

Physics Section-A

1. The mean free path and the average speed of oxygen molecules at 300 K and 1 atm are 3×10^{-7} m and 600 m/s, respectively. Find the frequency of its collisions.

- (a) $2 \times 10^{10}/s$ (b) $9 \times 10^5/s$
(c) $2 \times 10^9/s$ (d) $5 \times 10^8/s$

2. A small mirror of mass m is suspended by a massless thread of length l . Then the small angle through which the thread will be deflected when a short pulse of laser of energy E falls normal on the mirror.

(c = speed of light in vacuum and g = acceleration due to gravity)

- (a) $\theta = \frac{3E}{4mc\sqrt{gl}}$ (b) $\theta = \frac{E}{mc\sqrt{gl}}$
(c) $\theta = \frac{E}{2mc\sqrt{gl}}$ (d) $\theta = \frac{2E}{mc\sqrt{gl}}$

3. Two liquids A and B have θ_A and θ_B as contact angles in a capillary tube. If $K = \cos \theta_A / \cos \theta_B$, then identify the correct statement:

- (a) K is negative, then liquid A and liquid B have convex meniscus.
(b) K is negative, then liquid A and liquid B have concave meniscus.
(c) K is negative, then liquid A has concave meniscus and liquid B has convex meniscus
(d) K is zero, then liquid A has convex meniscus and liquid B has concave meniscus.

4. Which of the following are correct expression for torque acting on a body?

- A. $\vec{\tau} = \vec{r} \times \vec{L}$
B. $\vec{\tau} = \frac{d}{dt}(\vec{r} \times \vec{p})$
C. $\vec{\tau} = \vec{r} \times \frac{d\vec{p}}{dt}$
D. $\vec{\tau} = I\vec{\alpha}$
E. $\vec{\tau} = \vec{r} \times \vec{F}$

($\vec{r}$ = position vector; $\vec{p}$ = linear momentum;

$\vec{L}$ = angular momentum; $\vec{\alpha}$ = angular acceleration;

I = moment of inertia; $\vec{F}$ = force; t = time)

Choose the correct answer from the options given below:

- (a) B, D and E Only
(b) C and D Only
(c) B, C, D and E Only
(d) A, B, D and E Only

5. In a Young's double slit experiment, the slits are separated by 0.2 mm. If the slits separation is increased to 0.4 mm, the percentage change of the fringe width is:

- (a) 0% (b) 100%
(c) 50% (d) 25%

6. An alternating current is represented by the equation,

$i = 100\sqrt{2}\sin(100\pi t)$ ampere. The RMS value of current and the frequency of the given alternating current are.

- (a) $100\sqrt{2}$ A, 100 Hz (b) $\frac{100}{\sqrt{2}}$ A, 100 Hz
(c) 100 A, 50 Hz (d) $50\sqrt{2}$ A, 50 Hz

7. Consider the sound wave travelling in ideal gases of He, CH₄, and CO₂. All the gases have the same ratio $\frac{P}{\rho}$, where P is the pressure and ρ is the density. The ratio of the speed of sound through the gases

$v_{\text{He}} : v_{\text{CH}_4} : v_{\text{CO}_2}$ is given by.

- (a) $\sqrt{\frac{7}{5}} : \sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}}$ (b) $\sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}} : \sqrt{\frac{7}{5}}$
(c) $\sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}} : \sqrt{\frac{4}{3}}$ (d) $\sqrt{\frac{4}{3}} : \sqrt{\frac{5}{3}} : \sqrt{\frac{7}{5}}$

8. In an electromagnetic system, the quantity representing the ratio of electric flux

and magnetic flux has dimension of $M^P L^Q T^R A^S$, where value of ' Q ' and ' R ' are.

- (a) (3, -5) (b) (-2, 2)
(c) (-2, 1) (d) (1, -1)

9. When an object is placed 40 cm away from a spherical mirror an image of magnification $\frac{1}{2}$ is produced. To obtain an image with magnification of $\frac{1}{3}$, the object is to be moved :

- (a) 40 cm away from the mirror.
(b) 80 cm away from the mirror.
(c) 20 cm towards the mirror.
(d) 20 cm away from the mirror.

10. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: In photoelectric effect, on increasing the intensity of incident light the stopping potential increases.

