

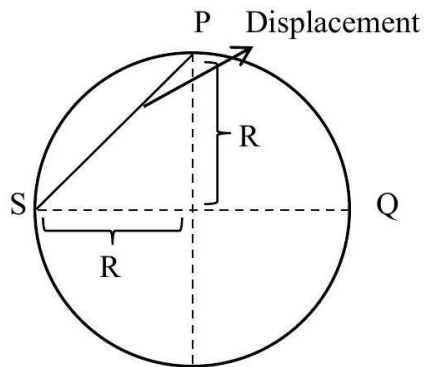
1. (b) Since CH_4 is polyatomic Non-Linear

D.O.F of CH_4

T. DOF = 3

R DOF = 3

2. (b)



$$\therefore \text{Displacement} = R\sqrt{2} = 2\sqrt{2} = \sqrt{8} \text{ km}$$

3. (b)

At stable equilibrium

$$U = -mB \cos 0^\circ = -mB$$

At unstable equilibrium

$$U = -mB \cos 180^\circ = +mB$$

$$W = \Delta U$$

$$\text{W.D.} = 2mB$$

$$= 2(0.5)8 \times 10^{-2} = 8 \times 10^{-2} \text{ J}$$

4. (b) Diode should be in forward biased to calculate dynamic resistance

Hence correct answer would be 2.

5. (c) $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$

6. (b) $Y = \overline{\overline{A}} \cdot \overline{B}$

By De-Morgan Law

$$Y = \overline{\overline{A} + \overline{B}}$$

$$Y = A + B$$

Hence OR gate

7. (a) Since, Intensity $\propto$ width of slit (ω)

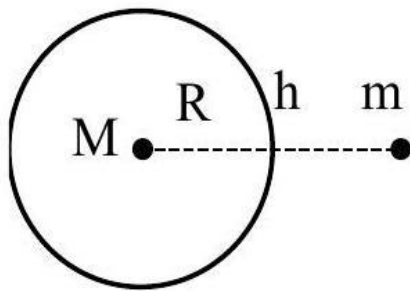
$$\text{so, } I_1 = I, I_2 = 4I$$

$$I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2 = I$$

$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 = 9I$$

$$\frac{I_{\max}}{I_{\min}} = \frac{9I}{I} = \frac{9}{1}$$

8. (b)



$$\Rightarrow \frac{GMm}{(R+h)^2} = \frac{mv^2}{(R+h)}$$

$$\Rightarrow \frac{GM}{(R+h)} = v^2 \dots \dots \dots (1)$$

$$\Rightarrow v = (R+h)\omega$$

$$\Rightarrow v = (R+h) \frac{2\pi}{T} \dots \dots \dots (2)$$

$$\Rightarrow \frac{GM}{R^2} = g$$

$$\Rightarrow GM = gR^2 \dots \dots \dots (3)$$

Put value from (2) & (c) in eq. (1)

$$\Rightarrow \frac{gR^2}{(R+h)} = (R+h)^2 \left(\frac{2\pi}{T} \right)^2$$

$$\Rightarrow \frac{T^2 R^2 g}{(2\pi)^2} = (R+h)^3$$

$$\Rightarrow \left[\frac{T^2 R^2 g}{(2\pi)^2} \right]^{1/3} - R = h$$

9. (d)

A - V lags by 90° from I hence option (I) is correct.

B - V lead by 90° from I hence option (IV) is correct

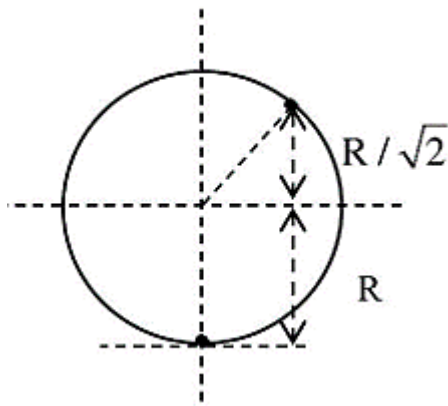
C - In LCR resonance $X_L = X_C$. Hence circuit is purely resistive so option (II) is correct

D - In LCR series V is at some angle from I hence (III) is correct

10. (c) Statement I is correct as we know contact angle depends on cohesive and adhesive forces.

Statement II is incorrect because height of liquid is given by $h = \frac{2 T \cos \theta_c}{\rho g r}$ where r is radius of Tube (assuming length of capillary is sufficient) Hence option (c) is correct.

11. (b)



Apply W.E.T. from A to B

$$\Rightarrow W_m = K_B - K_A$$

$$\Rightarrow mg \times \left(\frac{R}{\sqrt{2}} + R \right) = \frac{1}{2} m v_B^2 - 0 \{ v_A = 0 \text{ rest} \}$$

$$\Rightarrow mgR \frac{(\sqrt{2} + 1)}{\sqrt{2}} = \frac{1}{2} m v_B^2$$

$$\Rightarrow \sqrt{gR \frac{2(\sqrt{2} + 1)}{\sqrt{2}}} = v_B$$

$$\Rightarrow \sqrt{\frac{10 \times 14 \times 2(2.4)}{1.4}} = v_B$$

$$\Rightarrow 21.9 = v_B$$

12. (a) Rated power & voltage gives resistance

$$R = \frac{V^2}{P} = \frac{(200)^2}{50} = \frac{40000}{50}$$

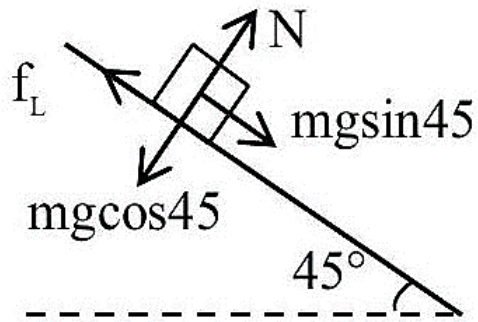
$$R = 800$$

$$P = \frac{(V_{\text{applied}})^2}{R} = \frac{(100)^2}{800}$$

$$P = 12.5 \text{ watt}$$

Hence option 1 is correct.

13. (a)



$$mg \sin 45 = f_L$$

$$mg \cos 45 = N$$

$$f_L = \mu_s N$$

$$\mu_s = \tan 45 = 1$$

or

$$\tan \theta = \mu_s (\theta \text{ is angle of repose})$$

$$\tan 45 = \mu_s = 1$$

$$14. (b) \text{ T.E.} = \frac{1}{2} k A^2$$

since A is same T.E. will be same correct option (b)

$$15. (d) \text{ Intensity of light } I = \frac{n h \nu}{A}$$

Here n is no. of photons per unit time.

$n = \frac{IA}{h\nu}$ so on increasing frequency ν , n decreases taking intensity constant.

$$K_{\max} = h\nu - \phi$$

So on increasing ν , kinetic energy increases.

$$16. (c) \text{ Moment of momentum is } \vec{r} \times \vec{P}$$

$$\vec{L} = \vec{r} \times m\vec{V}$$

$$L = mvr = \frac{nh}{2\pi} = \frac{4h}{2\pi} = \frac{2h}{\pi}$$

$$17. (d) W = \frac{nR\Delta T}{1-\gamma}$$

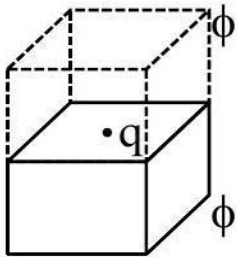
$$TV^{\gamma-1} = \text{const} \quad \tan t = T_f (2V)^{\gamma-1}$$

$$T_f = T \left(\frac{1}{2} \right)^{1/2} = \frac{T}{\sqrt{2}}$$

$$W = \frac{R \left(\frac{T}{\sqrt{2}} - T \right)}{1 - \frac{3}{2}} = 2RT \frac{(\sqrt{2} - 1)}{\sqrt{2}}$$

$$= RT(2 - \sqrt{2})$$

18. (b) From



$$2\phi = \frac{q}{\epsilon_0}$$

$$\phi = \frac{q}{2\epsilon_0}$$

19. (c) According to principle of homogeneity dimension of LHS should be equal to dimensions of RHS so option (c) is correct.

