

Physics Section-A

1. (d)

A : Since polar dielectrics are randomly oriental $\vec{P}_{\text{net}} = \vec{0}$.

R : If $\vec{E}$ is absent, polar dielectric remain polar & are randomly oriented.

2. (a)

$$\text{Voltage sensitivity} = \frac{\theta}{V} = \frac{NAB}{cR}$$

$$\text{Ratio} = \left(\frac{N_1 A_1 B_1}{N_2 A_2 B_2} \right) \frac{R_2}{R_1} = \frac{15 \times 3.6 \times 0.25}{21 \times 1.8 \times 0.5} \times \frac{7}{5} = \frac{1}{1}$$

3. (b)

Diameter is given as R.

$$\therefore \text{Radius} = R/2$$

$$I_{\text{tangent}} = \frac{3}{2} m \left(\frac{R}{2} \right)^2 = \frac{3}{8} m R^2$$

4. (a)

$$2 \times \frac{4}{3} \pi R^3 = \frac{4}{3} \pi r^3 \Rightarrow r = 2^{1/3} R$$

$$U_i = 2 \times 4 \pi R^2 T$$

$$U_f = 4 \pi r^2 T = 4 \pi R^2 T 2^{2/3}$$

$$\therefore \text{Heat lost} = u_i - u_f = 4 \pi R^2 T [2 - 2^{2/3}]$$

5. (d)

Magnetic field does not work

$\therefore$ Speed will not change, so De-Broglie wavelength remains same.

6. (a)

$$v = \frac{\text{distance}}{\text{time}}$$

$$v = \frac{12}{0.3} = 4 \text{ cm/s}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{7.5} = \frac{4\pi}{15} = 0.83v$$

$$= \frac{\omega}{k} \Rightarrow \omega = vk = 4 \times \frac{4\pi}{15} = 3.35$$

$$\text{So } y = A \cos(kx - \omega t)$$

7. (b)

$$Q = AT$$

$$I = A$$

$$\mu_0 = \text{MLT}^{-2} \text{A}^{-2}$$

$$P = Q^x \mu_0^y I^z = [AT]^x [\text{MLT}^{-2} \text{A}^{-2}]^y [A]^z$$

$$\text{MLT}^{-1} = \text{M}^y \text{L}^y \text{T}^{x-2y} \text{A}^{-2y+z+x}$$

$$\text{Now; } y = 1$$

$$x - 2y = -1$$

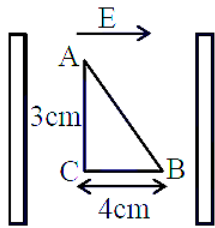
$$-2y + z = 0$$

$$\therefore x = y = z = 1$$

8. (d)

$$\frac{U}{V} = \frac{B^2}{2\mu_0 u_0} \Rightarrow U = \frac{B^2}{4\mu_0} V = \frac{B^2}{4\mu_0} A \ell$$

9. (b)



Using $\Delta V = E(\Delta d)$

$$V = E(10)$$

$$V_{AB} = E \cdot 4 = \frac{V}{10} \times 4 = \frac{2V}{5}$$

10. (c)

$$Q = \Delta U + W = 0 \Rightarrow -\Delta U = W$$

$$WD = -nC_v \Delta T \Rightarrow |WD| = nC_v \Delta T \propto T_2 - T_1$$

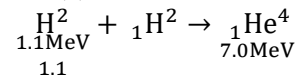
$\therefore$ B & E [Only possibility]

11. (c)

$$r = r_0 \frac{n^2}{Z} \text{ \& } Z = 3 \text{ for } L_i^{+2} \text{ and } n = 1$$

$$\therefore r = r_0 \frac{1^2}{3} = \frac{r_0}{3} \therefore x = 3$$

12. (c)

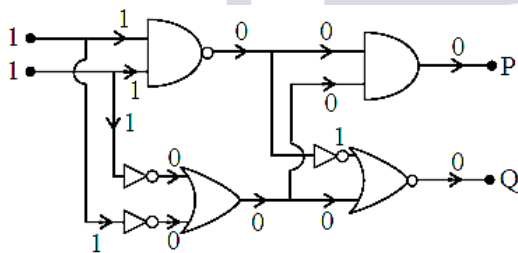


$$E_B = BE_{\text{reactant}} - BE_{\text{product}}$$

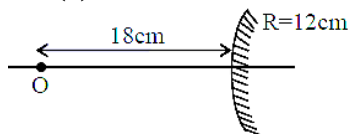
$$= 1.1 \times 2 + 1.1 \times 2 - 7 \times 4 = -23.6 \text{ MeV}$$

$$= Q = 23.6 \text{ MeV}$$

13. (b)