Reason R : Increase in intensity of light increases the rate of photoelectrons emitted, provided the frequency of incident light is greater than threshold frequency.

In the light of the above statements, choose the correct answer from the options given below

- (a) Both A and R are true but R is NOT the correct explanation of A
(b) A is false but R is true
(c) A is true but R is false
(d) Both A and R are true and R is the correct explanation of A

11. If $\vec{L}$ and $\vec{P}$ represent the angular momentum and linear momentum respectively of a particle of mass ' m ' having position vector $\vec{r} = a(\hat{i}\cos\omega t + \hat{j}\sin\omega t)$. The direction of force is.

- (a) Opposite to the direction of $\vec{r}$
(b) Opposite to the direction of $\vec{L}$
(c) Opposite to the direction of $\vec{P}$
(d) Opposite to the direction of $\vec{L} \times \vec{P}$

12. A body of mass m is suspended by two strings making angles θ_1 and θ_2 with the horizontal ceiling with tensions T_1 and T_2 simultaneously. T_1 and T_2 are related by $T_1 = \sqrt{3}T_2$. the angles θ_1 and θ_2 are

- (a) $\theta_1 = 30^\circ$ $\theta_2 = 60^\circ$ with $T_2 = \frac{3mg}{4}$
(b) $\theta_1 = 60^\circ$ $\theta_2 = 30^\circ$ with $T_2 = \frac{mg}{2}$
(c) $\theta_1 = 45^\circ$ $\theta_2 = 45^\circ$ with $T_2 = \frac{3mg}{4}$
(d) $\theta_1 = 30^\circ$ $\theta_2 = 60^\circ$ with $T_2 = \frac{4mg}{5}$

13. Current passing through a wire as function of time is given as $I(t) = 0.02t + 0.01$ A. The charge that will flow through the wire from $t = 1$ s to $t = 2$ s is :

- (a) 0.06 C
(b) 0.02 C
(c) 0.07 C
(d) 0.04 C

14. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R .

Assertion A : The kinetic energy needed to project a body of mass m from earth surface to infinity is $\frac{1}{2}mgR$, where R is the radius of earth.

Reason R : The maximum potential energy of a body is zero when it is projected to infinity from earth surface.

In the light of the above statements, choose the correct answer from the option given below

- (a) A False but R is true
(b) Both A and R are true and R is the correct explanation of A
(c) A is true but R is false
(d) Both A and R are true but R is NOT the correct explanation of A

15. The Boolean expression $Y = \bar{A}\bar{B}C + \bar{A}C$ can be realised with which of the following gate configurations.

- A. One 3-input AND gate, 3 NOT gates and one 2input OR gate, One 2-input AND gate,
B. One 3-input AND gate, 1 NOT gate, One 2input NOR gate and one 2-input OR gate
C. 3-input OR gate, 3 NOT gates and one 2-input AND gate

Choose the correct answer from the options given below

- (a) B, C Only (b) A, B Only
(c) A, B, C Only (d) A, C Only

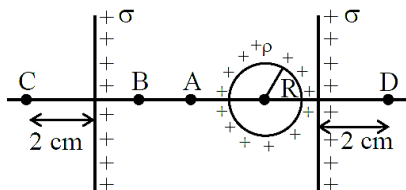
16. In an experiment with a closed organ pipe, it is filled with water by $\left(\frac{1}{5}\right)$ th of its volume. The frequency of the fundamental note will change by.

- (a) 25% (b) 20%
(c) -20% (d) -25%

17. Two simple pendulums having lengths l_1 and l_2 with negligible string mass undergo angular displacements θ_1 and θ_2 , from their mean positions, respectively. If the angular accelerations of both pendulums are same, then which expression is correct ?

