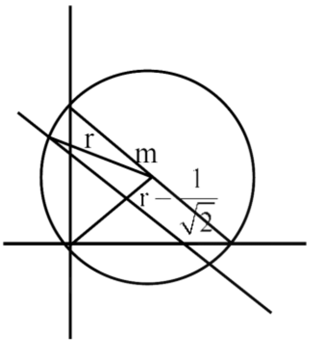
73. (81) $8\,b^2 - 4ac < 0 \Rightarrow 2\,b^2 < ac$

b	a	c		
1	1	3,4	→	2
1	2	2,3,4,	→	3
1	3	1,2,3,4	→	4
1	4	1,2,3,4	→	4
2	3	3,4	→	2
2	4	3,4	→	2
				17

Required probability $= \frac{17}{64} \equiv \frac{m}{n}$

$\therefore m + n = 17 + 64 = 81$

74. (8)



$$r^2 = \left(\frac{\sqrt{14}}{2}\right)^2 + \left(r - \frac{1}{\sqrt{2}}\right)^2$$

$$r^2 = \frac{7}{2} + r^2 - \sqrt{2}r + \frac{1}{2}$$

$$r^2 = \frac{7}{2} + r^2 - \sqrt{2}r + \frac{1}{2}$$

$\sqrt{2}r = 4$

$r = 2\sqrt{2}$

$r^2 = 8$

75. (192) As we know that

$$\int_a^b f(x)dx + \int_{f(a)}^{f(b)} f^{-1}(x)dx = bf(b) - af(a)$$

$\therefore \alpha = (2\sqrt{3})(4) - (0)(2)$

$= 8\sqrt{3}$

$\Rightarrow \alpha^2 = 192$