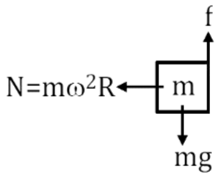
1. **(a)** Energy $= \frac{hc}{\lambda} = \frac{GM_1M_2}{r}$

$[ML^2\,T^{-2}] = [h][T^{-1}] = [G][M^2\,L^{-1}]$

$[G] = [M^{-2}\,L^{+1}\,T^{-1}\,h^{+1}]$

2. **(a)**



$N = m\omega^2 R$

$f_s = mg$

$f_s \le f_\ell$

$mg \le \mu N$

$mg \le \mu m\omega^2 R$

$\mu \ge \frac{g}{\omega^2 R}$

$\mu_{min} = \frac{g}{\omega^2 R} = \frac{10}{10^2 \times 1} = 0.1$

3. **(c)** $a = \frac{(m_2-m_1)}{m_1+m_2}\,g = \frac{(2-1)}{3} \times 10 = \frac{10}{3}\ m/s^2$

$a_{cm} = \frac{(1)(10/3) - (2)(10/3)}{3} = \frac{-10}{9}\ m/s^2(\downarrow)$

$S_{cm} = u_{cm}t + \frac{1}{2}a_{cm}t^2$

$= 0 + \frac{1}{2}\left(\frac{10}{9}\right)(4) = \frac{20}{9}$ m (downward)

$S_{COM} = \frac{20}{9}$ (Downward) $= 2.22$

4. **(a)** $\vec{F} = q\vec{E} + q(\vec{V} \times \vec{B})$

$(4\hat{\imath} + 2\hat{\jmath}) \times 10^{-10}$

$= 10^{-9}\left[0.4\hat{\jmath} + \left(v_x\hat{\imath} + v_y\hat{\jmath}\right) \times (4 \times 10^{-3})\hat{k}\right]$

$(4\hat{\imath} + 2\hat{\jmath}) \times 10^{-10}$

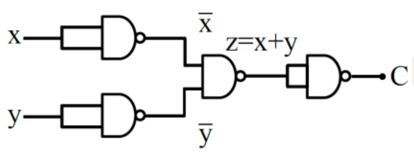
$= 10^{-9}\left(0.4\hat{\jmath} - 4 \times 10^{-3}v_x\hat{\jmath} + 4 \times 10^{-3}v_y\hat{\imath}\right)$

On comparing

$v_y = 100, v_x = 50$

$\hat{V} = 50\hat{\imath} + 100\hat{\jmath}$

5. **(d)**



$C = \overline{x + y}$

6. **(a)** M.E.C

$K_1 + U_i = K_r + U_r$

$\Rightarrow 0 - \frac{GM_e\,m}{\infty} = \frac{1}{2}mv^2 - \frac{GMm}{Re}$

$V = \sqrt{\frac{2GM}{Re}} = \sqrt{2gRe} = \sqrt{2 \times 9.8 \times 6400 \times 1000}$

$= 11.2$ km/s

Kinetic energy $= \frac{1}{2}mv^2$

$= 6.27 \times 10^7$ J

7. **(a)** Least count : $\frac{0.1}{100}$ mm $= 0.001$ mm

Reading = MSD + (CSD) × LC - zero error

$= 5\text{ mm} + 50 \times 0.001 - 5 \times 0.001 = 5.045\text{ mm}$

8. **(c)** $W = \Delta U = 2T[4\pi R_2^2 - 4\pi R_1^2]$

$W = 2 \times 3 \times 10^{-2} \times 4\pi[9 \times 10^{-4} - 1 \times 10^{-4}]$

$W = 1.92\pi \times 10^{-4} J$

$\alpha = 1.92$

9. **(c)** First, we determine the total number of moles (n_{total}) in the mixture using the Ideal Gas Law: PV = nRT.

Given the values:

- Pressure (P): $100\text{kPa} = 10^5$ Pa
- Volume (V): $8310\text{ cm3} = 8.31 \times 10^{-3}\text{ m}^3$
- Temperature (T): 300 K
- Gas Constant (R): $8.31\text{ J/mol} \cdot \text{K}$

$\frac{PV}{RT} = \frac{10^5 \times 8.31 \times 10^{-3}}{8.31 \times 300} = \frac{100}{300} \approx 0.333$ mole

1. Mole equation: $n_1 + n_2 = 0.333$

2. Mass equation: $44n_1 + 32n_2 = 13.2$ (where 44 and 32 are the molar masses of CO_2 and O_2 respectively)

3. Solve for individual moles

Using the first equation to express n_2 as $0.333 - n_1$, we substitute this into the mass equation:

$44n_1 + 32(0.333 - n_1) = 13.2$

$44n_1 + 10.667 - 32n_1 = 13.2$

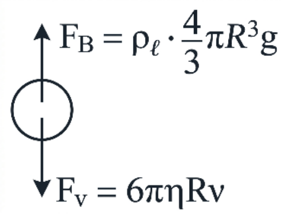
$12n_1 = 2.533$

$n_1 \approx 0.211$ mol

Then, calculate n_2 :

$n_2 = 0.333 - 0.211 = 0.122$ mol

10. **(b)**



For constant speed

$F_B = F_v$

$\Rightarrow \rho_\ell \frac{4}{3}\pi R^3 g = 6\pi\eta Rv$

$\frac{2}{9}\frac{\rho_\ell R^2 g}{V} = \eta$

$\Rightarrow \frac{2}{9} \times \frac{2000 \times 10^{-6} \times 10}{5 \times 10^{-3}} = \eta$

$\Rightarrow \eta = \frac{8}{9} \times 10$

$\Rightarrow \eta = \frac{80}{9}$ Poise $= 8.88$

11. **(b)** $(\text{W.D.})_g + (\text{W.D.})_{sand} = \Delta K$

$2 \times 10 \times \left(10 + \frac{10}{100}\right) - F_{sand} \times \left(\frac{10}{100}\right) = 0 = 0$