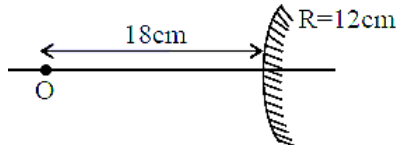


14. (a)



$$\text{Using } m = \frac{f}{u-f}$$

$$m_1 = \frac{6}{18-6} = \frac{1}{2}$$



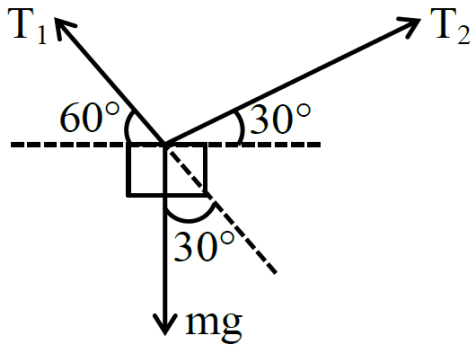
$$m_2 = \frac{6}{18+6} = \frac{1}{4} \therefore \frac{m_2}{m_1} = \frac{1}{2}$$

15. (d)

$$\text{Displacement} = 2r$$

$$\text{Distance} = 2\pi r + \pi r = 3\pi r$$

16. (b)



$$T_1 = mg \cos 30^\circ$$

$$T_2 = mg \sin 30^\circ$$

17. (b)

This will happen when

$$\frac{1}{f_1} = \frac{1}{f_2}$$

$$(\mu - 1) \left(\frac{1}{R_1} - \frac{1}{-R_2} \right) = (\mu - 1) \left(\frac{2}{R} \right)$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{R}$$

18. (b)

$$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \Rightarrow \frac{1}{\mu_0 \epsilon_0} = C^2 = L^2 T^{-2}$$

19. (d)

$$C' = \frac{\Delta Q}{\Delta T} = JK^{-1}$$

$$S = \frac{\Delta Q}{m \Delta T} = J kg^{-1} K^{-1}$$

$$L = \frac{\Delta Q}{\Delta T} = J kg^{-1}$$

$$\Delta Q = \frac{L}{KA \Delta T} \Rightarrow K = \frac{\Delta Q (L)}{A \Delta T} = J m^{-1} K^{-1} s^{-1}$$

20. (c)

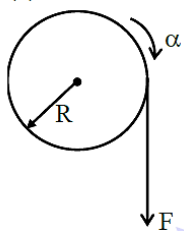
$$E = \frac{KQr}{(x^2 + R^2)^{3/2}}$$

$$\frac{dE}{dx} = 0$$

$$\therefore x = \frac{R}{\sqrt{2}} = \frac{\sqrt{2}a}{\sqrt{2}} = a$$

SECTION-B

21. (1)



$$FR = I\alpha$$

$$\Rightarrow I = \frac{FR}{\alpha} = \frac{10 \times 0.2}{2} = 1 \text{ kg} \cdot \text{m}^2$$

22. (12)

To find the internal energy of gas in the room.

$$U = nC_v T = n \frac{5RT}{2}$$

$$= \frac{5}{2} PV = \frac{5}{2} \times 10^5 \times 48 = 12 \times 10^6 \text{ J}$$

23. (45)

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}, \text{ since } A = 60^\circ \therefore \delta_m = 30^\circ$$

$$\delta_m = 2i - A \text{ [as } i = e \text{]}$$

$$\Rightarrow i = 45^\circ$$

24. (1)

$$T = K(\ell - \ell_0)$$

$$\Rightarrow 5 = K(1.4 - \ell_0)$$

$$\Rightarrow 7 = K(1.56 - \ell_0)$$

$$\Rightarrow \frac{5}{1.4 - \ell_0} = \frac{7}{1.56 - \ell_0}$$

$$\therefore \ell_0 = 1 \text{ m}$$

25. (3)

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} m \frac{GM_E}{r} = \frac{GM_E m}{2r} = \frac{GM_E m}{2(R_E + h)}$$

$$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 6.4 \times 10^6}{2(6.4 \times 10^6 + 2.7 \times 10^5)} = 3 \times 10^{10} \text{ J}$$

Chemistry
Section-A

26.(c)