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$[T^2] = \frac{[L^3]}{[M^{-1} L^3 T^{-2}][M]}$$

(Dimension of G is $[M^{-1} L^3 T^{-2}]$)

$$[T^2] = \frac{[L^3]}{[L^3 T^{-2}]} = [T^2]$$

20. (d) Value of $g = g_s \left(1 + \frac{h}{R} \right)^{-2}$

$$= g_s (1 + 2)^{-2} = \frac{g_s}{9}$$

Here g_s = gravitational acceleration at surface

$$\text{Force} = mg = 90 \times \frac{g_s}{9} = 100 \text{ N}$$

21. (10) Given,

$$T = 3.14 = \frac{2\pi}{\omega}$$

$$\omega = 2\text{rad/s}$$

$$x = 10\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$v = \frac{dx}{dt} = 10\omega \cos\left(\omega t + \frac{\pi}{3}\right)$$

$$\text{at } t = 0$$

$$v = 10\omega \cos\left(\frac{\pi}{3}\right) = 10 \times 2 \times \frac{1}{2} [\text{using } \omega = 2\text{rad/s}]$$

$$v = 10 \text{ m/s}$$

$$22. (40) \text{ Initial velocity } u = 72 \text{ km/h} = 20 \text{ m/s}$$

$$v = u + at$$

$$\Rightarrow 0 = 20 + a \times 4$$

$$a = -5 \text{ m/s}^2$$

$$v^2 - u^2 = 2as$$

$$\Rightarrow 0^2 - 20^2 = 2(-5).s$$

$$s = 40 \text{ m}$$

$$23. (750) \text{ Before inserting dielectric capacitance is given } C_0 = 12.5\text{pF} \text{ and charge on the capacitor } Q = C_0 V \text{ After inserting dielectric capacitance will become } \epsilon C_0.$$

$$\text{Change in potential energy of the capacitor} = E_i - E_f$$

$$= \frac{Q^2}{2C_i} - \frac{Q^2}{2C_f} = \frac{Q^2}{2C_0} \left[1 - \frac{1}{\epsilon_r}\right]$$

$$= \frac{(C_0 V)^2}{2C_0} \left[1 - \frac{1}{\epsilon_r}\right] = \frac{1}{2} C_0 V^2 \left[1 - \frac{1}{\epsilon_r}\right]$$

$$\text{Using } C_0 = 12.5\text{pF}, V = 12 \text{ V}, \epsilon_r = 6$$

$$= \frac{1}{2} (12.5) \times 12^2 \left[1 - \frac{1}{6}\right] = \frac{1}{2} (12.5) \times 12^2 \times \frac{5}{6}$$

$$= 750\text{pJ} = 750 \times 10^{-12} \text{ J}$$

$$24. (3)$$

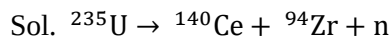
$$\begin{array}{cc} m_1=3\text{kg} & m_2=2\text{kg} \\ \bullet \xrightarrow{2\text{cm}} & \xleftarrow{x} \bullet \end{array}$$

$$\Delta X_{\text{C.O.M.}} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

$$\Rightarrow 0 = \frac{3 \times 2 + 2(-x)}{3 + 2}$$

$$\Rightarrow x = 3 \text{ cm}$$

25. (208)



Disintegration energy

$$Q = (m_R - m_P) \cdot c^2$$

$$m_R = 235.0439\text{u}$$

$$m_P = 139.9054\text{u} + 93.9063\text{u} + 1.0086\text{u}$$

$$= 234.8203\text{u}$$

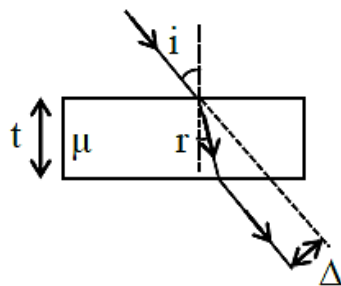
$$Q = (235.0439\text{u} - 234.8203\text{u})c^2$$

$$= 0.2236c^2$$

$$= 0.2236 \times 931$$

$$Q = 208.1716$$

26. (2)



$$i = \theta_c$$

$$\Rightarrow i = \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\Rightarrow i = 45^\circ$$

and according to snell's law

$$1 \sin 45^\circ = \sqrt{2} \sin r$$

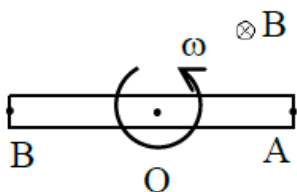
$$\Rightarrow r = 30^\circ$$

$$\text{Lateral displacement } \Delta = \frac{t \sin(i-r)}{\cos r}$$

$$\Rightarrow \Delta = \frac{4\sqrt{3} \times \sin 15^\circ}{\cos 30^\circ}$$

$$\Rightarrow \Delta = 2 \text{ cm}$$

27. (0)



$$\therefore V_0 - V_A = \frac{B\omega\ell^2}{2}$$

$$V_0 - V_B = \frac{B\omega\ell^2}{2}$$

$$\therefore V_A = V_B \therefore V_A - V_B = 0$$

28. (32)

$$\therefore R = \frac{\rho\ell}{A} = \frac{\rho V}{A^2}$$

$$\therefore \frac{R_A}{R_B} = \frac{A_B^2}{A_A^2} = \frac{r_B^4}{r_A^4}$$

$$\Rightarrow \frac{R_A}{2} = \left[\frac{4 \times 10^{-3}}{2 \times 10^{-3}} \right]^4$$

$$\Rightarrow R_A = 32\Omega$$

29. (5) $B_A = \frac{\mu_0 i}{2\pi r} + \frac{\mu_0 (2i)}{2\pi (3r)} = \frac{5\mu_0 i}{6\pi r}$

$$B_C = \frac{\mu_0 (2i)}{2\pi r} + \frac{\mu_0 i}{2\pi (3r)} = \frac{7\mu_0 i}{6\pi r}$$

$$\therefore \frac{B_A}{B_C} = \frac{5}{7}$$

$$\therefore x = 5$$

30. (177)

$$F = P_0 A + \rho_m ghA$$

$$= 10^5 \times \frac{22}{7} \times (2 \times 10^{-2})^2$$

$$+ 1.36 \times 10^4 \times 10 \times (30 \times 10^{-2}) \left(\frac{22}{7} \times (2 \times 10^{-2})^2 \right)$$

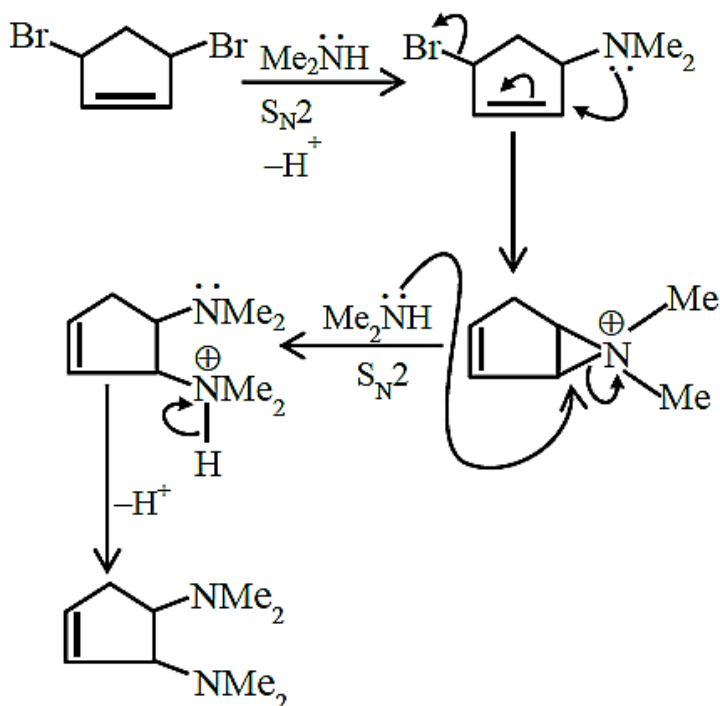
$$F = 51.29 + 125.71 = 177 \text{ N}$$

31. (d)

$$K'_C = \left(\frac{1}{K_C} \right)^2 = \left(\frac{1}{4.9 \times 10^{-2}} \right)^2$$

