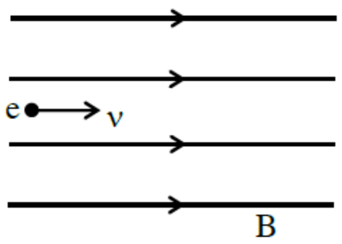
1. (b)



Since $\vec{v} \parallel \vec{B}$ so force on electron due to magnetic field is zero. So it will move along axis with uniform velocity.

2. (d) $\vec{E} = \hat{i}40\cos\omega\left(t - \frac{z}{c}\right)$

$\vec{E}$ is along $+x$ direction

$\vec{v}$ is along $+z$ direction

So, direction of $\vec{B}$ will be along $+y$ and magnitude of B will be $\frac{E}{c}$

So answer is $\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)\hat{j}$

3. (d) Balancing mass number and atomic number ${}_{92}^{235}U + {}_{0}^{1}n \rightarrow {}_{56}^{144}Ba + {}_{36}^{89}Kr + 3\,{}_{0}^{1}n$

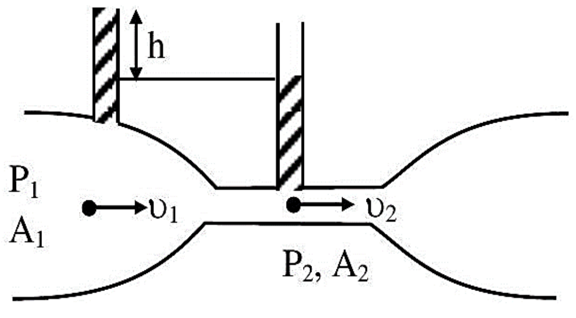
4. (b) $f^{-1} = v^{-1} - u^{-1}$

$-f^{-2}df = -v^{-2}dv - u^{-2}du$

$\frac{df}{f^2} = \frac{dv}{v^2} + \frac{du}{u^2}$

$df = f^2\left[\frac{dv}{v^2} + \frac{du}{u^2}\right]$

5. (d)



Applying Bernoulli's equation

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

[h_1&h_2 are height of point from any reference level]

Given $V_1 = V_2 = 0$ (for statement-1)

$\therefore P_1 - P_2 = \rho g(h_2 - h_2)$

For statement-2

$P_1+\frac{1}{2}\rho v_1^2=P_2+\frac{1}{2}\rho v_2^2$ $P_1-P_2=\rho gh$

$P_1-P_2=\dfrac{1}{2}\rho v_2^2-\dfrac{1}{2}\rho v_1^2$

$\rho gh=\dfrac{1}{2}\rho v_2^2-\dfrac{1}{2}\rho v_1^2$

$2gh=v_2^2-v_1^2$

Hence answer (d)

6. (b) Given $R_0=8\Omega, R_{100}=10\Omega$

$\therefore R_{100}=R_0(1+\alpha\Delta T)$

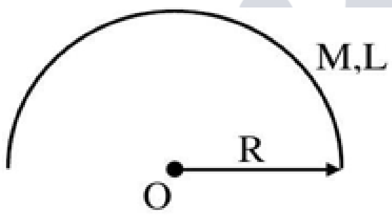
Also, $R_{400}=R_0(1+\alpha\Delta T^1)$

$\therefore 10=8(1+\alpha\times 100)\Rightarrow 100\alpha=\dfrac{1}{4}$

$\therefore R_{400}=8(1+400\alpha)=8(1+1)=16\Omega$

Hence option (b)

7. (d)



We have $R=\frac{L}{\pi}$

$g_0=\dfrac{2G\frac{M}{L}}{R}=\dfrac{2GM\pi}{L^2}$

$\therefore F_m=mg_0=\dfrac{2GM\pi m}{L^2}$

8. (c) We know that per $^\circ C$ change is equivalent to 1.8° change in $^\circ F$.

$\therefore$ 40° change on celcius scale will corresponds to 72° change on Fahrenheit scale.

9. (a) We know that $P_{eq}=\Sigma P_i$

$\therefore$ given all lenses are identical

$\therefore 5P=25D$

$\therefore P=5D$

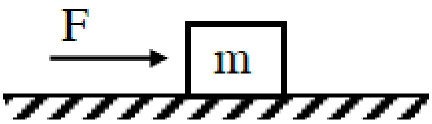
$\therefore \frac{1}{f} = 5 \Rightarrow f = \frac{1}{5}\text{ m} = 20\text{ cm}$

10. (c) Given lights are of same wavelength.

Hence stopping potential will remain same.

Since $I_2 > I_1$, hence saturation current corresponding to I_2 will be greater than that corresponding to I_1.

11. (b)



$F = ma \Rightarrow a = \frac{F}{m} = \frac{kt}{m}$

a vs t will be straight line passing through origin.

12. (d) Velocity just before collision $= \sqrt{2gh}$

Velocity just after collision $= \sqrt{2\,g\left(\frac{h}{2}\right)}$

$\therefore \Delta KE = \frac{1}{2}m(2gh) - \frac{1}{2}mgh$

$= \frac{1}{2}mgh$

$\therefore$ % loss in energy

$= \frac{\Delta KE}{KE_i} \times 100 = \frac{\frac{1}{2}mgh}{\frac{1}{2}mg2\,h} \times 100 = 50\%$

13. (c) Comparing the given equation with standard equation of standing $\frac{2\pi n}{\lambda} = \omega \& \frac{2\pi}{\lambda} = k$

$\left[\frac{n}{\lambda}\right] = [\omega] = T^{-1}$

$[nt] = [\lambda] = L$

$[n] = [\lambda\omega] = LT^{-1}$

$[x] = [\lambda] = L$

14. (b) Given, $102.5 = u + \frac{a}{2}(2n - 1)$ &

$115 = u + \frac{a}{2}(2n + 3)$

$\Rightarrow 102.5 = u + an - \frac{a}{2}$ &

$115 = u + an + \frac{3a}{2}$

$12.5 = 2a \Rightarrow a = 6.25\ m/s^2$

15. (d) Let potential gradient be λ.