27.(a)

28.(b)

29.(c)

30. (c)

31.(a)

32.(d)

33.(b)

34.(d)

35.(d)

36.(d)

37.(b)

38.(b)

39. (a)

40. (a)

41.(b)

42.(b)

43.(a)

44.(d)

45. (d)

Section-B

46.(43)

47.(23)

48. (184)

49.(25)

50.(o)

Mathematics
Section-A

51. (b)

$$P(1,0,3)$$

$$A(4,7,1), B(3,5,3)$$

$$\text{Line AB} \Rightarrow \frac{x-3}{1} = \frac{y-5}{2} = \frac{z-3}{-2} = \lambda$$

Let foot of perpendicular of P on AB be

$$R \equiv (\lambda + 3, 2\lambda + 5, -2\lambda + 3)$$

$$\Rightarrow (\lambda + 3 - 1)(1) + (2\lambda + 5 - 0)(2) + (-2\lambda + 3 - 3)$$

$$(-2) = 0$$

$$\Rightarrow \lambda + 2 + 4\lambda + 10 + 4\lambda = 0$$

$$\Rightarrow \lambda = -\frac{4}{3}$$

$$\Rightarrow R \equiv \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$$

$$Q \equiv \left(\frac{10}{3} - 1, \frac{14}{3} - 0, \frac{34}{3} - 3\right) \equiv \left(\frac{7}{3}, \frac{14}{3}, \frac{25}{3}\right)$$

$$\Rightarrow \alpha + \beta + \gamma = \frac{7 + 14 + 25}{3} = \frac{46}{3}$$

52. (b)

$$10 \frac{d}{dx} \int_1^x f(t) dt = \frac{d}{dx} (5xf(x) - x^5 - 9)$$

$$\Rightarrow 10f(x) = 5f(x) + 5xf'(x) - 5x^4$$

$$\Rightarrow f(x) + x^4 = xf'(x)$$

$$\Rightarrow y + x^4 = x \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} + y \left(-\frac{1}{x}\right) = x^3$$

$$\Rightarrow ye^{\int -\frac{1}{x} dx} = \int x^3 e^{\int -\frac{1}{x} dx} + c$$

$$\Rightarrow ye^{-|n|x|} = \int x^3 e^{-|n|x|} + c$$

$$\Rightarrow \frac{y}{|x|} = \int \frac{x^3}{|x|} + c$$

$$\Rightarrow \frac{y}{x} = \int x^2 + c$$

$$\Rightarrow \frac{y}{x} = \frac{x^3}{3} + c$$

Put $x = 1$ in given equation

$$\Rightarrow 0 = 5f(1) - 1 - 9 \Rightarrow f(1) = 2$$

$$\Rightarrow \frac{2}{1} = \frac{1}{3} + c \Rightarrow c = \frac{5}{3}$$

$$\Rightarrow \frac{f(3)}{3} = \frac{27}{3} + \frac{5}{3}$$

$$\Rightarrow f(3) = 32$$

53. (a)

$$a_2 + a_4 + \dots + a_n = 30$$

$$a_1 + a_3 + \dots + a_{n-1} = 24$$

$$(1) - (2)$$

$$(a_2 - a_1) + (a_4 - a_3) \dots (a_n - a_{n-1}) = 6$$

$$\Rightarrow \frac{n}{2}d = 6 \Rightarrow nd = 12$$

$$a_n - a_1 = (n - 1)d = \frac{21}{2}$$

$$\Rightarrow nd - d = \frac{21}{2} \Rightarrow 12 - \frac{21}{2} = d$$

$$\Rightarrow d = \frac{3}{2}, n = 8$$

$$\text{Sum of odd terms} = \frac{4}{2}[2a + (4 - 1)3] = 24$$

$$\Rightarrow a = \frac{3}{2}$$

$$\text{A.P.} \Rightarrow \frac{3}{2}, 3, \frac{9}{2}, 6, \frac{15}{2}, 9, \frac{21}{2}, 12$$

$$\text{no. of integer terms} = 4$$

54. (c)

$$a = 2b + 1$$

$$2b = a - 1$$

$$R = \{(3,1), (5,2), \dots, (99,49)\}$$

Let $(2m + 1, m), (2\lambda - 1, \lambda)$ are such ordered pairs.

According to the condition

Let $(2m + 1, m), (2\lambda - 1, \lambda)$ are such ordered pairs.

