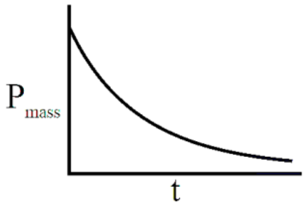
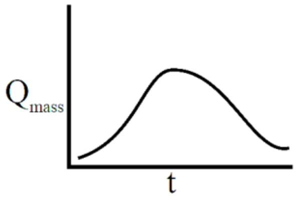
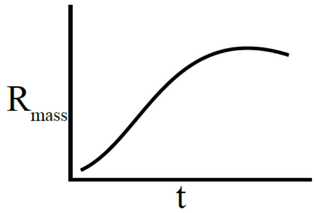
Physics
Section-A

1. (b)







2. (c)

$R_s = R_1 + R_2 + R_3 + \cdots \ldots\ldots + R_n$

$\dfrac{V^2}{P_s} = \dfrac{V^2}{P} + \dfrac{V^2}{P} + \cdots \ldots. + \dfrac{V^2}{P_n}$

$P_s = \dfrac{P}{n}$

3. (a)

$\Delta Q = ms\Delta T$

$8.1 \times 10^2 = 0.1 \times 900 \times \Delta T$

$\Delta A = A_0 2\alpha\Delta T = 2.0 \times 10^{-6}\ m^2$

4. (c)

$E = hf$

$ML^2T^{-2} = [h] \times [T^{-1}]$

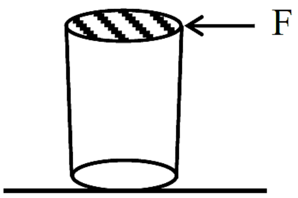
$[h] = [ML^2T^{-1}]$

$L = [MVR] = [ML^2T^{-1}]$

$L = \dfrac{nh}{2\pi}$

L is integral multiple of $\frac{h}{2\pi}$

5. (a)



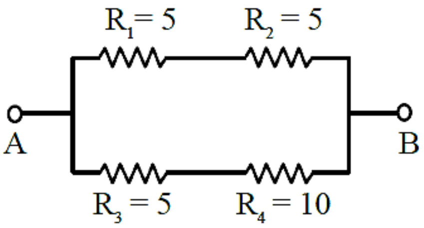
Shear moduli $= \dfrac{\sigma_{shear}}{\theta}$

$10^{10} = \dfrac{10^5}{\pi \times 16 \times 10^{-4}} \times \dfrac{1}{\theta}$

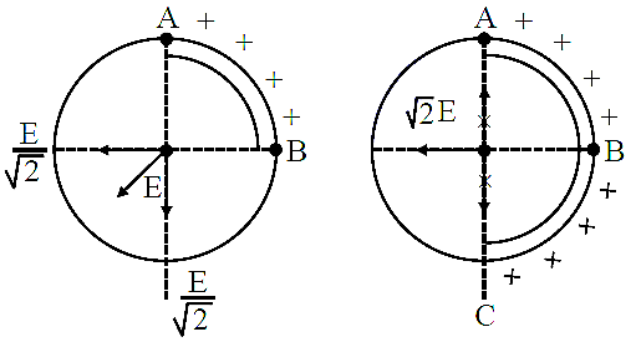
$\theta = \dfrac{1}{160\pi}$ Radian

6. (a)

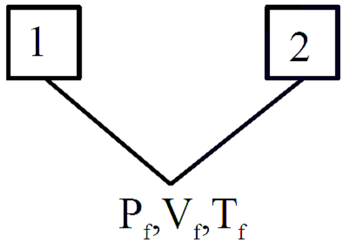
$\frac{1}{R_P} = \frac{1}{10} + \frac{1}{15} = \frac{3+2}{30} = \frac{1}{6}$



7. (b)



8. (b)



Number of masses will remain constant

$n_1 + n_2 = n_f$

$\frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2} = \frac{P_fV_f}{RT_f}$

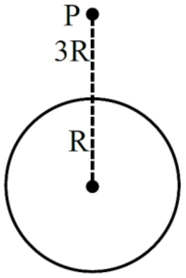
$\frac{8 \times 2\text{ V}}{R \times 1000} + \frac{7 \times \text{V}}{R \times 500} = \frac{P_f(3\text{ V})}{R \times 600}$

$\frac{16}{1000} + \frac{14}{1000} = \frac{P_f}{R \times 600}$

$\frac{30}{1000} = \frac{P_f}{200}$

$P_f = 6\text{kPa}$

9. (a)



$$P_P + k_P = P_O + k_0$$
$$-\frac{GMm}{4R} + \frac{1}{2}mV_P^2 = 0$$
$$V_P = \sqrt{\frac{GM}{2R}}$$

10. (c)

After dielectric

$$C_1 = 4C$$
$$C_1 = 4 \times 5 = 20\mu\text{ F}$$
$$C_2 = C_3 = 5\mu\text{ F}$$

C_1&C_2 are in series which is parallel to C_3So

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} + C_3 \Rightarrow \frac{20 \times 5}{20 + 5} + 5$$
$$= 4 + 5 = 9\mu\text{ F}$$

11. (d)

$$\langle\vec{v}\rangle = \frac{\Delta\vec{s}}{\Delta t} = \frac{S_f - S_i}{t_f - t_i}$$
$$\vec{v} = \frac{ds}{dt} = \text{ slope}$$