$\therefore i \times 10 = \lambda \times 500 = \varepsilon - ir_s$

$\Rightarrow 500\lambda = \varepsilon - 50\lambda r_s$

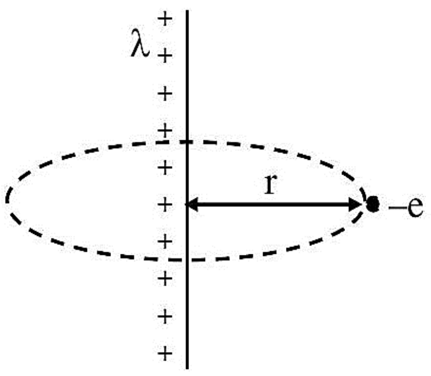
Also,

$i' \times 1 = \lambda \times 400 = \varepsilon - i'r_s$

$\Rightarrow 400\lambda = \varepsilon - 400\lambda r_s$

$\therefore 100\lambda = 350\lambda r_s \Rightarrow r_s = \frac{10}{35} \approx 0.3\Omega$

16. (b)



Electric field E at a distance r due to infinite long wire is $E = \frac{2k\lambda}{r}$

Force of electron $\Rightarrow F = eE$

$F = e\left(\frac{2k\lambda}{r}\right)$

$F = \frac{2k\lambda e}{r}$

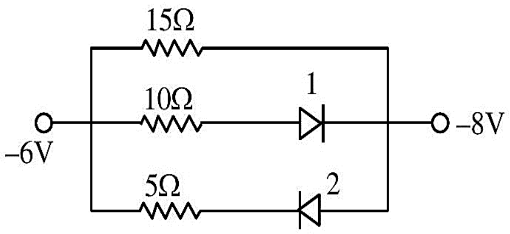
This force will provide required centripetal force

$F = \frac{mv^2}{r} = \frac{2k\lambda e}{r}$

$v = \sqrt{\frac{2k\lambda e}{m}}$

$KE = \frac{1}{2}mv^2 = \frac{1}{2}\ m\left(\frac{2k\lambda e}{m}\right)$
$= k\lambda e$

17. (c)

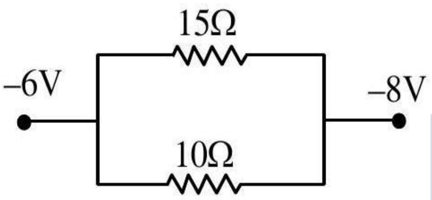


Diode 2 is in reverse bias

So current will not flow in branch of 2nd diode, So we can assume it to be broken wire.

Diode 1 is in forward bias

So it will behave like conducting wire. So new circuit will be



$$R_{eq} = \frac{15 \times 10}{15 + 10} = \frac{15 \times 10}{25} = 6\Omega$$

18. (b) For ideal gas

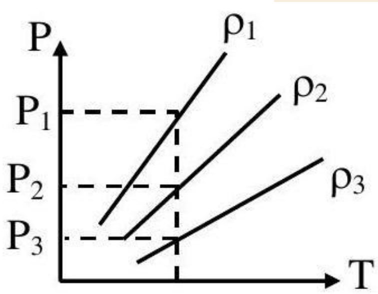
$$PV = nRT$$

$$PV = \frac{m}{M}RT$$

$$P = \left(\frac{M}{V}\right)\frac{RT}{M}$$

$$P = \frac{\rho TT}{M}$$

(Where m is mass of gas and M is molecular mass of gas)



for same temperature $P_1 > P_2 > P_3$

So $\rho_1 > \rho_2 > \rho_3$

So correct answer is (b)

19. (d)

$x = 2 + 4t$

$\frac{dx}{dt} = v_x = 4$

$\frac{dv_x}{dt} = a_x = 0$

$y = 3t + 8t^2$

$\frac{dy}{dt} = v_y = 3 + 16t$

$\frac{dv_y}{dt} = a_y = 16$

the motion will be uniformly accelerated motion.

For path

$x = 2 + 4t$

$\frac{(x-2)}{4} = t$

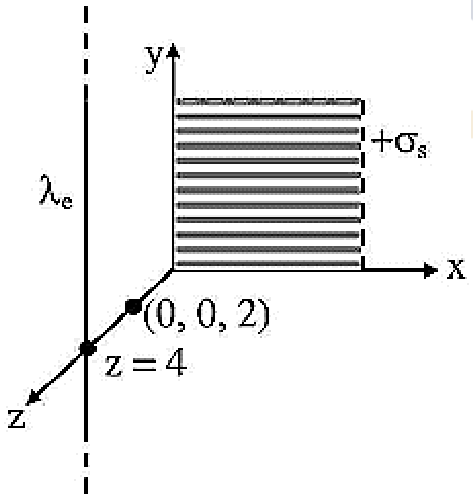
Put this value of t is equation of y

$y = 3\left(\frac{x-2}{4}\right) + 8\left(\frac{x-2}{4}\right)^2$

this is a quadratic equation so path will be parabola.

20. (d) This is possible when phase difference is $\frac{\pi}{2}$ between current and voltage so correct answer will be (d)

21. (16)



$\frac{E_S}{E_\ell} = \frac{\sigma}{2\epsilon_0} \times \frac{2\pi\epsilon_0 r}{\lambda}$

$$= \frac{\pi \times \sigma r}{\lambda}$$

$$= \frac{\pi \times 2\lambda \times 2}{\lambda} = \frac{4\pi}{1}$$

$\therefore n = 16$

22. (7) $\Delta E = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.9\text{eV}$

$$\Delta E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{\Delta E}$$

$$P_i = P_f$$

$$0 = -mv + \frac{h}{\lambda'}$$

$$\Rightarrow v = \frac{h}{m\lambda'}$$

$$\Delta E = \frac{1}{2}mv^2 + \frac{hc}{\lambda'}$$

$$= \frac{1}{2}\, m\left(\frac{h}{m\lambda'}\right)^2 + \frac{hc}{\lambda'}$$

Now

$$\Delta E = \frac{h^2}{2\, m\lambda'^2} + \frac{hc}{\lambda'}$$

$$\lambda'^2\Delta E - hc\lambda' - \frac{h^2}{2\, m} = 0$$

$$\lambda' = \frac{hc \pm \sqrt{h^2c^2 + \frac{4\Delta E h^2}{2m}}}{2\Delta E}$$

$$\lambda' = \frac{hc \pm hc\sqrt{1+\frac{2\Delta E}{mc^2}}}{2\Delta E}\ \frac{\lambda'}{\lambda} = \frac{1+\left(1+\frac{2\Delta E}{mc^2}\right)^{\frac{1}{2}}}{2} = \frac{1+1+\frac{\Delta E}{mc^2}}{2}$$

$$\frac{\lambda'}{\lambda} = 1 + \frac{\Delta E}{2mc^2}$$

$$\frac{\lambda' - \lambda}{\lambda} = \frac{\Delta E}{2mc^2} = \frac{1.9 \times 1.6 \times 10^{-19}}{2 \times 1.67 \times 10^{-27} \times 9 \times 10^{16}} = 10^{-9}$$

$\therefore$ % change $\approx 10^{-7}$

Correct answer 7

23. (50) Force on segment parallel to x-axis will cancel each other. Hence F_{net} will be due to portion parallel to y-axis.