According to the condition

$$m = 2\lambda - 1 \Rightarrow m = \text{odd number}$$

$\Rightarrow 1^{\text{st}}$ element of ordered pair (a, b)

$$a = 2(2\lambda - 1) + 1 = 4\lambda - 1$$

$$\text{Hence } a \in \{3, 7, \dots, 99\}$$

$$\Rightarrow \lambda \in \{1, 2, \dots, 25\}$$

$\Rightarrow$ set of sequence

$$\left\{ (4\lambda - 1, 2\lambda - 1), (2\lambda - 1, \lambda - 1) \right. \\ \left. , \left(\lambda - 1, \frac{\lambda - 2}{2} \right), \dots \dots \right\}$$

$$2^{\text{nd}} \text{ element of each ordered pair} = \frac{\lambda - 2^{r-2}}{2^{r-2}}$$

For maximum number of ordered pairs in such sequence

$$\frac{\lambda - 2^{r-2}}{2^{r-2}} = 1 \text{ or } 2; 1 \leq \lambda \leq 25$$

$$\lambda = 2^{r-1} \text{ or } \lambda = 3 \cdot 2^{r-2}$$

Case-I: $\lambda = 2^{r-1}$

$$\lambda = 2, 2^2, 2^3, 2^4$$

$$r = 2, 3, 4, 5$$

Hence maximum value of r is 5 when $\lambda = 16$

Case-II : $\lambda = 3 \cdot 2^{r-2}$

$$\lambda = 3, 6, 12,$$

$$r = 2, 3, 4, 5$$

Hence maximum value of r is 5 when $\lambda = 24$

55. (a)

$$2b = \frac{1}{4}(2ae)$$

$$\frac{b}{a} = \frac{e}{4}$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$e = \sqrt{1 - \frac{e^2}{16}}$$

$$e^2 \left(1 + \frac{1}{16}\right) = 1$$

$$e = \frac{4}{\sqrt{17}}$$

56. (a)

$$L_1: (7\hat{i} + 6\hat{j} + 2\hat{k}) + \lambda(-3\hat{i} + 2\hat{j} + 4\hat{k})$$

$$L_2: (5\hat{i} + 3\hat{j} + 4\hat{k}) + \lambda(2\hat{i} + \hat{j} + 3\hat{k})$$

Distance between skew lines

$$= \frac{(2\hat{i} + 3\hat{j} - 2\hat{k}) \cdot (2\hat{i} + 17\hat{j} - 7\hat{k})}{\sqrt{342}}$$

$$= \frac{69}{\sqrt{342}} = \frac{69}{3\sqrt{38}} = \frac{23}{\sqrt{38}}$$

57. (a)

$$x^2 = 2y \text{ and } y = 2x + 6$$

$$x^2 = 4x + 12$$

$$x^2 - 4x - 12 = 0 \Rightarrow \begin{matrix} x = 6 \\ y = 18 \end{matrix} \text{ if } x = -2$$

$$y = 2$$

$$\therefore (6, 18) \text{ and } (-2, 2)$$

Here (6, 18) Rejected because (a, b) lies in 2nd quadrant

$$\therefore a = -2 \text{ and } b = 2$$

$$\therefore I = \int_{-2}^2 \frac{9x^2}{1 + 5^x} dx = \int_{-2}^2 \frac{9 \cdot 5^x \cdot x^2}{1 + 5^x} dx$$

$$\therefore 2I = \int_{-2}^2 9x^2 dx = 18 \int_0^2 x^2 dx = 18 \left(\frac{x^3}{3} \right)_0^2$$

$$2I = 48$$

$$I = 24$$

58. (c)

$$\Delta = 0 \Rightarrow \begin{vmatrix} 2 & \lambda & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \mu \end{vmatrix} = 0$$

$$\Rightarrow 2(2\mu + 5) + \lambda(-4 - 3\mu) + 3(7) = 0$$

$$\Rightarrow 4\mu - 3\lambda\mu - 4\lambda + 31 = 0 \dots \dots (1)$$

$$\Delta_3 = 0 \Rightarrow \begin{vmatrix} 2 & \lambda & 5 \\ 3 & 2 & 7 \\ 4 & 5 & 9 \end{vmatrix} = 0$$

$$\Rightarrow 2(-17) + \lambda(1) + 5(7) = 0$$

$$\Rightarrow \lambda = -1$$

from equation (1)

$$4\mu + 3\mu + 4 + 31 = 0 \Rightarrow \mu = -5$$

$$\therefore \lambda^2 + \mu^2 = 26$$

59. (a)

