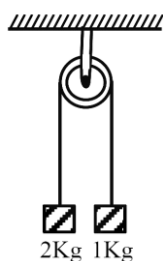
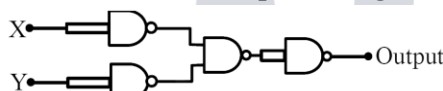


PHYSICS

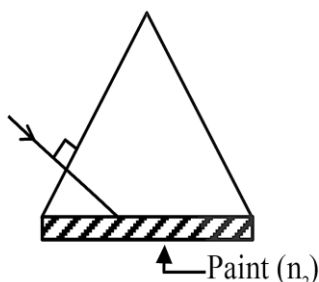
SECTION-A

1. Dimensions of universal gravitational constant (G) in terms of Planck's constant (h), distance (L), mass (M) and time (T) are ____ .
 (a) $[hTLM^{-2}]$ (b) $[hT^{-1}LM^{-2}]$
 (c) $[hTL^2M^{-2}]$ (d) $[h^{-1}T^{-1}LM^{-2}]$
2. A 0.5 kg mass is in contact against the inner wall of a cylindrical drum of radius 4 m rotating about its vertical axis. The minimum rotational speed of the drum to enable the mass to remain stuck to the wall (without falling) is 5 rad/s. The coefficient of friction between the drum's inner wall surface and mass is ____ . (Take $g = 10 \text{ m/s}^2$)
 (a) 0.1 (b) 0.5
 (c) 0.7 (d) 0.3
3. Two blocks of masses 2 kg and 1 kg respectively, are tied to the ends of a string which passes over a light frictionless pulley as shown in the figure below. The masses are held at rest at the same horizontal level and then released. The distance traversed by the centre of mass in 2 s is ____ m. (Take $g = 10 \text{ m/s}^2$)



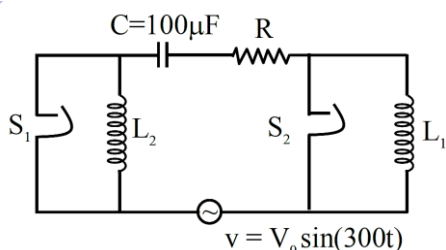
- (a) 3.33 (b) 3.12
 - (c) 2.22 (d) 1.42
4. A particle having charge 10^{-9} C moving in $x - y$ plane in fields of 0.4 N/C and $4 \times 10^{-3} \text{ kT}$ experiences a force of $(4\hat{i} + 2\hat{j}) \times 10^{-10} \text{ N}$. The velocity of the particle at that instant is ____ m/s.
 (a) $50\hat{i} + 100\hat{j}$ (b) $100\hat{i} + 50\hat{j}$
 (c) $-50\hat{i} + 100\hat{j}$ (d) $50\hat{i} - 100\hat{j}$
5. If X and Y are the inputs, the given circuit works as ____ .

 - (a) OR gate (b) AND gate
 - (c) NAND gate (d) NOR gate
6. If a body of mass 1 kg falls on the earth from infinity, it attains velocity (v) and kinetic energy (k) on reaching the surface of earth. The values of v and k respectively are ____ .
 (Take radius of earth to be 6400 km and $g = 9.8 \text{ m/s}^2$)
 (a) 11.2 km/s; $6.27 \times 10^7 \text{ J}$
 (b) 11.2 km/s; $12.54 \times 10^7 \text{ J}$
 (c) 8.8 km/s; $6.27 \times 10^7 \text{ J}$
 (d) 8.8 km/s; $12.54 \times 10^7 \text{ J}$
7. In a screw gauge the zero of main scale reference line coincides with the fifth division of the circular scale when two studs are in contact. There are 100 divisions in circular scale and pitch of screw gauge is 0.1 mm . When diameter of a sphere is measured, the reading of main scale is 5 mm and 50th division of circular scale coincides with the reference line of main scale. The diameter of sphere is ____ mm.
 (a) 5.045 (b) 5.055
 (c) 5.450 (d) 5.550
8. The surface tension of a soap bubble is 0.03 N/m . The work done in increasing the diameter of bubble from 2 cm to 6 cm is $\alpha\pi \times 10^{-4} \text{ J}$. The value of α is ____ . (Take $\pi = 3.14$)