$F = 0.5 \times 0.5 \times 6 \times 0.2 - 0.5 \times 0.5 \times 0.2 \times 5$

$= 0.5 \times 0.5 \times 0.2$

$= 0.25 \times 0.2$

$= 50 \times 10^{-3}$ N

$= 50$mN

24. (8)

$$I_{rms} = \sqrt{\frac{\int i^2 dt}{\int dt}}$$

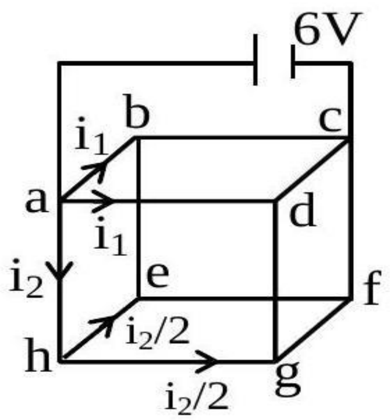
$$I_{rms} = \sqrt{(6)^2 + \frac{(\sqrt{56})^2}{2}}$$

$= \sqrt{36 + 28}$

$= \sqrt{64}$

$= 8$ A

25. (1)



From symmetry, current through e-b & g-d = 0

$\therefore R_{eq} = \frac{3}{4} \times R = \frac{3}{2} \Omega$

$\therefore$ Current through battery $= \frac{6 \times 2}{3} = 4$ A

$i_2 = \frac{4}{8} \times 2 = 1$ A

$\therefore \Delta V$ across e-f $= \frac{i_2}{2} \times R = \frac{1}{2} \times 2 = 1$ V

26. (7) $\omega = \Delta U = S\Delta A$

$$36960 \text{ erg} = \frac{40\text{dyne}}{\text{cm}} 8\pi\left[(r)^2 - \left(\frac{7}{2}\right)^2\right] \text{cm}^2$$

$r = 7$ cm

27. (9)

$$n_2\lambda_2 = n_1\lambda_1$$

$$\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} = \frac{450}{650} = \frac{9}{13}$$

$n_2 = 9$

28. (5) $3 = K(a - \ell)$

$2 = K(b - \ell)$

$T = K(3a - 2b - \ell)$

$T = K(3(a - \ell) - 2(b - \ell)$

$$= K\left[3\left(\frac{3}{K}\right) - 2\left(\frac{2}{K}\right)\right]$$

$= 9 - 4$

$= 5$ N

29. (6)

$|\vec{F}_1| = F$

$|\vec{F}_R| = |\vec{F}_2| = 3F$

$F_R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$

$9F^2 = F^2 + 9F^2 + 6F^2\cos\theta$

$$\cos\theta = -\frac{1}{6}$$

$$\theta = \cos^{-1}\left(\frac{1}{-6}\right)$$

$n = -6$

$|n| = 6$

30. (7) Gain in P.E. = Loss in K.E.

$$mgh = \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$$

$h \propto 1 + \frac{K^2}{R^2}$

$\frac{h_1}{h_2} = \frac{1+\frac{2}{5}}{1+1} = \frac{7}{5 \times 2} = \frac{7}{10}$

$n = 7$

31. (c)

$2e^- + 2H^+(aq) \rightarrow H_2(\,g)$

$E = E^o - \frac{0.059}{n} \log \frac{P_{H_2}}{[H^+]^2}$

$0 = 0 - \frac{0.059}{2} \log \frac{P_{H_2}}{(10^{-7})^2}$

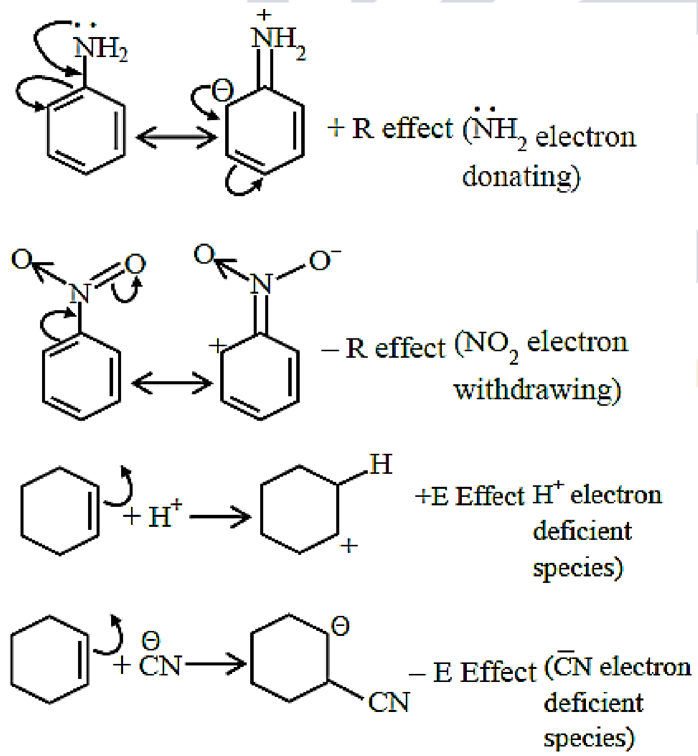
$\log \frac{P_{H_2}}{(10^{-7})^2} = 0$

$\frac{P_{H_2}}{10^{-14}} = 1$

$P_{H_2} = 10^{-14}\text{bar}$

32. (a) According to spectrochemical series ligand field strength is $CO > H_2O > F^- > S^{2-}$

33. (a)



34. (b) Strong acid have weak conjugate base

Acidic strength:

$H-Cl > CH_3COOH > H_2O > R-OH$

Conjugate base strength:

$Cl^- < CH_3COO^- < \bar{O}H < RO^-$

35. (b)

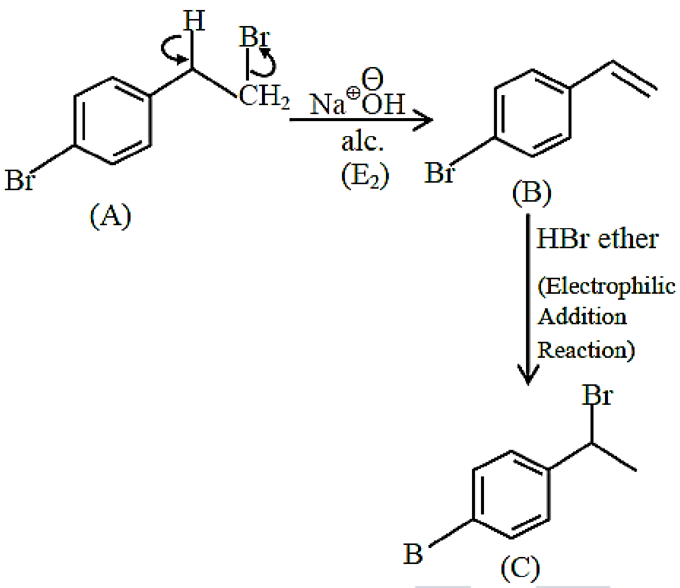
$NH_4OH \rightleftharpoons NH_4^+ + OH^-$

$NH_4Cl \rightarrow NH_4^+ + Cl$

Due to common ion effect of NH_4^+,

$[OH^-]$decreases in such extent that only group-III cation can be precipitated, due to their very low K_{sp} in the range of 10^{-38}.

36. (c)



A and C are position isomer.