$$\operatorname{cosec} \theta = \frac{2(\sqrt{3}-1) \pm \sqrt{4(3+1-2\sqrt{3})+16\sqrt{3}}}{2\sqrt{3}}$$

$$= \frac{2(\sqrt{3}-1) \pm \sqrt{16+8\sqrt{3}}}{2\sqrt{3}}$$

$$= \frac{2(\sqrt{3}-1) \pm (2+2\sqrt{3})}{2\sqrt{3}}$$

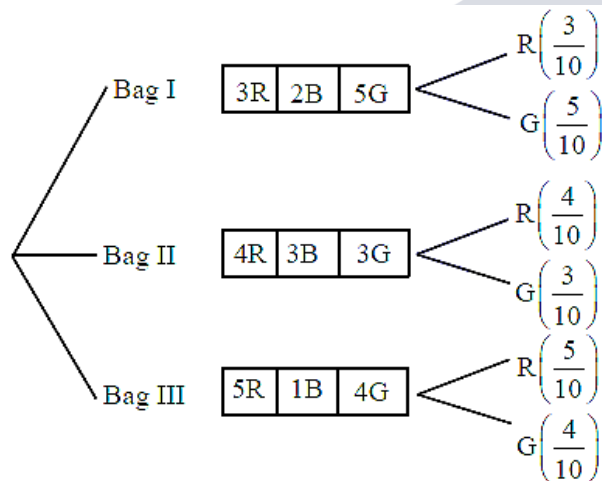
$$\operatorname{cosec} \theta = 2 \text{ or } \frac{-2}{\sqrt{3}}$$

$$\therefore \sin \theta = \frac{1}{2} \text{ or } \frac{-\sqrt{3}}{2}$$

$$\therefore \sin \theta = \frac{1}{2} \text{ has 3 solutions \& also } \sin \theta = \frac{-\sqrt{3}}{2}$$

$$\text{has 3 solutions in } \left[\frac{-7\pi}{6}, \frac{4\pi}{3} \right]$$

60. (c)



$$p = P\left(\frac{B_I}{R}\right) = \frac{\frac{1}{3}\left(\frac{3}{10}\right)}{\frac{1}{3}\left(\frac{3}{10} + \frac{4}{10} + \frac{5}{10}\right)} = \frac{1}{4}$$

$$q = P\left(\frac{B_{III}}{G}\right) = \frac{\frac{1}{3}\left(\frac{4}{10}\right)}{\frac{1}{3}\left(\frac{5}{10} + \frac{3}{10} + \frac{4}{10}\right)} = \frac{1}{3}$$

$$\therefore \frac{1}{p} + \frac{1}{q} = 7$$

61. (b)

$$\therefore \text{mean} = 9$$

$$\therefore 53 + a + b = 72$$

$$\Rightarrow a + b = 19$$

$$\therefore \sigma^2 = \frac{37}{4} \text{ and } (\bar{X})^2 + \sigma^2 = \frac{\sum x_1^2}{N}$$

$$\Rightarrow 81 + \frac{37}{4} = \frac{529 + a^2 + b^2}{8}$$

$$\Rightarrow 648 + 74 = 529 + a^2 + b^2$$

$$\Rightarrow a^2 + b^2 = 193$$

$$\therefore a + b = 19 \Rightarrow a^2 + b^2 + 2ab = 361$$

$$\Rightarrow 2ab = 168$$

$$\Rightarrow ab = 84$$

$$\therefore a + b + ab = 103$$

62.(a)

$$x + |x| > 0 \Rightarrow x \in (0, p_0)$$

$$10 + 3x - x^2 > 0$$

$$\Rightarrow x^2 - 3x - 10 < 0$$

$$\Rightarrow x \in (-2, 5)$$

$$\text{from (1) \& (2) } x \in (0, 5)$$

$$\therefore a = 0 \& b = 5$$

$$\therefore (1 + a^2) + b^2 = 1 + 25 = 26$$

63.(b)

