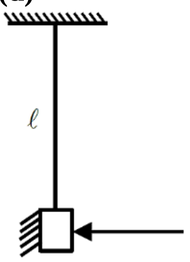
1. (c)

Frequency $= \frac{1}{T} = \frac{V_{avg}}{\lambda}$

$= \frac{600}{3 \times 157} = 2 \times 10^9 \sec^{-1}$

2. (d)

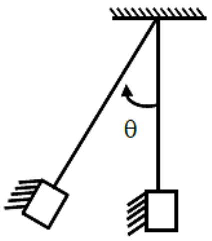


Force due to beam assuming complete reflection $F = \frac{2P}{C} = \frac{2}{C}\frac{dE}{dt}$; P is power

So change in momentum of mirror.

$$m(V-0) = \int Fdt = \frac{2}{C}\int dE = \frac{2E}{C}$$

Now using work energy theorem ….(1)



$W_g = \Delta k$

$-mg\ell(1-\cos\theta) = 0 - \frac{1}{2}mv^2$

$g\ell\left(2\sin^2\frac{\theta}{2}\right) = \frac{v^2}{2}$

as θ is small

$g\ell 2\left(\frac{\theta}{2}\right)^2 = \frac{1}{2}\frac{4E^2}{m^2c^2}$ (from eq. (1))

$g\ell\theta^2 = \frac{4E^2}{m^2c^2}$

$\theta = \frac{2E}{mc\sqrt{g\ell}}$

3. (c)

$k = \frac{\cos\theta_A}{\cos\theta_B}$

It is negative when $\cos\theta_A$&$\cos\theta_B$ are of opposite sign. so option (3)

4. (c)

conceptual

5. (c)

$\beta = \frac{D\lambda}{d} \propto \frac{1}{d}$

If d is doubled then β is half so 50% decrement.

6. (c)

$i_r = \frac{i_0}{\sqrt{2}} = 100\ \mathrm{A}$

$f = \frac{w}{2\pi} = \frac{100\pi}{2\pi} = 50\ \mathrm{Hz}$

7. (c)

$v_{sound} = \sqrt{\frac{\gamma p}{\rho}} \Rightarrow \gamma = 1 + \frac{2}{f}$

$\gamma_{He} = \frac{5}{3}; \gamma_{CH_4} = \gamma_{CO_2} \approx 1.33 = \frac{4}{3}$ (Experimental data)

8. (4)

$\frac{\phi_E}{\phi_M} = \frac{EA}{BA} = \frac{E}{B}$

$B = \frac{M/T^{-2}}{ATLT^{-1}}$

So $\left[\frac{E}{B}\right] = \frac{ML^{-3}A^{-1}}{MT^{-2}A^{-1}} = LT^{-1}$

Or

$E = c \cdot B$ ($c =$ Speed of light) $\left[\frac{E}{B}\right] = LT^{-1}$

9. (a)

$m = \frac{1}{2} = \frac{f}{f-u}$

$\frac{1}{2} = \frac{f}{f-(-40)}$

$f + 40 = 2f \Rightarrow f = 40\ \mathrm{cm}$

now $m = \frac{1}{3} = \frac{40}{40-u}$

$40 - u = 120 \Rightarrow u = -80$

10. (b)

$V_S = \frac{h\nu-\phi}{e}$

so stopping potential doesn't depend on Intensity

$I = \frac{\eta h\nu}{A}$

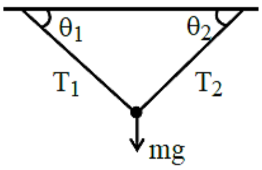
On increasing intensity no. of photons per sec. n increases so the no. of electrons.

11. (b)

$\vec{a} = -\omega^2 \vec{r}$

$\therefore \vec{F}$ opposite to $\vec{r}$ –

12. (b)



$T_1 \sin\theta_1 + T_2 \sin\theta_2 = mg \ \& \ T_1 = \sqrt{3}\, T_2$

$\Rightarrow T_2\left[\sqrt{3}\sin\theta_1 + \sin\theta_2\right] = mg$

for $\theta_1 = 60^\circ \ \& \ \theta_2 = 30^\circ$

$T_2 = \frac{mg}{2}$

13. (d)

$$q = \int idt$$

$$\int_0^2 (0.02t + 0.01)dt$$

$$q = \left[0.02\frac{t^2}{2} + 0.01t\right]_1^2$$

$$= 0.01(3) + 0.01(1) = 0.04C$$

14. (a)

$$KE = \frac{1}{2} m \left(\frac{2Gm}{R}\right) = mgR$$

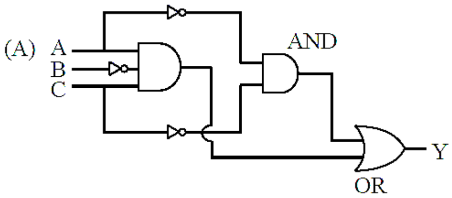
Assertion wrong

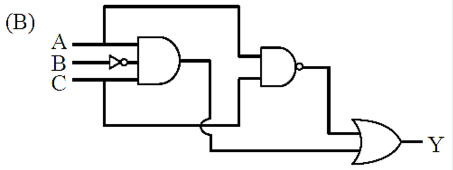
at ∞

$U = 0$

$\therefore$ Reason correct.

15. (b)



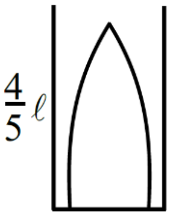


$\because\ \overline{A}\cdot\overline{C} + \overline{A + C} \equiv$ NOR gate

16. (a)

$\lambda_1 = 4\ell$

$f_1 = \frac{v}{4\ell}$



$$\lambda_2 = \frac{16\ell}{5}$$

$$f_2 = \frac{5\ V}{16\ell}$$

$$\frac{\Delta f}{f} = \frac{\frac{V}{\ell}\left(\frac{1}{16}\right)}{\frac{V}{4\ell}} \times 100 = 25\%$$

17. (d)

$$\omega = \sqrt{\frac{g}{\ell}}$$

$$\alpha = -\omega^2\theta$$

$$\therefore \frac{g}{\ell_1}\theta_1 = \frac{g}{\ell_2}\theta_2$$

$$\Rightarrow \theta_1\ell_2 = \theta_2\ell_2$$

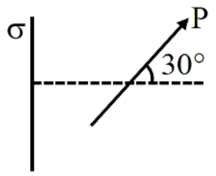
18. (c)

