Physics
Section-A

1. (d)

$\tau = M \times B = MB\sin 60 = \frac{\sqrt{3}}{2}MB = 80\sqrt{3}$

$MB = 160$

$U = -M \cdot B = -MB\cos 60$

$U = -160 \times 1/2 = -80J$

2. (d)

Fundamental frequency in close/organ pipe

$(f) = \frac{v}{4\ell}$

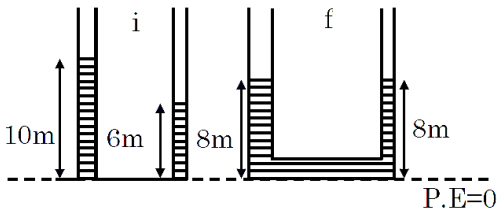
$f_1 = \frac{v}{4\ell_1} \& f_2 = \frac{v}{4\ell_2}$

Beat $= (f_1 - f_2) = \frac{v}{4}\left(\frac{1}{\ell_1} - \frac{1}{\ell_2}\right)$

$15 = \frac{v}{4}\left(\frac{1}{1} - \frac{1}{1.2}\right)$

$v = \left(\frac{15 \times 4 \times 1.2}{0.2}\right) = 60 \times 6 = 360$ m/s

3. (d)



$U_1 = (\rho A \times 10)g \times 5 + (\rho A6)g \times 3$

$U_i = \rho Ag(50 + 18)$

$U_i = 68\rho Ag$

$U_f = (\rho A \times 16)g \times 4$

$= (\rho Ag) \times 64$

$\omega = \Delta U = 4 \times \rho Ag$

$= 4 \times 1000 \times 2 \times 10 = 8 \times 10^4$ J

4. (b)

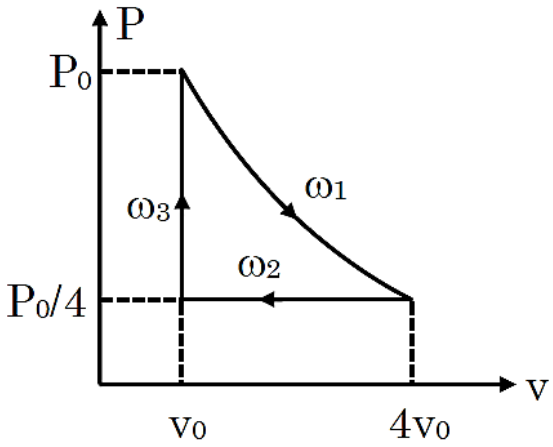
I $\propto$ width

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$I_0\ I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)^2} \Rightarrow \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$

5. (a)



$\omega_1 = P_0 v_0 \ell n 4$

$\omega_2 = \frac{P_0}{4}(-3v_0) = -\frac{3P_0 v_0}{4}$

$\omega_3 = 0$

$Q_T = \Delta U_{cyclic} + \omega$

$Q_T = \omega \left(\Delta U_{cyclic} = 0\right)$

$Q_T = P_0 v_0 \left(\ell n 4 - \frac{3}{4}\right)$

$= P_0 v_0 (2\ell n 2 - 0.75)$

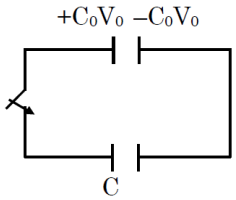
6. (d)

$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} \Rightarrow \frac{(4)^2}{(2)^2} \Rightarrow \frac{16}{4} = 4$

7. (c)

Coneptual

8. (b)



New potential $= \frac{C_0 V_0}{C_0 + C} = \frac{V_0}{3}$

$3C_0V_0 = C_0V_0 + CV_0$

$2C_0V_0 = CV_0$

$C \Rightarrow 2C_0$

9. (a)

$f\lambda = v\,\lambda_{medium} = \frac{\lambda_{vacuum}}{\mu}$

$\lambda_{medium} \Rightarrow \frac{3 \times 10^8}{2 \times 5 \times 10^{14}} \Rightarrow 0.3 \times 10^{-6}$

$\Rightarrow 300$ nm

10. (c)

$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_{cm}$

$v_{cm} \Rightarrow \frac{10 \times 3}{10+5} \Rightarrow \frac{30}{15} = 2\ m/s$

$\frac{1}{2}kx^2 = \frac{1}{2}(10)(3)^2 - \left[\frac{1}{2}(15)(2)^2\right]$

$\Rightarrow 90 - 60 = 30 = 3000x^2$

$x^2 \Rightarrow \frac{30}{3000} = \frac{1}{100}$

$x \Rightarrow \frac{1}{10}\ m$

11. (d)

$\text{Range} = 3H_{max}$

$\frac{u^2 \sin 2\theta}{g} = \frac{3u^2 \sin^2\theta}{2g}$

$2\sin\theta\cos\theta = \frac{3}{2}\sin^2\theta$

$\tan\theta = \frac{4}{3} \Rightarrow \theta = 53^\circ$

$R = \frac{u^2\left(2 \times \frac{3}{5} \times \frac{4}{5}\right)}{g} \Rightarrow \frac{24u^2}{25g}$