- (a) $\theta_1 l_2^2 = \theta_2 l_1^2$
(b) $\theta_1 l_1 = \theta_2 l_2$
(c) $\theta_1 l_1^2 = \theta_2 l_2^2$
(d) $\theta_1 l_2 = \theta_2 l_1$

18. Two infinite identical charged sheets and a charged spherical body of charge density ' ρ ' are arranged as shown in figure. Then the correct relation between the electrical fields at A, B, C and D points is :



- (a) $\vec{E}_A = \vec{E}_B$; $\vec{E}_C = \vec{E}_D$
(b) $\vec{E}_A > \vec{E}_B$; $\vec{E}_C = \vec{E}_D$
(c) $\vec{E}_C \neq \vec{E}_D$; $\vec{E}_A > \vec{E}_B$
(d) $|\vec{E}_A| = |\vec{E}_B|$; $\vec{E}_C > \vec{E}_D$

19. Two small spherical balls of mass 10 g each with charges $-2\mu\text{C}$ and $2\mu\text{C}$, are attached to two ends of very light rigid rod of length 20 cm. The arrangement is now placed near an infinite nonconducting charge sheet with uniform charge density of $100\mu\text{C}/\text{m}^2$ such that length of rod makes an angle of 30° with electric field generated by charge sheet. Net torque acting on the rod is:

(Take $\epsilon_0 : 8.85 \times 10^{-12} \text{C}^2/\text{Nm}^2$)

- (a) 112 Nm (b) 1.12 Nm
(c) 2.24 Nm (d) 11.2 Nm

20. Considering the Bohr model of hydrogen like atoms, the ratio of the ratio of the radius 5th orbit of the electron in Li^{2+} and He^+ is.

- (a) $\frac{3}{2}$ (b) $\frac{4}{9}$
(c) $\frac{9}{4}$ (d) $\frac{2}{3}$

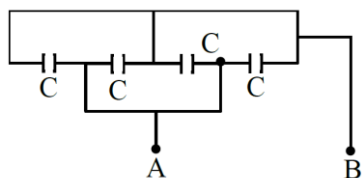
Section-B

21. A circular ring and a solid sphere having same radius roll down on an inclined plane from rest without slipping. The ratio of their velocities when reached at the bottom of the plane is $\sqrt{\frac{x}{5}}$ where $x =$

22. Two slabs with square cross section of different materials (1,2) with equal sides (l) and thickness d_1 and d_2 such that $d_2 = 2d_1$ and $l > d_2$. Considering lower edges of these slabs are fixed to the floor, we apply equal shearing force on the narrow faces. The angle of deformation is $\theta_2 = 2\theta_1$. If the shear moduli of material 1 is $4 \times 10^9 \text{N/m}^2$, then shear moduli of material 2 is $x \times 10^9 \text{N/m}^2$, where value of x is .

23. Distance between object and its image (magnified by $-\frac{1}{3}$) is 30 cm. The focal length of the mirror used is $\left(\frac{x}{4}\right)$ cm, where magnitude of value of x is .

24. Four capacitor each of capacitance $16\mu\text{F}$ are connected as shown in the figure. The capacitance between points A and B is : (in μF).

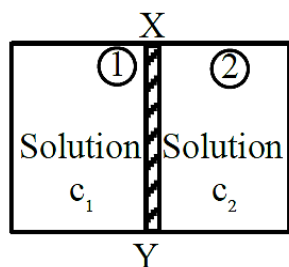


25. Conductor wire ABCDE with each arm 10 cm in length is placed in magnetic field of $\frac{1}{\sqrt{2}}$ Tesla, perpendicular to its plane. When conductor is pulled towards right with constant velocity of 10 cm/s, induced emf between points A and E is _____ mV .

Chemistry

Section-A

26. XY is the membrane / partition between two chambers 1 and 2 containing sugar solutions of concentration c_1 and c_2 ($c_1 > c_2$) molL⁻¹. For the reverse osmosis to take place identify the correct condition.



(Here p_1 and p_2 are pressures applied on chamber 1 and 2)

- (A) Membrane/Partition; Cellophane, $p_1 > \pi$
 (B) Membrane/Partition ; Porous. $p_2 > \pi$
 (C) Membrane/Partition ; Parchment paper, $p_1 > \pi$
 (D) Membrane/Partition : Cellophane, $p_2 > \pi$

Choose the correct answer from the option given below:

- (a) B and D only (b) A and D only
 (c) A and C only (d) C only

27. Let us consider a reversible reaction at temperature, T .

In this reaction, both ΔH and ΔS were observed to have positive values. If the equilibrium temperature is T_e , then the reaction becomes spontaneous at :

- (a) $T = T_e$
 (b) $T_e > T$
 (c) $T > T_e$
 (d) $T_e = 5 T$

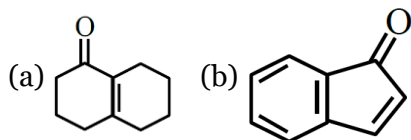
28. Which of the following molecules(s) show/s paramagnetic behavior ?