(A) 0 to 3sec; $\langle\vec{v}\rangle = \frac{5-0}{3} = 5/3\text{ m/s}$

(B) 0 to 5sec; $\langle\vec{v}\rangle = \frac{5-5}{2} = 0$

(C) $\text{t} = 2$; slope $= \vec{\text{v}} = 5\text{ m/s}$

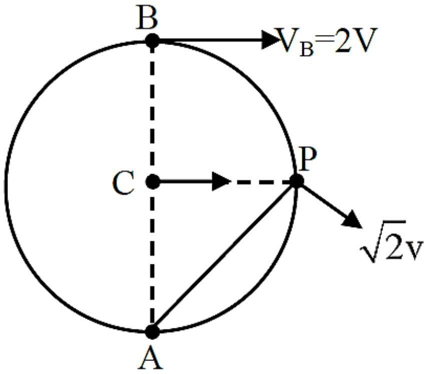
(D) $\text{t} = 5$ to 7sec; $\langle\vec{\text{v}}\rangle = \frac{0-5}{2} = -2.5\text{ m/s}$

At $\text{t} = 6.5$sec; $\vec{v} = 10$

(E) $\text{t} = 0$ to $\text{t} = 9$; $\langle\vec{v}\rangle = 0$

12. (a)

If $V_B = 2\text{ V}$



Point A is instantaneous center of rotation

Given $V_B = 8\text{ m/s}$

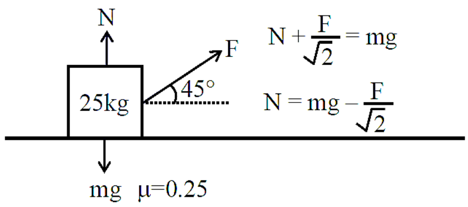
$$V = 4\text{ m/s}$$
$$V_P = \sqrt{2}\text{v} \Rightarrow V_P = 4\sqrt{2}\text{ m/s}$$

13. (b)

$$300\text{msd} = 15\text{ cm}$$

$$1\text{msd} = \frac{15}{300}\text{ cm} = 0.05\text{ cm}$$
$$25\text{vsd} = 24\text{msd}$$
$$1\text{vsd} = \frac{24}{25}\text{msd}$$
$$\text{LC} = 1\text{msd} - 1\text{vsd}$$
$$\text{LC} = 1\text{msd} - \frac{24}{25}\text{msd} = \frac{1}{25}\text{msd}$$
$$\text{LC} = \frac{1}{25} \times 0.05 = 0.002\text{ cm}$$

14. (c)

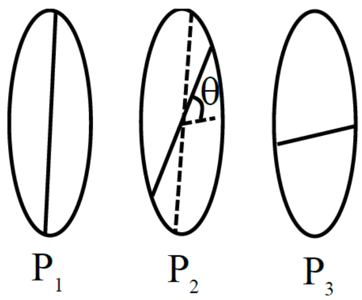


Block travels with uniform velocity
So $a = 0 \Rightarrow$ $F\cos 45^\circ$ = friction

$$\frac{F}{\sqrt{2}} = \mu\left[\text{mg} - \frac{F}{\sqrt{2}}\right]$$
$$\frac{F}{\sqrt{2}} = 0.25\left[25 \times 9.8 - \frac{F}{\sqrt{2}}\right]$$
$$\Rightarrow 1.25\frac{F}{\sqrt{2}} = 61.25$$
$$F = \frac{61.25 \times \sqrt{2}}{1.25} = 49\sqrt{2}$$
$$W_{\text{ext}} = FS\cos 45^\circ$$
$$= 49\sqrt{2} \times 5 \times \frac{1}{\sqrt{2}} = 245\text{ J}$$

15. (a)

Through $P_2 I_1 = I_0 \sin^2\left(\frac{\pi}{2} - \theta\right)$



$I_1 = I_0 \cos^2\theta$
Through P_3
$I_{\text{net}} = (I_0\cos^2\theta)\sin^2\theta$
$I_{\text{net}} = \frac{I_0}{4}[\sin(2\theta)]^2$ for max I_{net} $\theta = 45^\circ$
So angle between P_2 and $P_3 = \frac{\pi}{4}$

16. (c)

(A) n type semiconductor holes are minority carriers and e^- are majority carriers
(B) Dopant are pentavalent atom.
(C) $n_e \cdot n_h = n_i^2$ for intrinsic semiconductor
(E) In n type semiconductor primary source of holes generation are thermal excitation.

17. (c)

(A) Isobaric (P = C)
$\Delta Q = \Delta U + P\Delta V$
(B) Isochoric (V = C)
$\Delta Q = \Delta U$
(C) Adiabatic ($\Delta Q = 0$)
$\Delta Q = 0$
(D) Isothermal ($\Delta U = 0$)
$\Delta Q = \Delta W$

18. (b)

$$x(t) = 5\cos\left[628t + \frac{\pi}{2}\right] \text{ m}$$

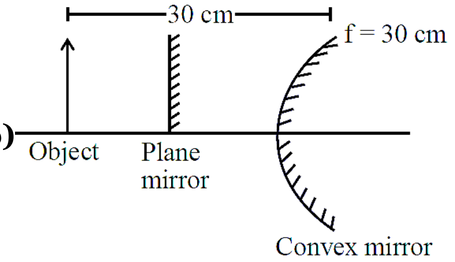
velocity (v_ω) = 300 m/s

$$v_w = \frac{\omega}{K}$$

$$300 = \frac{628}{K} \Rightarrow K = \frac{628}{300}$$

$$\frac{2\pi}{\lambda} = \frac{628}{300} \Rightarrow \lambda = \frac{2 \times 3.14 \times 300}{628} \lambda = 2 \text{ m}$$

19. (b)



For Convex mirror

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{30} = \frac{1}{30}$$

$$\frac{1}{v} = \frac{2}{30} = \frac{1}{15} \Rightarrow v = 15 \text{ cm}$$

Image formed by convex mirror is at 45cm from object so plane mirror should be placed midway at 22.5 cm from object so that both of their images may coinside,
Therefore distance between both mirrors
= 30 – 22.5 = 7.5 cm