- (a) 0.86 (b) 0.64
(c) 1.92 (d) 7.68
9. A mixture of carbon dioxide and oxygen has volume 8310 cm^3 , temperature 300 K , pressure 100 kPa and mass 13.2 g . The number of moles of carbon dioxide and oxygen gases in the mixture respectively are ____ . (Assume both carbon dioxide and oxygen gases behave like ideal gases) [$R = 8.31 \text{ J/mol.K}$]
(a) 0.15 and 0.18 (b) 0.25 and 0.08
(c) 0.21 and 0.12 (d) 0.13 and 0.20
10. If an air bubble of diameter 2 mm rises steadily through a liquid of density 2000 kg/m^3 at a rate of 0.5 cm/s , then the coefficient of viscosity of liquid is ____ Poise. (Take $g = 10 \text{ m/s}^2$)
(a) 0.88 (b) 8.8
(c) 88.8 (d) 0.088
11. A spherical ball of mass 2 kg falls from a height of 10 m and is brought to rest after penetrating 10 cm into sand. The average force exerted by sand on the ball is ____ N.
(a) 1980 (b) 2020
(c) 2000 (d) 1000
12. An electromagnetic wave travels in free space along the x -direction. At a particular point in space and time, $\vec{B} = 2 \times 10^{-7} \hat{j} \text{ T}$ is associated with this wave. The value of corresponding electric field $\vec{E}$ at this point is ____ V/m.
(a) $60 \hat{k}$ (b) $-60 \hat{k}$
(c) $30 \hat{k}$ (d) $-600 \hat{k}$
13. Two resistors of 200Ω and 400Ω are connected in series with a battery of 100 V . A bulb rated at 200 V , 100 W is connected across the 400Ω resistance. The potential drop across the bulb is ____ V.
(a) 25 (b) 50
(c) 66.6 (d) 100
14. Two metal plates (A, B) are kept horizontally with separation of $\left(\frac{12}{\pi}\right) \text{ cm}$, with plate A on the top. An atomizer jet sprays oil (density 1.5 g/cm^3) droplets of radius 1 mm horizontally. All oil droplets carry a charge 5 nC . The potentials V_A and V_B are required on plates A and B respectively in order to ensure the droplets do not descend. The values of V_A and V_B are ____ . (Neglect the air resistance to the droplets and take $g = 10 \text{ m/s}^2$)
(a) 100 V and 580 V (b) 580 V and 100 V
(c) 60 V and 400 V (d) 0 V and -200 V
15. Two-point charges $8 \mu\text{C}$ and $-2 \mu\text{C}$ are located at $x = 2 \text{ cm}$ and $x = 4 \text{ cm}$, respectively on the x -axis. The ratio of electric flux due to these charges through two spheres of radii 3 cm and 5 cm with their centers at the origin is ____ .
(a) 4:1 (b) 3:4
(c) 4:3 (d) 4:5
16. One side of an equilateral prism is painted by a transparent material of refractive index n_2 . The refractive index of prism is 1.6 . The minimum value of n_2 required for total internal reflection from painted face is ____ .



- (a) $\frac{3\sqrt{3}}{1.6}$ (b) $\sqrt{3}$
(c) $\frac{3.2}{\sqrt{3}}$ (d) $\frac{4\sqrt{3}}{5}$

17. The figure given below shows an LCR series circuit with two switches S_1 and S_2 . When switch S_1 is closed keeping S_2 open, the phase difference (ϕ) between the current and source voltage is 30° and phase difference is 60° when S_2 is closed keeping S_1 open. The value of $(3L_1 - L_2)$ is ____ H.



- (a) $\frac{9}{2}$ (b) $\frac{2}{9}$
(c) $\frac{1}{3}$ (d) 3

18. A circular current loop of radius R is placed inside square loop of side length L ($L \gg R$) such that they are co-planar, and their centers coincide. The permeability of free space is μ_0 . The mutual inductance between circular loop and square loop is ____.