Conceptual

$$E_C \neq E_D$$

$$E_A > E_B$$

19. (b)



$$\mathrm{E} = \frac{\sigma}{2\varepsilon_0}$$

$$\tau = PE\sin\theta$$

$$= \left[(2\times10^{-6})\left(\frac{2}{10}\right)\right]\left[\frac{100\times10^{-6}}{2\times8.85\times10^{-12}}\right]\left(\frac{1}{2}\right)$$

$$= \frac{10}{8.85} = 1.12\mathrm{Nm}$$

20. (d)

$$\mathrm{r} = \mathrm{r}\cdot\frac{\mathrm{n}^2}{2}$$

for Li^{2+}

$$\mathrm{r}_5 = \mathrm{r}\cdot\frac{25}{3}$$

for He^{+}

$$\mathrm{r}_5 = \mathrm{r}\cdot\frac{25}{2}$$

$$\therefore \frac{\mathrm{r}_{\mathrm{Li}^{2+}}}{\mathrm{r}_{\mathrm{He}^{+}}} = \frac{2}{3}$$

Section-B

21. (4)

Applying Mechanical Energy conservation :

$$\mathrm{k_i} + \mathrm{U_i} = \mathrm{k_f} + \mathrm{U_f}$$

$$\Rightarrow 0 + \mathrm{Mgh} = \frac{1}{2}\mathrm{mv}^2\left(1 + \frac{\mathrm{k}^2}{\mathrm{R}^2}\right) + 0$$

$$\Rightarrow \mathrm{V} = \sqrt{\frac{2\mathrm{gh}}{1 + \frac{\mathrm{k}^2}{\mathrm{R}^2}}}$$

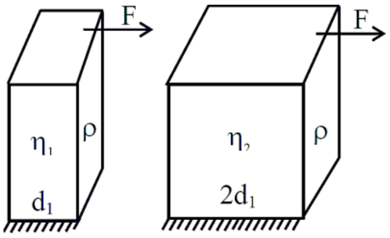
So Ratio of velocities

$$\frac{\mathrm{V_{Ring}}}{\mathrm{V_{solids\ sphere}}} = \sqrt{\frac{1 + \frac{2}{5}}{1 + 1}} = \sqrt{\frac{7}{10}}$$

$\mathrm{x} = 3.5$ Rounding off $\mathrm{x} = 4$

22. (1)

Deformation angle



$2\theta_1 = \theta_2$

$\Rightarrow 2\dfrac{\sigma_1}{\eta_1} = \dfrac{\sigma_2}{\eta_2}$

$\Rightarrow 2\left(\dfrac{F}{\ell d_1 \eta_1}\right) = \dfrac{F}{\ell d_2 \eta_2}$

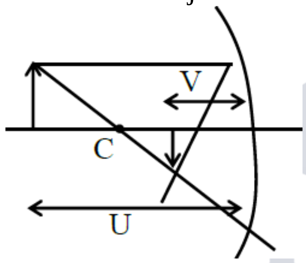
$\Rightarrow \eta_2 = \dfrac{\eta_1}{4} = 1 \times 10^9 \Rightarrow x = 1$

23. (45)

$M = -\dfrac{1}{3}$

$-\dfrac{-V}{-U} = \dfrac{-1}{3} \Rightarrow V = \dfrac{U}{3}$

Distance b/w object and image :



$U - V = 30$

$U - \dfrac{\mu}{3} = 30$

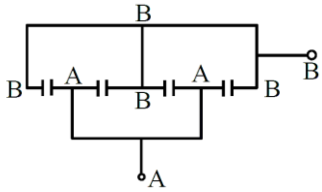
$\Rightarrow U = 45\ V = 15$

$\dfrac{1}{f} = \dfrac{1}{V} + \dfrac{1}{U} = -\dfrac{1}{15} - \dfrac{1}{45}$

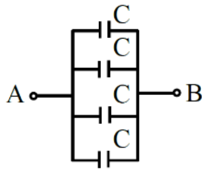
$\Rightarrow F = \dfrac{45}{4}$

$x = 45$

24.(64)



Redrawing



$C_{eq} = 4C = 64$

$C_{eq} = 4C = 64$

25. (45)

$$M = -\frac{1}{3}$$

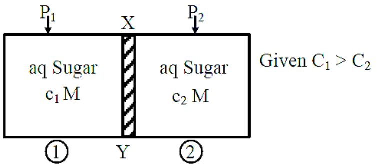
$$-\frac{-V}{-U} = \frac{-1}{3} \Rightarrow V = \frac{U}{3}$$

Distance b/w object and image :

Chemistry

Section-A

26. (c)



Normal osmosis occurs from (2) to (1)

For reverse osmosis from (1) to (2)

Pressure : $P_1 > \pi$

∴ Answer [A & C] only

27. (c)

For reaction to be spontaneous according to 2^{nd} law:

$\Delta G < 0$

$\Rightarrow \Delta H - T\Delta S < 0$

$\Rightarrow T > \left(\frac{\Delta H}{\Delta S}\right) = T_e$

$\Rightarrow T > T_e$

28. (d)

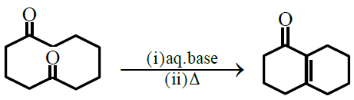
No. of unpaired e^-

		No. of unpaired e^-
(A)	O_2	2
(B)	N_2	0
(C)	F_2	0
(D)	S_2	2
(E)	Cl_2	0

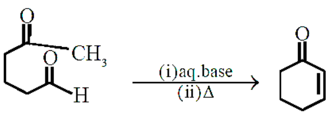
If species contain unpaired electron than it is paramagnetic.

So A&D are paramagnetic.