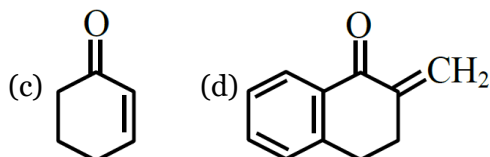
- (A) O_2 (B) N_2
 (C) F_2 (D) S_2
 (E) Cl_2

Choose the correct answer from the options given below :

- (a) B only
 (b) A & C only
 (c) A & E only
 (d) A & D only

29. Aldol condensation is a popular and classical method to prepare α, β -unsaturated carbonyl compounds. This reaction can be both intermolecular and intramolecular. Predict which one of the following is not a product of intramolecular aldol condensation?





30. One mole of an ideal gas expands isothermally and reversibly from 10dm^3 to 20dm^3 at 300K . ΔU , q and work done in the process respectively are

Given : $R = 8.3\text{JK}^{-1}\text{mol}^{-1}$

$\ln 10 = 2.3$

$\log 2 = 0.30$

$\log 3 = 0.48$

- (a) $0, 21.84\text{ kJ}, -1.26\text{ kJ}$
 (b) $0, -17.18\text{ kJ}, 1.718\text{ J}$
 (c) $0, 21.84\text{ kJ}, 21.84\text{ kJ}$
 (d) $0, 178\text{ kJ}, -1.718\text{ kJ}$

31. Which one of the following complexes will have $\Delta_0 = 0$ and $\mu = 5.96\text{ B.M.}$?

- (a) $[\text{Fe}(\text{CN})_6]^{4-}$
 (b) $[\text{Co}(\text{NH}_3)_6]^{3+}$
 (c) $[\text{FeF}_6]^{4-}$
 (d) $[\text{Mn}(\text{SCN})_6]^{4-}$

32. For $\text{A}_2 + \text{B}_2 \rightleftharpoons 2\text{AB}$

E_a for forward and backward reaction are 180 and 200 kJ mol^{-1} respectively

If catalyst lowers E_a for both reaction by 100 kJ mol^{-1} . Which of the following statement is correct?

- (a) Catalyst does not alter the Gibbs energy change of a reaction.
 (b) Catalyst can cause non-spontaneous reactions to occur.
 (c) The enthalpy change for the reaction is $+20\text{ kJ mol}^{-1}$.
 (d) The enthalpy change for the catalysed reaction is different from that of uncatalyzed reaction

33. Rate law for a reaction between A and B is given by

$R = k[A]^n[B]^m$ If concentration of A is doubled and concentration of B is halved from their initial value, the ratio of new rate of reaction to the initial rate of reaction $\left(\frac{r_2}{r_1}\right)$ is.

- (a) $2^{(n-m)}$ (b) $(n-m)$
 (c) $(m+n)$ (d) $\frac{1}{2^{m+n}}$

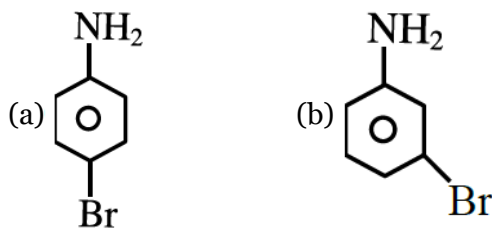
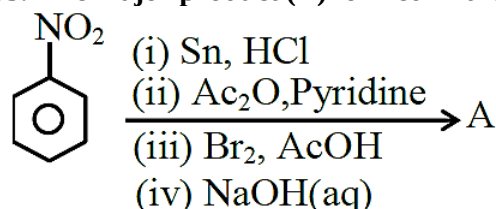
34. Number of stereoisomers possible for the complexes, $[\text{CrCl}_3(\text{py})_3]$ and

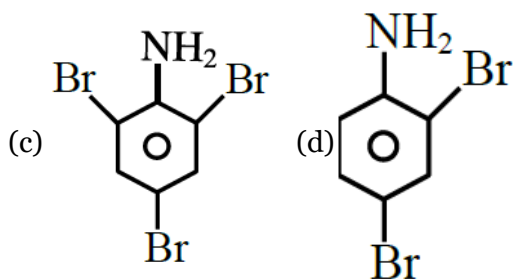
$[\text{CrCl}_2(\text{ox})_2]^{3-}$ are respectively.