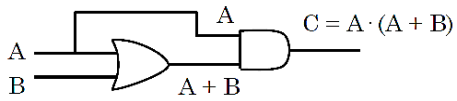
12. (d)

$v_T \Rightarrow \frac{2}{9}\frac{(\rho_0 - \rho_\ell)r^2\, g}{\eta}$

$\eta = \frac{2}{9}\left(\frac{7825 - 925}{2.45 \times 10^{-2}}\right) \times (1.8)^2 \times 10^{-6} \times 9.8$

$\eta \approx 1.99$

13. (b)



A	B	A + B	C
0	0	0	0
1	0	1	1
0	1	1	0
1	1	1	1

14. (d)

$v = \frac{dx}{dt} = 2tC_0 + 2C(t-2)$

$a = \frac{dv}{dt} = 2C_0 + 2C$

15. (a)

$P = v_{rms} i_{rms}$

$i_{rms} = \frac{100}{220}$

$i_0 = \sqrt{2} i_{rms} = 0.64\ A$

16. (a)

(A) $[k] = \frac{PV}{NT} = \frac{ML^2\,T^{-2}}{K} = ML^2\,T^{-2}\,K^{-1}$

(B) $[\eta] = \frac{F}{6\pi rv} = \frac{MLT^{-2}}{L^2\,T^{-1}} = ML^{-1}\,T^{-1}$

(C) $[h] = \frac{E}{f} = \frac{ML^2\,T^{-2}}{T^{-1}} = ML^2\,T^{-1}$

(D) $\frac{dQ}{dt} = k\frac{AdT}{dx}$

$k = \frac{(ML^2\,T^{-3})L}{L^2 \cdot K} = MLT^{-3}\,K^{-1}$

17. (c)

Isochoric process

$P \propto T$

$\frac{\Delta P}{P} = \frac{\Delta T}{T}$

$\frac{0.4}{100} = \frac{1}{T}$

$T = 250\ K$

18. (c)

$P_{input} = Vi = 100\ W$

$\eta = \frac{P_{out}}{P_{input}} = 0.916$

$P_{out} = 91.6\ W$

Loss $= 100 - 91.6$

$= 8.4\ J/s = 2cal/s$

19. (d)

$a = \frac{F}{m} = -10x$

$v\frac{dv}{dx} = -10x$

$\int_{10}^{v} vdv = -10\int_{0.1}^{1.9} xdx$

$\frac{v^2 - 100}{2} = -10\left(\frac{1.9^2 - 0.1}{2}\right)^2$

$v = 8\ m/s$

20. (a)

$r = \frac{mv}{qB} = \frac{p}{qB}$

$r \propto \frac{1}{q}$

Assertion is true reason is false

Section-B

21. (d)

$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\frac{1.5}{200} - \frac{1}{-x} = \frac{1.5 - 1}{50}$

$\frac{1}{x} = \frac{1}{100} - \frac{3}{400}$

$x = 400\ cm$

$x = 4\ m$

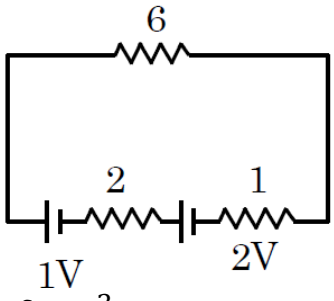
22. (8)

$$\Delta P = \frac{4\,T}{R}$$

$$\frac{R_A}{R_B} = \frac{\Delta P_B}{\Delta P_A} = 2$$

$$\frac{V_A}{V_B} = \left(\frac{R_A}{R_B}\right)^3 = 8$$

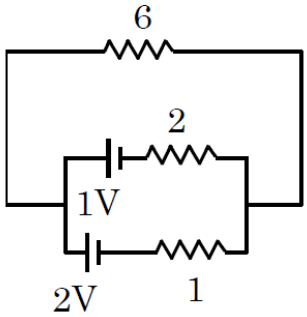
23.(4)



$\varepsilon_{eq} = 3$

$R_{eq} = 9$

$i_1 = \frac{3}{9} = \frac{1}{3}$



$$\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}$$

$$\varepsilon_{eq} = \frac{\frac{1}{2} + \frac{2}{1}}{\frac{1}{2} + \frac{1}{1}} = \frac{5}{3}$$

$$r_{equ} = \frac{2 \times 1}{3} + 6 = \frac{20}{3}$$

$$i_2 = \frac{1}{4} \Rightarrow \frac{i_1}{i_2} = \frac{4}{3}$$

24.(b)

$$E = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_1^2}\right)$$

$$2.86 = 13.6\left(\frac{1}{n^2} - \frac{1}{5^2}\right)$$

$$\frac{1}{n^2} = 0.21 + \frac{1}{2.5}$$

$$n^2 = 4$$

$$n = 2$$

25.(15)

$$C = P^1 q^2 r^{-3} s^{1/2}$$

$$\left(\frac{dC}{C}\right)_{max} = \frac{dP}{P} + \frac{2dq}{q} + \frac{3dr}{r} + \frac{1}{2}\frac{ds}{s}$$

$$= \left(1 + 2 \times 2 + 3 \times 3 + \frac{1}{2} \times 2\right)\%$$

$$= 15\%$$

CHEMISTRY
Section-A

26.(b)

From the given graph 2 ml NaOH solution is used for neutralisation of HCl and 3 ml NaOH solution is used for neutralisation of CH_3COOH.