- (a) $2\sqrt{2} \frac{\mu_0 L^2}{R}$ (b) $\sqrt{2} \frac{\mu_0 L^2}{R}$
(c) $\sqrt{2} \frac{\mu_0 R^2}{L}$ (d) $2\sqrt{2} \frac{\mu_0 R^2}{L}$

19. The binding energy per nucleon of $^{209}_{83}\text{Bi}$ is ____ MeV. [Take $m(^{209}_{83}\text{Bi}) = 208.980388\text{u}$, $m_p = 1.007825\text{u}$, $m_n = 1.008665\text{u}$, $1\text{u} = 931\text{MeV}/c^2$]

- (a) 7.48 (b) 7.84
(c) 8.79 (d) 6.94

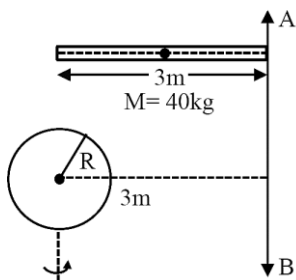
20. The equation of motion of a particle is given by $x = a \sin(50t + \pi/3)\text{cm}$. The particle will come to rest at time t_1 and it will have zero acceleration at time t_2 . The t_1 and t_2 respectively are ____.

- (a) $\frac{\pi}{300}\text{ s}, \frac{\pi}{75}\text{ s}$ (b) $\frac{\pi}{75}\text{ s}, \frac{\pi}{300}\text{ s}$
(c) $\frac{\pi}{300}\text{ s}, \frac{\pi}{25}\text{ s}$ (d) $\frac{\pi}{50}\text{ s}, \frac{\pi}{100}\text{ s}$

SECTION-B

21. In a Young's double slit experiment, the intensity at some point on the screen is found to be $\frac{3}{4}$ times of the maximum of the interference pattern. The path difference between the interfering waves at this point is $\frac{\lambda}{x}$ where λ is wavelength of the incident light. The value of x is ____.
22. Using Bohr's model, calculate the ratio of the magnetic fields generated due to the motion of the electrons in the 2nd and 4th orbits of hydrogen atom ____.
23. 5 moles of unknown gas is heated at constant volume from 10°C to 20°C . The molar specific heat of this gas at constant pressure $c_p = 8\text{cal/mol. }^\circ\text{C}$ and $R = 8.36\text{ J/mol. }^\circ\text{C}$. The change in the internal energy of the gas is ____ calorie.

24. If sunlight is focused on a paper using convex lens, it starts burning the paper in shortest time when the lens is kept at 30 cm above the paper. If the radius of curvature of the lens is 60 cm then the refractive index of the lens material is $\frac{\alpha}{10}$. The value of α is ____.
25. Moment of inertia about an axis AB for a rod of mass 40 kg and length 3 m is same as that of a solid sphere of mass of 10 kg and radius R about an axis parallel to AB axis with separation of 3 m as shown in figure below. The value of R is given as $\sqrt{\frac{\alpha}{2}}$. The value of α is ____.



CHEMISTRY

SECTION-A

26. The ratio of mass percentage (w/w) of C: H in a hydrocarbon is 12: 1. It has two carbon atoms. The weight (in g) of CO_2 (g) formed when 3.38 g of this hydrocarbon is completely burnt in oxygen is :
(Given : Molar mass in gmol^{-1} C: 12, H: 1, O: 16)
(a) 5.68 (b) 11.44
(c) 22.74 (d) 17.05
27. The first and second ionization constants of a weak dibasic acid H_2A are 8.1×10^{-8} and 1.0×10^{-13} respectively. 0.1 mol of H_2A was dissolved in 1L of 0.1 M HCl solution. The concentration of HA^- in the resultant solution is :
(a) 0.1 M (b) $9.53 \times 10^{-6}\text{M}$
(c) $8.1 \times 10^{-8}\text{M}$ (d) $1.0 \times 10^{-13}\text{M}$
28. SF_4 is isostructural with :
(A) $\text{BrF}_4^\ominus$ (B) CH_4
(C) $\text{IF}_4^\oplus$ (D) XeF_4
(E) XeO_2F_2
Choose the correct answer from the options given below:
(a) (C) only
(b) (C and (E) only
(c) (A) and (D) only
(d) (B) and (E) only
29. Gas 'A' undergoes change from state 'X' to state 'Y'. In this process, the heat absorbed, and work done by the gas is 10 J and 18 J respectively. Now gas is brought back to state 'X' by another process during which 6 J of heat is evolved. In the reverse process of 'Y' to 'X',
(a) 18 J of the work is done by the gas 'A'.
(b) 2 J of the work is done by the gas 'A'.
(c) 12 J of the work is done on the gas 'A' by the surrounding.
(d) 14 J of the work is done on the gas 'A' by the surrounding.
30. Solution A is prepared by dissolving 1 g of a protein (molar mass = 50000 g mol^{-1}) in 0.5 L of water at 300 K. Its osmotic pressure is x bar. Solution B is made by dissolving 2 g of same protein in 1 L of water at 300 K. Osmotic pressure of solution B is y bar. Entire solution of A is mixed with entire solution of B at same temperature. The osmotic pressure of resultant solution is z bar. x, y and z respectively are :
(R = $0.083 \text{ L bar mol}^{-1} \text{ K}^{-1}$)