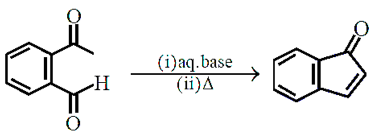
29. (d)

(1) 

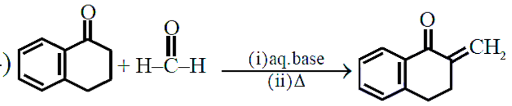
(Intramolecular aldol)

(2) 

(Intramolecular aldol)

(3) 

(Intramolecular aldol)

(4) 

(Intermolecular aldol)

30. (d)

$(10\text{ L}, 300\text{ K}) \overset{n=1}{\rightarrow} (20\text{ L}, 300\text{ K})$

$-q = w = -nRT\ln\frac{V_2}{V_1}$

$= -8.3 \times 300 \times \ln\left(\frac{20}{10}\right)$

$= -1.718\text{ kJ}$

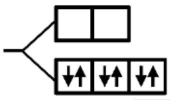
$\Rightarrow q = 1.718\text{ kJ}$

$w = -1.718\text{ kJ}$

$\Delta U = 0(\because \Delta T = 0)$

31. (d)

(1) $[Co(NH_3)_6]^{3+}$ $Co^{3+} \Rightarrow 3d^6 4s^0$

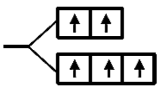
NH_3 is strong field ligand  $\mu = 0$

$= [-0.4 \times 6 + 0.6 \times (0)]\Delta_0 = -2.4\Delta_0$

(2)

$[Mn(SCN)_6]^{4-}$

$Mn^{2+} \Rightarrow 3d^5 4s^0$

$\overset{\ominus}{S}CN$ is weak field 

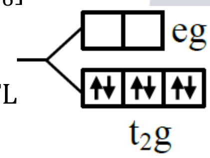
$\mu = \sqrt{35}$ B.M. $= 5.96$ B.M.

CFSE $= (-0.4 \times 3 + 0.6 \times 2)\Delta_0$

So $\Delta_0 = 0$

(3)

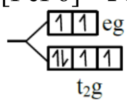
$[Fe(CN)_6]^{-4}$ $Fe^{2+} \Rightarrow 3d^6 4s^0$

$\overset{\ominus}{C}N$ is SFL  $\mu = 0$

CFSE $= -2.4\Delta_0$

(4)

$[FeF6]^{4-}$ $Fe^{2+} \Rightarrow 3d64s^0$



$\mu = 24$ B.M. $= 4.89$B.M.

CFSE $= (-0.4 \times 4 + 0.6 \times 2)\Delta_0 = -1.2\Delta_0$

32. (a)

$A_2 + B_2 \rightleftharpoons 2AB$

$E_f = 180\text{ kJ mol}^{-1}$

$E_b = 200\text{ kJ mol}^{-1}$

$\Delta H = E_f - E_b = -20\text{ kJ mol}^{-1}$

In presence of catalyst :

$E_f = 180 - 100 = 80\text{ kJ mol}^{-1}$

$E_b = 200 - 100 = 100\text{ kJ mol}^{-1}$

Catalyst does not change ΔH or ΔG of a reaction.

33. (a)

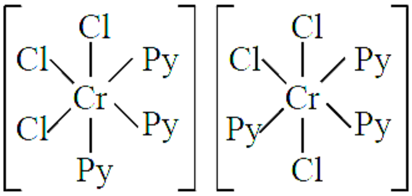
$r_1 = k[A]^n[B]^m$

Now A is doubled & B is halved in concentration

$\Rightarrow r_2 = k2^n[A]^n \cdot \frac{[B]^m}{2^m}$

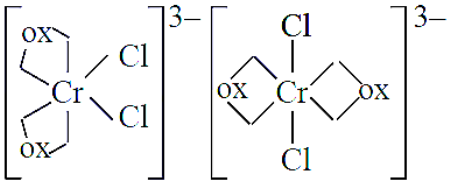
Now $\frac{r_2}{r_1} = 2^{(n-m)}$

34. (c)

[CrCl3(Py)3] 

Facial Meridional

So total stereo isomer = 2

$[CrCl3(ox)2]^{3-}$ 

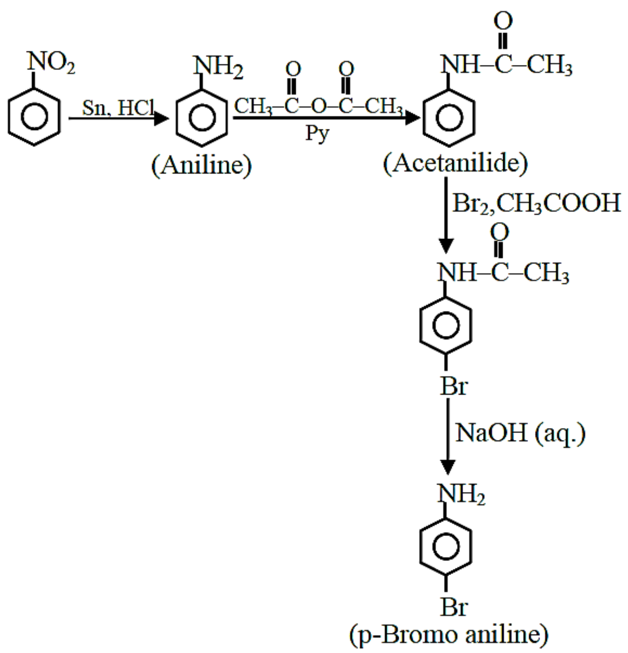
cis isomer trans isomer

Geometrical isomer = 2 (1cis + 1 trans)

Optical isomer = 3 (2 optically active +1 optically inactive)

Stereoisomer = 3

35. (a)



36. (b)

For charging of lead storage battery cell reaction is

$2PbSO_4(s) + 2H_2O(l) \rightarrow Pb(s) + PbO_2(s) + 2H_2SO_4(aq)$

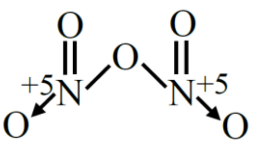
At anode $PbSO_4$ reduced back to Pb and at cathode $PbSO_4$ oxidised back to PbO_2.