(py = pyridine, ox = oxalate)

- (a) 3&3 (b) 2&2
 (c) 2&3 (d) 1&2

35. The major product (A) formed in the following reaction sequence is.





36. On charging the lead storage battery, the oxidation state of lead changes from x_1 to y_1 at the anode and from x_2 to y_2 at the cathode. The values of x_1, y_1, x_2, y_2 are respectively:

- (a) +4, +2, 0, +2
(b) +2, 0, +2, +4
(c) 0, +2, +4, +2
(d) +2, 0, 0, +4

37. Given below are two statements :

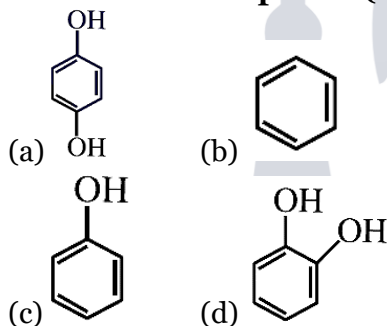
Statement I : Nitrogen forms oxides with +1 to +5 oxidation states due to the formation of $p\pi - p\pi$ bond with oxygen .

Statement II : Nitrogen does not form halides with +5 oxidation state due to the absence of d-orbital in it.

In the light of given statements, choose the correct answer from the options given below.

- (a) Statement I is true but Statement II is false
(b) Both Statement I and Statement II are false
(c) Statement I is false but Statement II is true
(d) Both Statement I and Statement II are true

38. Benzene is treated with oleum to produce compound (X) which when further heated with molten sodium hydroxide followed by acidification produces compound (Y). The compound Y is treated with zinc metal to produce compound (Z). Identify the structure of compound (Z) from the following option.



39. Identify the pair of reactants that upon reaction, with elimination of HCl will give rise to the dipeptide Gly-Ala.

- (a) $\text{NH}_2 - \text{CH}_2 - \text{COCl}$ and $\begin{array}{c} \text{CH}_3 \\ | \\ \text{NH}_2 - \text{CH} - \text{COOH} \end{array}$
- (b) $\text{NH}_2 - \text{CH}_2 - \text{COCl}$ and $\begin{array}{c} \text{CH}_3 \\ | \\ \text{NH}_3^+ - \text{CH} - \text{COCl} \end{array}$
- (c) $\text{NH}_2 - \text{CH}_2 - \text{COOH}$ and $\begin{array}{c} \text{CH}_3 \\ | \\ \text{NH}_2 - \text{CH} - \text{COCl} \end{array}$
- (d) $\text{NH}_2 - \text{CH}_2 - \text{COOH}$ and $\begin{array}{c} \text{CH}_3 \\ | \\ \text{NH}_2 - \text{CH} - \text{COOH} \end{array}$

40. Given below are the pairs of group 13 elements showing their relation in terms of atomic radius. ($\text{B} < \text{Al}$), ($\text{Al} < \text{Ga}$), ($\text{Ga} < \text{In}$) and ($\text{In} < \text{Tl}$)

Identify the elements present in the incorrect pair and in that pair find out the element (X) that has higher ionic radius (M^{3+}) than the other one. The atomic number of the element (X) is

- (a) 31 (b) 49
(c) 13 (d) 81

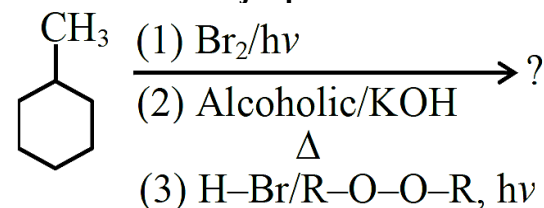
41. An organic compound (X) with molecular formula C_3H_6O is not readily oxidised.