$$\begin{aligned} & 4 \int_0^1 \frac{1}{\sqrt{3+x^2} + \sqrt{1+x^2}} dx - 3 \ln \sqrt{3} \\ &= 4 \int_0^1 \frac{\sqrt{3+x^2} - \sqrt{1+x^2}}{(3+x^2) - (1+x^2)} dx - \frac{3}{2} \ln 3 \\ &= 2 \left[\left\{ \frac{x}{2} \sqrt{3+x^2} + \frac{3}{2} \ln(x + \sqrt{3+x^2}) \right\}_0^1 \right. \\ &\quad \left. - \left\{ \frac{x}{2} \sqrt{1+x^2} + \frac{1}{2} \ln(x + \sqrt{1+x^2}) \right\}_0^1 \right] - \frac{3}{2} \ln 3 \\ &= 2 \left[\left\{ \frac{x}{2} \sqrt{1+x^2} + \frac{1}{2} \ln(x + \sqrt{1+x^2}) \right\}_0^1 \right] - \frac{3}{2} \ln 3 = \\ & 2 \left[\left\{ \frac{1}{2} \sqrt{4} + \frac{3}{2} \ln(1 + \sqrt{4}) \right\} - \left\{ 0 + \frac{3}{2} \ln \sqrt{3} \right\} \right] \\ &= - \left\{ \frac{1}{2} \sqrt{2} + \frac{1}{2} \ln(1 + \sqrt{2}) \right\} + \left\{ 0 + \frac{1}{2} \ln(0) \right\} - \frac{3}{2} \ln 3 \\ &= 2 \left[1 + \frac{3}{2} \ln 3 - \frac{3}{4} \ln 3 - \frac{1}{\sqrt{2}} - \frac{1}{2} \ln(1 + \sqrt{2}) \right] - \frac{3}{2} \ln 3 \\ &= 2 + 3 \ln 3 - \frac{3}{2} \ln 3 - \sqrt{2} - \ln(1 + \sqrt{2}) - \frac{3}{2} \ln 3 \\ &= 2 - \sqrt{2} - \ln(1 + \sqrt{2}) \end{aligned}$$

64. (a)

$$\lim_{x \rightarrow 0} \frac{\cos 2x + a \cos 4x - b}{x^4} = \text{finite}$$

$$L = \frac{\left\{ 1 - \frac{(2x)^2}{2} + \frac{(2x)^4}{4} \dots \right\} + a \left\{ 1 - \frac{(4x)^2}{2} + \frac{(4x)^4}{4} \dots \right\} - b}{x^4}$$

$$L = \frac{(1 + a - b) - x^2(2 + 8a) + x^4 \left(\frac{2}{3} + \frac{32}{3}a \right) + x^6(\dots)}{x^4}$$

$$\therefore 1 + a - b = 0 \text{ and } 2 + 8a = 0 \Rightarrow a = -\frac{1}{4}$$

$$b = a + 1$$

$$= -\frac{1}{4} + 1 = \frac{3}{4}$$

$$\therefore a + b = -\frac{1}{4} + \frac{3}{4} = \frac{1}{2}$$

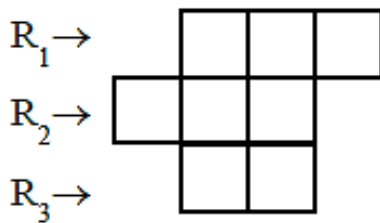
65.(d)

$$\begin{aligned} & \sum_{r=0}^{10} \left(\frac{10^{r-1} - 1}{10^r} \right) {}^{11}C_{r+1} \\ &= \sum_{r=0}^{10} \left(10 - \frac{1}{10^r} \right) {}^{11}C_{r+1} \end{aligned}$$

$$\begin{aligned}
 &= 10 \sum_{r=0}^{10} {}^{11}C_{r+1} - 10 \sum \left({}^{11}C_{r+1} \left(\frac{1}{10} \right)^{r+1} \right) \\
 &= 10[{}^{11}C_1 + {}^{11}C_2 + \dots + {}^{11}C_{11}] \\
 &\quad - 10 \left[{}^{11}C_1 \left(\frac{1}{10} \right)^1 + {}^{11}C_2 \left(\frac{1}{10} \right)^2 + \dots + {}^{11}C_{11} \left(\frac{1}{10} \right)^{11} \right] \\
 &= 10[2^{11} - 1] - 10 \left[\left(1 + \frac{1}{10} \right)^{11} - 1 \right] \\
 &= 10(2)^{11} - 10 - \frac{11^{11}}{10^{10}} + 10 \\
 &= \frac{(20)^{11} - 11^{11}}{10^{10}}
 \end{aligned}$$