37. (a)

38. (a) $[V(H_2O)_6]^{3+} \rightarrow d^2sp^3$

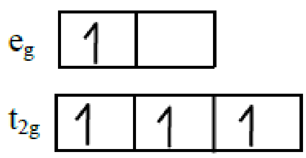
$_{23}V: -[Ar]3\,d^3 4\,s^2$

$V^{+3}: -[Ar]3\,d^2, n = 2$ (even number of unpaired e^-)

$[Cr(H_2O)_6]^{2+} \rightarrow sp^3\,d^2$

$_{24}Cr: -[Ar]3\,d^5 4\,s^1$

Cr^{+2} :- $[Ar]3\,d^4, n = 4$ (even number of unpaired e^-)



$[Fe(H_2O)_6]^{3+} \rightarrow sp^3\ d^2$

$Fe^{3+}: -[Ar]3\ d^5 4\ s^0$

$n = 5$ (odd number of unpaired e^-)

$[Ni(H_2O)_6]^{3+} \rightarrow sp^3\ d^2$

$Ni: -[Ar]3\ d^8 4\ s^2$

$Ni^{+3}: -[Ar]3\ d^7, n = 3$ (odd number of unpaired e^-)

$[Cu(H_2O)_6]^{2+} \rightarrow sp^3\ d^2$

$Cu: -[Ar]3\ d^9 4\ s^0$

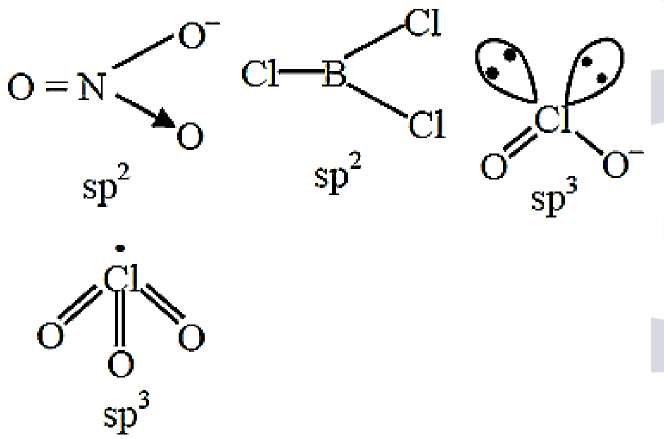
$n = 1$ (odd number of unpaired e^-)

39. (c)

CH_4 & PF_5 , $\mu_{net} = 0$ (non polar)

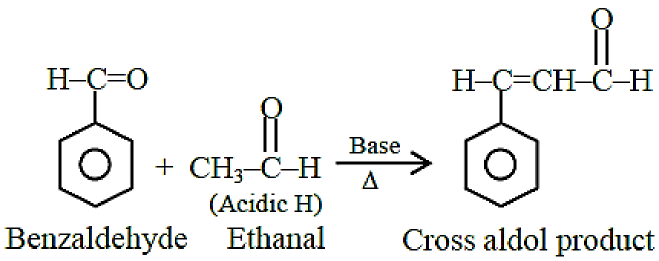
μ_{NH_3}	$>$	μ_{NF_3}
Vector addition of bond moment & lone pair moment		Vector subtraction of bond moment & lone pair moment

40. (a)



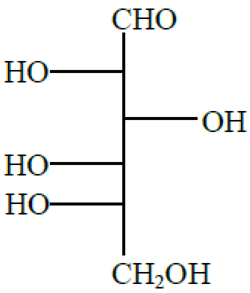
41. (d) Hydrogen ion (H^+)shows positive electromeric effect.

42. (d) Aldehyde and ketones having acidic α-hydrogen show aldol reaction



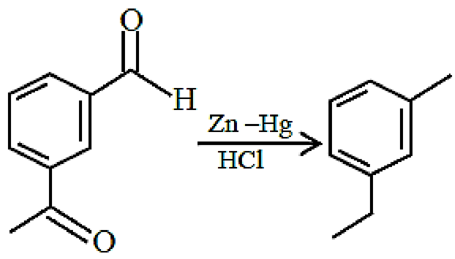
43. (d) Hydrazine $(NH_2 - NH_2)$ have no carbon so does not show Lassaigne's test.

44. (a) Structure of L-Glucose is



45. (b) Co, Ti, Ni can show +2, +3 and +4 oxidation state, But 'Sc' only shows +3 stable oxidation state.

46. (d)



(Clemmensen reduction)

47. (d) N, O, F can't extend their valencies upto their group number due to the non-availability of vacant 2d like orbital.

48. (b) $M = \frac{n_{NaCl}}{V_{sol}\ (\text{in L})}$

$$M = \frac{\frac{5.85}{58.5}}{0.5} = 0.2M$$

49. (c)

$$CH_3{-}CH_2{-}CH_2{-}Br \xrightarrow[80\,^{o}C]{X = \text{conc. alc. NaOH}}$$

$$CH_3{-}CH{=}CH_2 \xrightarrow{Y \equiv HBr/\text{Acetic acid}} CH_3{-}CHBr - CH_3$$

50. (b) Correct order of I^{st} IE

$$\begin{array}{ccccccccccccccc} Li & < & B & < & Be & < & C & < & O & < & N & < & F & < & Ne \\ & & \downarrow & & \downarrow & & & & \downarrow & & \downarrow & & \downarrow & & \\ & & E & < & C & & < & & A & < & B & < & D & & \end{array}$$

51. (125)

$C_2H_4(g) + H_2(g) \rightarrow C_2H_6(g)$

$\Delta H = BE(C = C) + 4BE(C - H) + BE(H - H)$

$-BE(C - C) - 6BE(C - H)$

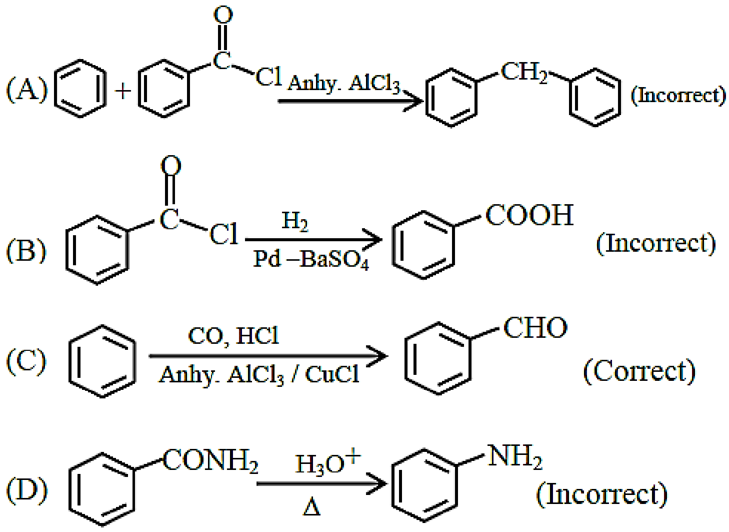
$\Delta H = BE(C = C) + BE(H - H) - BE(C - C)$