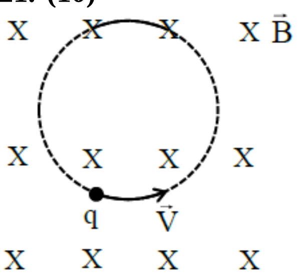
20. (d)

Electric dipole moment $(\vec{P}) = q \times 2\ell$
Magnetic dipole moment $(\vec{M}) = IA$

$$\left[\frac{P}{M}\right] = \left[\frac{LTA}{L^2 A}\right] = L^{-1}\, T = M^0\, L^{-1}\, T^1\, A^0$$

After compering values of P&Q are $0, -1$
Correct Answer : Option 4

21. (10)



Angle between $\vec{V}$ of charge & $\vec{B}$ is 90° motion will be uniform circular motion time period is given by $T = \frac{2\pi m}{qB} = \frac{2\pi \times 16 \times 10^{-9}\ kg}{1.6 \times 10^{-6} \times 6.28}$

T = 0.01 seconds

Answer is 10

22. (32)

When sphere is of radius R , its mass is M , when radius is reduced to $\frac{R}{2}$, mass will reduced to $\frac{M}{8}$

Now by conservation of angular momentum

$(\tau_{ext} = 0)$

$L_1 = L_2$

$I_1\omega_1 = I_2\omega_2$

$$\left(\frac{2}{5}M^2\right)\omega_1 = \left(\frac{2}{5}\left(\frac{M}{8}\right)\left(\frac{R}{2}\right)^2\right)\omega_2$$

$\omega_2 = 32\omega_1$ value of x is 32

23. (6)

Since velocity of light in terms of μ&E is

$$V = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\mu_r}} \times \frac{1}{\sqrt{\epsilon_0\epsilon_r}}$$

$$= \frac{1}{\sqrt{\mu_r\epsilon_r}} \times \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$= \frac{C}{\sqrt{\mu_r\epsilon_r}} = \frac{C}{\sqrt{\frac{10}{\pi} \times \frac{1}{0.0885}}}$$

$$= \frac{C}{\sqrt{36}} = \frac{C}{6}$$

$V = \frac{C}{6}$

C = 6V

Velocity of light in vacuum is greater by 6 times
the velocity of light in medium

24. (15)

Width of 20 maxima of double slit = width of central maxima of single slit

$$\frac{20\lambda D}{d} = \frac{2\lambda D}{a}$$

$$\frac{10}{d} = \frac{1}{a}$$

$$a = \frac{d}{10} = \frac{1.5 \times 10^{-1}}{10}\ cm = 15 \times 10^{-3}\ cm$$

Value of x is 15

25. (a)

Impedance of circuit

$$Z = \sqrt{R^2 + (X_L)^2} = \sqrt{R^2 + (\omega_L)^2}$$

$$= \sqrt{(100\pi)^2 + (2\pi \times 50 \times 1)^2}$$

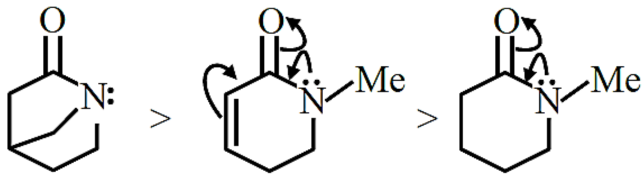
$$= \sqrt{(100\pi)^2 + (100\pi)^2}$$

$$= \sqrt{2} \times 100\pi$$

$$I_{ms} = \frac{V}{2} = \frac{100\pi}{\sqrt{2} \times 100\pi} = \frac{1}{\sqrt{2}}$$

$$I_{max} = \sqrt{2} I_{rms} = \sqrt{2} \times \frac{1}{\sqrt{2}} = 1\ \text{Ampere}$$

26.(d)



(R) (Q) (P)

According Bredts Rule it is Localised lone pair

cross Conjugation

27. (d)

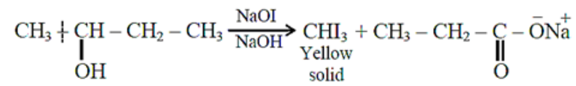
$Mn^{2+}: [Ar]3d^5$

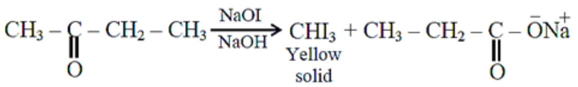
Half filled stability

More 1E than Fe^{2+}

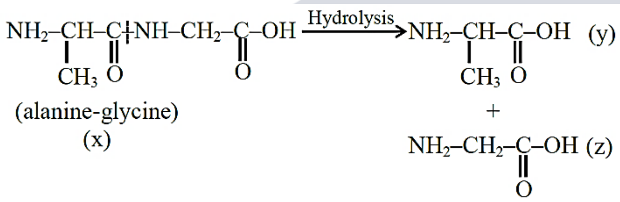
$Fe^{2+}: [Ar]3d^6$

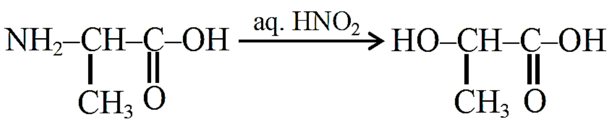
28. (b)



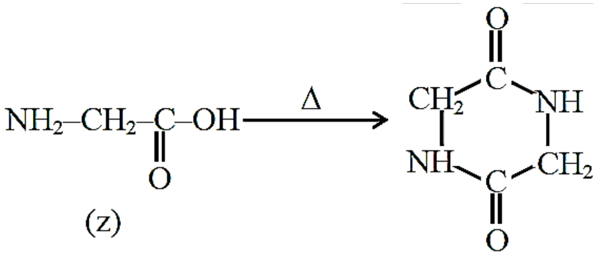


29. (b)