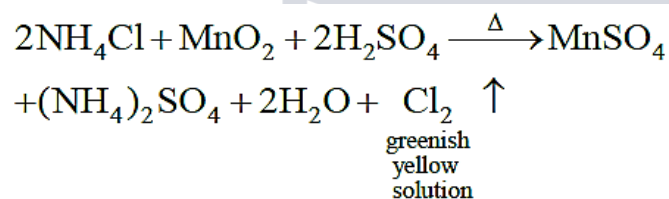
$$K'_C = 416.49$$

32. (b)



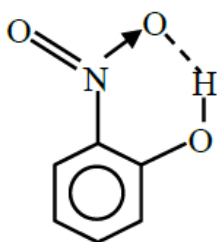
The above mechanism is valid for both cis and trans isomers. So the products are the same for both cis and trans isomers.

33. (d)



34. (b) (A) Generally hydrogen bonding exists when H is covalently bonded to the highly electronegative atom like F, O, N.

(B) Intramolecular H bonding is present in

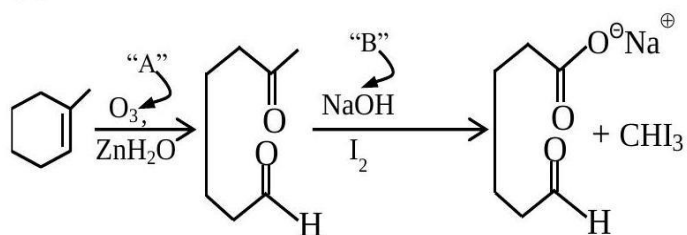


(C) Intermolecular Hydrogen bonding is present in HF

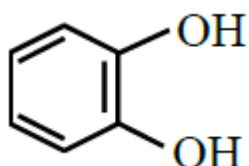
(D) The magnitude of hydrogen bonding in solid state is greater than liquid state.

(E) Hydrogen bonding has a powerful effect on the structure & properties of compounds like melting point, boiling point, density etc.

35. (a)



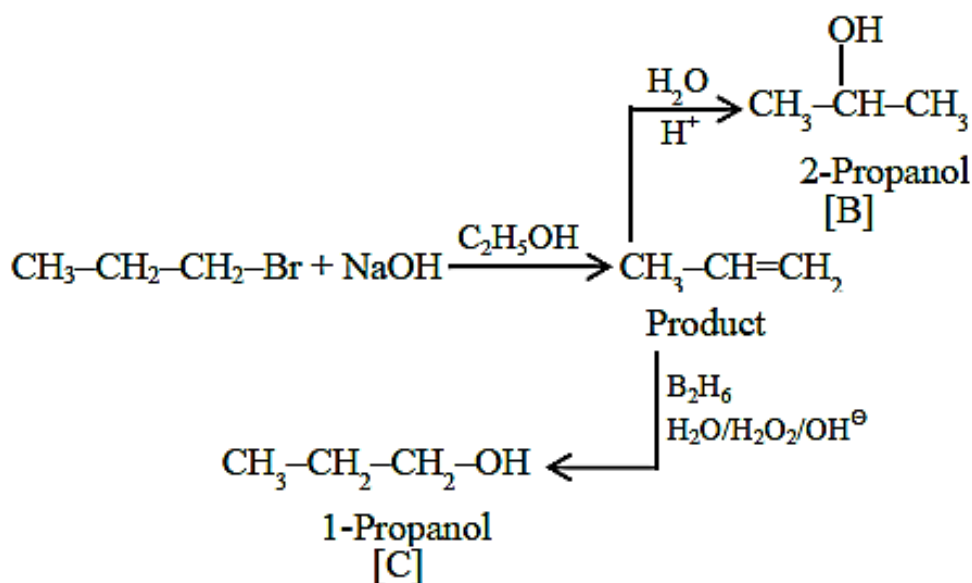
36. (c)



IUPAC name: Benzene-1,2-diol

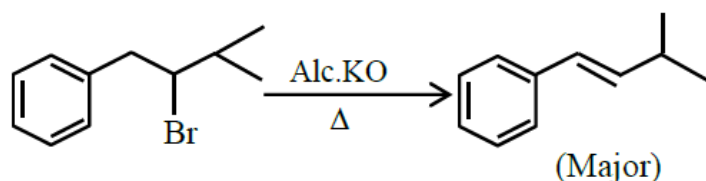
Common name: catechol

37. (b)

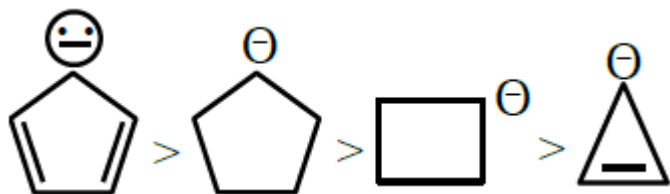


38. (c) The most common polar and acidic support used in adsorption chromatography is silica. The surface silanol groups on their supported to adsorb polar compound and work particularly well for basic substances. Alumina is the example of polar and basic adsorbent that is used in adsorption chromatography.

39. (b)



40. (d) As we know compound (d) is aromatic and the compound (a) is anti-aromatic. Hence compound (d) is most stable and compound (a) is least stable among these in compound (b) and (c) carbon atom of that positive charge is sp^3 hybridised they on the basis of angle strain theory compound (c) is more stable than compound (b).



41. (d) (i) due to lanthanide contraction Tl has more I.E.

as compared to Ga and Al

(ii) due to scandide contraction Ga has more I.E. as compared to Al

42. (b)

Complex is $[Fe(NH_3)_2(CN)_4]^{3-}$

$$x = 2$$

$$y = 4$$

$$\text{so } x + y = 6$$

43. (d) Fuel cell is used in spaceship and it is type of galvanic cell.

44. (a) In compound atoms of different elements combine in fixed ratio by mass.

45. (c) Based on biomolecules theory and structure of these named compounds -

(A) α -Glucose and α -Galactose (IV) Epimers.