∴ Mole of HCl = Mole of NaOH used

$M \times 40 = 0.1 \times 2$

$M = 0.005$

∴ Mole of CH_3COOH = Mole of NaOaH used

$M \times 40 = 0.1 \times 3$

$M = 0.0075$

HCl is strong acid and will be neutralized first.

27. (b)

When 10ml, 2MNaOH solution is added to 20 ml of 1 M HCl solution :

$NaOH + HCl \rightarrow NaCl + H_2O$

Initial : $MV = 2 \times 0.1$ $MV = 1 \times 0.2$

$= 0.2$ mole $= 0.2$ mole

Final 0 0

∴ Resulting solution becomes neutral.

Now when 10 mol of above solution is poured into a flask containing 2 mole HCl and made solution 100 ml will distilled water.

Molarity of HCl $= \frac{2}{100} \times 1000 = 20$

28. (d)

Vit D, E, K. A are fat soluble vitamins.

29. (b)

(A) Pnictogen ⇒ Mc (Moscovium),

Atomic No. = 115

(B) Chalcogen ⇒ Lv (Livermorium),

Atomic No. = 116

(C) Halogen ⇒ Ts (Tennessine),

Atomic No. = 117

(D) Noble gas ⇒ 0g (Oganesson),

Atomic No. = 118

30. (d)

Orbital angular momentum $= \sqrt{\ell(\ell+1)}\frac{h}{2\pi}$

∴ For 2s orbital : $\ell = 0$

Orbital angular momentum = 0

∴ For 2 p orbital : $\ell = 1$

Orbital angular momentum $= \sqrt{1(1+2)}\frac{h}{2\pi}$

$= \sqrt{2}\frac{h}{2\pi}$

31. (d)

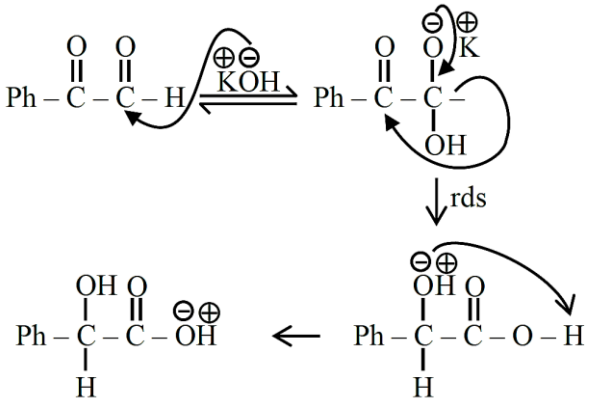
The primary standard is a highly pure stable compound with a known exact composition that can be accurately weighed and dissolved to creat a solution of known concentration.

NaOH is hygroscopic and can't be used.

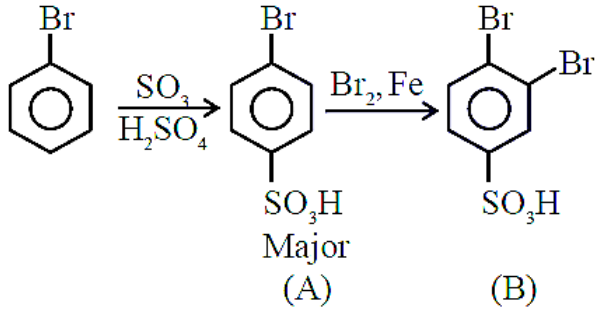
$FeSO_4 \cdot 6H_2O$ is unstable and can be easily oxidised.

$Na_2Cr_2O_7$ is hygroscopic and can't be used.

32.(b)



33.(b)



Both reactions are electrophilic substitution reaction, Ist is sulphonation and IInd is halogenatio

34. (a)

$\because E^{\circ}_{cell} = E^{\circ}_{cathode} - E^{\circ}_{Anode}$

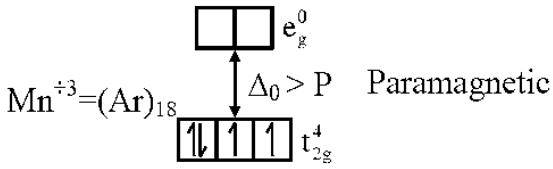
$1.21 = 1.229 - E^{\circ}_{Anode}$

$\because$ Fuel cell involves oxidation of methanol which will occur at anode and reduction of O_2 will occur at cathode.