- (a) 9.96×10^{-4} ; 9.96×10^{-4} ; 9.96×10^{-4}
 (b) 9.96×10^{-4} ; 9.96×10^{-4} ; 19.92×10^{-4}
 (c) 4.98×10^{-4} ; 4.98×10^{-4} ; 9.96×10^{-4}
 (d) 4.98×10^{-4} ; 4.98×10^{-4} ; 4.98×10^{-4}

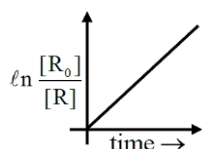
31. At 25°C, 20.0 mL of 0.2 M weak monoprotic acid HX is titrated against 0.2 M NaOH. The pH of the solution (a) at the start of the titration (when NaOH has not been added) and (b) when 10 mL of NaOH is added respectively, are :

Given : $K_a = 5 \times 10^{-4}$, $pK_a = 3.3$, $\alpha \ll 1$

- (a) 0.7, 2.0 (b) 2.0, 3.3
 (c) 1.1, 2.2 (d) 3.0, 2.2

32. Consider the reaction $aX \rightarrow bY$, for which the rate constant at 30°C is $1 \times 10^{-3} \text{ mol}^{-1} \text{ L s}^{-1}$. Which of the following statements are true ?

- (A) When concentration of 'X' is increased to four times, the rate of reaction becomes 16 times.
 (B) The reaction is a second order reaction.
 (C) The half-life period is independent of the concentration of X.
 (D) Decomposition of N_2O_5 is an example of the above reaction.
 (E)



is valid for the above reaction.

Choose the correct answer from the options given below :

- (a) (A) and (B) only
 (b) (A), (B) and (C) only
 (c) (A), (B), (D) and (E) only
 (d) (C) and (D) only

33. The correct set that contains all kinds (basic, acidic, amphoteric and neutral) of oxides is :

- (a) Na_2O , K_2O , Al_2O_3 and As_2O_3
 (b) Al_2O_3 , As_2O_3 , CO and NO
 (c) K_2O , Cl_2O_7 , As_2O_3 and NO
 (d) Na_2O , N_2O , Al_2O_3 and CO

34. Given below are two statements :

Statement-I : The second ionization enthalpy of B, Al and Ga is in the order of $B > Al > Ga$.

Statement-II : The correct order in terms of first ionization enthalpy is $Si < Ge < Pb < Sn$.

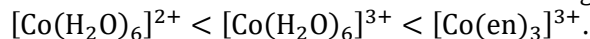
In the light of the above statements, choose the correct answer from the options given below :

- (a) Both Statement-I and Statement-II are true.
 (b) Both Statement-I and Statement-II are false.
 (c) Statement-I is true but Statement-II is false.
 (d) Statement-I is false but Statement-II is true.

35. Given below are two statements :

Statement-I : Among Zn, Mn, Sc and Cu, the energy required to remove the third valence electron is highest for Zn and lowest for Sc.