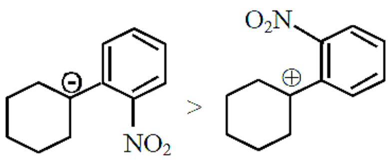


(y) Lactic acid



(z)

30. (b)

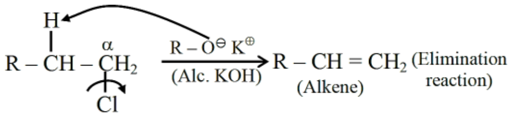


Stable by –M, –I effect of NO_2 group

31. (b)

Statement (I) : $R-Cl \xrightarrow{\text{(aq. KOH)}} R-OH$ (S_N reaction)

Statement (II) :



32. (d)
Statement-I
Molar depression constant $k_f = \frac{M_1RT_f^2}{\Delta H_{fus}}$

$$k_f = \frac{M_1RT_f}{\left[\frac{\Delta H_{fus}}{T_f}\right]}$$

$$k_f = \frac{M_1RT_f}{\Delta S_{fus}}$$

Hence statement-I is correct
but k_f for benzene $= 5.12 \frac{^\circ C}{\text{molal}}$
k_f for water $= 1.86 \frac{^\circ C}{\text{molal}}$ Hence statement- II is incorrect

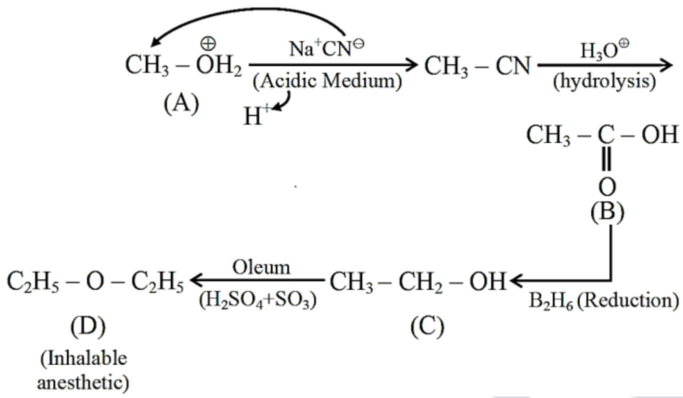
33. (d)

$$\overset{7}{HC}\equiv\overset{6}{C}-\overset{5}{CH_2}-\overset{4}{CH}(OH)-\overset{3}{CH_2}-\overset{2}{CH}=\overset{1}{CH_2}$$

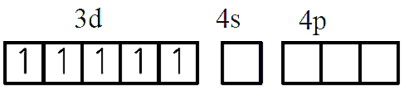
Hept-1-en-6-yn-4-ol

34. (a)
(A) Aniline - H_2O : Steam Distillation
(B) Glycerol from spent-lye in soap industry
Distillation under reduced pressure
(C) Different fraction of crude oil in petroleum industry - Fractional distillation
(D) $CHCl_3$ – Aniline - Simple distillation

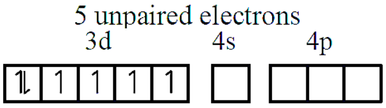
35. (c)



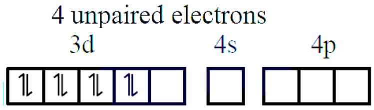
36. (d)

$[FeF_6]^{3-}$: [Ar]


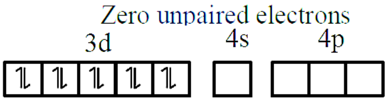
5 unpaired electrons

$[CoF_6]^{3-}$: [Ar]


4 unpaired electrons

$[Ni(CN)_4]^{2-}$: [Ar]


Zero unpaired electrons

$[Ni(CO)_4]$: [Ar]


Zero unpaired electrons

37. (b)
From the given information
ΔH is negative so it means dissolution of gas HCl(g) is exothermic.

$HCl(g) + 10H_2O(l) \rightarrow HCl \cdot 10H_2O$ ……(1)

$\Delta H_1 = -69.01 \frac{kJ}{mol}$

$HCl(g) + 40H_2O(l) \rightarrow HCl \cdot 40H_2O$ ………(2)

$\Delta H_2 = -72.79 \frac{kJ}{mol}$

Hence heat of solution depends upon amount of solvent
By equation.
(2) - equation
$HCl.10H_2O + 30H_2O(\ell) \rightarrow HCl.40H_2O$
So Heat of dilution $= -72.79 - (-69.01)$
$= -3.78 \frac{kJ}{mol}$
Hence option (3) is incorrect.
For heat of formation reactant should be in elemental form hence option (4) is incorrect

38. (c)

Cr: $1s^2 2s^2 2p^6 3s^2 3p^6 3d^5 4s^1$
$\ell=1$ $\ell=1$ $\ell=2$
electrons having $\ell = 1 \Rightarrow 12$
electrons having $\ell = 2 \Rightarrow 5$

39. (a)

Statement 1 is correct since left to right 1E increases in general in periodic table.
Statement 2 is incorrect since M.P. of group 14 elements is more than group 13 elements.

40. (a)

$K = Ae^{-EaRT}$

$\log k = \log A - \frac{Ea}{2.303RT}$

For graph between logk with $\frac{1}{T}$

$\left| \text{Slope of curve} \right| = \frac{Ea}{2.303R}$

From given graph
Magnitude of slope $\Rightarrow (2) > (1) > (3)$
Hence $Ea_2 > Ea_1 > Ea_3$

41. (d)

Most stable is $[Fe(C_2O_4)_3]^{3-}$ due to Chelation effect.