$F_{sand} = 2(10 + 0.1) \times 100 = 2020$

12. **(b)** $\vec{E} = \vec{B} \times \vec{C}$

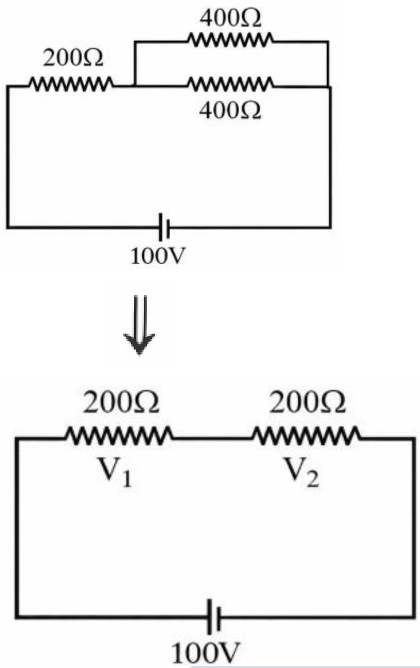
$= 2 \times 10^{-7}\hat{j} \times 3 \times 10^8\hat{i}$

$\vec{E} = -60\hat{k}$ V/m

13. (b) For resistance of bulb

$$R=\frac{V^2}{P}=\frac{200\times 200}{100}=400\Omega$$

Now circuit will look like this

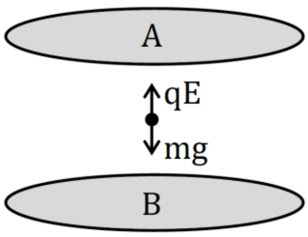


V_1 & V_2 will be same as resistances are same

$V_1=V_2=50\text{ V}$

$V_{bulb}=V_2=50\text{ V}$

14. (a)



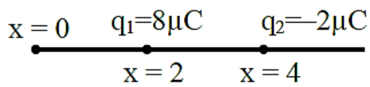
$$\frac{\Delta V}{(12/\pi)\times 10^{-2}}\times 5\times 10^{-9}=\frac{4}{3}\pi(r^3)\rho(g)$$

$\Delta V\times 5\times 10^{-7}=16(r^3)\rho(g)$

$\Delta V=480,\ V_A<V_B$

Plate A will be lower potential.

15. (c)



For sphere with radius 3 cm and center origin

$$\phi_1=\frac{q_{en}}{\varepsilon_0}=\frac{8\mu C}{\varepsilon_0}$$

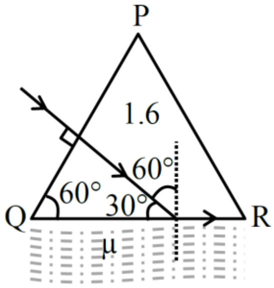
for sphere of radius 5 cm

$$\phi_2=\frac{q_{en}}{\varepsilon_0}=\frac{(8-2)\mu C}{\varepsilon_0}=\frac{6\mu C}{\varepsilon_0}$$

$$\frac{\phi_1}{\phi_2}=\frac{8}{6}=\frac{4}{3}=1.33$$

$$\frac{\phi_1}{\phi_2}=1.33$$

16. (d)



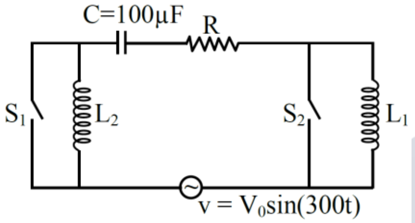
For surface QR

$i = 60^\circ$

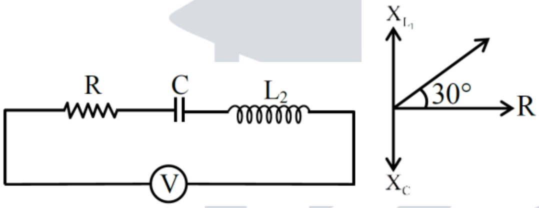
$1.6 \sin 60^\circ = \mu \sin 90^\circ$

$\mu = 1.6 \times \frac{\sqrt{3}}{2} = \frac{4}{5} \times \sqrt{3}$

17. (b)



For S_1 is closed and S_2 open

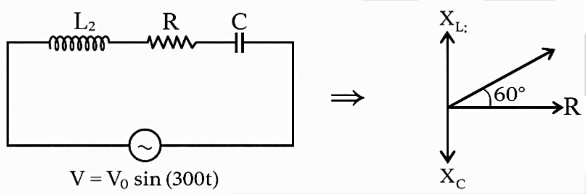


$V = V_0 \sin(300t)$

$\frac{X_{L_1} - X_C}{R} = \tan(30^\circ) = \frac{1}{\sqrt{3}}$

$\sqrt{3}(X_{L_1} - X_C) = R \qquad \ldots(1)$

For S_1 is open and S_2 is closed.



$\frac{|X_{L_2} - X_C|}{R} = \tan 60^\circ = \sqrt{3}$

$\frac{|X_{L_2} - X_C|}{\sqrt{3}} = R \qquad \ldots(2)$

from Eq. (1) and (2)

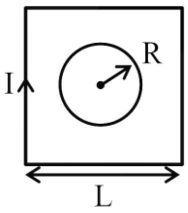
$\sqrt{3}(X_{L_1} - X_C) = \frac{(X_{L_2} - X_C)}{\sqrt{3}}$

$3\left(\omega L_1 - \frac{1}{\omega C}\right) = \left(\omega L_2 - \frac{1}{\omega C}\right)$

$|3L_1 - L_2| = \frac{3}{\omega^2 C} - \frac{1}{\omega^2 C} = \frac{2}{9 \times 10^4 \times 100 \times 10^{-6}}$

$|3L_1 - L_2] = \frac{2}{9}$

18. (d)