$$\therefore \alpha = 20$$

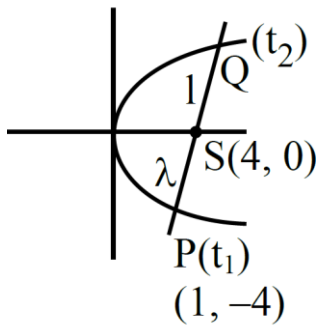
66.(d)



$$\begin{aligned}
 &= \text{Total} - [(\text{All in } R_1 \text{ and } R_3) + (\text{All in } R_2 \text{ and } R_3) + (\text{All in } R_1 \text{ and } R_2)] \\
 &= {}^8C_5 \cdot [5 - \{ [5 + [5 + {}^6C_5 \cdot [5] \} \\
 &= 5(56 - 1 - 1 - 6) = 120(48) \\
 &= 5760
 \end{aligned}$$

67.(a)

$$y^2 = 16x; a = 4 \text{ focus } S \equiv (4, 0)$$



$$2at_1 = -4$$

$$\Rightarrow 2(4)t_1 = -4$$

$$\Rightarrow t_1 = -\frac{1}{2}$$

$$\therefore t_1 t_2 = -1$$

$$\Rightarrow t_2 = 2$$

$$\therefore Q(at_2^2, 2at_2) = (16, 16)$$

Let, S divides PQ internally in $\lambda : 1$ ratio

$$\therefore \frac{16\lambda - 4}{\lambda + 1} = 0\lambda$$

$$= \frac{1}{4} = \frac{m}{n}$$

$$\therefore m^2 + n^2 = 1 + 16 = 17$$

68. (d)

$$\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}, \vec{b} = 3\hat{i} + 2\hat{j} + 5\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 3 & 2 & 5 \end{vmatrix}$$

$$= -17\hat{i} - 7\hat{j} + 13\hat{k}$$

$$(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}$$

$$\Rightarrow (\vec{a} \times \vec{b}) - (\vec{c} \times \vec{b}) = -18\hat{i} - 3\hat{j} + 12\hat{k}$$

$$\Rightarrow \vec{b} \times \vec{c} = (-18\hat{i} - 3\hat{j} + 12\hat{k}) - (\vec{a} \times \vec{b})$$

$$= (-18\hat{i} - 3\hat{j} + 12\hat{k}) - (-17\hat{i} - 7\hat{j} + 13\hat{k})$$

$$\vec{b} \times \vec{c} = -\hat{i} + 4\hat{j} - \hat{k}$$

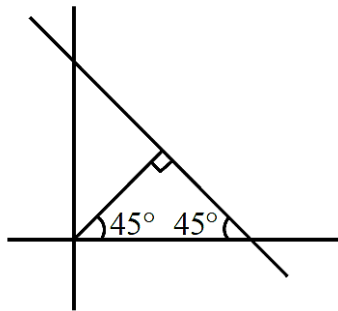
$$\therefore \vec{a} \cdot \vec{d} = \vec{a} \cdot (\vec{b} \times \vec{c}) = (2\hat{i} - 3\hat{j} + \hat{k}) \cdot (-\hat{i} + 4\hat{j} - \hat{k})$$

$$= -2 - 12 - 1 = -15$$

$$\therefore |\vec{a} \cdot \vec{d}| = 15$$

69. (c)

$$\frac{x}{-c} + \frac{y}{-c/b} = 1$$



$$\therefore \text{area of triangle} = \frac{1}{2} \left| \frac{c^2}{b} \right| = 48$$

$$\left| \frac{c^2}{b} \right| = 96$$

$$\therefore -c = -\frac{c}{b}$$

$$\Rightarrow b = 1 \therefore c^2 = 96$$

$$\therefore b^2 + c^2 = 97$$

70. (a)

$$A^3 - 2A^2 - 4A + 4I = 0$$

$$A^3 = 2A^2 + 4A - 4I$$

$$A^4 = 2A^3 + 4A^2 - 4A$$

$$= 2(2A^2 + 4A - 4I) + 4A^2 - 4A$$

$$A^4 = 8A^2 + 4A - 8I$$

$$A^5 = 8A^3 + 4A^2 - 8A$$

$$= 8(2A^2 + 4A - 4I) + 4A^2 - 8A$$

$$A^5 = 20A^2 + 24A - 32I$$

$$\therefore \alpha = 20, \beta = 24, \gamma = -32$$

$$\therefore \alpha + \beta + \gamma = 12$$

Section-B

71. (21)

$$\text{I.F.} = e^{\int 2\sec^2 x dx}$$

$$= e^{2\tan x}$$

Solution of diff. eq.