$\because x_1 = +2, y_1 = 0$

$x_2 = +2, y_2 = 4$

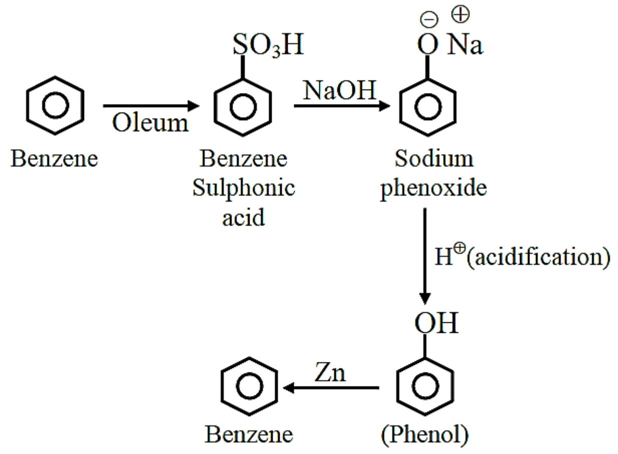
37. (d)

In oxide of nitrogen it can achieve +5 oxidation state because it can form $p\pi - p\pi$ bond with oxygen e.g. N_2O_5

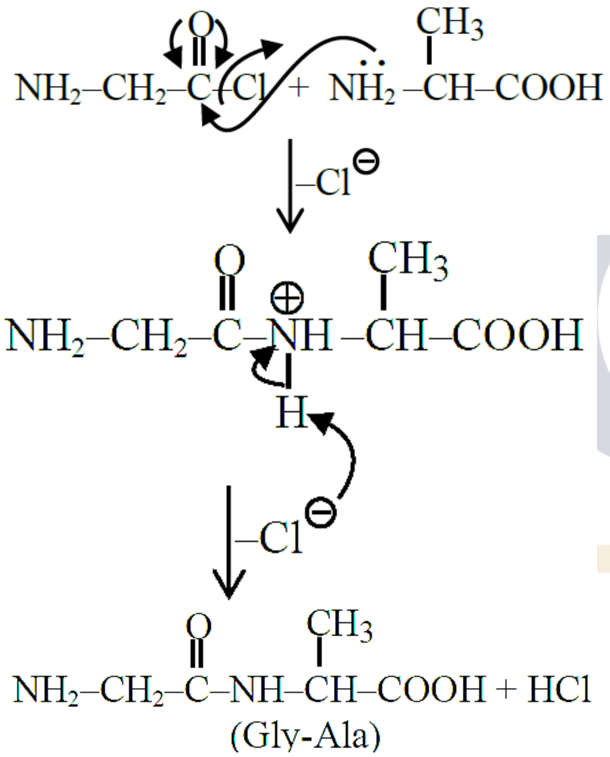


Nitrogen cannot form halide in +5 oxidation state because it does not contain d-orbital.
e.g. NX_5 does not exist
X = halide

38. (b)

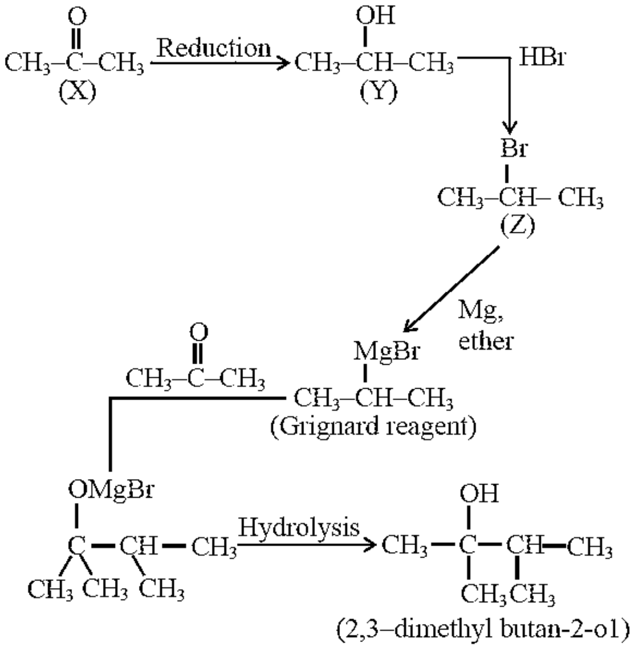


39.(a)



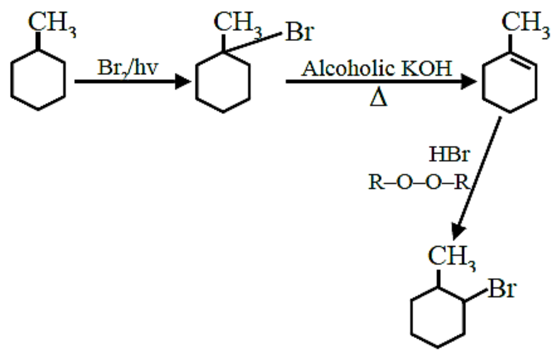
40. (a)
Size order
Al > Ga
$Al^{3+} < Ga^{3+}$
Atomic number of Ga is 31

41. (b)



O
CH3–C–CH3
(X)
Reduction
OH
CH3–CH–CH3
(Y)
HBr
Br
CH3–CH– CH3
(Z)
Mg,
ether
MgBr
CH3–CH–CH3
(Grignard reagent)
O
CH3–C–CH3
OMgBr
C–CH–CH3
CH3 CH3 CH3
Hydrolysis
OH
CH3–C–CH–CH3
CH3 CH3
(2,3–dimethyl butan-2-o1)

42. (b)



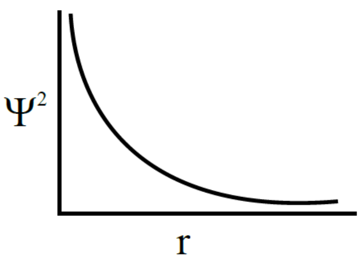
43. (c)

44. (a)

Ion	Configuration	No. of unpaired e^-
(1) V^{3+}	[Ar] $3d^3\ 4s^0$	3
Co^{2+}	[Ar] $3d^7\ 4s^0$	3
(2) Ti^{2+}	[Ar] $3d^2\ 4s^0$	2
Co^{2+}	[Ar] $3d^7\ 4s^0$	3
(3) Fe^{3+}	[Ar] $3d^5\ 4s^0$	5
Cr^{2+}	[Ar] $3d^4\ 4s^0$	4
(4) Ti^{3+}	[Ar] $3d^1\ 4s^0$	1
Mn^{2+}	[Ar] $3d^5\ 4s^0$	5

So V^{2+} & Co^{2+} same number of unpaired electron.

45. (d)



Ψ^2 = Probability density is maximum at nucleus.
Electron can exist upto infinity from nucleus.
True
Energy of electron is maximum at infinite distance from nucleus.