Magnetic field due to square

$B = 4 \times \frac{\mu_0 I}{4\pi \frac{L}{2}} \{2\sin 45°\}$

$= \frac{4\mu_0 I}{\pi L} \times \frac{1}{\sqrt{2}} = \frac{2\sqrt{2}\mu_0 I}{\pi L}$

$Q = B \times A_{circle}$

$Q = \frac{2\sqrt{2}\mu_0 I}{\pi L} \times \pi R^2$

$Q = \frac{2\sqrt{2}\mu_0 R^2}{L} I$

Inductance $= \frac{2\sqrt{2}\mu_0 R^2}{L}$

19. (b) $83\ {}_1P^1 + 126\ {}_0n^1 \rightarrow {}_{83}Bi^{209}$

$\Delta m = 83\ m_p + 126\ m_n - M_{Bi}$

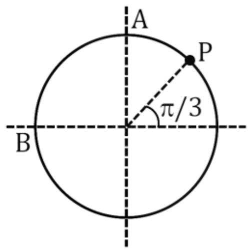
$= 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$

$= 1.760877u$

$BE = 1.760877 \times 931 MeV = 1639.3764$

$\frac{BE}{A} = \frac{1639.3764}{209} MeV = 7.8439 MeV$

20. (a) Phase of the motion of particle is as follows :



$V = 0$, when particle cross A .

$\therefore t_1 = \frac{\pi}{6\omega} = \frac{\pi}{300}$

$a = 0$, when particle cross B .

$\therefore t_2 = \frac{2\pi}{3\omega} = \frac{\pi}{75}$

21. (6) $I_r = I_{max} \cos^2 \frac{\phi}{2}$

$\frac{3I_{max}}{4} = I_{max} \cos^2 \frac{\phi}{2}$

$\cos \frac{\phi}{2} = \frac{\sqrt{3}}{2}$

$\frac{\phi}{2} = 30° = \frac{\pi}{6}$

$\left(\phi = \frac{\pi}{3}\right)$

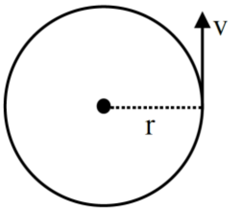
Path difference $\Delta x = \frac{\phi}{2\pi} \times \lambda$

$\Rightarrow \frac{\pi}{3 \times 2\pi} \times \lambda$

$\Delta x = \frac{\lambda}{6}$

$x = 6$

22. (64)



$B = \frac{\mu_0 I}{2\pi r}$

$I = \frac{ev}{2\pi r}$

$B \propto \frac{V}{r^2}$

$V \propto \frac{Z}{n}$

$r \propto \frac{n^n}{Z}$

$B \propto \frac{\left(\frac{Z}{n}\right)}{\left(\frac{n^2}{Z}\right)^2}$

$B \propto \frac{Z^3}{n^5}$

$\frac{B_1}{B_2} = \frac{n_2^5}{n_1^5} = \frac{4^5}{2^5} = 64$

23. (300) $C_P - C_v = R \Rightarrow C_v = (C_p - R)$

$C_v = 6 \text{cal/mol}^\circ\text{C}$

$\Delta U = nC_v\Delta T$

$= 5 \times 6 \times 10 = 300 \text{cal}$

24. (20) To burn in minimum time distance of paper must be at focus.

So $f = 30$ cm

$\frac{1}{30} = (\mu - 1)\left(\frac{1}{60} - \frac{1}{-60}\right)$

$\frac{1}{30} = \frac{\mu - 1}{30}$

$\mu = 2$

$\alpha = 20$

25. (15) $I_{Rod} = I_{SS}$

$\frac{M_1 L^2}{3} = \left(\frac{2}{5} m_2 R^2 + m_2 L^2\right)$

$\frac{40 \times (3)^2}{3} = \frac{2}{5} \times 10 \times R^2 + 10 \times (3)^2$

$120 = 4R^2 + 90$

$4R^2 = 30$

$R = \sqrt{\frac{15}{2}} = \sqrt{\frac{\alpha}{2}}$

$\alpha = 15$

26. **(b)** Molecular formula of hydrocarbon is C_2H_2

Moles of $C_2H_2 = \frac{3.38}{26}$

$C_2H_2(g) + \frac{5}{2}O_2(g) \rightarrow 2CO_2(g) + H_2O(l)$

Moles of CO_2 produced $= \left(\frac{3.38}{26} \times 2\right)$

Mass of CO_2 produced $= \left(\frac{3.38}{26} \times 2\right) \times 44 = 11.44$

27. **(c)** $\underset{0.1M}{HCl} \rightarrow \underset{0.1\,m}{\underline{H^+}} + \underset{0.1\,m}{\underline{Cl^-}}$

$\underset{0.1-x}{H_2A} \rightleftharpoons \underset{X+0.1}{H^+} + \underset{X}{HA^-}$

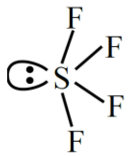
(Due to common ion effect x is very less)

$Ka_1 = \frac{[H^+][HA^-]}{[H_2A]}$

$Ka_1 = \frac{0.1 \times [HA^-]}{0.1}$

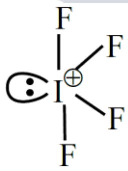
$[HA^-] = Ka_1 = 8.1 \times 10^{-8}$

28. **(b)** Species Structure / Shape



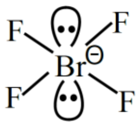
$\rightarrow$ see-saw

SF_4



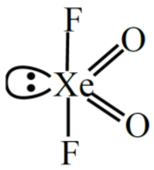
$\rightarrow$ see-saw

$IF_4^{\oplus}$



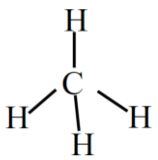
$\rightarrow$ square planar

$BrF_4^{\ominus}$



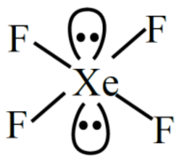
$\rightarrow$ see-saw

XeO_2F_2



$\rightarrow$ Tetrahedral

CH_4



$\rightarrow$ square planar

XeF_4

29. **(d)** For process $X \rightarrow Y$

$w = -18$ J

$q = +10$ J

$\Delta U = q + w$

$\Delta U_{X\rightarrow Y} = 10 + (-18) = -8$ J

For reverse process :