$$y \cdot e^{2\tan x} = \int e^{2\tan x} (2\sec^2 x + 3\tan x \cdot \sec^2 x) dx$$

$$y \cdot e^{2\tan x} = \int e^{2\tan x} \cdot (2\sec^2 x) dx + \int e^{2\tan x} \cdot (3\tan x \cdot \sec^2 x) dx$$

$$y \cdot e^{2\tan x} = e^{2\tan x} \cdot 2\tan x - \int e^{2\tan x} \cdot 2\sec^2 x \times 2\tan x dx + \int e^{2\tan x} \cdot 3\tan x \cdot \sec^2 x dx$$

$$y \cdot e^{2\tan x} = 2\tan x \cdot e^{2\tan x} - \int e^{2\tan x} \cdot \tan x \sec^2 x dx$$

$$y \cdot e^{2\tan x} = 2\tan x \cdot e^{2\tan x} - \frac{\tan x \cdot e^{2\tan x}}{2} + \frac{e^{2\tan x}}{4} + C$$

$$y = 2\tan x - \frac{\tan x}{2} + \frac{1}{4} + C e^{-2\tan x}$$

$$= 0, y = \frac{5}{4}x$$

$$c = 1$$

$$y\left(\frac{\pi}{4}\right) = \frac{7}{4} + e^{-2}$$

$$\text{Then } 12\left(y\left(\frac{\pi}{4}\right) - e^{-2}\right) = 12\left(\frac{7}{4}\right) = 21$$

72. (441)

$$T_r = \frac{4 \cdot r}{1 + 4 \cdot r^4}$$

$$T_r = \frac{4 \cdot r}{(2r^2 + 2r + 1)(2r^2 - 2r + 1)}$$

$$T_r = \frac{(2r^2 + 2r + 1) - (2r^2 - 2r + 1)}{(2r^2 + 2r + 1)(2r^2 - 2r + 1)}$$

$$T_r = \frac{1}{2r^2 - 2r + 1} - \frac{1}{2r^2 + 2r + 1}$$

$$T_1 = \frac{1}{1} - \frac{1}{5}$$

$$T_2 = \frac{1}{5} - \frac{1}{13}$$

$$T_{10} = \frac{1}{181} - \frac{1}{221}$$

$$S_{10} = 1 - \frac{1}{221} = \frac{220}{221} = \frac{m}{n}$$

$$m + n = 441$$

73. (c)

$$y = \cos\left(\cos^{-1}\frac{1}{2} + \cos^{-1}\frac{x}{2}\right)$$

$$y = \frac{1}{2} \times \frac{x}{2} - \sqrt{1 - \frac{1}{4}} \sqrt{1 - \frac{x^2}{4}}$$

$$4y = x - \sqrt{3} \sqrt{4 - x^2}$$

$$3(4 - x^2) = x^2 + 16y^2 - 8xy$$

$$12 - 3x^2 = x^2 + 16y^2 - 8xy$$

$$4x^2 + 16y^2 - 8xy = 12$$

$$x^2 + 4y^2 - 2xy = 3$$

$$x^2 + y^2 - 2xy - 3y^2 = 3$$

$$(x - y)^2 + 3y^2 = 3$$

74. (c)

$$\text{Area of } \triangle ABC = \frac{1}{2} \begin{vmatrix} 4 & -2 & 1 \\ 1 & 1 & 1 \\ 9 & -3 & 1 \end{vmatrix} = 6 \text{ square units}$$

$$\text{Maximum area of AFDE} = \frac{1}{2} \times 6 = 3 \text{ sq. units}$$

75. (7)

Both the roots are positive

$$D \geq 0$$

$$4(a-3)^2 - 4 \times 9(1-a) \geq 0$$

$$a^2 - 6a + 9 - 9 + 9a \geq 0$$

$$a^2 + 3a \geq 0$$

$$a(a+3) \geq 0$$

$$a \in (-\infty, -3] \cup [0, \infty)$$

$$-\frac{b}{2a} > 0$$

$$\frac{2(a-3)}{2(a-1)} > 0$$

$$a \in (-\infty, 1) \cup (3, \infty)$$

$$f(0) = 9 > 0$$

Equation (i) $\cap$ (ii)

$$a \in (-\infty, -3] \cup [0, 1)$$

$$2\alpha + \beta + \gamma - 6 + 0 + 1 = 7$$