Section-B

46. (20)

Partial pressure of $N_2 = (900 - 15) = 885$ mmHg Mole of $N_2 = \frac{\left(\frac{885}{760} \times 0.15\right)}{(0.0821 \times 300)} = 0.0071$moles % of nitrogen in organic compound

$$= \frac{(0.0071 \times 28)}{1} \times 10$$
$$= 19.85\%$$

47.(10)

$$\underset{(O.A)}{\overset{+7}{KMnO}} \xrightarrow{Acidic\ Medium} Mn^{2+}$$

X is difference in oxidation state.
$7 - 2 = 5$
So $X = 5$

$$6CH_3COO^{\ominus} + Fe^{3+} + H_2O$$
$$\rightarrow [Fe_3(OH_2)(CH_3COO)_6]^{\oplus} + 2H^{\oplus}$$
$$[Fe_3(OH)_2(CH_3COO)_6]^{\oplus} + 4H_2O$$
$$\rightarrow \underset{\text{Brown red ppt}}{[Fe(OH)_2(CH_3COO]} + CH_3COOH + H^{\oplus}$$

$Fe^{3+} \Rightarrow 3\,d^5 4\,s^0$ contains 5 d electrons

So Y = 5

$X + Y = 5 + 5 = 10$

48. (9)

Let mass of iron = wg

$$\Rightarrow \frac{W}{150 \times 10^3} \times 10^6 = 12$$
$$\Rightarrow \ W = 150 \times 12 \times 10^{-3} = 1.8\text{gm}$$

Let mass of $FeSO_4 \cdot 7H_2O = w_1$gm

$$\Rightarrow \ \text{Moles of Fe} = \frac{1.8}{56} = \left(\frac{w_1}{56 + 96 + 7 \times 18}\right)$$
$$\Rightarrow w_1 = 8.935\text{gm}$$

49. (25)

After dilution

$$pH = 6\ [H^+] = 10^{-6}$$
$$[H^+] = 10^{-6} = \sqrt{K_a C}$$
$$10^{-6} = \sqrt{4 \times 10^{-10} \times C}$$
$$\frac{10^{-12}}{4 \times 10^{-10}} = C$$
$$0.25 \times 10^{-2} = C$$
$$20 \times 10^{-4} = C \ \therefore x = 25$$

50. (33)

Two nucleic acid chains are wound about each other and held together by H bonds between pair of bases. Adenine from two hydrogen bonds with thymine and Guanine form three hydrogen bond with cytosine.

$5'G - G - C - A - A - A - T - C -$

$G - G - C - T - A - 3'$

In given DNA strand total seven guanine and cytosine bases which form total 21 H -bonds and six adenine and thymine base which will form total 12 H -bonds with other DNA strand.

Total no. of H bonds $= 7 \times 3 + 6 \times 2 = 33$

Mathematics

Section-A

51. (d)

$$\text{fog}(x) = f(g(x))$$
$$= f\left(\frac{2-3x}{1-x}\right) = \frac{2\left(\frac{2-3x}{1-x}\right) + 3}{5\left(\frac{2-3x}{1-x}\right) + 2}$$
$$= \frac{4 - 6x + 3 - 3x}{10 - 15x + 2 - 2x} = \left(\frac{7-9x}{12-17x}\right)$$
$$\therefore \left[\begin{array}{l} 12 - 7x \neq 0 \\ x \neq \dfrac{12}{17} \end{array}\right.$$

$$\left[\begin{array}{l} f \circ g(2) = \dfrac{7-9(2)}{12-17(2)} = \dfrac{-11}{-22} = \dfrac{1}{2} \\ f \circ g(4) = \dfrac{7-9(4)}{12-17(4)} = \dfrac{-29}{-56} = \dfrac{29}{56} \end{array}\right.$$

Range of fog: $[\alpha, \beta] = \left[\frac{1}{2}, \frac{29}{56}\right]$

$\therefore (\beta-\alpha)=\frac{29}{56}-\frac{1}{2}=\frac{29-28}{56}=\frac{1}{56}$

$\frac{1}{(\beta-\alpha)}=56$

52. (c)

$A: x^2+y^2=25$………(1)

$B: \frac{x^2}{144}+\frac{y^2}{16}=1$……….(2)

$C: x^2+y^2\leq 4$……….(3)

Solve (1) & (2)

$x^2+9(25-x^2)=144$

$-8x^2=144-225=-81$

$x=\pm\frac{9}{2\sqrt{2}}$

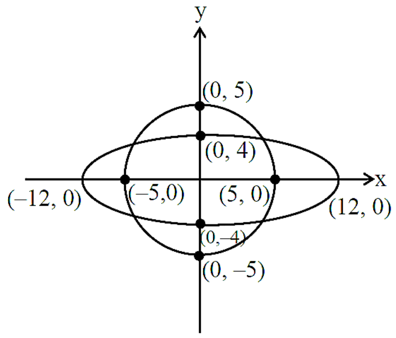
By(1) $\Rightarrow y=\pm\sqrt{25-x^2}$

$=\pm\sqrt{25-\frac{81}{8}}=\pm\frac{\sqrt{119}}{2\sqrt{2}}$

$\therefore D=A\cap B=$

$$\left\{\begin{array}{l}\left(\frac{9}{2\sqrt{2}},\frac{\sqrt{119}}{2\sqrt{2}}\right),\left(\frac{9}{2\sqrt{2}},-\frac{\sqrt{119}}{2\sqrt{2}}\right),\\ \left(\frac{-9}{2\sqrt{2}},\frac{\sqrt{119}}{2\sqrt{2}}\right),\left(\frac{-9}{2\sqrt{2}},\frac{-\sqrt{119}}{2\sqrt{2}}\right)\end{array}\right\}$$

No. of elements in set D=4



$\because C=\{(x,y)\in Z\times Z: x^2+y^2\leq 4\}$

$=\{(0,2),(2,0),(0,-2),(-2,0),(1,1),(-1,-1)$

$(1,-1),(-1,1),(1,0),(0,1),(-1,0),(0,-1)$

$(0,0)\}$

No. of elements in set C = 13

Total no. of one-one function from

Set D to sec $C \Rightarrow 13\times 12\times 11\times 10=17160$

53. (c)