$\Delta U_{Y\to X} = 8\ J$
$q = -6\ J$
$\Delta U = q + w$
$8 = -6 + w$
$w = 14\ J$

30. **(a)** $'x' = \frac{1}{50,000\times0.5}\times R\times 300 = 9.96\times10^{-4}$ bar

$y' = \frac{2}{50,000\times1}\times R\times300 = 9.96\times10^{-4}$ bar $'z' = \frac{'x'\times\frac{1}{2}+'y'\times1}{\frac{3}{2}} = 9.96\times10^{-4}$ bar

31. **(b)** For 0.2M, 20mlHX solution,

pH of weak acid $= \frac{1}{2}[pk_a - \log c]$
$= \frac{1}{2}[4 - \log 5 - \log 0.2]$
$= \frac{1}{2}[4 - \log 5 + \log 5]$
$= 2$

$HX + + \underset{}{\overset{(LR)}{NaOH}} \longrightarrow NaA + H_2O$

20ml, 0.2M   10ml, 0.2M
4 m mole   2 m mole
2 m mole –   2 m mole
[HX + NaA] ⇒ acidic buffer

$pH = pK_a + \log\frac{salt}{acid}$
$pH = 3.3 + \log\frac{2}{2}$
$pH = 3.3$

32. **(a)** Rate of constant (k) $= 10^{-3}M^{-1}sec^{-1}$
$\because 1 - n = -1$ [unit of $k = M^{1-n}sec^{-1}$]
$n = 2 \Rightarrow 2^{nd}$ order reaction
rate $= k[A]^2$
(A) If concentration of A is 4 times then reaction will become 16 times.
(B) Order of reaction is 2
(C) Half-life of the reaction is independent of the concentration of the reactant for 1^{st} order, not for 2^{nd} order reaction.
(D) Decomposition of N_2O_5 is example of 1^{st} order reaction
(E) Graph of $\ell\frac{[R]_0}{[R]_t}$ vs t is straight line for 1^{st} order not for 2^{nd} order reaction.
So, statement (A) & (B) are correct

33. **(c)** Basic Oxides : Na_2O, K_2O
Acidic Oxides : Cl_2O_7
Neutral Oxides : N_2O, NO & CO
Amphoteric Oxides : Al_2O_3, As_2O_3

34. **(b)** Order of II ionization energy for boron family elements is :
$\underset{(2427KJ/mol)}{B} > \underset{(1979KJ/mol)}{Ga} > \underset{(1816KJ/mol)}{Al}$
Order of I ionization energy for carbon family elements is :
$\underset{(786KJ/mol)}{Si} > \underset{(761KJ/mol)}{Ge} > \underset{(715KJ/mol)}{Pb} > \underset{(708KJ/mol)}{Sn}$

35. **(a)**

Statement I	**Statement II**
Sc	2393

Mn 3260

Cu 3556

Zn 3837

Sc < Mn < Cu < Zn ($3^{rd} IE$)

CFSE $\Rightarrow$ SFL $>$ WFL

($en > H_2O$)

CFSE $\propto$ Charge on metal

$Co^{+2} < Co^{+3}$

$[Co(H_2O)_6]^{2+} < [Co(H_2O)_6]^{3+} < [Co(en)_3]^{3+}$

36. **(a,b,d)** Compound, $[Ag(NH_3)_2][Ag(CN)_2]$ cannot show coordination isomerism. All other complexes (B, C , D and E) will show co-ordination isomerism.

37. **(a)** Mass of carbon $= \frac{0.25}{44} \times 12$

Mass% of carbon $= \frac{\text{Mass of carbon}}{x} \times 100x = \frac{\frac{0.25\times12}{44}\times100}{25}$

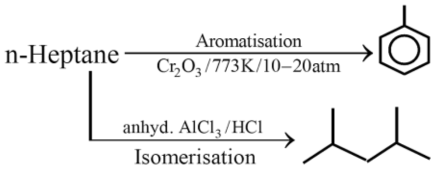
$x = \frac{12}{44}$

$x = 0.2727 = 272.7 \times 10^{-3}$

$\simeq 273 \times 10^{-3}$

38. **(a)**

39. **(c)**



40. **(c)** Ar-Cl and R-Cl shows different chemical properties

Rate of $S_N{}^1 \propto$ stability of first formed carbocation

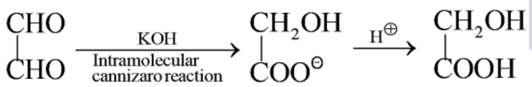
Water is more polar than alcohol

Vinyl alcohol, allyl alcohol, both are alkene unsaturated alcohols

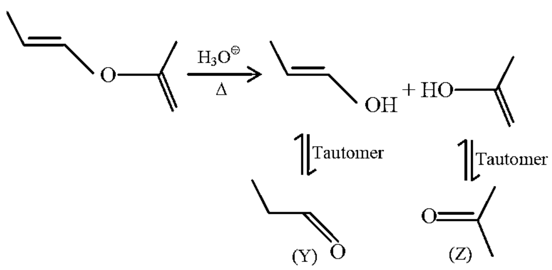
$R-OH \xrightarrow{SOCl_2} R-Cl$

Phenol does not give $Ph-Cl$ with $SOCl_2$.