$-2BE(C - H)$

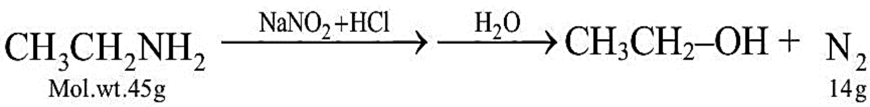
$= 615 + 435 - 347 - 2 \times 414$

$= -125$ kJ

52. (1)



53. (45)



given: N_2 evolved is 2.24 L i.e. 0.1 mole.

i.e. $CH_3CH_2NH_2$ (ethyl amine) will be 4.5 g

$(= 0.1$ mole)

Hence the answer $= 45 \times 10^{-1}$ g

54. (8)

$2\pi r_n = n\lambda_d$

$$2\pi a_0 \frac{n^2}{Z} = n\lambda_d$$
$$2\pi a_0 \frac{4^2}{1} = 4\lambda_d$$
$$\lambda_d = 8\pi a_0$$

55. (50)

eq. $(KMnO_4) =$ eq. $(H_2C_2O_4)$

$M \times 2 \times 5 = 2 \times 20 \times 2$

$M = 8M$

56. (4)

$MnO_2 + KOH + O_2 \rightarrow K_2MnO_4 + H_2O$

(A)

$K_2MnO_4 \xrightarrow{\text{Neutral/acidic solution}} KMnO_4 + MnO_2$

$Mn^{+4}: -[Ar]3\,d^3$

$n = 3, \mu = \sqrt{3(3+2)} = 3.87$ B.M.

Nearest integer is (4)

57. (100)

$$K = \frac{K_1 K_2}{K_3}$$

$$Ae^{\frac{-E_a}{RT}} = \frac{A_1 e^{\frac{-E_{a_1}}{RT}} A_2 e^{\frac{-E_{a_2}}{RT}}}{A_3 e^{\frac{-E_{a_3}}{RT}}}$$

$$Ae^{\frac{-E_2}{RT}} = \frac{A_1 A_2}{A_3} e^{\frac{-(E_{a_1}+E_{a_2}-E_{a_3})}{RT}}$$

$$E_a = E_{a_1} + E_{a_2} - E_{a_3}$$

$$400 = 300 + 200 - E_{a_3}$$

$$E_{a_3} = 100 \text{ kJ/mole}$$

58. (707)

$$2 = 0.52 \times m$$

$$m = \frac{2}{0.52}$$

According to question, solution is much diluted

so $\frac{\Delta P}{P^\circ} = \frac{n_{solute}}{n_{solvent}}$

$$\frac{\Delta P}{P^\circ} = \frac{m}{1000} \times M_{solvent}$$

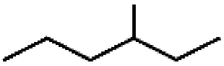
$$\Delta P = P^o \times \frac{m}{1000} \times M_{solvent}$$

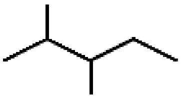
$$= 760 \times \frac{\frac{2}{0.52}}{1000} \times 18 = 52.615$$

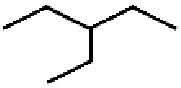
$$P_5 = 760 - 52.615 = 707.385 \text{ mm of Hg}$$

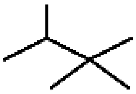
59. (9)

(i)  (ii) 

(iii)  (iv) 

(v)  (vi) 

(vii)  (viii) 

(ix) 

60. (2)

According to M.O.T.

$O_2 \rightarrow$ no. of unpaired electrons $= 2$

$O_2^- \rightarrow$ no. of unpaired electron $= 1$

$NO \rightarrow$ no. of unpaired electron $= 1$

$CN^- \rightarrow$ no. of unpaired electron $= 0$

$O_2^{2-} \rightarrow$ no. of unpaired electron $= 0$

61. (b)

$$f(0^-) = \lim_{x\to 0^-} \frac{2\sin^2 x}{x^2} = 2 = \alpha$$

$$f(0^+) = \lim_{x\to 0^+} \beta \times \sqrt{2}\frac{\sin\frac{x}{2}}{2\frac{x}{2}} = \frac{\beta}{\sqrt{2}} = 2$$

$\Rightarrow \beta = 2\sqrt{2}$

$\alpha^2 + \beta^2 = 4 + 8 = 12$

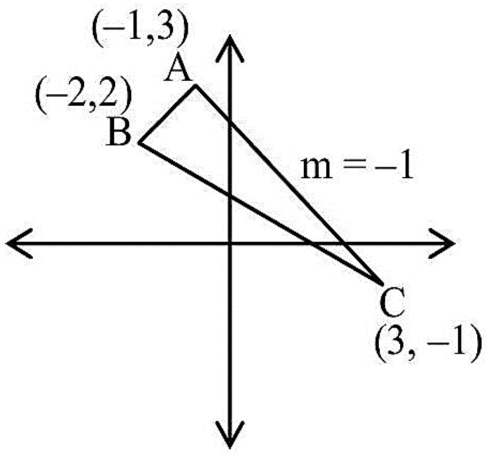
62. (b)

A	B	C
7R, 5 B	5R, 7 B	6R, 6 B

$$P(B) = \frac{1}{3}\cdot\frac{5}{12} + \frac{1}{3}\cdot\frac{7}{12} + \frac{1}{3}\cdot\frac{6}{12}$$

$$\text{required probability } = \frac{\frac{1}{3}\cdot\frac{5}{12}}{\frac{1}{3}\cdot\left[\frac{5}{12}+\frac{7}{12}+\frac{6}{12}\right]} = \frac{5}{18}$$

63. (c)



equation of AC $\rightarrow x+y=2$

equation of line parallel to AC$x+y=d$

$$\left|\frac{d-2}{\sqrt{2}}\right|=1$$

$d=2-\sqrt{2}$

eq^{n} of new required line

$x+y=2-\sqrt{2}$

64. (c)

$$\int dy=\int \frac{(2x^2+2x+3)}{x^4+2x^3+3x^2+2x+2}dx$$

$$y=\int \frac{(2x^2+2x+3)}{(x^2+1)(x^2+2x+2)}dx$$

$$y=\int \frac{dx}{x^2+2x+2}+\int \frac{dx}{x^2+1}$$

$y=\tan^{-1}(x+1)+\tan^{-1}x+C$

$$y(-1)=\frac{-\pi}{4}$$

$$\frac{-\pi}{4}=0-\frac{\pi}{4}+C \Rightarrow C=0$$

$\Rightarrow y=\tan^{-1}(x+1)+\tan^{-1}x$

$$y(0)=\tan^{-1}1=\frac{\pi}{4}$$

65. (d)