$A=\{1,6,11,16,21,26,31,36,41,46,51,56,61, 66,71,76,81,86,91, \ldots\ldots\}$

$B=\{9,16,23,30,37,44,51,58,65,72,79,86, 93,100, \ldots\ldots\}$

$A\cap B=\{16,51,86, \ldots.$

For set ' A ' $\Rightarrow T_{2025}=1+(2025-1)(5)=10121$

For set ' B ' $\Rightarrow T_{2025}=9+(2025-1)(7)=14177$

So, for $(A\cap B)\Rightarrow T_n=16+(n-1)(35)\leq 10121$

$(n-1)\leq\frac{10121-16}{35}=288.71$

$n \leq 289.71 \Rightarrow n = 289$

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 2025 + 2025 - 289 = 3761$

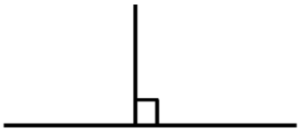
54. (d)

No. of terms in $(x+y)^{(2n-3)} \Rightarrow \left[\begin{matrix}(2n-3+1)\\(2n-2)\end{matrix}\right.$

$\therefore$ sum of all coefficients $= 2^{2n-3}$

(Put $x = y = 1$)

$\therefore$ Arithmetic mean of all coefficients



M

$$= \left(\frac{2^{2n-3}}{2n-2}\right) = 16$$

$\Rightarrow 2^{2n-3} = 2^5(n-1) \Rightarrow n = 5$

$\therefore P(2n-1, n^2-4n) = (9,5)$

$x + y = 8$

$$\therefore PM = \left|\frac{9+5-8}{\sqrt{2}}\right| = \frac{6}{\sqrt{2}} = \frac{3\times 2}{\sqrt{2}} = 3\sqrt{2}$$

55. (a)

3 engineering +1 doctor +8 Prof $\rightarrow\ {}^4C_3 \cdot {}^2C_1 \cdot {}^{10}C_8 = 360$

3 engineering +2 doctors +7 Prof $\rightarrow\ {}^4C_3 .\ {}^2C_2 .\ {}^{10}C_7$

$= 480$

4 engineering +1 doctor +7 Prof $\rightarrow\ {}^4C_4 \cdot {}^2C_1 \cdot {}^{10}C_7 = 240$

4 engineering +2 doctors +6 Prof $\rightarrow\ {}^4C_4 .\ {}^2C_2 .\ {}^{10}C_6$

$\underline{= 210}$

Total $= 1290$

Req. probability $= \frac{1290}{{}^{16}C_{12}} = \frac{1290}{1820} = \frac{129}{182}$

56. (b)

$A(3, \alpha, 3)$ & $B(-3, -7, \beta)$

$\overrightarrow{BA} = 6\hat{i} + (\alpha+7)\hat{j} + (3-\beta)\hat{k}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}$$

$$\frac{|\overrightarrow{BA} \cdot (\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|} = 3\sqrt{30}$$

$36 + 15(\alpha+7) - 3(3-\beta) = (3\sqrt{30})^2$

$36 + 15\alpha + 105 - 9 + 3\beta = 270$

$15\alpha + 3\beta = 138$

$5\alpha + \beta = 46$

57. (a)

Put $x = 1 + h$

$$\lim_{h\to 0} \frac{h(6 + \lambda\cos h) - \mu\sin h}{h^3} = -1$$

$$\lim_{h\to 0} \frac{h\left(6 + \lambda\left(1 - \frac{h^2}{2!}\right)\right) - \mu\left(h - \frac{h^3}{3!}\right)}{h^3} = -1$$

$6 + \lambda - \mu = 0$ and $-\frac{\lambda}{2} + \frac{\mu}{6} = -1$

$\lambda + \mu = 18$

58. (b)

$$y=1-2x+e^{x}\int_0^x e^{-t}f(t)dt$$

$$\frac{dy}{dx}=-2+e^{-x}\cdot e^{x}f(x)+e^{x}\int_0^x e^{-t}f(t)dt$$

$$\frac{dy}{dx}=-2+y+y+2x-1$$

$$\frac{dy}{dx}-2y=(2x-3)$$

$$ye^{-2x}=\int (2x-3)dx\cdot e^{-2x}$$

$$ye^{-2x}=\frac{-(2x-3)}{2}e^{-2x}+\int e^{-2x}dx$$

$$ye^{-2x}=\frac{-(2x-3)}{2}e^{-2x}-\frac{1}{2}e^{-2x}+c$$

$$f(0)=1\Rightarrow c=1-\frac{3}{2}+\frac{1}{2}=0$$

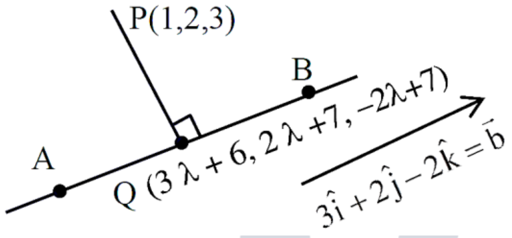
$$y=-\frac{(2x-3)}{2}-\frac{1}{2}$$

$$y=-x+1$$

$$x+y=1$$

$$\text{area }=\frac{1}{2}(1)(1)=\frac{1}{2}$$

59. (b)



$$\overrightarrow{PQ}\cdot\vec{b}=0$$

$$\Rightarrow 3(3\lambda+5)+2(2\lambda+5)-2(-2\lambda+4)$$

$$\Rightarrow 17\lambda=-17\Rightarrow\lambda=-1$$

Q(3,5,9)

Let $A(3\mu+6,2\mu+7,-2\mu+7)$

$$(3\mu+3)^2+(2\mu+2)^2+(-2\mu-2)^2=68$$

$$\Rightarrow \mu^2+2\mu-3=0\mu=-3 \text{ or } \mu=1$$

A(−3,1,13) and B(9,9,5)

$$\overrightarrow{OA}\cdot\overrightarrow{OB}=-27+9+65=47$$

60. (b)

$$f(2x)-f(x)=x$$

$$f(x)-f\left(\frac{x}{2}\right)=\frac{x}{2}$$

$$f\left(\frac{x}{2}\right)-f\left(\frac{x}{4}\right)=\frac{x}{4}$$

$$f\left(\frac{x}{4}\right)-f\left(\frac{x}{8}\right)=\frac{x}{8}$$

$$f\left(\frac{x}{2^{n-1}}\right)-f\left(\frac{x}{2^n}\right)=\frac{x}{2^n}$$

$$f(2x)-f\left(\frac{x}{2^n}\right)=x\left\{\frac{1-\left(\frac{1}{2}\right)^{n-1}}{1-\frac{1}{2}}\right\}$$

$$f(x)-f\left(\frac{x}{2^n}\right)=2x\left(1-\left(\frac{1}{2}\right)^{n+1}\right)$$

$$f(x)+x-f\left(\frac{x}{2^n}\right)=2x\left(1-\left(\frac{1}{2}\right)^{n+1}\right)$$

$$\lim_{n\to\infty}\left(f(x)-f\left(\frac{x}{2^n}\right)\right)=\lim_{n\to\infty}\left(2x\left(1-\left(\frac{1}{2}\right)^{n+1}\right)-x\right)$$