(B) α -Glucose and β -Glucose (III) Anomers

(C) α -Glucose and α -Fructose (I) Functional isomers

(D) α -Glucose and α -Ribose (II) Homologous

46. (a)

(i)	Na	<	Li	<	Cl	<	F
	↓		↓		↓		↓
I.E. ₁ in kJ/mol	496		520		1256		1681

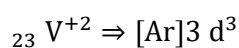
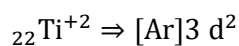
(ii)	Na	<	Li	<	F	<	Cl
	↓		↓		↓		↓
$\Delta_{eg}H$ in kJ/mol	-53		-60		-328		-349

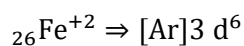
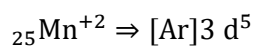
$$47. (a) \Lambda_m = \Lambda_m^0 - A\sqrt{C}$$

$$\text{Units of } A\sqrt{C} = \text{Scm}^2 \text{ mole}^{-1}$$

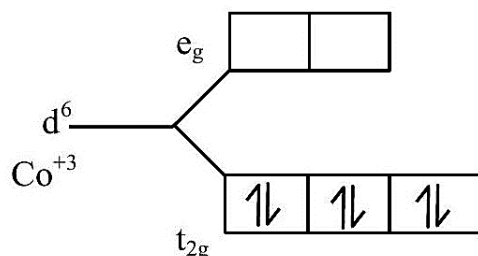
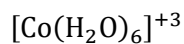
$$\text{Units of } A = \text{Scm}^2 \text{ mole}^{-3/2} \text{ L}^{1/2}$$

48. (d)



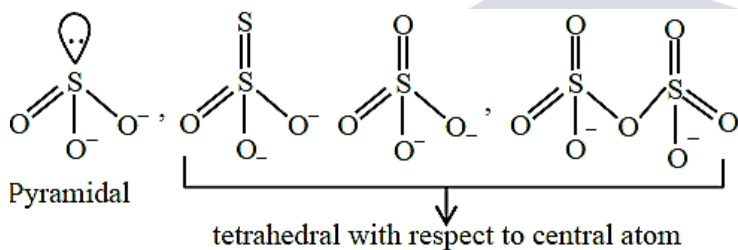


49. (c)

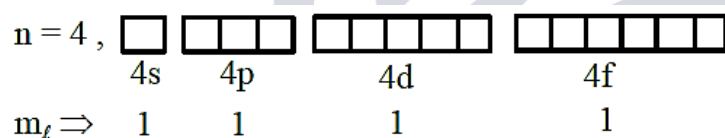


No unpaired electrons

50. (c)



51. (4)



52. (5)

Polar molecule : $\text{NF}_3, \text{H}_2\text{O}, \text{H}_2\text{S}, \text{HBr}, \text{HCl}$
($\mu \neq 0$)

Non Polar molecule : $\text{BeCl}_2, \text{BCl}_3, \text{XeF}_4, \text{CCl}_4, \text{CO}_2, \text{H}_2$
($\mu = 0$)

53. (200)

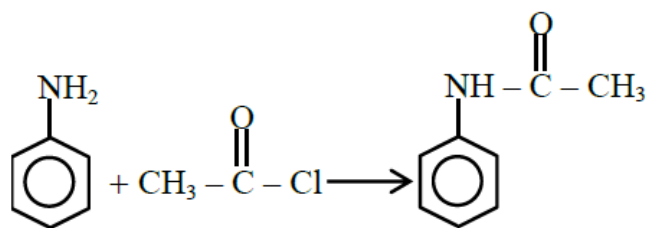
As isothermal $\Delta U = 0$

and process is irreversible

$$Q = -W = -[-P_{\text{ext}}(V_2 - V_1)]$$

$$Q = 5(20 - 60) = -200 \text{ atm} - \text{L}$$

54. (95)



93 g aniline form 135gm acetanlide

so 6.55 g anilne form $\frac{135}{93} \times 6.55 = 9.5$

$$95 \times 10^{-1}$$

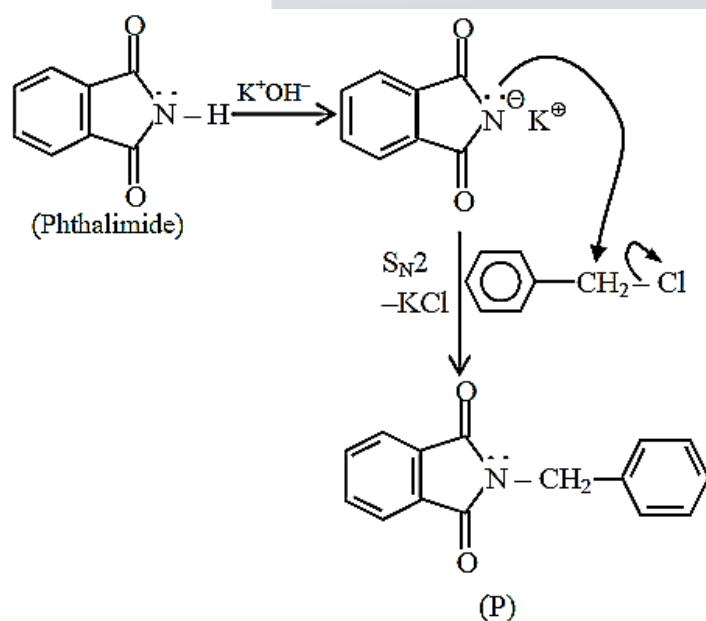
55. (50)

$$K = \frac{2.303}{t} \log \frac{1}{0.1}$$

$$4.6 \times 10^{-2} = \frac{2.303}{t}$$

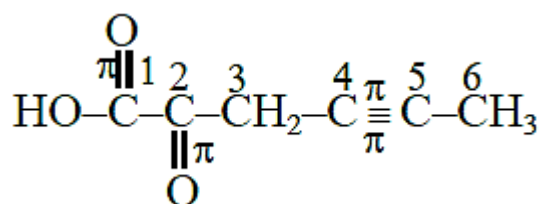
$$t = 50\text{sec.}$$

56. (8)



Total number of π -bonds present in product P is 8

57. (18)



2-Oxohex-4-ynoic acid

Number of σ -bonds = 14

Number of π -bonds = 4

= 18

58. (0) 'V' has highest enthalpy of atomisation (515 kJ/mol) among first row transition elements.

V_2O_5

Here 'V' is in +5 oxidation state

$V^{+5} \Rightarrow 1s^2 2s^2 2p^6 3s^2 3p^6$ (no unpaired electrons)

59. (31) As moles of water > moles of CH_3COOH water is solvent.

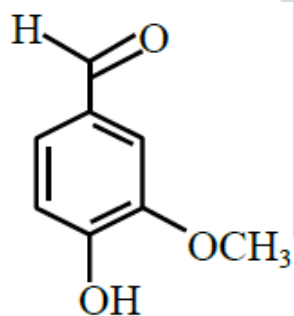
$$T_F^\circ - (T_F)_S = K_F \times M$$

$$0 - (T_F)_S = 1.86 \times \frac{2700/60}{2700/1000}$$

$$(T_F)_S = -31^\circ C.$$

60. (11)

Vanillin compound is an organic compound molecular formula $C_8H_8O_3$. It is a phenolic aldehyde. Its functional compounds include aldehyde, hydroxyl and ether. It is the primary component of the extract of the vanilla beans.



Total sum of oxygen atoms and π -electrons is $3 + 8 = 11$

Total number of oxygen atoms = 3

Total number of π -bonds = 4

$\therefore$ Total number of π -electrons = 8

61. (b) $\lim_{x \rightarrow 0} f(x) = \ln 2 \ln 3$

$$\lim_{n \rightarrow 0} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1 + \cos x}} = \lim_{x \rightarrow 0} \frac{(8^x - 1)(9^x - 1)}{\sqrt{2} - \sqrt{1 + \cos x}}$$

$$\lim_{n \rightarrow 0} \left(\frac{8^x - 1}{x} \right) \left(\frac{9^x - 1}{x} \right) \left(\frac{x^2}{1 - \cos x} \right) (\sqrt{2} + \sqrt{1 + \cos x})$$

$$\therefore \ln 8 \times \ln 9 \times 2 \times 2\sqrt{2} = 24\sqrt{2} \ln 2 \ln 3$$

$$\therefore a = 24\sqrt{2}, a^2 = 576 \times 2 = 1152$$

62. (a)