Statement-II : The correct order of the following complexes in terms of CFSE is

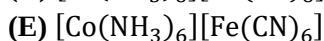
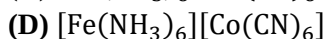
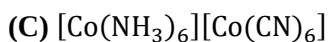


In the light of the above statements, choose the correct answer from the options given below :

- (a) Both Statement-I and Statement-II are true.
 (b) Both Statement-I and Statement-II are false.
 (c) Statement-I is true but Statement-II is false.
 (d) Statement-I is false, but Statement-II is true.

36. Which of the following complexes will show coordination isomerism?

- (A) $[Ag(NH_3)_2][Ag(CN)_2]$
 (B) $[Co(NH_3)_6][Cr(CN)_6]$



Choose the correct answer from the options given below :

(a) B, C and D Only

(b) B, D and E Only

(c) A, C and D Only

(d) C, D and E Only

37. Complete combustion of X g of an organic compound gave 0.25 g of CO_2 and 0.12 g of H_2O . If the % of carbon is 25% and of hydrogen is 4.89%, then $X = ___ \times 10^{-3}$ g. (Nearest integer) (Molar mass of C, H and O are 12, 1 and 16 g mol^{-1} respectively)

(a) 273

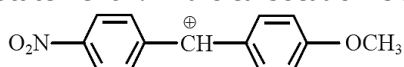
(b) 27

(c) 2730

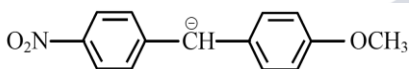
(d) 227

38. Given below are two statements :

Statement I: In the carbocation is stabilized by +R effect of $-\text{OCH}_3$ group.



Statement II : In the carbanion is stabilized by -R effect of $-\text{NO}_2$ group. In the light of the above statements, choose the correct answer from the options given below :



(a) Both Statement I and Statement II are true

(b) Both Statement I and Statement II are false

(c) Statement I is true but Statement II is false

(d) Statement I is false but Statement II is true

39. The compound (X) on

(i) on heating in the presence of anhydrous AlCl_3 and HCl gas gives 2, 4-dimethyl pentane

(ii) aromatization gives toluene and

(iii) cyclisation gives methyl cyclohexane

The correct name of compound (X) is :

(a) Hept-2-ene

(b) Hept-1,3,5-triene

(c) Heptane

(d) Hept-2,4,6-triene

40. Correct statements regarding alkyl halides (R – X) among the following are :

(A) Alcohol being less polar solvent as compared to water, alcoholic KOH favours elimination reaction with R – X.

(B) Order of reactivity towards $\text{S}_{\text{N}}1$ mechanism is $\text{C}_6\text{H}_5 - \text{CH}_2 - \text{Cl} > \text{C}_6\text{H}_5 - \text{CHCl} - \text{C}_6\text{H}_5$.

(C) Non substituted aryl halides exhibit properties similar to alkyl halides.

(D) Vinyl chloride is an example of haloalkene and allyl chloride is an example of haloalkyne.

(E) R – Cl can be prepared by reacting R – OH with SOCl_2 but Ar – Cl cannot be prepared by reacting Ar – OH with SOCl_2 .

Choose the correct answer from the options given below :

(a) (A), (B) and (C) Only

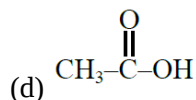
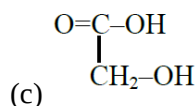
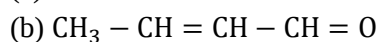
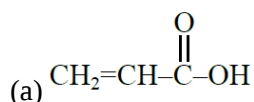
(b) (B) and (D) Only

(c) (A) and (E) Only

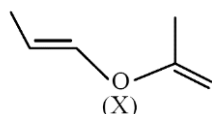
(d) (D) and (E) Only

41. An organic compound " x " where molar ratio of C , O and H are equal, on treatment with 50%KOH under reflux followed by acidification produced " y ". The most likely structure of " y " is:

[Molar mass of 'x' is 58 g mol^{-1}]



42. A molecule (X) with following structure under mild acidic condition is hydrolysed to produce (Y) and (Z). Identify the correct statements about (Y) and (Z).