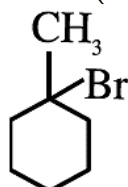
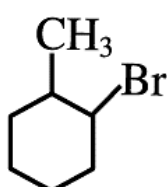
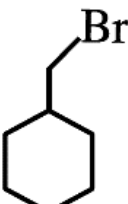
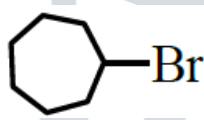
On reduction it gives $(C_3H_8O)(Y)$ which reacts with HBr to give a bromide (Z) which is converted to Grignard reagent. This Grignard reagent on reaction with (X) followed by hydrolysis give 2,3-dimethylbutan-2-ol.

Compounds (X), (Y) and (Z) respectively are :

- (a) CH_3COCH_3 , $CH_3CH_2CH_2OH$, $CH_3CH(Br)CH_3$
(b) CH_3COCH_3 , $CH_3CH(OH)CH_3$, $CH_3CH(Br)CH_3$
(c) CH_3CH_2CHO , $CH_3CH_2CH_2OH$, $CH_3CH_2CH_2Br$
(d) CH_3CH_2CHO , $CH_3CH=CH_2$, $CH_3CH(Br)CH_3$

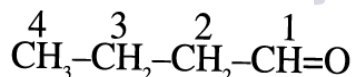
42. Predict the major product of the following reaction sequence :-



- (a)  (b) 
(c)  (d) 

43. Given below are two statements.

Statement I : The dipole moment of $CH_3-CH=CH-CH=O$ is greater than



Statement II : C_1-C_2 bond length of

$CH_3-CH=CH-CH=O$ is greater than C_1-C_2

bond length of $CH_3-CH_2-CH_2-CH=O$

In the light of the above statements, choose the correct answer from the options given below:

- (a) Statement I is false but Statement II is true
(b) Both Statement I and Statement II are false
(c) Statement I is true but Statement II is false
(d) Both Statement I and Statement II are true

44. Pair of transition metal ions having the same number of unpaired electrons is:

- (a) V^{2+} , Co^{2+}
(b) Ti^{2+} , Co^{2+}
(c) Fe^{3+} , Cr^{2+}
(d) Ti^{3+} , Mn^{2+}

45. Which one of the following about an electron occupying the 1 s orbital in a hydrogen atom is incorrect ? (Bohr's radius is represented by a_0)

- (a) The probability density of finding the electron is maximum at the nucleus

- (b) The electron can be found at a distance $2a_0$ from the nucleus
 (c) The 1 s orbital is spherically symmetrical
 (d) The total energy of the electron is maximum when it is at a distance a_0 from the nucleus

Section-B

46. In Dumas' method for estimation of nitrogen 1 g of an organic compound gave 150 mL of nitrogen collected at 300 K temperature and 900 mm Hg pressure. The percentage composition of nitrogen in the compound is % (nearest integer).
 (Aqueous tension at 300 K = 15 mmHg)
 47. KMnO_4 acts as an oxidising agent in acidic medium. 'X' is the difference between the oxidation states of Mn in reactant and product. 'Y' is the number of 'd' electrons present in the brown red precipitate formed at the end of the acetate ion test with neutral ferric chloride. The value of $X + Y$ is .
 48. Fortification of food with iron is done using $\text{FeSO}_4 \cdot 7\text{H}_2\text{O}$. The mass in grams of the $\text{FeSO}_4 \cdot 7\text{H}_2\text{O}$ required to achieve 12 ppm of iron in 150 kg of wheat is (Nearest integer)
 [Given : Molar mass of Fe, S and O respectively are 56, 32 and 16 g mol⁻¹]
 49. The pH of a 0.01 M weak acid HX ($K_a = 4 \times 10^{-10}$) is found to be 5 . Now the acid solution is diluted with excess of water so that the pH of the solution changes to 6 . The new concentration of the diluted weak acid is given as $x \times 10^{-4}$ M. The value of x is. (nearest integer)
 50. The total number of hydrogen bonds of a DNA-double Helix strand whose one strand has the following sequence of bases is .
 5' - G - G - C - A - A - A - T - C - G - G - C - T - A - 3,