41. **(c)**



42. **(d)**

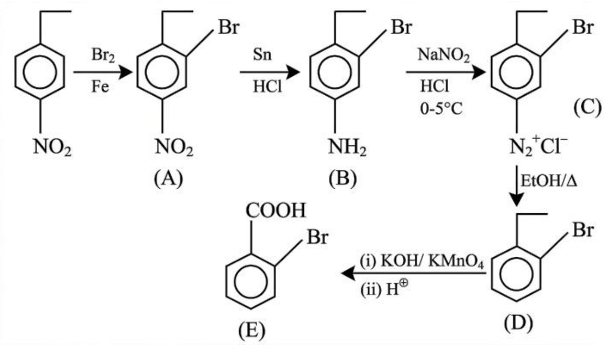


Y will give positive fehling's test

Molar mass of $[Z](C_3H_6O) = 58$

Molar mass of $[Y](C_3H_6O) = 58$

43. **(b)**



44. (c) Thyroxine is derivative of Tyrosine and single letter symbol of Tyrosine amino acid is Y .

45. (d) Zn^{2+} does not show Borax Bead Test.

Among remaining ions; Fe^{3+} has maximum ionization energy.

46. (5) for photoelectric effect :

$\frac{hc}{\lambda} = W + K.E.$

$\frac{6.6\times10^{-34}\times3\times10^{8}}{x} = 2.3 \times 1.6 \times 10^{-19} + 2.8 \times 10^{-20}$

$\frac{19.8\times10^{-26}}{x} = 3.96 \times 10^{-19}$

$x = 5 \times 10^{-7}$

$x = 500 \times 10^{-9}$ m

$x = 5 \times 10^{2}$ nm

47. (20) $2\,A(g) \rightarrow 4\,B(g) + C(g)$

$t = 30$ min $p^\circ - 2x\ 4x$

$t = \infty\ -\ 2p^\circ\ p^\circ/2$

$p_\infty = 2p^\circ + p^\circ/2 = 600$

$5p^\circ/2 = 600$

$P^\circ = 240$ mmHg

Att $= 30$ min, $p = 300 = p^\circ - 2x + 4x + x$

$300 = 240 + 3x$

$3x = 60$

$x = 20$

Pressure of $C(g) = 20$ mmHg

48. (560) $E^o_{cell} = [1.23 - 0.02] = 1.21$ V

$\Delta G^\circ = -6 \times 96500 \times 1.21$ J

$W = \frac{80}{100} \times \Delta G^\circ = -P\Delta V$

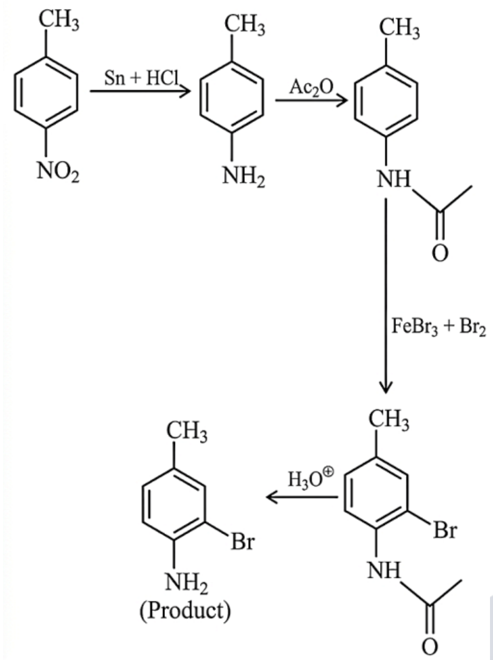
$0.8 \times 6 \times 96500 \times 1.21 = 1 \times 10^{3}(\Delta V)$

$\Delta V = 560.472\ m^3$

49. (3) Paramagnetic : $Mn^{+2}, Cu^{+2}, Gd^{+3}$

Diamagnetic: $Zn^{+2}, Yb^{+2}, Sc^{+3}, La^{+3}, Lu^{+3}, Ti^{+4}, Ce^{+4}$

50. (1)



Molar mass of product $= 186$

Mass of AgBr $= \frac{1}{186} \times 188 \approx 1$

51. (d) $\alpha + \beta = 3$

$\frac{\alpha}{2} + 2\beta = -3$

On solving, we get $\alpha = 6; \beta = -3$

Product of roots $= \alpha\beta = \mathrm{r} \Rightarrow \mathrm{r} = -18$

Now for $\mathrm{x}^2 + 6\mathrm{x} - \mathrm{m} = 0$

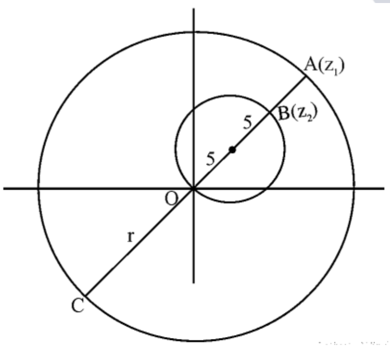
Product of roots $= -\mathrm{m}$

$= (2\alpha + \beta + 2\mathrm{r})\left(\alpha - 2\beta - \frac{\mathrm{r}}{2}\right)$

$\Rightarrow -\mathrm{m} = (-27)(21)$

$\Rightarrow \mathrm{m} = 567$

52. (c)



$|\mathrm{z}| = \mathrm{r}$

$\mathrm{OA} = \mathrm{r}$

minimum $|\mathrm{z}_1 - \mathrm{z}_2| = 2$

$\Rightarrow \mathrm{r} - 10 = 2 \Rightarrow \mathrm{r} = 12$

maximum value of CB

$|\mathrm{z}_1 - \mathrm{z}_2| = \mathrm{r} + 10 = 12 + 10 = 22$

53. (b) $D=\begin{vmatrix}1 & 5 & 6\\ 2 & 3 & 4\\ 1 & 6 & a\end{vmatrix}=0$

$\Rightarrow a=\frac{50}{7}$

$D_z=0\Rightarrow\begin{vmatrix}1 & 5 & 4\\ 2 & 3 & 7\\ 1 & 6 & b\end{vmatrix}=0$