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0, \lambda > 0$$

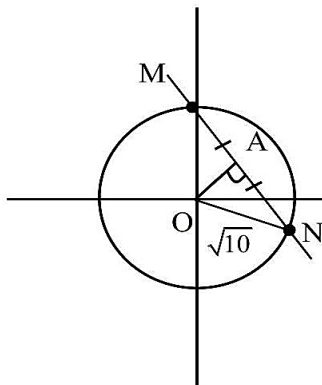
$$|\vec{a}|^2 - |\vec{b}|^2 = 0 \rightarrow 1 + \lambda^2 + 9 = 9 + 1 + 4$$

$$\therefore \lambda = 2, \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{3 - \lambda - 6}{\sqrt{14} \cdot \sqrt{14}}$$

$$14 \cos \theta = 3 - 8 = -5$$

$$\therefore (14 \cos \theta)^2 = 25$$

63. (b)



$$C: x^2 + y^2 = 10$$

$$AN = \frac{MN}{2} = 1$$

$$\therefore \text{In } \triangle OAN \rightarrow (ON)^2 = (OA)^2 + (AN)^2$$

$$10 = (OA)^2 + 1 \rightarrow OA = 3$$

$$\text{Perpendicular distance of center from PQ} = \frac{|0+0-2|}{\sqrt{2}} = \sqrt{2}$$

$$\text{Perpendicular distance between MN and PQ} = OA + \sqrt{2} \text{ or } |OA - \sqrt{2}|$$

$$= 3 + \sqrt{2} \text{ or } 3 - \sqrt{2}$$

64. (b) All $((x_1 y_1), (x_1, y_1))$ are in R where

$x_1, y_1 \in \mathbb{N} \therefore R$ is reflexive

$((1,1), (2,3)) \in R$ but $((2,3), (1,1)) \notin R$

$\therefore R$ is not symmetric

$((2,4), (3,3)) \in R$ and $((3,3), (1,3)) \in R$ but $((2,4), (1,3)) \notin R$

$\therefore R$ is not transitive

65. (a)

$$2b = a + c, b^2 = (a + 1)(c + 3),$$

$$\frac{a + b + c}{3} = 8 \rightarrow b = 8, a + c = 16$$

$$64 = (a + 1)(19 - a) = 19 + 18a - a^2$$

$$a^2 - 18a - 45 = 0 \rightarrow (a - 15)(a + 3) = 0, (a > 10)$$

$$a = 15, c = 1, b = 8$$

$$((abc)^{1/3})^3 = abc = 120$$

$$66. (c) \text{Adj}(A) = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

$$(\text{Adj } A)^2 = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$$

$$(\text{Adj } A)^{10} = \begin{bmatrix} 1 & -20 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} + \dots + \begin{bmatrix} 1 & -20 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 11 & -110 \\ 0 & 11 \end{bmatrix} \Rightarrow \text{sum of elements of } B$$

$$= -88$$

67. (b)

$$\frac{1 \times 2^2 + 2 \times 3^2 + \dots + 100 \times (101)^2}{1^2 \times 2 + 2^2 \times 3 + \dots + 100^2 \times 101} = \frac{\sum_{r=1}^{100} r(r+1)^2}{\sum_{r=1}^{100} r^2(r+1)}$$

$$= \frac{\sum_{r=1}^{100} (r^3 + 2r^2 + r)}{\sum_{r=1}^{100} (r^3 + r^2)} = \frac{\left(\frac{n(n+1)^2}{2}\right) + \frac{2 \cdot n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}}{\left(\frac{n(n+1)}{2}\right)^2 + \frac{n(n+1)(2n+1)}{6}}$$

$$= \frac{\frac{n(n+1)}{2} \left[\frac{n(n+1)}{2} + \frac{2}{3}(2n+1) + 1 \right]}{\frac{n(n+1)}{2} \left[\frac{n(n+1)}{2} + \frac{2n+1}{3} \right]}; \text{ Put } n = 100$$

$$= \frac{\frac{100(101)}{2} + \frac{2}{3}(201) + 1}{\frac{100 \times 101}{2} + \frac{201}{3}} = \frac{5185}{5117} = \frac{305}{301}$$

$$68. (b) \lim_{x \rightarrow 0} \frac{f(x)}{x^3}$$

Using L Hopital Rule.

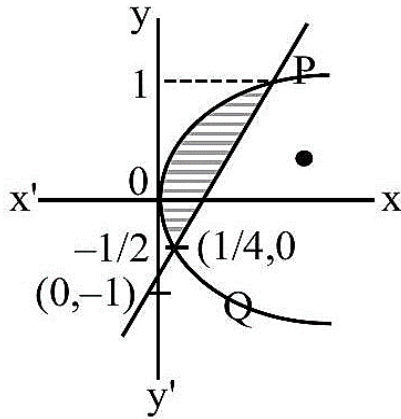
$$\lim_{x \rightarrow 0} \frac{f'(x)}{3x^2} = \lim_{x \rightarrow 0} \frac{x + \sin(1 - e^x)}{3x^2} \text{ (Again L Hopital)}$$

Using L.H. Rule

$$= \lim_{x \rightarrow 0} \frac{-[\sin(1 - e^x)(-e^x) \cdot e^x + \cos(1 - e^x) \cdot e^x]}{6}$$

$$= -\frac{1}{6}$$

69. (d)



$$\text{Shaded area} = \int_{-\frac{1}{2}}^1 (x_{\text{Right}} - x_{\text{Left}}) dy$$

$$\begin{cases} y^2 = 2x \\ y = 4x - 1 \\ y = 1, y = -\frac{1}{2} \end{cases} \quad \text{Solve}$$

$$\text{Shaded area} = \int_{-\frac{1}{2}}^1 \left(\frac{y+1}{4} - \frac{y^2}{2} \right) dy$$

$$= \left(\frac{1}{4} \left(\frac{y^2}{2} + y \right) - \frac{y^3}{6} \right) \Big|_{-\frac{1}{2}}^1 = \frac{9}{32}$$

70. (b)

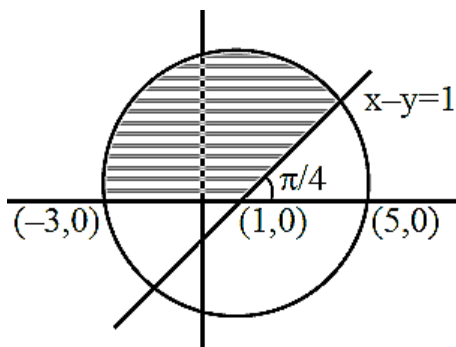
$$\text{Put } z = x + iy$$

$$|z - 1| \leq 2 \Rightarrow (x - 1)^2 + y^2 \leq 4$$

$$(z + \bar{z}) + i(z - \bar{z}) \leq 2 \Rightarrow 2x + i(2iy) \leq 2$$

$$\Rightarrow x - y \leq 1 \quad \dots (2)$$

$$\text{Im}(z) \geq 0 \Rightarrow y \geq 0$$



Required area

= Area of semi-circle – area of sector A

$$\frac{1}{2}\pi(2)^2 - \frac{\pi}{2}$$

$$= \frac{3\pi}{2}$$

71. (b) Let $I = \int_{-1}^{+1} \frac{\cos \alpha x}{1+3^x} dx$

$$I = \int_{-1}^{+1} \frac{\cos \alpha x}{1+3^{-x}} dx$$

(using $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$)