35. (d)

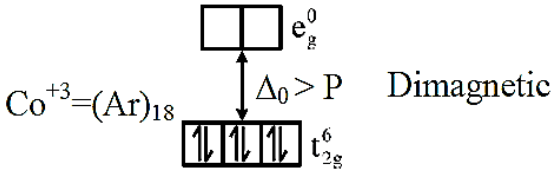
$[Mn(CN)_6]^{3-}$

(a)



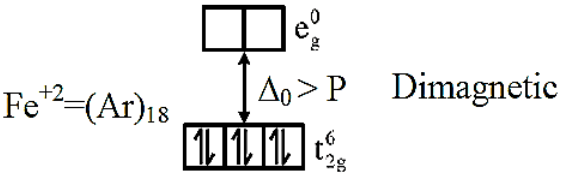
$[Co(NH_3)_6]^{3+}$

(b)



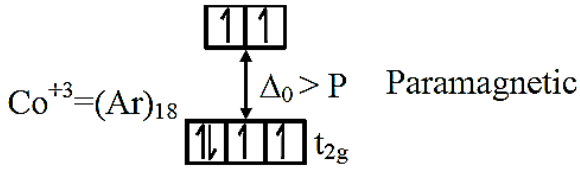
$[Fe(CN)_6]^{4-}$

(c)



$[Co(H_2O)_3F_3]$

(d)



36. (a)

Pressure of N_2 gas evolved $= 715 - 15$

$= 700$ mmHg

$= \frac{700}{760}$ atm.

$\therefore$ Mole of N_2 evolved $= \frac{PV}{RT}$

$= \frac{700 \times 60 \times 10^{-3}}{760 \times 0.0821 \times 300}$

$= 0.0022$ mole

$\therefore$ wt. of N_2 evolved $= 0.0022 \times 28 = 0.063$gm

$\therefore$ wt. % of nitrogen in compound

$= \frac{\text{wt. of nitrogen}}{\text{wt. of compound}} \times 100$

$= \frac{0.063}{0.4} \times 100$

$= 15.71\%$

37. (c)

$Mg + 2HCl \rightarrow MgCl_2 + H_2$

Volume H_2 evolved $= 220$ml

Mole of $H_2 = \frac{220 \times 10^{-3}}{22.4}$ = mole of Mg used

$\therefore$ Mass of Mg used $= \dfrac{220 \times 10^{-3}}{22.4} \times 24$

$= 235.7 \times 10^{-3}\text{gm}$

$= 235.7\text{mg}$

38. (b)

Cellulose derivcative has more number of hydroxy groups, so more H-bonding is present with water in cellulose derivatives cotton cloths.

39. (b)

Statement-I is true but statement II is false.

Cr (VI) is less stable than Mo (VI)

Hence, CrO_3 easily reduce into Cr^{+3} as compared to MoO_3 and show stronger oxidizing nature.

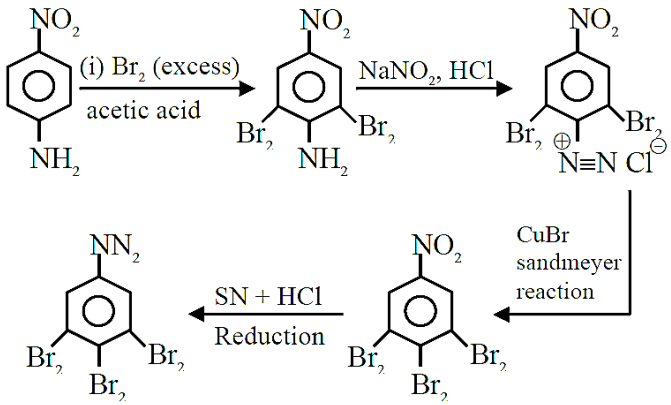
40. (c)

Hyper conjugation is permanent effect because external reagent is not required, so Statement-I is false and Statement-II is true because more alkyl group, more $\alpha - H$, so more hyperconjugation which results more stability of carbocation.

41. (c)

At melting point when ice melts, supplied heat is utilized to overcome intermolecular attraction within the molecules so temperature remain constant.

42.(c)



3,4,5-tri bromo aniline

43.(a)

	B	Al	Ga	In	Tl
Atomic radius (pm)	88	143	135	167	170
Electronegativity	2	1.5	1.6	1.7	1.8
Density (g/cm^3)	2.35	2.7	5.9	7.31	11.85
Ionisation Energy (kJ/mol)	801	577	579	558	589

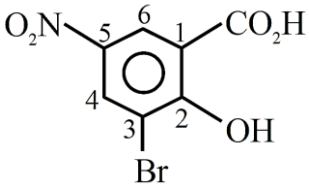
Radius Order $T\ell > In > A\ell > Ga > B$

EN Order $B > T\ell > In > Ga > Al$

Density Order $T\ell > In > Ga > A\ell > B$

IE_1 Order $B > T\ell > Ga > A\ell > In$

44.(a)



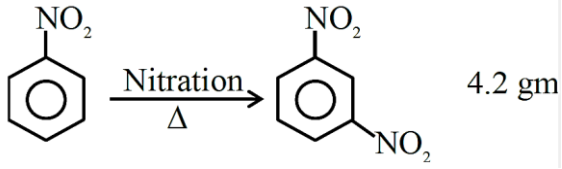
IUPAC 3-Bromo-2-hydroxy-5-nitro-Benzoic acid

45. (c)

Arrhenious equation hold true for elementary as well as complex reactions.
Unit of A is same as unit of k . Rate of reaction is high if activation energy is low,
A and Ea are temperature independent.