Mathematics

Section-A

51. Let $f, g: (1, \infty) \rightarrow \mathbb{R}$ be defined as $f(x) = \frac{2x+3}{5x+2}$ and $g(x) = \frac{2-3x}{1-x}$. If the range of the function $f \circ g: [2, 4] \rightarrow \mathbb{R}$ is $[\alpha, \beta]$, then $\frac{1}{\beta-\alpha}$ is equal to.
 (a) 68 (b) 29
 (c) 2 (d) 56
 52. Consider the sets $A = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 = 25\}$, $B = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + 9y^2 = 144\}$, $C = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \leq 4\}$, and $D = A \cap B$. The total number of one-one functions from the set D to the set C is:
 (a) 15120 (b) 19320
 (c) 17160 (d) 18290
 53. Let $A = \{1, 6, 11, 16, \dots\}$ and $B = \{9, 16, 23, 30, \dots\}$ be the sets consisting of the first 2025 terms of two arithmetic progressions. Then $n(A \cup B)$ is.
 (a) 3814 (b) 4027
 (c) 3761 (d) 4003
 54. For an integer $n \geq 2$, if the arithmetic mean of all coefficients in the binomial expansion of $(x + y)^{2n-3}$ is 16, then the distance of the point $P(2n - 1, n^2 - 4n)$ from the line $x + y = 8$ is:
 (a) $\sqrt{2}$ (b) $2\sqrt{2}$
 (c) $5\sqrt{2}$ (d) $3\sqrt{2}$
 55. The probability, of forming a 12 persons committee from 4 engineers, 2 doctors and 10 professors containing at least 3 engineers and at least 1 doctor, is:
 (a) $\frac{129}{182}$ (b) $\frac{103}{182}$
 (c) $\frac{17}{26}$ (d) $\frac{19}{26}$
 56. Let the shortest distance between the lines $\frac{x-3}{3} = \frac{y-a}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-\beta}{4}$ be $3\sqrt{30}$. Then the positive value of $5\alpha + \beta$ is.
 (a) 42 (b) 46
 (c) 48 (d) 40
 57. If $\lim_{x \rightarrow 1^+} \frac{(x-1)(6+\lambda \cos(x-1))+\mu \sin(1-x)}{(x-1)^3} = -1$, where $\lambda, \mu \in \mathbb{R}$, then $\lambda + \mu$ is equal to.
 (a) 18 (b) 20
 (c) 19 (d) 17
 58. Let $f: [0, \infty] \rightarrow \mathbb{R}$ be differentiable function such that $f(x) = 1 - 2x +$

$\int_0^x e^{x-t} f(t) dt$ for all $x \in [0, \infty)$. Then the area of the region bounded by $y = f(x)$ and the coordinate axes is

- (a) $\sqrt{5}$ (b) $\frac{1}{2}$
(c) $\sqrt{2}$ (d) 2

59. Let A and B be two distinct points on the line L: $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Both A and B are

at a distance $2\sqrt{17}$ from the foot of perpendicular drawn from the point $(1, 2, 3)$ on the line L. If O is the origin, then $\overrightarrow{OA} \cdot \overrightarrow{OB}$ is equal to:

- (a) 49 (b) 47
(c) 21 (d) 62

60. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function satisfying $f(0) = 1$ and $f(2x) - f(x) = x$

for all $x \in \mathbb{R}$. If $\lim_{n \rightarrow \infty} \left\{ f(x) - f\left(\frac{x}{2^n}\right) \right\} = G(x)$, then $\sum_{r=1}^{10} G(r^2)$ is equal to

- (a) 540 (b) 385
(c) 420 (d) 215

61. $1 + 3 + 5^2 + 7 + 9^2 + \dots$ upto 40 terms is equal to.

- (a) 43890 (b) 41880
(c) 33980 (d) 40870

62. In the expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$, $n \in \mathbb{N}$, if the ratio of 15th term from the beginning to the 15th term from the end is $\frac{1}{6}$, then the value of nC_3 is:

- (a) 4060 (b) 1040
(c) 2300 (d) 4960

63. Considering the principal values of the inverse trigonometric

functions, $\sin^{-1}\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{1-x^2}\right)$, $-\frac{1}{2} < x < \frac{1}{\sqrt{2}}$, is equal to.