$$y=\frac{2x^2-3x+8}{2x^2+3x+8}$$

$x^2(2y-2)+x(3y+3)+8y-8=0$

use $D \geq 0$

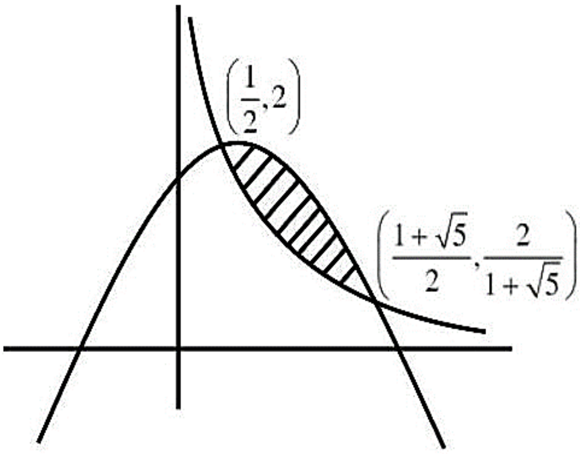
$(3y+3)^2-4(2y-2)(8y-8) \geq 0$

$(11y-5)(5y-11) \leq 0$

$\Rightarrow y \in \left[\frac{5}{11}, \frac{11}{5}\right]$

$y=1$ is also included

66. (b)



$$A=\int_{\frac{1}{2}}^{\frac{1+\sqrt{5}}{2}}\left(1+3x-2x^2-\frac{1}{x}\right)dx$$

$$A=\left[x+\frac{3x^2}{2}-\frac{2x^3}{3}-\ln x\right]_{\frac{1}{2}}^{\frac{1+\sqrt{5}}{2}}$$

$$A=\frac{1+\sqrt{5}}{2}+\frac{3}{2}\left(\frac{1+\sqrt{5}}{2}\right)^2-\frac{2}{3}\left(\frac{1+\sqrt{5}}{2}\right)^3-\ln\left(\frac{1+\sqrt{5}}{2}\right)$$

$$-\frac{1}{2}-\frac{3}{2}\left(\frac{1}{4}\right)+\frac{2}{3}\left(\frac{1}{8}\right)+\ln\left(\frac{1}{2}\right)$$

$$A=\frac{1}{2}+\frac{\sqrt{5}}{2}+\frac{3}{8}+\frac{3}{4}\sqrt{5}+\frac{15}{8}-\frac{4}{3}-\frac{2}{3}\sqrt{5}$$

$$-\frac{1}{2}-\frac{3}{8}+\frac{1}{12}-\ln(1+\sqrt{5})$$

$$=\sqrt{5}\left(\frac{1}{2}+\frac{3}{4}-\frac{2}{3}\right)+\frac{15}{8}-\frac{4}{3}+\frac{1}{12}-\ln(1+\sqrt{5})$$

$$=\frac{14}{24}\sqrt{5}+\frac{15}{24}-\ln(1+\sqrt{5})$$

67. (c)

$$\begin{vmatrix} 1 & \sqrt{2}\sin\alpha & \sqrt{2}\cos\alpha \\ 1 & \sin\alpha & -\cos\alpha \\ 1 & \cos\alpha & \sin\alpha \end{vmatrix} = 0$$

$$\Rightarrow 1 - \sqrt{2}\sin\alpha(\sin\alpha + \cos\alpha) + \sqrt{2}\cos\alpha(\cos\alpha - \sin\alpha) = 0$$

$$\Rightarrow 1 + \sqrt{2}\cos 2\alpha - \sqrt{2}\sin 2\alpha = 0$$

$$\cos 2\alpha - \sin 2\alpha = -\frac{1}{\sqrt{2}}$$

$$\cos\left(2\alpha + \frac{\pi}{4}\right) = -\frac{1}{2}$$

$$2\alpha + \frac{\pi}{4} = 2n\pi \pm \frac{2\pi}{3}$$

$$\alpha + \frac{\pi}{8} = n\pi \pm \frac{\pi}{3}$$

$n = 0,$

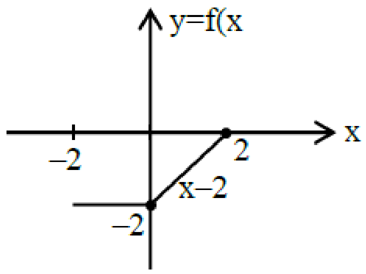
$$x = \frac{\pi}{3} - \frac{\pi}{8} = \frac{5\pi}{24}$$

68. (b)

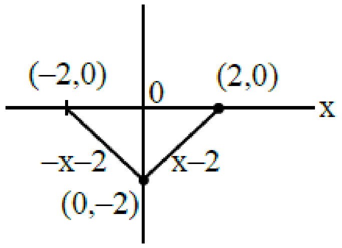
$${}^{18}C_3 - {}^{5}C_3 - {}^{6}C_3 - {}^{7}C_3$$

$= 751$

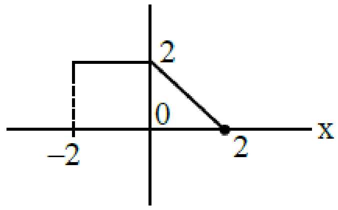
69. (a)



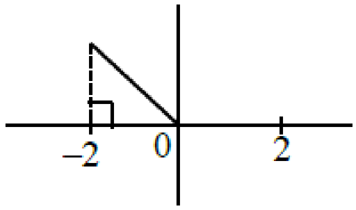
f(|x|) →



|f(x)|



$$h(x) = \begin{cases} x - 2 + 2 - x = 0, & 0 \le x \le 2 \\ -x - 2 + 2 = -x & -2 \le x < 0 \end{cases}$$



70. (a)

$T_{r+1} = {}^{15}C_r \left(5^{\frac{1}{3}}\right)^r \left(2^{\frac{1}{5}}\right)^{15-r}$

$= {}^{15}C_r 5^{\frac{r}{3}} \cdot 2^{\frac{15-r}{5}}$

$R = 3\lambda, 15\mu$

$\Rightarrow r = 0,15$

2 rational terms

$\Rightarrow {}^{15}C_0 2^3 + {}^{15}C_{15}(5)^5$

$= 8 + 3125 = 3133$

71. (d)

$\vec{C} = C_1\hat{i} + C_2\hat{j} + C_3\hat{k}$

$C_1{}^2 + C_2{}^2 + C_3{}^2 = 1$

$\vec{C}.(2\hat{i} + 2\hat{j} - \hat{k}) = |C|\sqrt{9}\cos 60^\circ$

$2C_1 + 2C_2 - C_3 = \frac{3}{2}$

$C_1 - C_3 = 1$

$C_1 + 2C_2 = \frac{1}{2}$

$C_1 = \frac{\sqrt{2}}{3} + \frac{1}{2}$

$C_2 = \frac{-1}{3\sqrt{2}}$

$C_3 = \frac{\sqrt{2}}{3} - \frac{1}{2}$

72. (d) $p^2 = 2q\ 2 = a + 6d\ \ldots(i)$

$p = a + 7\ d$

$q = a + 12\ d$

$p - 2 = d$

((ii) - (i))