Section-B

46. (c)



4.2 gm

$C_6H_5NO_2$ $\quad$ MF = $C_6H_4N_2O_4$
MW = 123 $\quad$ MW = 168

$\therefore \frac{4.2}{168} = 0.025$ mol

$\because$ required gm of nitro benzene
$= 123 \times 0.025$
$= 3.075$

$\therefore$ Nearest integer is 3

47. (304)

$W_{1\to2} = 0$
$W_{2\to3} = -P\Delta V$
$= -3[2-1]$
$= -3 \text{ atm} - \ell$
$W_{3\to4} = 0$
Total work done
$= -3 \text{ atm} - \ell$
$= -3 \times 101.3$ Joule
$= -304$ Joule

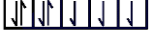
48. (2500)

Mole of octane $= \frac{1.14}{114} = 0.01$ mole
Heat evolved $= C \times \Delta T$
$= 5 \times 5$ kJ
$= 25$ kJ
$\therefore$ Magnitude of Heat of combustion $= \frac{25}{0.01} = 2500 \text{kJ/ mole}$

49. (d)

	Sc	Mn	Co	Cu
Enthalpy of Atomisation (kJ/ mole)	326	281	425	339

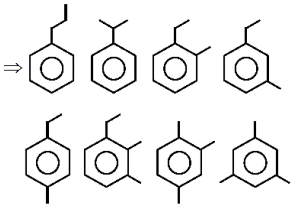
Highest Co
$Co^{+2} = (Ar)3\,d^7$



$n = 3$
$\mu = \sqrt{15} = 3.87$

50. (8)

MF = C_9H_{12}

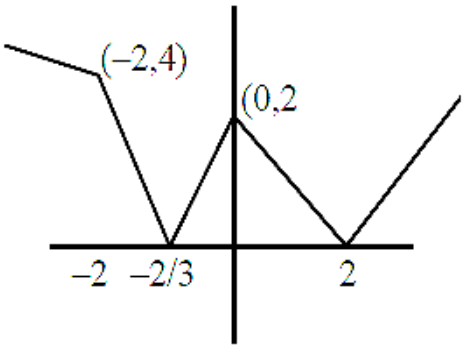


Mathematics
Section-A

51. (b)

$f(x) = ||x+2| - 2|x||$

Critical points, $0, -2, 2, -\frac{2}{3}$



No of Maxima = 1

No of Maxima = 2

52. (a)

$\beta = \frac{\alpha}{2}, \gamma = \frac{\alpha}{2}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\alpha + 2\cos^2\frac{\alpha}{2} = 1$

$\cos^2\alpha + \cos\alpha = 0$

$\cos\alpha(\cos\alpha + 1) = 0$

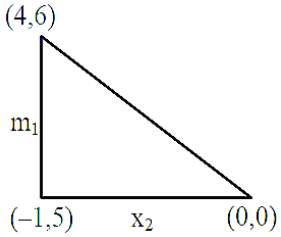
$\cos\alpha = 0, -1$

$\alpha = \frac{\pi}{2}, \pi$

Now $\beta = \frac{\alpha}{2} \Rightarrow \frac{\pi}{4}, \frac{\pi}{2}$

so sum is $\frac{3\pi}{4}$

53. (d)



$m_1\, m_2 = -1$ so right angle equation circle is

$(x-4)(x-0) + (y-6)(y-0) = 0$

$x^2 + y^2 - 4x - 6y = 0$

$(k, 3k)$ lies on it so

$k^2 + 9k^2 - 4k - 18k = 0$

$10k^2 - 22k = 0$

$k = 0, \frac{11}{5}$

$k = 0$ is not possible so $k = \frac{11}{5}$

also $r = \sqrt{4+9} = \sqrt{13}$

so $10k + r^2 = 10 \cdot \frac{11}{5} + (\sqrt{13})^2 = 35$

54. (d)

$\bar{x} = \frac{\sum x_i}{n} = \frac{1+3+a+7+b}{5} = 5$

$a + b = 14$

$\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$

$\Rightarrow \frac{1^2+3^2+a^2+7^2+b^2}{5} - 25 = 10$

$a^2 + b^2 = 116$

$a > b\ a = 10\ b = 4$

$n + x_n$: 2,5,13,11,9

$\sigma^2 = \frac{2^2+5^2+13^2+11^2+9^2}{5} - \left(\frac{2+5+13+11+9}{5}\right)^2$