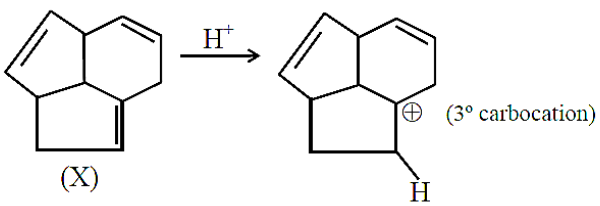
$[Fe(C_2O_4)_3]^{3-}$: [Ar] 3d: 1 1 1 1 1 ; 4s: [ ] ; 4p: [ ] [ ] [ ]
5 unpaired electrons ; x = 5

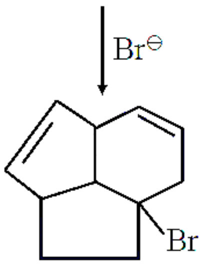
V_2O_5 is amphoteric.

42.(d)

IE order
$B > T\ell > Ga > Al > In$

43. (b)





(Major Product)

44.(b)

Statement 1 is correct.

Statement 2 is incorrect since in sp^3 d hybridization; lone pair cannot occupy axial position.

45. (c)

For zero order reaction

$$\text{Half life } = \frac{A_o}{2k}$$

$$60\text{ min} = \frac{2}{2k}$$

$$k = \frac{1}{60}\text{M/min}$$

Now

$$A_t = A_o - kt$$

$$t = \frac{A_o - A_t}{k}$$

$$= \frac{0.5 - 0.25}{1/60}$$

$$0.25 \times 60$$

$$t = 15\text{ min}$$

Section-B

46. (2)

Sea water is 6 Molar in NaCl , So 1000 ml of sea water contains 6 mol of NaCl .

$$\text{mass of solution } = \text{ Volume } \times \text{ density}$$

$$= 1000 \times 2$$

$$\text{mass of solution } = 2000\text{ g}$$

$$\text{ppm } = \frac{\text{mass of } O_2}{2000} \times 10^6$$

$$\text{mass of } O_2 = 5.8 \times 2 \times 10^{-3}$$

$$= 1.16 \times 10^{-2}\text{ g}$$

$$\text{molality for } O_2 = \frac{1.16\times10^{-2}/32}{(2000-6\times58.5)} \times 1000$$

$$= \frac{1.16 \times 10}{32 \times 1649}$$

$$= 0.000219$$

$$= 2.19 \times 10^{-4}$$

Correct answer $\Rightarrow$ 2

47. (63)

$$CaCO_3 \rightarrow CaO + CO_2$$

$$\text{mass of } CaCO_3 = \frac{150 \times 75}{100} = 112.5\text{ kg}$$

$$= 112500\text{ g}$$

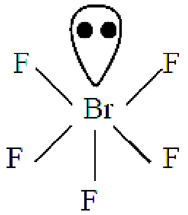
$$n_{CaCO_3} = 1125$$

So moles of CaO = 1125

mass of CaO $= \frac{1125 \times 56}{1000} = 63$ kg

Correct answer $\Rightarrow 63$

48. (2)



1 lone pair hence 1 mole AgCl.

Complex is $[M(NH_3)_3Cl_3]Cl$.

It shows 2 geometrical isomers ($Ma_3\ b_3$ type) facial (fac) & meridional (Mer)

49.(3)

λ_m° of $NH_4Cl = 185$

$(\lambda_m^\circ)_{NH_4^+} + (\lambda_m^\circ)_{Cl^-} = 185$

$(\lambda_m^\circ)_{NH_4^+} = 185 - 70 = 115\text{Scm}^2\ \text{mol}^{-1}$

$(\lambda_m^\circ)_{NH_4OH} = (\lambda_m^\circ)_{NH_4^+} + (\lambda_m^\circ)_{OH^-}$

$= 115 + 170$

$(\lambda_m^\circ)_{NH_4OH} = 285\text{Scm}^2\ \text{mol}^{-1}$

degree of dissociation $= \frac{(\lambda_m)_{NH_4OH}}{(\lambda_m^\circ)_{NH_4OH}}$

$= \frac{85.5}{285}$

$= 0.3$

$= 3 \times 10^{-1}$

50.(3)

$\text{pH} = 10$

$\text{pOH} = 4$

$[OH^-] = 10^{-4}$

no. of moles of $OH^- = 10^{-4}$

no. of moles of $Mg(OH)_2 = \frac{10^{-4}}{2} = 5 \times 10^{-5}$

mass of $Mg(OH)_2 = 5 \times 10^{-5} \times 58 \times 10^3\text{mg}$

$= 2.9$

Mathematics

Section-A

51. (b)

. $f'(x) = 18x^2 - 90ax + 108a^2 = 0$

$x = 2a \& x = 3a$

$x_1 = 2a\ x_2 = 3a$

$x_1 x_2 = 54$

$6a^2 = 54$

$a = 3$

$a + x_1 + x_2$

$3 + 2 \times 3 + 3 \times 3 = 18$

52. (a)

$\lim_{x\to 0}(f(2+x))\dfrac{3}{x}$

$e^{\lim_{x\to 0}\frac{(f(2+x)-1)3}{x}}$

$e^{3f'(2)}=(e)^{12}=(e)^{a}\Rightarrow a=12$

$y=4x^3-4x^2-4(a-7)x-a$

$y=4x^3-4x^2-20x-12$

roots$x=-1,-1,3$

53. (d)

$T_n=\tan^{-1}\left(\dfrac{4}{4n^2+3}\right)$

$T_n=\tan^{-1}\left(\dfrac{\left(n+\frac{1}{2}\right)-\left(n-\frac{1}{2}\right)}{1+\left(n+\frac{1}{2}\right)\left(n-\frac{1}{2}\right)}\right)$