$G(x)=x$

$$\sum_{r=1}^{10} G(r^2)=\sum_{r=1}^{10} r^2=385$$

61. (b)

$(1^2+5^2+9^2+\cdots$ .... upto 20 terms $)+(3+7+11+$ ...upto 20 terms$)$

$$=\sum_{\mathrm{r}=1}^{20}(4\mathrm{r}-3)^2+\sum_{\mathrm{r}=1}^{20}(4\mathrm{r}-1)$$

$$=\sum_{\mathrm{r}=1}^{20}(4\mathrm{r}-3)^2+(4\mathrm{r}-1)$$

$$=4\sum_{\mathrm{r}=1}^{20}(4\mathrm{r}^2-5\mathrm{r}+2)$$

$=16\sum_{\mathrm{r}=1}^{20}\mathrm{r}^2-20\sum_{\mathrm{r}=1}^{20}\mathrm{r}+8\sum_{\mathrm{r}=1}^{20}1=41880$

62. (c)

$$T_{r+1}={}^{n}C_r\left(2^{1/3}\right)^{n-r}\left(\frac{1}{3^{1/3}}\right)^r$$

$r=14$

$$T_{15}={}^{n}C_{14}\left(2^{1/3}\right)^{n-14}\left(\frac{1}{3^{1/3}}\right)^{14}$$

$T'_{15}=15^{\text{th}}$ term from last is $(n-13)^{\text{th}}$ term from beginning.

$$T'_{15}={}^{n}C_{n-14}\left(2^{1/3}\right)^{14}\left(\frac{1}{3^{1/3}}\right)^{n-14}$$

$$\Rightarrow\frac{T_{15}}{T'_{15}}=\frac{{}^{n}C_{14}\left(2^{1/3}\right)^{n-14}\left(\frac{1}{3^{1/3}}\right)^{14}}{{}^{n}C_{n-14}(2^{1/3})^{14}\left(\frac{1}{3^{1/3}}\right)^{n-14}}=\frac{1}{6}$$

$$=\left(2^{1/3}\right)^{n-28}\left(3^{1/3}\right)^{n-28}=\frac{1}{6}$$

$$=6^{\frac{n-28}{3}}=6^{-1}$$

$=n=25$

So, ${}^{\mathrm{n}}\mathrm{C}_3={}^{25}\mathrm{C}_3=2300$

63.(b)

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}x+\frac{1}{2}\sqrt{1-x^2}\right), \frac{-1}{2}<x<\frac{1}{\sqrt{2}}$$

$\Rightarrow$ Let $\sin^{-1}(x)=\theta \ \frac{-\pi}{6}<\theta<\frac{\pi}{4}$

$\Rightarrow x=\sin\theta$, then

$$\Rightarrow \sin^{-1}\left(\frac{\sqrt{3}}{2}\sin\theta+\frac{1}{2}\cos\theta\right)$$

$$\Rightarrow \sin^{-1}\left(\sin\left(\theta+\frac{\pi}{6}\right)\right)=\theta+\frac{\pi}{6}$$

$$\Rightarrow \sin^{-1}(x)+\frac{\pi}{6}$$

64.(b)

$\vec{u}=3\hat{\imath}-\hat{\jmath}, \vec{v}=2\hat{\imath}+\hat{\jmath}-\lambda\hat{k}$

$$\Rightarrow \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}=\cos\theta$$

$$\Rightarrow \frac{5}{\sqrt{10}\sqrt{5+\lambda^2}}=\frac{\sqrt{5}}{2\sqrt{7}}$$

$\Rightarrow \lambda^2=9 \Rightarrow \lambda=3 (\because \lambda>0)$

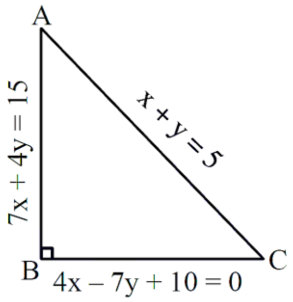
$\vec{v}=\vec{v}_1+\vec{v}_2$

$\Rightarrow |\vec{v}|^2=\vec{v}_1^2+\vec{v}_2^2+2\vec{v}_1\cdot\vec{v}_2$

$\Rightarrow 14=\vec{v}_1^2+\vec{v}_2^2+0 \ (\because \vec{v}_1\perp\vec{v}_2)$

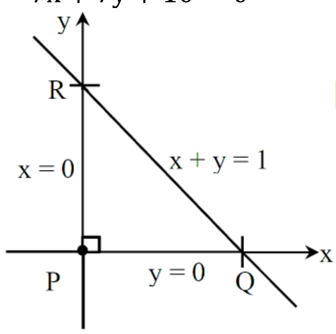
$\Rightarrow \left|\vec{v}_1^2\right|+\left|\vec{v}_1^2\right|=14$

65. (b)



Since triangle is right angle so orthocentre will be at right angle vertex

$$\left.\begin{array}{l}7x+4y=15\\ 7x+7y+10=0\end{array}\right\} \text{P.O.I } B\equiv(1,2)$$



distance between P and B $=\sqrt{5}$

66. (d)

$$I=\int_{-1}^{1}\frac{(1+\sqrt{|-x|-(-x)})e^{-x}+(\sqrt{|-x|-(-x)})e^{-(-x)}}{e^{-x}+e^{-(-x)}}dx$$

$$\Rightarrow I=\int_{-1}^{1}\frac{(1+\sqrt{|x|+x})e^{-x}+(\sqrt{|x|+x})e^{x}}{e^{x}+e^{x}}dx$$

$$\Rightarrow 2I = \int_{-1}^{1} \frac{(1+\sqrt{|x|+x}+\sqrt{|x|-x})(e^{x}+e^{-x})}{(e^{x}+e^{-x})} dx$$

$$\Rightarrow 2I = \int_{-1}^{1} (1+\sqrt{|x|+x}+\sqrt{|x|-x}) dx$$

$$\Rightarrow 2I = 2\int_{0}^{1} (1+\sqrt{|x|+x}+\sqrt{|x|-x}) dx$$

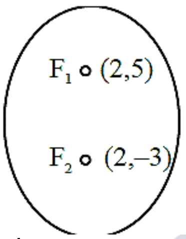
$$\Rightarrow 2I = 2\int_{0}^{1} (1+\sqrt{2x}+\sqrt{0}) dx$$

$$\Rightarrow I = \int_{0}^{1} (1+\sqrt{2x}) dx = \left[x+\frac{2\sqrt{2}}{3}x^{3/2}\right]_{0}^{1}$$

$$\Rightarrow I = \frac{2\sqrt{2}}{3}+1$$

67. (d)