Add (a) and (II)

$$2I = \int_{-1}^{+1} \cos(\alpha x) dx = 2 \int_0^1 \cos(\alpha x) dx$$

$$I = \frac{\sin \alpha}{\alpha} = \frac{2}{\pi} \text{ (given)}$$

$$\therefore \alpha = \frac{\pi}{2}$$

72. (b)

$$f(x) = 3\sqrt{x-2} + \sqrt{4-x}$$

$$x-2 \geq 0 \text{ and } 4-x \geq 0$$

$$\therefore x \in [2, 4]$$

$$\text{Let } x = 2\sin^2 \theta + 4\cos^2 \theta$$

$$\therefore f(x) = 3\sqrt{2}|\cos \theta| + \sqrt{2}|\sin \theta|$$

$$\therefore \sqrt{2} \leq 3\sqrt{2}|\cos \theta| + \sqrt{2}|\sin \theta| \leq \sqrt{9 \times 2 + 2}$$

$$\sqrt{2} \leq 3\sqrt{2}|\cos \theta| + \sqrt{2}|\sin \theta| \leq \sqrt{20}$$

$$\therefore \alpha = \sqrt{2} \quad \beta = \sqrt{20}$$

$$\alpha^2 + 2\beta^2 = 2 + 40 = 42$$

73. (a) Coeff. of $x^4 = {}^nC_4$

$$\text{Coeff. of } x^5 = {}^nC_5$$

$$\text{Coeff. of } x^6 = {}^nC_6$$

$${}^nC_4, {}^nC_5, {}^nC_6 \dots \text{AP}$$

$$2. {}^nC_5 = {}^nC_4 + {}^nC_6$$

$$2 = \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5} \left\{ \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r} \right\}$$

$$2 = \frac{5}{n-4} + \frac{n-5}{6}$$

$$12(n-4) = 30 + n^2 - 9n + 20$$

$$n^2 - 21n + 98 = 0$$

$$(n-14)(n-7) = 0$$

$$n_{\max} = 14 \quad n_{\min} = 7$$

74. (a) Let H: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ($b^2 = a^2(e^2 - 1)$)

$$\therefore \text{eq}^n \text{ of } C_1 = x^2 + y^2 = a^2$$

$$\text{Ar.} = 36\pi$$

$$\pi a^2 = 36\pi$$

$$a = 6$$

Now radius of C_2 can be $a(e-1)$ or $a(e+1)$

for $r = a(e-1)$	for $r = a(e+1)$
------------------	------------------

Ar. = 4π	$\pi r^2 = 4\pi$
--------------	------------------

$\pi a^2(e-1)^2 = 4\pi$	$a^2(e+1)^2 = 4$
-------------------------	------------------

$36\pi(e-1)^2 = 4\pi$	$36(e+1)^2 = 4$
-----------------------	-----------------

$e-1 = \frac{1}{3}$	$e+1 = \frac{1}{3}$
---------------------	---------------------

$e = \frac{4}{3}$	$-\frac{2}{3}$
-------------------	----------------

Not possible

$$\therefore b^2 = 36\left(\frac{16}{9} - 1\right) = 28$$

$$\therefore \text{LR} = \frac{2b^2}{a} = \frac{2 \times 28}{6} = \frac{28}{3}$$

75. (b) $\sum P_i = 1$

$$a + 2a + a + b + 2b + 3b = 1$$

$$4a + 6b = 1 \dots\dots\dots (I)$$

$$E(x) = \text{mean} = \frac{46}{9} \sum P_i X_i = \frac{46}{9} \Rightarrow 4a + 4a + 4b + 12b + 24b = \frac{46}{9}$$

$$8a + 40b = \frac{46}{9}$$

$$4a + 20b = \frac{23}{9} \dots\dots\dots (II)$$

Subtract (I) from (II) we get

$$b = \frac{1}{9} \text{ \& } a = \frac{1}{12}$$

$$\text{Variance} = E(x_i^2) - E(x_i)^2$$

$$E(x_i^2) = 0^2 \times 9^2 + 2^2 \times 2a + 4^2(a + b) + 6^2(2b) + 8^2(3b)$$

$$= 24a + 280b$$

$$\text{Put } a = \frac{1}{12} \quad b = \frac{1}{9}$$

$$E(x_i^2) = 2 + \frac{280}{9} = \frac{298}{9}$$

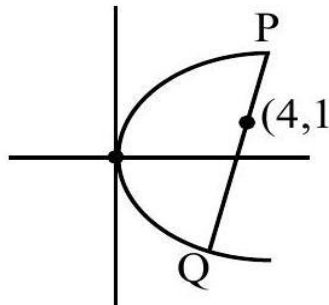
$$\therefore \sigma^2 = E(x_i^2) - E(x_i)^2$$

$$= \frac{298}{9} - \left(\frac{46}{9}\right)^2$$

$$\sigma^2 = \frac{298}{9} - \frac{2116}{81}$$

$$= \frac{566}{81}$$

76. (d)



$$T = S_1$$

$$y - 6(x + 4)$$

$$= 1 - 48$$

$$6x - y = 23$$

$$\left(\frac{1}{2}, -20\right) \text{ will satisfy}$$

77. (b)

$$\cos^{-1} x - \left(\frac{\pi}{2} - \cos^{-1} y\right) = \alpha$$

$$\cos^{-1} x + \cos^{-1} y = \frac{\pi}{2} + \alpha$$

$$\alpha \in \left[-\frac{\pi}{2}, \pi\right], \frac{\pi}{2} + \alpha \in \left[0, \frac{3\pi}{2}\right]$$

$$\cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2}) = \frac{\pi}{2} + \alpha$$

$$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -\sin \alpha$$

$$(xy + \sin \alpha) = (1-x^2)(1-y^2)$$

$$x^2y^2 + 2xysin \alpha + \sin^2 \alpha = 1 - x^2 - y^2 + x^2y^2$$

$$x^2 + y^2 + 2xysin \alpha = 1 - \sin^2 \alpha$$

$$x^2 + y^2 + 2xysin \alpha = \cos^2 \alpha$$

$$\text{Min. value of } \cos^2 \alpha = 0$$

$$\text{At } \alpha = \frac{\pi}{2}$$

Option (b) is correct

$$78. (d) \quad \frac{dy}{dx} + y \left(\frac{2x^3+8x}{(x^2+4)^2} \right) = \frac{2}{(x^2+4)^2}$$

$$\frac{dy}{dx} + y \left(\frac{2x}{x^2+4} \right) = \frac{2}{(x^2+4)^2}$$

$$\text{IF} = e^{\int \frac{2x}{x^2+4} dx}$$

$$\text{IF} = x^2 + 4$$

$$y \times (x^2 + 4) = \int \frac{2}{(x^2 + 4)^2} \times (x^2 + 4)$$

$$y(x^2 + 4) = 2 \int \frac{dx}{x^2 + 2^2}$$

$$y(x^2 + 4) = \frac{2}{2} \tan^{-1} \left(\frac{x}{2} \right) + c$$

$$0 = 0 + c = c = 0$$

$$y(x^2 + 4) = \tan^{-1} \left(\frac{x}{2} \right)$$

$$y \text{ at } x = 2$$

$$y(4 + 4) = \tan^{-1}(a)$$

$$y(b) = \frac{\pi}{32}$$

Option (d) is correct

79. (d)

$$\vec{d} = \lambda(\vec{b} + \vec{c})$$

$$\vec{a} \cdot \vec{d} = \lambda(\vec{b} \cdot \vec{a} + \vec{c} \cdot \vec{a})$$

$$1 = \lambda(1 + x + 5)$$

$$1 = \lambda(x + 6) \dots \dots \dots (1)$$

$$|\vec{d}| = 1 \frac{1}{\lambda} = x + 6$$

$$|\lambda(\vec{b} + \vec{c})| = 1$$

$$|\lambda((x + 2)\hat{i} + 6\hat{j} - 2\hat{k})| = 1$$

$$\lambda^2((x + 2)^2 + 6^2 + 2^2) = 1$$

$$x^2 + 4x + 4 + 36 + 4 = (x + 6)^2$$

$$x^2 + 4x + 44 = x^2 + 12x + 36$$

$$8x = 8, x = 1$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -5 \\ x & 2 & 3 \end{vmatrix} = (\vec{a} \times \vec{b}) \cdot \vec{c}$$