$T_n=\tan^{-1}\left(n+\dfrac{1}{2}\right)-\tan^{-1}\left(n-\dfrac{1}{2}\right)$

$T_1+T_2+\cdots+T_n=\tan^{-1}\left(n+\dfrac{1}{2}\right)-\tan^{-1}\left(\dfrac{1}{2}\right)$

$S_\infty=\dfrac{\pi}{2}-\tan^{-1}\left(\dfrac{1}{2}\right)$

54. (b)

$2x-y=0$

$\{0,0\}\{-1,-2\}\{1,2\}$

$2x-y=1$

$\{0,-1\}\{1,1\}\{2,3\}\{-1,-3\}$

Total $(0,0)(-1,-2),(1,2)(0,-1),(1,1)(2,3)(-1,-3)$

Reflexive $m=5\ \&\ell=7$

Symm. $n=5\ \ell+m+n=17$

55. (b)

$\omega_1=(8\sin\theta+7\cos\theta)+i(\sin\theta+4\cos\theta)$

$\omega_2=(\sin\theta+4\cos\theta)+i(8\sin\theta+7\cos\theta)$

$\omega_1\omega_2=8\sin^2\theta+7\sin\theta\cos\theta+32\sin\theta\cos\theta+$
$28\cos^2\theta-8\sin^2\theta-32\sin\theta\cos\theta-7\sin\theta\cos\theta$
$-28\cos^2\theta+i(\sin^2\theta+4\sin\theta\cos\theta+4\sin\theta\cos\theta$
$+16\cos^2\theta+64\sin^2\theta+56\sin\theta\cos\theta+56\sin\theta$
$\cos\theta+49\cos^2\theta)$

$\omega_1\omega_2=0+i(65\sin^2\theta+120\sin\theta\cos\theta+65\cos^2\theta)$

$\alpha+\beta=65+60\sin 2q$

$\alpha+\beta|_{\max}=125$

$\alpha+\beta|_{\min}=5$

Ans. $=125+5=130$

56. (c)

shortest distance $=\frac{|(\bar{a}-\bar{b})|\cdot(\bar{p}\times\bar{q})}{|\bar{p}\times\bar{q}|}$

where

$\bar{a}=-\hat{\imath}+0\hat{\jmath}+0\hat{k}$

$>\bar{a}-\bar{b}=(-1-p)\hat{\imath}-2\hat{\jmath}-\hat{k}$

$\bar{b} = p\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\bar{p} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$

$\bar{q} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$

$$\frac{1}{16} = \frac{|-1-p+4-1|}{\sqrt{6}}$$

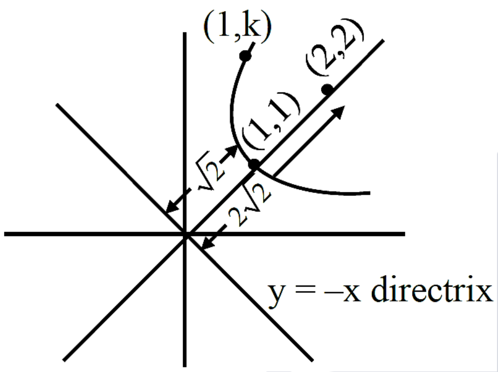
$|-p+2| = 1$

$p = 3 \,\&\, q = 1$

$$\frac{x^2}{1^2} + \frac{y^2}{3^3} = 1$$

$$\text{L.R} = \frac{2a^2}{b} = \frac{2 \times 1}{3} = \frac{2}{3}$$

57. (b)



Directrix $x + y = 0$

$PS = PM$

$$\sqrt{(1-2)^2 + (K-2)^2} = \frac{(1+K)}{\sqrt{2}}$$

$2K^2 + 8 - 8K + 2 = K^2 + 1 + 2K$

$K^2 - 10K + 9 = 0$

$K = 9$

58. (a)

$$\log_3\left(\log_7\left(8 - \log_2(x^2 + 4x + 5)\right)\right) > 0$$

$\log_2(x^2 + 4x + 5) < 1$

$x^2 + 4x + 3 < 0$

$\Rightarrow x \in (-3, -1)$

$$-1 \le \frac{7x + 10}{x - 2} \le 1$$

$\Rightarrow x \in [-2, -1]$

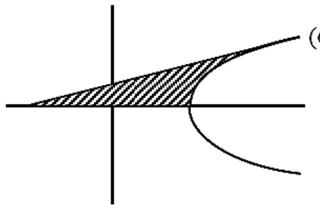
$\alpha = -3, \beta = -1, \gamma = -2, \delta = -1$

$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 15$

59. (c)

Tangent

$y = m(x+2)$
$y^2 = x - 2$
$(m(n+2))^2 = n - 2$
$m^2x^2 + (4m^2 - 1)x + (4m^2 + 2) = 0$
$D = 0$
$(4m^2 - 1)^2 - 4m^2(4m^2 + 2) = 0$
$m = \frac{1}{4}$
$y = \frac{1}{4}(n+2)$
and point of tangency (6,2)



Area $\mathrm{A} = \int_0^2 \left((\mathrm{y}^2 + 2) - (4\mathrm{y} - 2)\right)\mathrm{dy}$
$\mathrm{A} = \frac{8}{3}$

60. (c)

$ex + a + ex - a = 8\sqrt{\frac{5}{3}}$

$2ex = 8\sqrt{\frac{5}{3}}$

$2e \times 4 = 8\sqrt{\frac{5}{3}}$

$e = \sqrt{\frac{5}{3}}$

$b^2 = a^2\left(\left(\frac{\sqrt{5}}{3}\right)^2 - 1\right)$

$b^2 = \frac{2}{3}a^2$

$\frac{16}{\mathrm{a}^2} - \frac{9}{\mathrm{b}^2} = 1$
and $\mathrm{b}^2 = \frac{2}{3}\mathrm{a}^2$
$\Rightarrow \mathrm{a}^2 = \frac{5}{2}\ \ \mathrm{b}^2 = \frac{5}{3}$
Now,