$\Rightarrow b=\frac{29}{7}$

$\therefore (a,b)$ lies on $x-y=3$

54. (d) A.P. : $1, a_2, a_3, \ldots$

G.P. : $1,\ g_2,\ g_3, \ldots$

$a_2+g_2=1\Rightarrow 1+d+r=1\Rightarrow\ d+r=0$

$1+2\,d+r^2=4\Rightarrow r^2-2r-3=0$

$\Rightarrow r=3,-1$ (reject)

$\therefore d=-3$

$\therefore a_{10}=1+9d=1+9(-3)=-26$

$g_5=1.r^4=3^4=81$

$\therefore a_{10}+g_5=55$

55. (b) $T_r=\frac{1^3+2^3+3^3+\cdots\ldots r^3}{1+3+5+\cdots\ldots+(2r-1)}=\frac{\left(\frac{r(r+1)}{2}\right)^2}{r^2}$

$=\frac{r^2+2r+1}{4}$

$S_n=\sum_{r=1}^{n}T_r$

$S_n=\frac{1}{4}\sum(r^2+2r+1)$

$=\frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6}+2\frac{n(n+1)}{2}+n\right]$

$S_8=\frac{1}{4}\left[\frac{8\times 9\times 17}{6}+8\times 9+8\right]$

$=\frac{1}{4}[204+72+8]=71$

56. (c) ${}^{3}C_3\cdot{}^{30}C_{30-r}+{}^{3}C_2\cdot{}^{30}C_{31-r}+{}^{3}C_1\cdot{}^{30}C_{32-r}+{}^{3}C_0\cdot{}^{30}C_{33-r}$

$={}^{33}C_{33-r}\equiv{}^{m}C_r\equiv{}^{m}C_{m-r}$

$\Rightarrow m=33$

57. (c) $P_n={}^{n}C_3$

$P_{n+1}-P_n=66$

${}^{n+1}C_3-{}^{n}C_3=66$

$\Rightarrow\frac{(n+1)n(n-1)}{6}-\frac{n(n-1)(n-2)}{6}=66$

$\Rightarrow\frac{n(n-1)}{6}[n+1-n+2]=66$

$\Rightarrow n(n-1)=132$

$n=12$

Prime divisors of 12 are 2 & 3

sum $=2+3=5$

58. (c) $P=(6\text{ tail })+(4\text{ tail }+1\text{ head })+(2\text{ tail }+2\text{ head })+(3\text{ head })$

$P=\left(\frac{1}{2}\right)^6+\frac{5!}{1!\,4!}\left(\frac{1}{2}\right)^5+\frac{4!}{2!\,2!}\left(\frac{1}{2}\right)^4+\left(\frac{1}{2}\right)^3$

$P = \frac{1}{64} + \frac{5}{32} + \frac{3}{8} + \frac{1}{8}$

$P = \frac{1 + 10 + 24 + 8}{64} = \frac{43}{64} = \frac{m}{n}$

$m + n = 107$

59. (d) $x_1 + x_2 + \cdots + x_{n-1} + x_n = 8n$

$48 + x_n = 8n$

$\Rightarrow x_n = 8n - 48 \qquad \ldots(1)$

$\Rightarrow 16 = \frac{496 + x_n^2}{n} - (8)^2$

$\Rightarrow 80n = 496 + x_n^2$

$\Rightarrow x_n^2 = 80n - 496 \qquad \ldots(2)$

$\Rightarrow (8n - 48)^2 = 80n - 496$

$\Rightarrow 64(n - 6)^2 = 8(10n - 62)$

$\Rightarrow 8(n - 6)^2 = 10n - 62$

$\Rightarrow 4(n - 6)^2 = 5n - 31$

$\Rightarrow 4(n^2 - 36 - 12n) = 5n - 31$

$\Rightarrow 4n^2 + 144 - 48n = 5n - 31$

$\Rightarrow 4n^2 - 53n + 175 = 0$

$\Rightarrow 4n^2 - 28n - 25n + 175 = 0$

$\Rightarrow 4n(n - 7) - 25(n - 7) = 0$

$n = 7$

60. (c) $x + (k - 1)y + 3 = 0$

$2x + k^2y - 4 = 0$

$\left(\frac{1}{1-k}\right)\left(\frac{2}{k^2}\right) = 1$

$2 = k^2 - k^3$

$k^3 - k^2 + 2 = 0$

$k = -1$

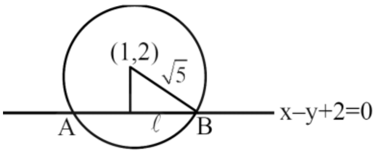
Solving :

$2(x - 2y + 3) = 0$

$2x + y - 4 = 0$

$-5y + 10 = 0$

$y = 2$



$x = 1$

Centre (1,2)

$r = \sqrt{5}$

So circle is $(x - 1)^2 + (y - 2)^2 = 5$

Chord $x - y + 2 = 0$

$p = \frac{1}{\sqrt{2}}$

$\ell = \sqrt{5 - \frac{1}{2}} = \frac{3}{\sqrt{2}}$

$AB = \frac{6}{\sqrt{2}} \therefore AB^2 = 18$

61. (c) Equation of chord whose midpoint is $\left(\frac{1}{2},\frac{-1}{2}\right)$

$\frac{x}{x_1}+\frac{y}{y_1}=2$

$2x-2y=2$

$x-y=1$

solving with $xy=12$

$P(-3,-4), Q(4,3)$

Area $(\triangle OPQ)=\frac{1}{2}|-3\times 3-(-4)(4)|=\frac{7}{2}$

62. (b) parabola passes through $(1,-1)$ so

$-1=1+p+q$

$p+q=-2$

$q=-2-p \qquad \ldots(1)$

Distance from x -axis

$\frac{-D}{4a}=\frac{-(p^2-4q)}{4(1)}=\frac{4q-p^2}{4}$

$=\frac{4(-2-p)-p^2}{4}$ from (1)