$= 80 - 64 = 16$

55. (a)

$x(3\lambda+1) + y(7\lambda+2) = 17\lambda + 5$

$(x + 2y - 5) + \lambda(3x + 7y - 17) = 0$

intersection of family of lines

P(1,2)

Let Q(3,6)

$d = PQ = \sqrt{2^2+4^2} = \sqrt{20}$

$d^2 = 20$

56. (a)

$A = \{-2,-1,0,1,2,3\}$

$R = \{(-2,1),(-1,1),(0,1),(1,1),(2,2),(3,3)\}$

$\ell = 6$

$m = 3$

$n = 3$

$\ell + m + n = 12$

57. (a)

$(x^2 + 2x)(12 - k) = 2$

$\lambda x^2 + 2\lambda x - 2 = 0\ k \neq 12$ Let $12 - k = \lambda$

$D = 0$

$4\lambda^2 + 8\lambda = 0$

$\lambda = 0$ or $\lambda = -2$

$\Rightarrow 12 - k = -2$

$k = 14$

So $P\left(k, \frac{k}{2}\right) = (14,7)$

$d = \left|\frac{3 \times 14 + 4 \times 7 + 5}{5}\right| = 15$

58. (b)

Total number of Δ are

$= {}^9C_1\ {}^{12}C_2 + {}^9C_2\ {}^{12}C_1 + {}^1C_1\ {}^9C_1\ {}^{12}C_1$

$= 594 + 432 + 108$

$= 1134$

59. (a)

$$I = \int_0^{\pi} \frac{8xdx}{4\cos^2 x + \sin^2 x}$$

$$I = \int_0^{\pi} \frac{8(\pi - x)dx}{4\cos^2 x + \sin^2 x}$$

$$2I = 8\pi \int_0^{\pi} \frac{dx}{4\cos^2 x + \sin^2 x}$$

$$2I = 8\pi \times 2 \int_0^{\pi/2} \frac{\sec^2 x}{4 + \tan^2 x} dx$$

$$I = 8\pi \int_0^{\infty} \frac{dt}{4 + t^2} = 8\pi \times \frac{1}{2}\left(\tan^{-1}\frac{t}{2}\right]_0^{\infty}$$

$$= 4\pi \times \frac{\pi}{2} = 2\pi^2$$

60. (a)

$f(x) + 3f\left(\frac{24}{x}\right) = 4x$

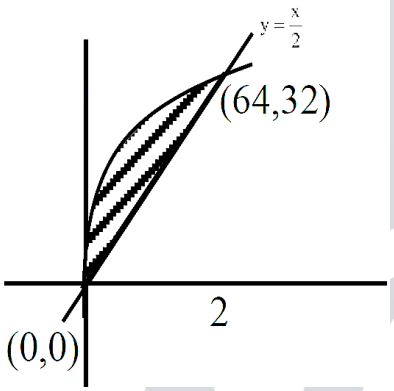
Put $x = 3$ $f(3) + 3f(8) = 12$

Put $x = 8$ $f(8) + 3f(3) = 32$

Add both $4(f(3) + f(8)) = 44$

$f(3) + f(8) = 11$

61. (b)



$|x - y| \le y \le 4\sqrt{x}$

Now $y = |x - y|$

$y^2 = (x - y)^2$

$\Rightarrow y = \frac{x}{2}$ and $x = 0$

Now area $= \int_0^{64} \left(4\sqrt{x} - \frac{x}{2}\right) dx$

$= \left[\frac{4x^{3/2}}{3/2} - \frac{x^2}{4}\right]_0^{64} = \frac{8}{3} \cdot 8^3 - \frac{64^2}{4} = 64^2\left(\frac{1}{12}\right)$

$= \frac{1024}{3}$

62. (a)

Domain $1 - \log_4(x^2 - 9x + 18) > 0$

Also $x^2 - 9x + 18 > 0$

$(x - 3)(x - 6) > 0$

$x \in (-\infty, 3) \cup (6, \infty)$

also $x^2 - 9x + 18 < 4$

$x^2 - 9x + 14 < 0$

$x \in (2,7)$

$(1) \cap (2)$ $(2,3) \cup (6,7) = (\alpha, \beta) \cup (\gamma, \delta)$

$\Rightarrow \alpha + \beta + \gamma + \delta = 18$

63. (d)

$$\sum_{x=0}^{\infty} k(x+1)3^{-x} = 1$$

$$\Rightarrow \frac{1}{k} = 1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \cdots \text{ (i)}$$

$$\frac{1}{3k} = \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \cdots \ldots \text{ (ii)}$$