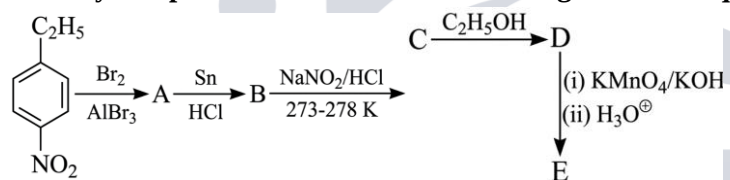


- (A) Both (Y) and (Z) have same molar mass
 (B) (Y) and (Z) can be distinguished from each other by NaHCO_3
 (C) (Y) and (Z) react with HCN with same rates
 (D) (Y) and (Z) undergo addition reaction with 2, 4-DNP

Choose the correct answer from the options given below :

- (a) (A), (B) and (C) Only
 (b) (B) and (C) Only
 (c) (C) and (D) Only
 (d) (A) and (D) Only

43. Identify compounds A and E in the following reaction sequence.



- (a) $\text{A} = \text{1-ethyl-3-bromo-4-nitrobenzene}$, $\text{E} = \text{3-bromo-4-nitrobenzoic acid}$
- (b) $\text{A} = \text{1-ethyl-2-bromo-4-nitrobenzene}$, $\text{E} = \text{2-bromo-4-nitrobenzoic acid}$
- (c) $\text{A} = \text{1-(2-bromoethyl)-4-nitrobenzene}$, $\text{E} = \text{4-nitrobenzoic acid}$
- (d) $\text{A} = \text{1-ethyl-3-bromo-4-nitrobenzene}$, $\text{E} = \text{3-bromo-4-nitrophenol}$

44. Identify the correct pair having amino acid (A) and the hormone (B) that is iodinated derivative of the amino acid (A).

(T and Y represent one letter code for amino acids)

Amino acid (A) Hormone (B)

- (a) T Insuline
 (b) T Thyroxine

- (c) Y Thyroxine
(d) Y Insuline

45. Among Fe^{2+} , Fe^{3+} , Cr^{2+} and Zn^{2+} , the ion that shows positive borax bead test and with highest ionisation enthalpy is:

- (a) Fe^{2+} (b) Zn^{2+}
(c) Cr^{2+} (d) Fe^{3+}

SECTION-B

46. The surface of sodium metal is irradiated with radiation of wavelength x nm. The kinetic energy of ejected electrons is 2.8×10^{-20} J. The work function of sodium is 2.3 eV. The value of x is $___ \times 10^2$ nm. (Nearest integer)

(Given : $h = 6.6 \times 10^{-34}$ J s; $1\text{eV} = 1.60 \times 10^{-19}$ J; $c = 3.0 \times 10^8$ ms $^{-1}$)

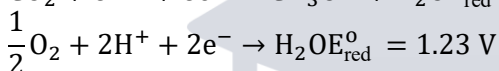
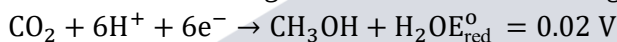
47. Consider the following gas phase reaction being carried out in a closed vessel at 25°C.



time (min)	total pressure of the system (mm Hg)
30	300
∞	600

The pressure of C(g) at 30 minutes time interval would be $___ \text{ mm Hg}$. (Nearest integer)

48. Consider the following two half-cell reactions along with the standard reduction potential given :

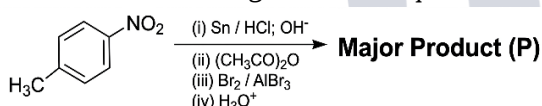


The fuel cell was set up using the above two reactions such that the cell operates under the standard condition of 1 bar pressure and 298 K temperature. The fuel cell works with 80% efficiency. If the work derived from the cell using 1 mol of CH_3OH is used to compress an ideal gas isothermally against a constant pressure of 1 kPa, then the change in the volume of the gas, $\Delta V = ___ \text{ m}^3$. (Nearest integer) Given : $F = 96500 \text{ Cmol}^{-1}$

49. Number of paramagnetic ions among the following d- and f-block metal ions is $___$.

Mn^{2+} , Cu^{2+} , Zn^{2+} , Yb^{2+} , Sc^{3+} , La^{3+} , Gd^{3+} , Lu^{3+} , Ti^{4+} , Ce^{4+} (Atomic number of Mn = 25, Cu = 29, Zn = 30, Yb = 70, Sc = 21, La = 57, Gd = 64, Lu = 71, Ti = 22, Ce = 58)