$\ell = \frac{2\,b^2}{a}$

$\ell^2 = \frac{4\,b^4}{a^2}$

$9\ell^2 = 36 \times \frac{25}{9 \times 5} \times 2$

$9\ell^2 = 40$

$m = (ex + a)(ex - a)$

$m = e^2x^2 - a^2$

$= \frac{5}{3} \times 16 - \frac{5}{2} = \frac{145}{6}$

$= 6\,m = 145$

$9\ell^2 + 6\,m$

$40 + 145 = 185$

option (3)

61. (a)

$A^{50} = A^{48} + A^2 - I$

$= A^{46} + 2(A^2 - I)$

$= A^{44} + 3(A^2 - I)$

$= A^2 + 24(A^2 - I)$

$= 25A^2 - 24I$

$= 25\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} - 24\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$

Sum $= 53$

62.(c)

$\sum_{r=1}^{20} \frac{4r}{4+3r^2+r^4}$

$\sum_{r=1}^{20} \frac{4r}{(r^2+r+2)(r^2-r+2)}$

$2\sum_{r=1}^{20} \left(\frac{1}{r^2-r+2} - \frac{1}{r^2+r+2}\right)$

$2\left(\frac{1}{2} - \frac{1}{4}\right)$

$\frac{1}{4} - \frac{1}{8}$

$\frac{1}{8} - \frac{1}{14}$

$\left(\frac{1}{382} - \frac{1}{422}\right)$

$= 2\left(\frac{1}{2} - \frac{1}{422}\right)$

$= \frac{420}{422}$

$= \frac{210}{211}$

63.(a)

$$\sum_{r=1}^{15} r^2({}^{15}C_r) \Rightarrow 15\sum_{r=1}^{15} r\,{}^{14}C_{r-1}$$

$$15\sum_{r=1}^{15}(r-1+1)\,{}^{14}C_{r-1}$$

$$15\cdot 14\sum_{r=1}^{15}{}^{13}C_{r-2}+15\sum_{r=1}^{15}{}^{14}C_{r-1}$$

$15\cdot 14\cdot 2^{13}+15\cdot 2^{14}$

$3^1\cdot 2^{13}(70+10)$

$3^1\cdot 2^{13}\cdot 80$

$3^1\cdot 5^1\cdot 2^{17}$

$m=17\ n=1\ k=1$

64. (b)

Point of contact are $\left(\frac{\mp a^2 m}{\sqrt{a^2m^2+b^2}},\frac{\pm b^2}{\sqrt{a^2m^2+b^2}}\right)$ A $\left(\frac{-16}{5},\frac{9}{5}\right)$ B $\left(\frac{16}{5},\frac{-9}{5}\right)$

Point D is $\left(\frac{12}{5},\frac{12}{5}\right)$

Area of ABD $=\frac{1}{2}\begin{vmatrix}-\frac{16}{5} & \frac{9}{5} & 1\\ \frac{16}{5} & \frac{-9}{5} & 1\\ \frac{12}{5} & \frac{12}{5} & 1\end{vmatrix}$

$=12$

Area of ABCD is $=24$

65. (d)

Let $A(a-d,a,a+d)$ $B(b-D,b,b+D)$

$a=12$ $\quad b=12$

$p=12(144-d^2)$

$q=12(144-D^2)$

$$\frac{p+q}{p-q}=\frac{19}{5}$$

$$\frac{p}{q}=\frac{24}{14}=\frac{12}{7}$$

$$\frac{144-d^2}{144-(d^2+6d+9)}=\frac{12}{7}$$

$1008-7d^2=-12d^2-72d+1620$

$5d^2+72d-612=0$

$d=6$

$D=9$

$p-q=12(D^2-d^3)$

$=12(81-36)$

$=12(45)$

$=540$

66.(c)

$$\frac{dy}{dx}=\frac{7\cot y}{x}-\frac{e^x\operatorname{cosec}y}{x^5}$$

$$\frac{dy}{dx}=\frac{7\cot y}{\sin y\cdot x}-\frac{e^x}{\sin y x^5}$$

$$\sin y\frac{dy}{dx}-\cos y\cdot\frac{7}{x}=\frac{-e^x}{x^5}$$

let $-\cos y=t$

$\sin y\frac{dy}{dx}=\frac{dt}{dx}$

$\frac{dt}{dx}+\frac{7t}{x}=\frac{-e^x}{x^5}$

I.F. $=x^7$

t. $x^7=-\int x^2e^x dx$

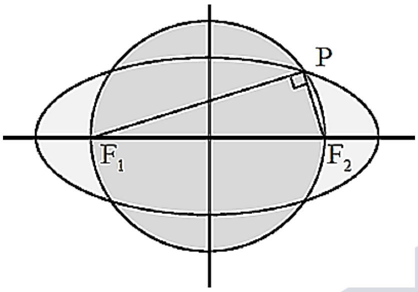
$\cos yx^7=x^2e^x-2\int xe^x dx$

$\cos yx^7=x^2e^x-2xe^x+2e^x+c$

$x=1, y=\frac{\pi}{2}, c=-e$

$\cos y=\frac{2e^2-e}{128}$

67. (b)



$\frac{1}{2}\mathrm{PF}_1\cdot\mathrm{PF}_2=30$

$\mathrm{PF}_1+\mathrm{PF}_2=17$

$\mathrm{PF}_1=12\mathrm{PF}_2=5$

$F_1F_2=13$

option (2)

68. (d)