(i)- (ii) $\Rightarrow \frac{1}{k} - \frac{1}{3k} = 1 + \frac{1}{3} + \frac{1}{3^2} + \cdots$

$$\Rightarrow k = \frac{4}{9}$$

$$P(x \geq 3) = 1 - P(x = 0) - P(x = 1) - P(x = 2)$$

$$= 1 - k\left(1 + \frac{2}{3} + \frac{3}{9}\right) = \frac{1}{9}$$

64.(b)

$$\frac{dy}{dx} + 3(\sec^2 x)y = \sec^2 x, y(0) = \frac{1}{3} + e^3$$

$$\text{If } = e^{3\int \sec^2 x dx} = e^{3\tan x}$$

$$e^{3\tan x} y = \int e^{3\tan x} \sec^2 x dx$$

$$e^{3\tan x} y = \frac{e^{3\tan x}}{3} + c$$

$$\because y(0) = \frac{1}{3} + e^3 \Rightarrow c = e^3$$

$$\therefore y\left(\frac{\pi}{4}\right) = \frac{\frac{e^3}{3} + e^3}{e^3} = \frac{4}{3}$$

65.(a)

$$z_1 + z_2 + z_3 = 3z_0$$

$$(z_1 + z_2 + z_3)^2 = 9z_0^2$$

$$\Rightarrow z_1^2 + z_2^2 + z_3^2 + 2(z_1^2 + z_2^2 + z_3^2) = 9z_0^2$$

$$\Rightarrow z_1^2 + z_2^2 + z_3^2 = 3z_6^2$$

$$\sum_{k=1}^{3} (z_k - z_0)^2 = (z_1 - z_0)^2 + (z_2 - z_0)^2 + (z_3 - z_0)^2$$

$$= z_1^2 + z_2^2 + z_3^2 + 3z_0^2 - 2(z_1 + z_2 + z_3)z_0$$

$$= 6z_0^2 - 6z_0^2$$

$$= 0$$

66. (d)

$$(4 - \sqrt{3})\sin x - 2\sqrt{3}\cos^2 x = \frac{-4}{1 + \sqrt{3}}, x \in \left[-2\pi, \frac{5\pi}{2}\right]$$

$$\Rightarrow (4 - \sqrt{3})\sin x - 2\sqrt{3}(1 - \sin^2 x) = 2(1 - \sqrt{3})$$

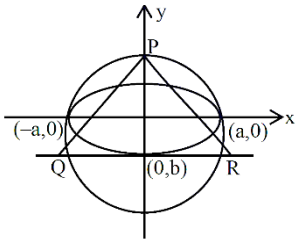
$$\Rightarrow 2\sqrt{3}\sin^2 x + 4\sin x - \sqrt{3}\sin x - 2 = 0$$

$$\Rightarrow (2\sin x - 1)(\sqrt{3}\sin x + 2) = 0$$

$$\Rightarrow \sin x = \frac{1}{2}$$

$\therefore$ Number of solution $= 5$

67. (d)



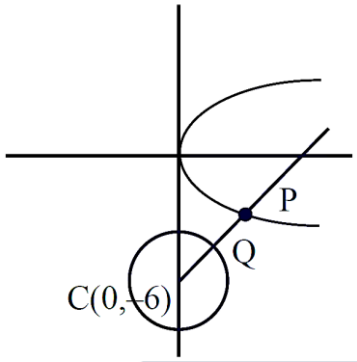
Area of $\triangle PQR$

$= \frac{1}{2}(2a)(a\sin\theta + b)$

$\therefore$ maximum area $= a(a+b)$

$= 4(4+2\sqrt{3}) = 8(2+\sqrt{3})$

68. (d)



Equation of normal to parabola $y^2 = 8x$ is $y = mx - 4m - 2m^3$ passes through $(0,-6)$ we get

$-6 = -4m - 2m^3$

$\Rightarrow m^3 + 2m - 3 = 0$

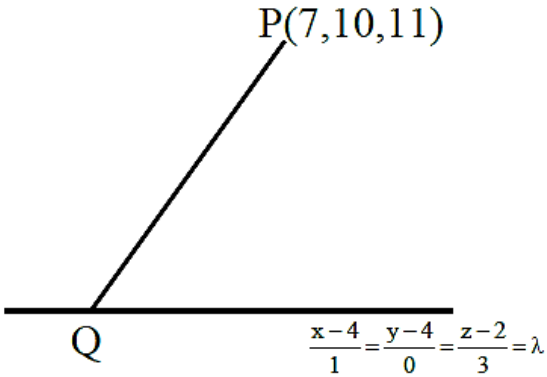
$\Rightarrow (m-1)(m^2+m+3) = 0 \Rightarrow m = -1$

$P = (am^2, -2am) = (2,-4)$

$\therefore$ Shortest distance $= PC - r$

$= (2\sqrt{2} - 1)$

69. (b)



$(\lambda + 4, 4, 3\lambda + 2)$

$\because$ line PQ is parallel to line $\frac{x-9}{2} = \frac{y-3}{3} = \frac{z-17}{6}$