50. Consider the following reactions sequence



When the product (P) is subjected to Carius analysis using AgNO_3 , 1.0 g of the product (P) will produce $___ \text{ g}$ of the precipitate of AgBr . (Nearest integer)

(Given : molar mass in gmol^{-1} : C: 12, H: 1, O: 16, N: 14, Br: 80, Ag: 108)

MATHEMATICS

SECTION-A

51. Let α, β be the roots of the equation $x^2 - 3x + r = 0$, and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 + 3x + r =$

0. If the roots of the equation $x^2 + 6x = m$ are $2\alpha + \beta + 2r$ and $\alpha - 2\beta - \frac{r}{2}$, then m is equal to :-

- (a) -135 (b) -567
(c) 135 (d) 567

52. Let the circle $C_1: |z| = r$ and $C_2: |z - 3 - 4i| = 5$, $z \in \mathbb{C}$, be such that C_2 lies within C_1 . If z_1 moves on C_1 , z_2 moves on C_2 and $\min|z_1 - z_2| = 2$, then $\max|z_1 - z_2|$ is equal to :-

- (a) 12 (b) 17
(c) 22 (d) 24

53. If the system of equations

$$x + 5y + 6z = 4$$

$$2x + 3y + 4z = 7$$

$$x + 6y + az = b$$

has infinitely many solutions, then the point (a, b) lies on the line

- (a) $y - x = 3$ (b) $x - y = 3$
(c) $x + y = 11$ (d) $x + y = 12$

54. Let $a_1, a_2, a_3, \dots$ be an A.P. and $g_1 = a_1, g_2, g_3, \dots$ be an increasing G.P. If $a_1 = a_2 + g_2 = 1$ and $a_3 + g_3 = 4$, then $a_{10} + g_5$ is equal to :

- (a) 81 (b) 76
(c) 62 (d) 55

55. The sum $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \dots$ up to 8 terms, is :

- (a) 70 (b) 71
(c) 72 (d) 73

56. If for $3 \leq r \leq 30$, $({}^{30}C_{30-r}) + 3({}^{30}C_{31-r}) + 3({}^{30}C_{32-r}) + ({}^{30}C_{33-r}) = {}^mC_r$, then m equals :

- (a) 31 (b) 32
(c) 33 (d) 34

57. Let p_n denote the total number of triangles formed by joining the vertices of an n -side regular polygon. If $p_{n+1} - p_n = 66$, then the sum of all distinct prime divisors of n is :

- (a) 7 (b) 8
(c) 5 (d) 6

58. A man throws a fair coin repeatedly. He gets 10 points for each head he throws and 5 points for each tail he throws. If the probability that he gets exactly 30 points is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to :

- (a) 53 (b) 55
(c) 107 (d) 105

59. The mean and variance of n observations are 8 and 16, respectively. If the sum of the first $(n - 1)$ observations is 48 and the sum of squares of the first $(n - 1)$ observations is 496, then the value of n is :

- (a) 21 (b) 16
(c) 13 (d) 7

60. Let a circle pass through the origin and its center be the point of intersection of two mutually perpendicular lines $x + (k - 1)y + 3 = 0$ and $2x + k^2y - 4 = 0$. If the line $x - y + 2 = 0$ intersects the circle at the points A and B, then $(AB)^2$ is equal to :

- (a) 10 (b) 27
(c) 18 (d) 34

61. Let O be the origin, and P and Q be two points on the rectangular hyperbola $xy = 12$ such that the mid point of the line segment PQ is $(\frac{1}{2}, -\frac{1}{2})$. Then the area of the triangle OPQ equals :

- (a) $\frac{3}{2}$ (b) $\frac{5}{2}$
(c) $\frac{7}{2}$ (d) $\frac{9}{2}$

62. Let the parabola $y = x^2 + px + q$ passing through the point $(1, -1)$ be such that the distance between its vertex and the x -axis is minimum. Then the value of $p^2 + q^2$ is :