$q - p = 5\,d$

((iii) - (ii))

$q - p = 5(p - 2)$

$q = 6p - 10$

$p^2 = 2(6p - 10)$

$p^2 - 12p + 20 = 0$

$p = 10,2$

$p = 10; q = 50$

$d = 8$

$a = -46$

$2,10,50,250,1250$

$ar^4 = a + (n - 1)d$

$1250 = -46 + (n - 1)8$

$n = 163$

73. (a)

$\frac{\sum x_i}{6} = 2$ and $\frac{\sum x_i^2}{N} - \mu^2 = 23$

$\alpha + \beta = 10$

$\alpha^2 + \beta^2 = 52$

solving we get $\alpha = 4, \beta = 6$

$$\frac{\sum|x_i - \bar{x}|}{6} = \frac{5 + 2 + 5 + 8 + 2 + 4}{6} = \frac{13}{3}$$

74. (d) Sum $= 8 = -\frac{b}{a}$

Product $= 12 = \frac{1}{a} \Rightarrow a = \frac{1}{12}$

$b = -\frac{2}{3}$

$$2a + b = \frac{2}{12} - \frac{2}{3} = -\frac{1}{2}$$

$6a + b = \frac{6}{12} - \frac{2}{3} = -\frac{1}{6}$

$\text{sum} = -8$

$P = 12$

$x^2 + 8x + 12 = 0$

75. (b)

$z = x + iy$
$\bar{z} = x - iy$
$\bar{z}^2 = x^2 - y^2 - 2ixy$

$\Rightarrow x^2 - y^2 - 2ixy + \sqrt{x^2 + y^2} = 0$

$\Rightarrow x = 0$ or	$y = 0$
$-y^2 + \lvert y\rvert = 0$	$x^2 + \lvert x\rvert = 0$
$\lvert y\rvert = \lvert y\rvert^2$	$\Rightarrow x = 0$
$y = 0, \pm 1$	$\Rightarrow \alpha = i - i = 0$
$\Rightarrow i, -i$	$\beta = i(-i) = 1$
are roots	$4(0 + 1) = 4$

76. (a) PQ line

$\frac{x - 1}{4} = \frac{y + 2}{-2} = \frac{z - 3}{4}$

pt $(4t + 1, -2t - 2, 4t + 3)$

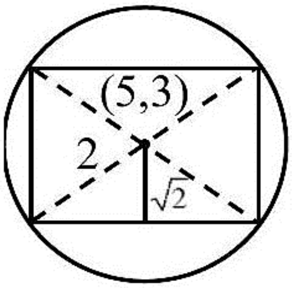
distance $^2 = 16t^2 + 4t^2 + 16t^2 = 81$

$t = \pm\frac{3}{2}$

pt$(7, -5, 9)$

$\alpha^2 + \beta^2 + \gamma^2 = 155$

77. (d)



$y = x + c$ & $x + y + d = 0$

$\left|\frac{5-3+c}{\sqrt{2}}\right| = \sqrt{2}$ $\left|\frac{8+d}{\sqrt{2}}\right| = \sqrt{2}$

$|c+2| = 2$ $8 + d = \pm 2$

$c = 0, -4$ $d = -10, -6$

pts$(5,5), (3,3), (7,3), (5,1)$

$\sum (x_i^2 + y_1^2) = 25 + 25 + 9 + 9 + 49 + 9 + 25 + 1$

$= 152$

78. (a)

$$-1 \le \frac{3x-22}{2x-19} \le 1 \quad \frac{3x^2-8x+5}{x^2-3x-10} > 0$$

$$x \in \left(5, \frac{41}{5}\right]$$

$3\alpha + 10\beta = 97$

79. (a)

$f(x) = 2$

when $x = 0$

$\because g'(f(x))f'(x) = 1$

$$g'(b) = \frac{1}{f'(0)}$$

$$\because f'(x) = 5x^4 + \frac{2}{4}e^{x/4}$$

$g'(b) = 2$

Ans $= 16$

80. (c)

$|\mathrm{adj}(A - 2A^T)(2A - A^T)| = 28$

$|(A - 2A^T)(2A - A^T)| = 24$

$|A - 2A^T||2A - A^T| = \pm 16$

$(A - 2A^T)^T = A^T - 2A$

$|A - 2A^T| = |A^T - 2A|$

$\Rightarrow |A - 2A^T|^2 = 16$

$|A - 2A^T| = \pm 4$

$$\begin{bmatrix}1 & 2 & \alpha\\1 & 0 & 1\\0 & 1 & 2\end{bmatrix}-\begin{bmatrix}2 & 2 & 0\\4 & 0 & 2\\2\alpha & 2 & 4\end{bmatrix}$$

$$\begin{vmatrix}-1 & 0 & \alpha\\-3 & 0 & -1\\-2\alpha & -1 & -2\end{vmatrix}$$

$1+3\alpha=4$

$3\alpha=3$

$\alpha=1$

$$|A|=\begin{vmatrix}1 & 2 & 1\\1 & 0 & 1\\0 & 1 & 2\end{vmatrix}=-1-3=-4$$

$|A|^2=16$

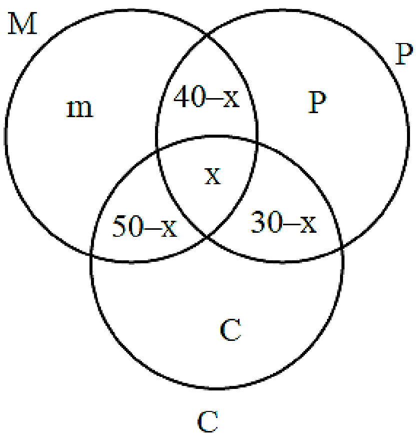
81. (100)

$$\lim_{x\to1}\frac{\frac{1}{3}(5x+1)^{-2/3}5-\frac{1}{3}(x+5)^{-2/3}}{\frac{1}{2}(2x+3)^{-1/2}\cdot2-\frac{1}{2}(x+4)^{-1/2}}$$

$$=\frac{8}{3}\frac{\sqrt{5}}{6^{2/3}}\quad\begin{matrix}m=8\\n=3\end{matrix}$$

$8m+12n=100$

82. (45)



$125\le m+90-x\le130$

$85\le P+70-x\le95$

$75\le C+80-x\le90$

$m+P+C+120-2x=210$

$\Rightarrow 15\le x\le45\ \&\ 30-x\ge0$

$\Rightarrow 15\le x\le30$

$30 + 15 = 45$

83. (7)