$\therefore \frac{\lambda - 3}{2} = \frac{-6}{3} = \frac{3\lambda - 9}{6} \Rightarrow \lambda = -1$

$Q = (3,4,-1)$

$\therefore PQ = \sqrt{16 + 36 + 144} = 14$

70. (d)

$$S = 1 + \frac{1+3}{2!} + \frac{1+3+5}{3!} + \cdots$$
$$= \sum_{r=1}^{\infty} \frac{r^2}{r!}$$
$$= \sum_{r=1}^{\infty} \frac{(r-1+1)}{(r-1)!} = \sum_{r=2}^{\infty} \frac{1}{(r-2)!} + \sum_{r=1}^{\infty} \frac{1}{(r-1)!}$$
$$= 2e$$

Section-B

71. (38)

$$|A| = \begin{vmatrix} \lambda & 2 & 3 \\ 4 & 5 & 6 \\ 7 & -1 & 2 \end{vmatrix} = -1$$
$$\lambda(16) - 2(-34) + 3(-39) = -1$$
$$16\lambda = 48 \Rightarrow \lambda = 3$$
$$B^{-1} = \text{adj}(A\text{adj}(A^2))$$

Let $C = A.\ \text{adj}\ (A^2)$

$$AC = A^2\text{adj}(A^2) = |A|^2.I = I \Rightarrow C = A^{-1}$$
$$\text{Now } B^{-1} = \text{adj}(A^{-1}) = B = \text{adj}(A)$$
$$\text{Now } \lambda B + I \Rightarrow 3B + I$$
$$\text{Let } P = 3B + I$$
$$P = 3\text{adj}(A) + I$$
$$AP = 3A\text{adj}(A) + A$$
$$AP = 3|A| \cdot I + A$$
$$AP = A - 3I$$
$$|AP| = |A - 3I|$$
$$|A| \cdot |P| = \begin{vmatrix} 0 & 2 & 3 \\ 4 & 2 & 6 \\ 7 & -1 & -1 \end{vmatrix} = 38$$

72. (239)

$$(1 + x + x^2)^{10} = a_0 + a_1x + a_2x^2 + \cdots . + a_{20}x^{20}$$
$$\therefore 3^{10} = a_0 + a_1 + a_2 + \cdots + a_{20} \ldots . \text{(i)}$$
$$1 = a_0 - a_1 + a_2 \ldots . . + a_{20} \ \ldots \text{(ii)}$$
$$\text{(i)} - \text{(ii)} \Rightarrow a_1 + a_3 + \cdots . + a_{19} = \frac{3^{10} - 1}{2} = 29524$$

Also $\{1 + x(1+x)\}^{10} = 1$
$$+ {}^{10}C_1x(1+x) + {}^{10}C_2x^2(1+x)^2 + \cdots$$
$$\therefore a_2 = {}^{10}C_1 + {}^{10}C_2 = 55$$
$$\therefore \frac{(a_1 + a_3 + \cdots + a_{19}) - 11a_2}{121} = 239$$

73. (32)

$$P = \lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$$
$$\Rightarrow P = e^{\lim_{x \to 0} \left(\frac{\tan x - x}{x^3}\right)}$$
$$= e^{\lim_{x \to 0} \frac{\left(x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots . . - x\right)}{x^3}}$$
$$= e^{1/3}$$
$$\therefore 96\log_e{}^{p} = 96 \times \frac{1}{3} = 32$$

74. (128)

$\vec{b} \times \vec{d} = \vec{c} \times \vec{d}$ and $\vec{a} \cdot \vec{d} = 4$

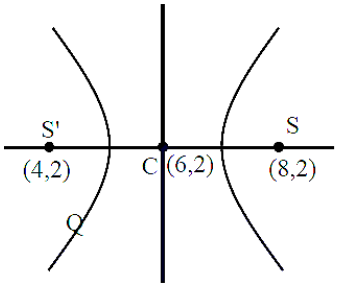
$\Rightarrow \vec{d} = \lambda(\vec{b} - \vec{c}) = \lambda(\hat{\imath} - 2\hat{\jmath} + \hat{k})$

$\because \vec{a} \cdot \vec{d} = 4 \Rightarrow \lambda = -2$

Also. $|\vec{a} \times \vec{d}|^2 + |\vec{a} \cdot \vec{d}|^2 = |\vec{a}|^2 |\vec{d}|^2$

$\Rightarrow |\vec{a} \times \vec{d}|^2 = 6 \times 4 \times 6 - 16 = 128$

75. (141)



Equation of hyperbola is

$$\frac{(x-6)^2}{a^2} - \frac{(y-2)^2}{4-a^2} = 1$$

$\Rightarrow (4-a^2)(x-6)^2 - a^2(y-2)^2 = a^2(4-a^2)$

comparing with $3x^2 - y^2 - \alpha x + \beta y + \gamma = 0$, we get

$a^2 = 1$ and $\alpha = 36, \beta = 4$ and $\gamma = 101$

$\therefore \alpha + \beta + \gamma = 141$