- (a) 2 (b) 4
(c) 5 (d) 8

63. Let $P = \{\theta \in [0, 4\pi] : \tan^2 \theta \neq 1\}$ and $S = \{a \in \mathbb{Z} : 2(\cos^8 \theta - \sin^8 \theta) \sec 2\theta = a^2, \theta \in P\}$. Then $n(S)$ is :

- (a) 0 (b) 1
(c) 2 (d) 3

64. Let the vectors $\vec{a} = -\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$. For some $\lambda, \mu \in \mathbb{R}$, let $\vec{c} = \lambda\vec{a} + \mu\vec{b}$. If $\vec{c} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 10$ and $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = -2$, then $|\vec{c}|^2$ is equal to :

- (a) 8 (b) 12
(c) 14 (d) 15

65. Let the point A be the foot of perpendicular drawn from the point $P(a, b, 0)$ on the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-\alpha}{3}$. If the midpoint of the line segment PA is $(0, \frac{3}{4}, \frac{-1}{4})$, then the value of $a^2 + b^2 + \alpha^2$ is equal to :
- (a) 1 (b) 2
(c) 6 (d) 9
66. Two adjacent sides of a parallelogram PQRS are given by $\overrightarrow{PQ} = \hat{j} + \hat{k}$ and $\overrightarrow{PS} = \hat{i} - \hat{j}$. If the side PS is rotated about the point P by an acute angle α in the plane of the parallelogram so that it becomes perpendicular to the side PQ, then $\sin^2\left(\frac{5\alpha}{2}\right) - \sin^2\left(\frac{\alpha}{2}\right)$ is equal to :
- (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$
(c) $\frac{\sqrt{3}}{4}$ (d) $\frac{2\sqrt{3}}{5}$
67. The value of $\int_0^{20\pi} (\sin^4 x + \cos^4 x) dx$ is equal to:
- (a) $\frac{15\pi}{2}$ (b) 25π
(c) 15π (d) $\frac{25\pi}{2}$
68. Let $f(x)$ be a polynomial of degree 5 and have extrema at $x = 1$ and $x = -1$. If $\lim_{x \rightarrow 0} \left(\frac{f(x)}{x^3}\right) = -5$, then $f(2) - f(-2)$ is equal to :
- (a) 0 (b) 50
(c) 92 (d) 112
69. Let $f(x) = \int \left(\frac{16x+24}{x^2+2x-15}\right) dx$. If $f(4) = 14\log_e(3)$ and $f(7) = \log_e(2^\alpha \cdot 3^\beta)$, $\alpha, \beta \in \mathbb{N}$, then $\alpha + \beta$ is equal to:
- (a) 31 (b) 37
(c) 39 (d) 41
70. Let $x = x(y)$ be the solution of the differential equation $2y^2 \frac{dx}{dy} - 2xy + x^2 = 0$, $y > 1$, $x(e) = e$. Then $x(e^2)$ is equal to:
- (a) $\frac{3}{2}e^2$ (b) $\frac{2}{3}e^2$
(c) e^2 (d) $2e^2$

SECTION-B

71. Let $A = \{2, 3, 4, 5, 6\}$. Let R be a relation on the set $A \times A$ given by $(x, y)R(z, w)$ if and only if x divides z and $y \leq w$. Then the number of elements in R is ____.
72. Consider the matrices $A = \begin{bmatrix} 2 & -2 \\ 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix}$. If matrices P and Q are such that $PA = B$ and $AQ = B$, then the absolute value of the sum of the diagonal elements of $2(P + Q)$ is ____.
73. Let A be the point (3,0) and circles with variable diameter AB touch the circle $x^2 + y^2 = 36$ internally. Let the curve C be the locus of the point B. If the eccentricity of C is e, then $72e^2$ is equal to ____.
74. If the area of the region bounded by $16x^2 - 9y^2 = 144$ and $8x - 3y = 24$ is A, then $3(A + 6\log_e(3))$ is equal to ____.
75. The number of points in the interval $[2, 4]$, at which the function $f(x) = \left[x^2 - x - \frac{1}{2}\right]$, where $[\cdot]$ denotes the greatest integer function, is discontinuous, is ____.