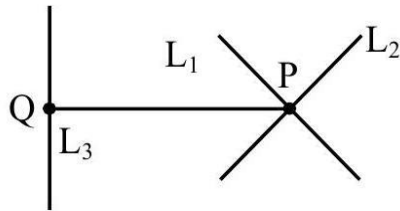
$$\begin{vmatrix} 0 & 0 & 1 \\ -2 & 9 & -4 \\ x - 2 & -1 & 3 \end{vmatrix} = 2 - 9(x - 2)$$

$$= 20 - 9x$$

$$\text{at } x = 1$$

$$20 - 9 = 11$$

80. (c)



$$L_1 \equiv \frac{x - 2}{1} = \frac{y - 4}{5} = \frac{z - 2}{1} = \lambda$$

$$P(\lambda + 2, 5\lambda + 4, \lambda + 2)$$

$$L_2 \equiv \frac{x - 3}{2} = \frac{y - 2}{3} = \frac{z - 3}{2}$$

$$P(2\mu + 3, 3\mu + 2, 2\mu + 3)$$

$$\lambda + 2 = 2\mu + 3 \quad 5\lambda + 4 = 3\mu + 2 \quad \lambda + 2 = 2\mu + 3$$

$$\lambda = 2\mu + 1 \quad 3\mu = 5\lambda + 2$$

$$3\mu = 5(2\mu + 1) + 2$$

$$3\mu = 10\mu + 7$$

$$\mu = -1 \quad \lambda = -1$$

Both satisfies (P)

$$P(1, -1, 1)$$

$$L_3 \equiv \frac{x}{1/4} = \frac{y}{1/2} = \frac{z}{1}$$

$$L_3 = \frac{x}{1} = \frac{y}{2} = \frac{z}{4} = k$$

Coordinates of $Q(k, 2k, 4k)$

$$\text{DR 's of PQ} = \langle k - 1, 2k + 1, 4k - 1 \rangle$$

$$PQ \perp \text{to } L_3$$

$$(k - 1) + 2(2k + 1) + 4(4k - 1) = 0$$

$$k - 1 + 4k + 2 + 16k - 4 = 0$$

$$k = \frac{1}{7}$$

$$Q\left(\frac{1}{7}, \frac{2}{7}, \frac{4}{7}\right)$$

$$PQ = \sqrt{\left(1 - \frac{1}{7}\right)^2 + \left(-1 - \frac{2}{7}\right)^2 + \left(1 - \frac{4}{7}\right)^2}$$

$$= \sqrt{\frac{36}{49} + \frac{81}{49} + \frac{9}{49}} = \frac{\sqrt{126}}{7}$$

$$PQ = \frac{3\sqrt{14}}{7}$$

Option-c will satisfy

81. (4)

$$D = (\sin 2\theta)^2 - 4 \left(1 - \frac{\sin^2 2\theta}{2}\right) \left(1 - \frac{3}{4} \sin^2 2\theta\right)$$

$$= (\sin 2\theta)^2 - 4 \left(1 - \frac{5}{4} \sin^2 2\theta + \frac{3}{8} \sin^4 2\theta\right)$$

$$D = -\frac{3}{2} \sin^4 2\theta + 6 \sin^2 2\theta - 4 > 0$$

$$3 \sin^4 2\theta - 12 \sin^2 2\theta + 8 < 0$$

$$\sin^2 2\theta = \frac{12 \pm \sqrt{12^2 - 12 \cdot 8}}{6} = \frac{12 \pm 4\sqrt{3}}{6} = \frac{6 \pm 2\sqrt{3}}{3}$$

$$\sin^2 2\theta = 2 \pm \frac{2}{\sqrt{3}}, \text{ but } \sin^2 2\theta \in [0, 1]$$

$$\therefore \alpha = 2 - \frac{2}{\sqrt{3}}, \beta = 1 \rightarrow (\alpha - 2)^2 = \frac{4}{3}, (\beta - 1)^2 = 0$$

$$3(\alpha - 2)^2 + (\beta - 1)^2 = 4$$

82. (1)

$$\int \operatorname{cosec}^3 x \cdot \operatorname{cosec}^2 x dx = I$$

By applying integration by parts

$$I = -\cot x \operatorname{cosec}^3 x + \int \cot x (-3 \operatorname{cosec}^2 x \cot x \operatorname{cosec} x) dx$$

$$I = -\cot x \operatorname{cosec}^3 x - 3 \int \operatorname{cosec}^3 x (\operatorname{cosec}^2 x - 1) dx$$

$$I = -\cot x \operatorname{cosec}^3 x - 3I + 3 \int \operatorname{cosec}^3 x dx$$

let

$$I_1 = \int \operatorname{cosec}^3 x dx = -\operatorname{cosec} x \cot x - \int \cot^2 x \operatorname{cosec} x dx$$

$$I_1 = -\operatorname{cosec} x \cot x - \int (\operatorname{cosec}^2 x - 1) \operatorname{cosec} x dx \quad 2I_1 = -\operatorname{cosec} x \cot x + \ln \left| \tan \frac{x}{2} \right|$$

$$I_1 = -\frac{1}{2} \operatorname{cosec} x \cot x + \frac{1}{2} \ln \left| \tan \frac{x}{2} \right|$$

$$4I = -\cot x \operatorname{cosec}^3 x - \frac{3}{2} \operatorname{cosec} x \cot x + \frac{3}{2} \ln \left| \tan \frac{x}{2} \right| + 4c$$

$$I = -\frac{1}{4} \operatorname{cosec} x \cot x \left(\operatorname{cosec}^2 x + \frac{3}{2} \right) + \frac{3}{8} \ln \left| \tan \frac{x}{2} \right| + c$$

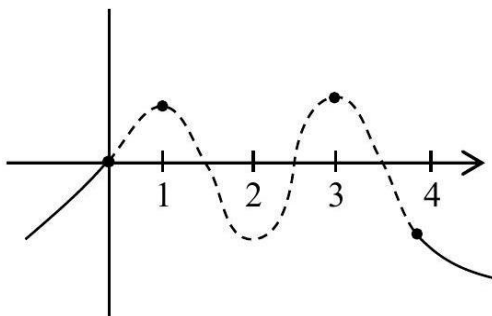
$$\therefore \alpha = \frac{-1}{4}, \beta = \frac{3}{8} \rightarrow 8(\alpha + \beta) = 1$$

83. (5)

$$(3f'f'' + ff''')(x) = ((f'f'') + (f')^2)(x))'$$

$$(ff'' + (f')^2)(x) = ((ff')(x))'$$

$$\therefore (3f'f'' + f''')(x) = (f(x) \cdot f'(x))''$$



min. roots of $f(x) \rightarrow 4$

$\therefore$ min. roots of $f'(x) \rightarrow 3$

$\therefore$ min. roots of $(f(x) \cdot f'(x)) \rightarrow 7$

$\therefore$ min. roots of $(f(x) \cdot f'(x))'' \rightarrow 5$

84. (1024)

$$f(f(x)) = \frac{2f(x)}{\sqrt{1+9f^2(x)}} = \frac{4x}{\sqrt{1+9x^2+9 \cdot 2^2x^2}}$$

$$f(f(f(x))) = \frac{2^3x/\sqrt{1+9x^2}}{\sqrt{1+9(1+2^2)\frac{2^2x^2}{1+9x^2}}} = \frac{2^3x}{\sqrt{1+9x^2(1+2^2+2^4)}}$$

$\therefore$ By observation

$$\alpha = 1 + 2^2 + 2^4 + \dots + 2^{18} = 1 \left(\frac{(2^2)^{10} - 1}{2^2 - 1} \right) = \frac{2^{20} - 1}{3}$$

$$3\alpha + 1 = 2^{20} \rightarrow \sqrt{3\alpha + 1} = 2^{10} = 1024$$

85. (5)

$$\text{Let } A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ b & d \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}, ad - b^2 = 1$$

$$a + b = 3, b + d = 7, (3 - b)(7 - b) - b^2 = 1$$

$$21 - 10b = 1 \rightarrow b = 2, a = 1, d = 5$$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}, A^{-1} = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \alpha A + \beta I$$

$$\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \alpha + \beta & 2\alpha \\ 2\alpha & 5\alpha + \beta \end{bmatrix}$$

$$\alpha = -1, \beta = 6 \rightarrow \alpha + \beta = 5$$

86. (5626)