- (a) $\frac{\pi}{4} + \sin^{-1}x$ (b) $\frac{\pi}{6} + \sin^{-1}x$
(c) $\frac{-5\pi}{6} - \sin^{-1}x$ (d) $\frac{5\pi}{6} - \sin^{-1}x$

64. Consider two vectors $\vec{u} = 3\hat{i} - \hat{j}$ and $\vec{v} = 2\hat{i} + \hat{j} - \lambda\hat{k}$, $\lambda > 0$. The angle between

them is given by $\cos^{-1}\left(\frac{\sqrt{5}}{2\sqrt{7}}\right)$. Let $\vec{v} = \vec{v}_1 + \vec{v}_2$, where $\vec{v}_1$ is parallel to $\vec{u}$ and $\vec{v}_2$ is perpendicular to $\vec{u}$. Then the value $|\vec{v}_1|^2 + |\vec{v}_2|^2$ is equal to.

- (a) $\frac{23}{2}$ (b) 14
(c) $\frac{25}{2}$ (d) 10

65. Let the three sides of a triangle are on the lines $4x - 7y + 10 = 0$, $x + y =$

5 and $7x + 4y = 15$. Then the distance of its orthocenter from the orthocenter of the triangle formed by the lines $x = 0$, $y = 0$ and $x + y = 1$ is.

- (a) 5 (b) $\sqrt{5}$
(c) $\sqrt{20}$ (d) 20

66. The value of $\int_{-1}^1 \frac{(1+\sqrt{|x|-x})e^x + (\sqrt{|x|-x})e^{-x}}{e^x + e^{-x}} dx$ is equal to.

- (a) $3 - \frac{2\sqrt{2}}{3}$ (b) $2 + \frac{2\sqrt{2}}{3}$
(c) $1 - \frac{2\sqrt{2}}{3}$ (d) $1 + \frac{2\sqrt{2}}{3}$

67. The length of the latus-rectum of the ellipse, whose foci are $(2, 5)$ and $(2, -3)$ and

eccentricity is $\frac{4}{5}$, is.

- (a) $\frac{6}{5}$ (b) $\frac{50}{3}$
(c) $\frac{10}{3}$ (d) $\frac{18}{5}$

68. Consider the equation $x^2 + 4x - n = 0$, where $n \in [20, 100]$ is a natural number. Then the number of all distinct values of n , for which the given equation has integral roots, is equal to.

- (a) 7 (b) 8
(c) 6 (d) 5

69. A box contains 10 pens of which 3 are defective. A sample of 2 pens is drawn at random and let X denote the number of defective pens. Then the variance of X is.

- (a) $\frac{11}{15}$ (b) $\frac{28}{75}$
(c) $\frac{2}{15}$ (d) $\frac{3}{5}$

70. If $10\sin^4 \theta + 15\cos^4 \theta = 6$, then the value of $\frac{27\operatorname{cosec}^6 \theta + 8\sec^6 \theta}{16\sec^8 \theta}$ is:

- (a) $\frac{2}{5}$ (b) $\frac{3}{4}$
(c) $\frac{3}{5}$ (d) $\frac{1}{5}$

Section-B

71. If the area of the region $\{(x, y): |x - 5| \leq y \leq 4\sqrt{x}\}$ is A , then $3A$ is equal to.

72. Let $A = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$. If for some $\theta \in (0, \pi)$, $A^2 = A^T$, then the sum of the

diagonal elements of the matrix $(A + I)^3 + (A - I)^3 - 6A$ is equal to.

73. Let $A = \{z \in \mathbb{C}: |z - 2 - i| = 3\}$, $B = \{z \in \mathbb{C}: \operatorname{Re}(z - iz) = 2\}$ and $S = A \cap B$.

Then $\sum_{z \in S} |z|^2$ is equal to .

74. Let C be the circle $x^2 + (y - 1)^2 = 2$, E_1 and E_2 be two ellipses whose centres lie at the origin and major axes lie on x -axis and y -axis respectively. Let the straight line $x + y = 3$ touch the curves C , E_1 and E_2 at $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$ respectively. Given that P is the mid-point of the line segment QR and $PQ = \frac{2\sqrt{2}}{3}$, the value of $9(x_1y_1 + x_2y_2 + x_3y_3)$ is equal to .

75. Let m and n be the number of points at which the function $f(x) = \max\{x, x^3, x^5, \dots, x^{21}\}$, $x \in \mathbb{R}$, is not differentiable and not continuous, respectively. Then $m + n$ is equal to .