$=\frac{-8-4p-p^2}{4}=\frac{-(p^2+4p+8)}{4}=\frac{-((p+2)^2+4)}{4}$

Minimum at $p=-2 \Rightarrow q=0$

$p^2+q^2=4+0=4$

63. (a) $2(\cos^8\theta-\sin^8\theta)\sec 2\theta=a^2$

$2(\cos^4\theta+\sin^4\theta)(\cos^2\theta+\sin^2\theta)(\cos^2\theta-\sin^2\theta)\sec 2\theta=a^2$

$2(\cos^4\theta+\sin^4\theta)=a^2$

$2(1-2\sin^2\theta\cos^2\theta)=a^2$

$2\left(1-\frac{\sin^2 2\theta}{2}\right)=a^2, a\in$

$a^2=2-\sin^2 2\theta\in[1,2]$

$a^2=1$ at $\sin^2 2\theta=1$

$2\theta=(2n+1)\frac{\pi}{2}$

$\theta=(2n+1)\frac{\pi}{4}\notin P$

$n(S)=0$

64. (b) $\vec{c}=\lambda\vec{a}+\mu\vec{b}$

$\vec{c}=(\mu-\lambda)\hat{i}+(\lambda+3\mu)\hat{j}+(3\lambda+\mu)\hat{k}$

$\vec{c}\cdot(3\hat{i}-6\hat{j}+2\hat{k})=10$

$3(\mu-\lambda)-6(\lambda+3\mu)+2(3\lambda+\mu)=10$

$-3\lambda-13\mu=10 \quad \ldots\ldots(1)$

$\vec{c}\cdot(\hat{i}+\hat{j}+\hat{k})=-2$

$5\mu+3\lambda=-2 \quad \ldots\ldots(2)$

Solving (1) & (2)

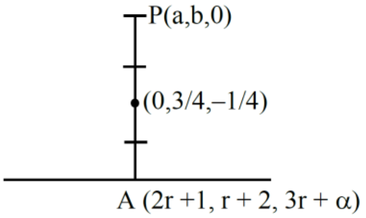
$\lambda=1, \mu=-1$

$\vec{c}=-2\hat{i}-2\hat{j}+2\hat{k}$

$|\vec{c}|^2=12$

65. (a) $\frac{2r+1+a}{2}=0\Rightarrow 2r+a=-1$

$\frac{r+2+b}{2}=\frac{3}{4}\Rightarrow r+b=-\frac{1}{2}$



$$\frac{3r+\alpha}{2}=\frac{-1}{4}\Rightarrow 3r+\alpha=\frac{-1}{2}$$

$$2\cdot a+1\cdot(b-3/4)+3\cdot\frac{1}{4}=0\Rightarrow a+b=0$$

$$\Rightarrow 2(-1-2r)+(-r-1/2)=0$$

$$\Rightarrow -3/2=3r=1\Rightarrow r=-1/2, a=0, b=0, \alpha=1$$

$$\therefore a^2+b^2+\alpha^2=1$$

66. (b) Let angle between $\overrightarrow{PQ}$ & $\overrightarrow{PS}$ is θ

$$\cos\theta=\frac{\overrightarrow{PQ}\cdot\overrightarrow{PS}}{|\overrightarrow{PQ}||\overrightarrow{PS}|}$$

$$\cos\theta=\frac{0-1+0}{\sqrt{2}\sqrt{2}}$$

$$\cos\theta=\frac{-1}{2}$$

$$\alpha=\frac{2\pi}{3}-\frac{\pi}{2}\text{ So }\alpha=\frac{\pi}{6}$$

$$\sin^2\frac{5\alpha}{2}-\sin^2\frac{\alpha}{2}=\sin\left(\frac{5\alpha}{2}+\frac{\alpha}{2}\right)\sin\left(\frac{5\alpha}{2}-\frac{\alpha}{2}\right)$$

$$=\sin\frac{6\alpha}{2}\times\sin\frac{4\alpha}{2}$$

$$=\sin3\alpha\times\sin2\alpha$$

$$=\sin\frac{\pi}{2}\times\sin\frac{\pi}{3}$$

$$=1\times\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{2}$$

67. (c) $I=\int_0^{20\pi}(1-2\sin^2x\cos^2x)\cdot dx$

$$=20\pi-\frac{1}{2}\int_0^{20\pi}\sin^2 2x dx$$

$$20\pi-\frac{1}{2}(40)\int_0^{\frac{\pi}{2}}\sin^2 2x$$

$$20\pi-\frac{20}{2}\left(x-\frac{\sin2x}{2}\right)_0^{\frac{\pi}{2}}$$

$$20\pi-10\left(\frac{\pi}{2}\right)=15\pi$$

68. (d) Given f(x) is a polynomial of degree 5

Also, $f'(1)=0; f'(-1)=0.$

Also $\because \lim_{x\to0}\frac{f(x)}{x^3}=-5$ (fixed and finite)

$\therefore f(0)=0; f'(0)=0; f''(0)=0$

$$\frac{f'''(0)}{6}=-5\Rightarrow f'''(0)=-30$$

Hence, let $f'(x)=(ax+b)(x-0)(x-1)(x+1)$

$\Rightarrow f'(x)=ax^4+bx^3-ax^2-bx$

$f''(x)=4ax^3+3bx^2-2ax-b$

$\because f'(0)=0\Rightarrow\ b=0$

$f'''(x)=12ax^2+6bx-2a$

$\because f'''(0) = -2a = -30$
$\Rightarrow a = 15$
$\therefore f'(x) = 15x.x.(x-1)(x+1)$
$\Rightarrow f'(x) = 15x^4 - 15x^2$
$\Rightarrow f(x) = 3x^5 - 5x^3 + C$
$\because f(0) = 0 \Rightarrow C = 0$
$\therefore f(x) = 3x^5 - 5x^3$
$\therefore f(2) - f(-2) = 112$