From Group A	From Group B	Ways of selection
4M	4 W	${}^4C_4 {}^4C_4 = 1$
3M1 W	1M3 W	${}^4C_3 {}^5C_1 C_1 {}^4C_3 = 400$
2M2 W	2M2 W	${}^4C_2 {}^5C_2 {}^5C_2 {}^4C_2 = 3600$
1M3 W	3M1 W	${}^4C_1 {}^5C_3 {}^5C_3 {}^4C_1 = 1600$

4 W	4M	${}^5C_4 {}^5C_4 = 25$
Total		5626

87. (8288)

$$P(W) = \frac{1}{3} \quad P(L) = \frac{2}{3}$$

x = number of matches that team wins

y = number of matches that team loses

$$|x - y| \leq 2 \text{ and } x + y = 10$$

$$|x - y| = 0, 1, 2 \quad x, y \in N$$

$$\text{Case-I: } |x - y| = 0 \Rightarrow x = y$$

$$\because x + y = 10 \Rightarrow x = 5 = y$$

$$P(|x - y| = 0) = {}^{10}C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^5$$

$$\text{Case-II : } |x - y| = 1 \Rightarrow x - y = \pm 1$$

$x = y + 1$	$x = y - 1$
$\because x + y = 10$	$\because x + y = 10$
$2y = 9$	$2y = 11$
Not possible	Not possible

$$\text{Case-III : } |x - y| = 2 \Rightarrow x - y = \pm 2$$

$$\begin{array}{ll} x - y = 2 & \text{OR} \quad x - y = -2 \\ \because x + y = 10 & \because x + y = 10 \\ x = 6, y = 4 & x = 4, y = 6 \end{array}$$

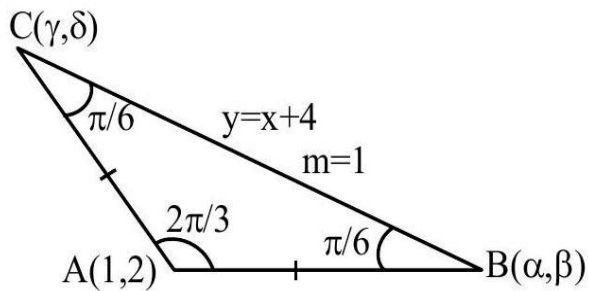
$$P(|x - y| = 2) = {}^{10}C_6 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^4 + {}^{10}C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^6$$

$$p = {}^{10}C_5 \frac{2^5}{3^{10}} + {}^{10}C_6 \frac{2^4}{3^{10}} + {}^{10}C_4 \frac{2^6}{3^{10}}$$

$$3^9 p = \frac{1}{3} ({}^{10}C_5 2^5 + {}^{10}C_6 2^4 + {}^{10}C_4 2^6)$$

$$= 8288$$

88. (14)



Equation of line passes through point A(1,2) which makes angle $\frac{\pi}{6}$ from $y = x + 4$ is

$$y - 2 = \frac{1 \pm \tan \frac{\pi}{6}}{1 \mp \tan \frac{\pi}{6}} (x - 1)$$

$$y - 2 = \frac{\sqrt{3} \pm 1}{\sqrt{3} \mp 1} (x - 1)$$

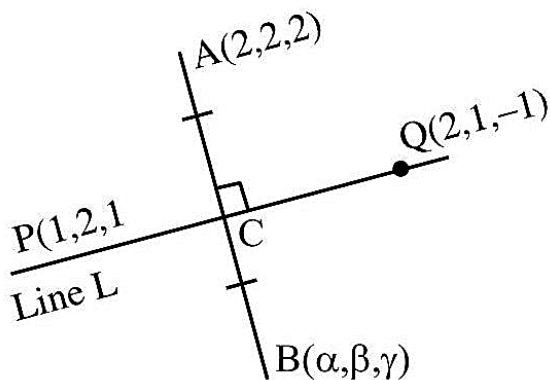
$$\begin{array}{ll} \oplus & \ominus \\ y - 2 = (2 + \sqrt{3})(x - 1) & y - 2 = (2 - \sqrt{3})(x - 1) \\ \text{solve with } y = x + 4 & \text{solve with } y = x + 4 \end{array}$$

$$\begin{array}{ll} +2 = (2 + \sqrt{3})x - 2 - \sqrt{3} & x + 2 = (2 - \sqrt{3})x - 2 + \sqrt{3} \\ x = \frac{4 + \sqrt{3}}{1 + \sqrt{3}} & x = \frac{4 - \sqrt{3}}{1 - \sqrt{3}} \end{array}$$

$$\alpha^2 + \gamma^2 = \left(\frac{4 + \sqrt{3}}{1 + \sqrt{3}} \right)^2 + \left(\frac{4 - \sqrt{3}}{1 - \sqrt{3}} \right)^2$$

$$\alpha^2 + \gamma^2 = 14$$

89. (6)



DR's of Line L $\equiv -1: 1: 2$

DR's of AB $\equiv \alpha - 2: \beta - 2: \gamma - 2$

$$AB \perp_{ar} L \Rightarrow 2 - \alpha + \beta - 2 + 2\gamma - 4 = 0$$

$$2\gamma + \beta - \alpha = 4$$

Let C is mid-point of AB

$$C\left(\frac{\alpha+2}{2}, \frac{\beta+2}{2}, \frac{\gamma+2}{2}\right)$$

$$\text{DR's of PC} = \frac{\alpha}{2} : \frac{\beta-2}{2} : \frac{\gamma}{2}$$

$$\text{line } L \parallel \text{PC} \Rightarrow \frac{-\alpha}{2} = \frac{\beta-2}{2} = \frac{\gamma}{4} = K (\text{let})$$

$$\alpha = -2K$$

$$\beta = 2K + 2$$

$$\gamma = 4K$$

$$\text{use in (a)} \Rightarrow K = \frac{1}{6}$$

$$\text{value of } \alpha + \beta + 6\gamma = 24K + 2 = 6$$

$$\mathbf{90. (31)} \frac{dy}{dx} = (x+y+2)^2 \dots (a), y(0) = -2$$

$$\text{Let } x+y+2 = v$$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\text{from (a)} \frac{dv}{dx} = 1 + v^2$$

$$\int \frac{dv}{1+v^2} = \int dx$$

$$\tan^{-1}(v) = x + C$$

$$\tan^{-1}(x+y+2) = x + C$$

$$\text{at } x=0, y=-2 \Rightarrow C=0$$

$$\Rightarrow \tan^{-1}(x+y+2) = x$$

$$y = \tan x - x - 2$$

$$f(x) = \tan x - x - 2, x \in \left[0, \frac{\pi}{3}\right]$$

$$f'(x) = \sec^2 x - 1 > 0 \Rightarrow f(x) \uparrow$$

$$f_{\min} = f(0) = -2 = \beta$$

$$f_{\max} = f\left(\frac{\pi}{3}\right) = \sqrt{3} - \frac{\pi}{3} - 2 = \alpha$$

$$\text{now } (3\alpha + \pi)^2 + \beta^2 = \gamma + 8\sqrt{3}$$

$$\Rightarrow (3\alpha + \pi)^2 + \beta^2 = (3\sqrt{3} - 6)^2 + 4$$

$$\gamma + 8\sqrt{3} = 67 - 36\sqrt{3}$$

$$\Rightarrow \gamma = 67 \text{ and } \delta = -36 \Rightarrow \gamma + \delta = 31$$