$ye^{-x} = \int (e^{-x} + 4e^{-x}\sin x)dx$

$ye^{-x} = -e^{-x} - 2(e^{-x}\sin x e^{-x}\cos x) + C$

$y = -1 - 2(\sin x + \cos x) + e^{x}$

$\because y(\pi) = 1 \Rightarrow c = 0$

$y(\pi/2) = -1 - 2 = -3$

$\text{Ans} = 10 - 3 = 7$

84. (48)

$$\frac{38}{3\sqrt{5}}\hat{k} = \frac{(5\hat{\imath} + 5\hat{\jmath} - 9\hat{k})}{\sqrt{5}} \cdot \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 4 \\ 1 & -3 & 2 \end{vmatrix}$$

$$\frac{38}{3\sqrt{5}}\hat{k} = \frac{19}{\sqrt{5}}$$

$$k = \frac{19}{\sqrt{5}}$$

$$k = \frac{3}{2}$$

$$\int_0^{3/2}[x^2] = \int_0^1 0 + \int_1^{\sqrt{2}} 1 + \int_{\sqrt{2}}^{3/2} 2$$

$$= \sqrt{2} - 1 + 2\left(\frac{3}{2} - \sqrt{2}\right)$$

$$= 2 - \sqrt{2}$$

$\alpha = 2$

$\Rightarrow 6\alpha^3 = 48$

85. (25)

Given $|A| = 2$

trace $A = -3$

and $A^2 + xA + yI = 0$

$\Rightarrow x = 3, y = 2$

so, information is incomplete to determine eccentricity of hyperbola (e) and length of latus rectum of hyperbola (ℓ)

86. (8)

$f(x) = 1 + \frac{(1+x)}{1!} + \frac{(1+x)^2}{2!} + \frac{(1+x)^3}{3!} + \cdots.$

$$\frac{e^{(1+x)}}{1+x} = \frac{1}{1+x} + 1 + \frac{(1+x)}{2!} + \frac{(1+x)^2}{3!} + \frac{(1+x)^2}{4!}$$

coef x^2 in RHS : $1 + \frac{{}^2C_2}{3} + \frac{{}^3C_2}{4} + \cdots = a$

coeff. x^2 in L.H.S.

$$e\left(1 + x + \frac{x^2}{2!}\right) \ldots \left(1 - x + \frac{x^2}{2!} \ldots \ldots\right)$$

is $e - e + \frac{e}{2!} = a$

$$b = 1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \cdots .. = e^2$$

$$\frac{2\,b}{a^2} = 8$$

87. (27)

Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$$A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\Rightarrow a_1 + a_2 + a_3 = 3 \ldots\ldots\ldots\ldots (1)$

$\Rightarrow b_1 + b_2 + b_3 = 3 \ldots\ldots\ldots\ldots (2)$

$\Rightarrow c_1 + ca_2 + c_3 = 3 \ldots\ldots\ldots\ldots. (3)$

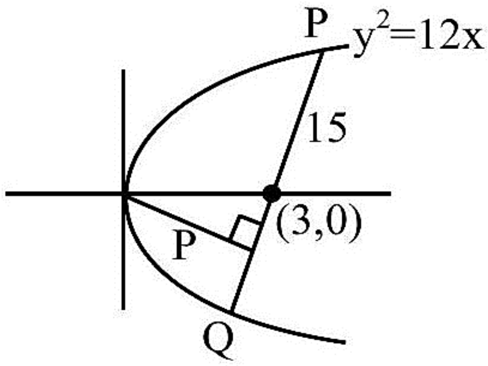
Now,

$|A| = (a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2)$

$-(a_3\, b_2c_1 + a_2\, b_1c_3 + a_1\, b_3c_2)$

$\therefore$ From above in formation, clearly $|A|_{max} = 27$, when $a_1 = 3,\ b_2 = 3, c_3 = 3$

88. (72)



length of focal chord $= 4a\operatorname{cosec}^2\theta = 15$

$12\operatorname{cosec}^2\theta = 15$

$\sin^2\theta = \frac{4}{5}$

$\tan^2\theta = 4$

$\tan\theta = 2$

equation $\frac{y-0}{x-3} = 2$

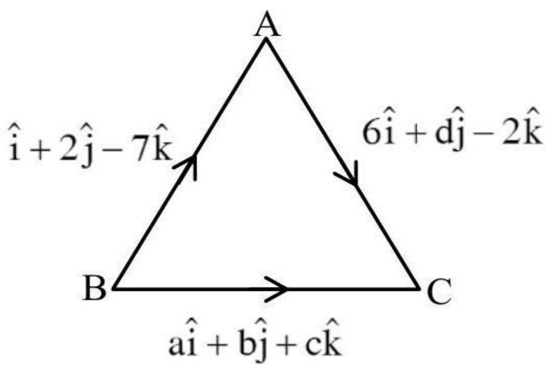
$y = 2x - 6$

$2x - y - 6 = 0$

$P = \frac{6}{\sqrt{5}}$

$10p^2 = 10 \cdot \frac{36}{5} = 72$

89. (54)



Area $= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -7 \\ 6 & d & -2 \end{vmatrix} = 15\sqrt{2}$

$(-4 + 7\,d)\hat{i} - \hat{j}(-2 + 42) + \hat{k}(d - 12)$

$(7d - 4)^2 + (40)^2 + (d - 12)^2 = 1800$

$50\ d^2 - 80\ d - 40 = 0$

$5\ d^2 - 8\ d - 4 = 0$

$5\ d^2 - 10\ d - 2\ d - 4$

$5\ d(\ d - 2) + 2(\ d - 2) = 0$

$d = 2$ or $d = -\frac{2}{5}$

$\because d > 0, d = 2$

$(a+1)\hat{\imath} + (b+2)\hat{\jmath} + (c-7)\hat{k} = 6\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$

$a + 1 = 6 \& b + 2 = 2, c - 7 = -2$

$a = 5\ \ b = 0\ c = 5$

$|AB| = \sqrt{1 + 4 + 49} = \sqrt{54}$

$|BC| = \sqrt{25 + 25} = \sqrt{50}$

$|AC| = \sqrt{86 + 4 + 4} = \sqrt{44}$

Ans. 54

90. (8)

$$\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + \frac{1}{2}\sin 2x} dx = \int_0^{\frac{\pi}{4}} \frac{1 - \cos 2x}{2 + \sin 2x} dx$$

$$\int \frac{1}{2 + \sin 2x} - \int \frac{\cos 2x}{2 + \sin 2x}$$

$(I_1) - (I_2)$

$$(I_1) = \int \frac{dx}{2 + \frac{2\tan x}{1 + \tan^2 x}}$$

$$\int_0^{\frac{\pi}{4}} \frac{\sec^2 xdx}{2\tan^2 x + 2\tan x + 2}$$

$\tan x = t$

$$\frac{1}{2}\int_0^1 \frac{dt}{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}} = \frac{\pi}{6\sqrt{3}}$$

$$I_2 = \int_0^{\pi/4} \frac{\cos 2x}{2 + \sin 2x} dx = \frac{1}{2}\left(\ln\frac{3}{2}\right)$$

$$I_1 - I_2 = \frac{1}{\sqrt{3}}\frac{\pi}{6} + \frac{1}{2}\ln\frac{2}{3}$$

$\Rightarrow a = 2, b = 6$