69. **(c)** $f(x) = \int \frac{8(2x+2)+8}{x^2+2x-15} dx$

$\Rightarrow f(x) = 8\ell n|x^2 + 2x - 15| + \ell n\left|\frac{x-3}{x+5}\right| + C$
$\Rightarrow f(4) = 14\ell n3 + C$
$\Rightarrow 14\ell n3 = 14\ell n3 + C$
$\therefore C = 0$
Now, $f(7) = 8\ell n48 - \ell n3$
$= \ell n\left(\frac{(48)^8}{3}\right)$
$\Rightarrow f(7) = \ln(2^{32} \times 3^7)$
$\alpha = 32, \beta = 7 \Rightarrow \alpha + \beta = 39$

70. **(b)** $2y(ydx - xdy) + x^2dy = 0$

$\Rightarrow -2yx^2 \, d\left(\frac{y}{x}\right) + x^2dy = 0$
$\Rightarrow -2yd\left(\frac{y}{x}\right) + dy = 0$
$\Rightarrow -2 \, d\left(\frac{y}{x}\right) + \frac{1}{y}dy = 0$
$\Rightarrow \frac{-2y}{x} + \log_e y = C$
Given $x(e) = e$
$\Rightarrow C = -1$
$\therefore \frac{-2y}{x} + \log_e y = -1$
$\Rightarrow \frac{2y}{x} - \log_e y = 1$
Put $y = e^2$, we get
$\Rightarrow \frac{2e^2}{x} - 2 = 1 \Rightarrow \frac{2e^2}{x} = 3 \Rightarrow x = \frac{2e^2}{3}$
$\Rightarrow x(e^2) = \frac{2e^2}{3}$

71. **(120)** For $x = 2, \ z = 2,4,6$

$x = 3, \quad z = 3, 6$
$x = 4, \quad z = 4$
$x = 5, \quad z = 5$
$x = 6, \quad z = 6$
Total case = 8
and no. of combination of y and w satisfying $y \le w$
$= 1 + 2 + 3 + 4 + 5 = 15$
No. of relations satisfying $(x, y)\, R(z, w)$ is $8 \times 15 = 120$.

72. **(34)** $A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 2 \\ -4 & 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}$

$\because PA = B$

$\Rightarrow P = BA^{-1}$

$= \frac{1}{2}\begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -21 & 12 \\ -7 & 4 \end{bmatrix}$

$\because AQ = B$

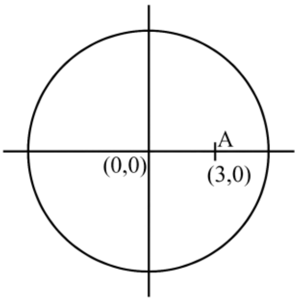
$\Rightarrow Q = A^{-1} B$

$= \frac{1}{2}\begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -2 & -6 \\ -5 & -15 \end{bmatrix}$

$\therefore |\mathrm{tr}(2(P+Q))| = |(-21) + (4) + (-2) + (-15)|$

$= |-34| = 34$

73. (18)



Let $B(h, k)$

Equation of circle with AB as diameter

$(x-h)(x-3) + (y-k)(y-0) = 0$

$x^2 + y^2 - (h+3)x - ky + 3h = 0$

Centre $\left(\frac{h+3}{2}, \frac{k}{2}\right)$

Circle touches internally

$\therefore C_1C_2 = |R - r|$

$\sqrt{\left(\frac{h+3}{2}\right)^2 + \left(\frac{k}{2}\right)^2} = \left|6 - \frac{1}{2}\left(\sqrt{(h-3)^2 + k^2}\right)\right|$

$\sqrt{(x+3)^2 + y^2} + \sqrt{(x-3)^2 + y^2} = 12$

$2a = 12 \;\therefore a = 6$

$(-3,0)$ and $(3,0)$ are foci

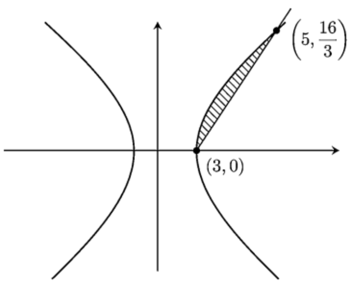
$2ae = 6$

$12e = 6$

$e = \frac{1}{2}$

$\therefore 72e^2 = 72\left(\frac{1}{4}\right) = 18$

74. (24)



$16x^2 - 9y^2 = 144$

$16x^2 - (8x-24)^2 = 144$

$16x^2 - 64(x-3)^2 = 144 \Rightarrow x^2 - 4(x-3)^2 = 9$

$3x^2 - 24x + 45 \Rightarrow x^2 - 8x + 15 = 0$

$x = 3,5$

Area $= \int_3^5 \sqrt{\frac{16x^2-144}{9}} - \frac{1}{2} \cdot 2 \cdot \frac{16}{3}$

$=\frac{4}{3}\int_{3}^{5}\sqrt{x^2-9}-\frac{16}{3}$

$=\frac{4}{3}\left(\frac{x}{2}\sqrt{x^2-9}-\frac{9}{2}\log_e\left(x+\sqrt{x^2-9}\right)\right)_3^5-\frac{16}{3}$

$=\frac{4}{3}\left(\frac{5}{2}\cdot 4-\frac{9}{2}\log_e 9-\frac{3}{2}\cdot 0+\frac{9}{2}\log_e 3\right)-\frac{16}{3}$

Area $=8-6\log_e 3=A$

$\therefore \mathrm{A}+6\ell\mathrm{n}3=8$

$\Rightarrow 3(\mathrm{A}+6\ell\mathrm{n}3)=24$

75. **(10)** In $\mathrm{x}\in[2,4]$, range of $\mathrm{x}^2-\mathrm{x}-\frac{1}{2}$ is [1.5, 11.5] f(x) is discontinuous at points where $\mathrm{x}^2-\mathrm{x}-\frac{1}{2}$ becomes integers.

$\therefore$ Integral values of $\mathrm{x}^2-\mathrm{x}-\frac{1}{2}$ are 2,3,4,5,6,7, 8,910,11

$\therefore$ Number of points where is discontinuous is 10