$2be = 8$
$be = 4$



$b\left(\frac{4}{5}\right) = 4 \Rightarrow b = 5$
$\because c^2 = b^2 - a^2$
$16 = 25 - a^2 \Rightarrow a = 3$
L.R. $= \frac{2a^2}{b} = \frac{18}{5}$

68. (c)

$x^2 + 4x + 4 = n + 4\ (x+2)^2 = n + 4$
$x = -2 \pm \sqrt{n+4}$
$\because 20 \le n \le 100$
$\sqrt{24} \le \sqrt{n+4} \le \sqrt{104}$
$\Rightarrow \sqrt{n+4} \in \{5,6,7,8,9,10\}$
$\therefore$ ' 6 ' integral values of ' n ' are possible

69.(b)

x	$x=0$	$x=1$	$x=2$
$P(x)$	$\frac{^{7}C_2}{^{10}C_2}$	$\frac{^{7}C_1\,^{3}C_1}{^{10}C_2}$	$\frac{^{3}C_2}{^{10}C_2}$

$\mu = \Sigma x_i P(x_i) = 0 + \frac{7}{15} + \frac{2}{15} = \frac{3}{5}$

Variance $(x) = \Sigma P_i (x_i - \mu)^2 = \frac{28}{75}$

70. (a)

$10(\sin^2\theta)^2 + 15(1-\sin^2\theta)^2 = 6$

Let $\sin^2\theta = t \Rightarrow 10t^2 + 15(1-t)^2 = 16$

$10t^2 + 15 - 30t + 15t^2 = 6$

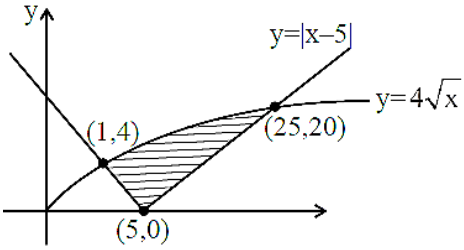
$25t^2 - 30t + 9 = 0$

$(5t-3)^2 = 0$

$\sin^2\theta = \frac{3}{5}$ and $\cos^2\theta = \frac{2}{5}$

$$\frac{27 \times \frac{125}{27} + 8 + \frac{125}{8}}{16\left(\frac{5}{2}\right)^4} = \frac{250}{125 \times 5} = \frac{2}{5}$$

71. (368)



$A = \int_1^{25} 4\sqrt{x}dx - \frac{1}{2} \times 4 \times 4 - \frac{1}{2} \times 20 \times 20$

$$A = \left[\frac{4x^{3/2}}{\frac{3}{2}}\right]^{25} - 8 - 200$$

$A = \frac{8}{3}(125-1) - 208$

$A = \frac{368}{3} \Rightarrow 3A = 368$

72. (6)

$\because$ A is orthogonal matrix

$\therefore A^T = A^{-1}$

$\Rightarrow A^2 = A^{-1}$

$\Rightarrow A^3 = I$

let$B = (A+I)^3 + (A-I)^3 - 6A$

$= 2(A^3 + 3A) - 6A$

$= 2A^3$

$$B = 2I = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Now sum of diagonal elements $= 2 + 2 + 2 = 6$

73. (22)

Let $z = x + iy$

$A: |z - 2 - i| = 3$

$|(x-2) + (y-1)i| = 3$

$(x-2)^2 + (y-1)^2 = 9$

$B = \mathrm{Re}(z - iz) = 2$

$\mathrm{Re}((x+y) + i(y-x)) = 2$

$x + y = 2$

On solving (1) and (2) we get

$x=\frac{3\pm\sqrt{17}}{2}, y=\frac{1\mp\sqrt{17}}{2}$

$\sum_{zes}|z|^2=\frac{1}{4}[2\times 26+2\times 18]$

$\Rightarrow \frac{88}{4}=22$

74. (46)

Let $E_1: \frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (a>b)$

$E_2: \frac{x^2}{c^2}+\frac{y^2}{d^2}=1, (c<d)$

$C: x^2+(y-1)^2=2$

Equation of tangent at $P(x_1, y_1)$

$xx_1+y(y_1-1)=(y_1+1)$

comparing with $x+y=3$ we get $P(1,2)$

$\because$ Now parametric equation of $x+y=3$

$\frac{(x-1)}{\left(\frac{-1}{\sqrt{2}}\right)}=\frac{(y-2)}{\left(\frac{1}{\sqrt{2}}\right)}=\pm\frac{2\sqrt{2}}{3}\left(\because PQ=\frac{2\sqrt{2}}{3}\right)$

On solving we get $Q\left(\frac{5}{3},\frac{4}{3}\right), R\left(\frac{1}{3},\frac{8}{3}\right)$

So, $9(x_1y_1+x_2y_2+x_3y_3)$

$9\left(2+\frac{5}{3}\times\frac{4}{3}+\frac{1}{3}\times\frac{8}{3}\right)$

$\Rightarrow 46$

75. (3)

$$f(x)=\begin{cases} x, & x<-1 \\ x^{21}, & -1\le x<0 \\ x, & 0\le x<1 \\ x^{21}, & x\ge 1 \end{cases}$$

$f(x)$ is continuous everywhere.

$\therefore n=0$

$$f'(x)=\begin{cases} 1, & x<-1 \\ 21x^{20}, & -1\le x<0 \\ 1, & 0<x<1 \\ 21x^{20}, & x\ge 1 \end{cases}$$

$\therefore f(x)$ is non-differentiable at $x=-1,0,1$

$\therefore m=3$