$f(x)+2f\left(\frac{1}{x}\right)=x^2+5$

$f\left(\frac{1}{x}\right)+2f(x)=\frac{1}{x^2}+5$

$f(x)=\frac{2}{3x^2}-\frac{x^2}{3}+\frac{5}{3}$

$\alpha=\int_1^2\left(\frac{2}{3x^2}-\frac{x^2}{3}+\frac{5}{3}\right)dx$

$\left(-\frac{2}{3x}-\frac{x^3}{9}+\frac{5x}{3}\right)_1^2$

$-\frac{1}{3}-\frac{8}{9}+\frac{10}{3}+\frac{2}{3}+\frac{1}{9}-\frac{5}{3}$

$\alpha=2-\frac{7}{9}=\frac{11}{9}$

$2g(x)-3g\left(\frac{1}{2}\right)=x$

$g\left(\frac{1}{2}\right)=-\frac{1}{2}$

$g(x)=\frac{x}{2}-\frac{3}{4}$

$\beta=\int_1^2\left(\frac{x}{2}-\frac{3}{4}\right)dx$

$\left(\frac{x^2}{4}-\frac{3x}{4}\right)_1^2=1-\frac{3}{2}-\frac{1}{4}+\frac{3}{4}=0$

$9\alpha+\beta=11$

69. (a)

Angle between both lines

$\cos\theta=\left|\frac{3+0-5}{\sqrt{2}\sqrt{50}}\right|$

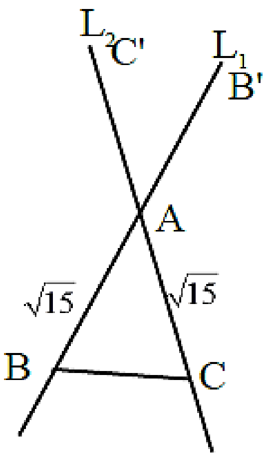
$\sin\theta=\frac{2}{10}=\frac{1}{5}$

$\sin\theta=\frac{\sqrt{24}}{5}$

area $=\frac{1}{2}$ absin θ

$\frac{1}{2}\sqrt{15}\sqrt{15}\frac{\sqrt{24}}{5}$

square of area $\frac{15.15.24}{4.25}$



70. (a)

$\frac{24+a+b}{8}=4$

$a+b=8$

$2=\frac{4+1+1+0+1+9+(a-4)^2+(b-4)^2}{8}$

$16=48+a^2+b^2-8a-8b$

$a^2+b^2=32$

$32=2ab$

$ab=16$

$a=4$ $b=4$

mode $=4$

mean deviation $=\frac{2+1+1+0+1+3+0+0}{8}=1$

71. (11)

$\alpha = \omega$

$$\therefore \left(\omega^k + \frac{1}{\omega^k}\right)^2 = \omega^{2k} + \frac{1}{\omega^{2k}} + 2$$

$$= \omega^{2k} + \omega^k + 2 \because \omega^{3k} = 1$$

$$\therefore \sum_{k=1}^{n} \left(\omega^{2k} + \omega^k + 2\right) = 20$$

$\Rightarrow (\omega^2 + \omega^4 + \omega^6 + \cdots + \omega^{2n})$
$+(\omega + \omega^2 + \omega^3 + \cdots +$
$\omega^n) + 2n = 20$

Now if $n = 3m, \ m \in I$

Then $0 + 0 + 2n = 20 \Rightarrow n = 10$ (not satisfy)

if $n = 3m + 1$, then

$\omega^2 + \omega + 2n = 20$

$-1 + 2n = 20 \Rightarrow n = \frac{21}{2}$ (not possible)

if $n = 3m + 2$,

$(\omega^8 + \omega^{10}) + (\omega^4 + \omega^5) + 2n = 20$

$\Rightarrow (\omega^2 + \omega) + (\omega + \omega^2) + 2n = 20$

$2n = 22$

$n = 11$ satisfy $n = 3m + 2$

$\therefore n = 11$

72. (379)

rationalise

$$\Rightarrow \int \frac{\left(\sqrt{1+x^2}+x\right)^{10}}{\left(\sqrt{1+x^2}-x\right)^9} \times \frac{\left(\sqrt{1+x^2}+x\right)^9}{\left(\sqrt{1+x^2}+x\right)^9} dx$$

$$\Rightarrow \int \frac{\left(\sqrt{1+x^2}+x\right)^{19}}{1} dx$$

Put $\sqrt{1+x^2} + x = t$

$$\left(\frac{x}{\sqrt{1+x^2}} + 1\right) dx = dt$$

$$dx = \frac{dt}{t}\sqrt{1+x^2}$$

Now as $\sqrt{1+x^2} + x = t$

$$\text{so } \sqrt{1+x^2} - x = \frac{1}{t}$$

$$\therefore \sqrt{1+x^2} = \frac{1}{2}\left(t + \frac{1}{t}\right)$$

Thus $I = \int \ t^{19} \cdot \frac{dt}{t} \cdot \frac{1}{2}\left(t + \frac{1}{t}\right)$

$$\Rightarrow \frac{1}{2}\int \ (t^{19} + t^{17})dt$$

$$= \frac{1}{2}\left(\frac{t^{20}}{20} + \frac{t^{18}}{18}\right) + C$$

$$= \frac{t^{19}}{360}\left[9t + \frac{10}{t}\right] + C$$

$$= \frac{t^{19}}{360}\left[9\left(t + \frac{1}{t}\right) + \frac{1}{t}\right] + C$$

$$\Rightarrow \frac{\left(\sqrt{1+x^2}+x\right)^{19}}{360}\left[9\left(2\sqrt{1+x^2}\right) + \left(\sqrt{1+x^2} - x\right)\right] + C$$

$\Rightarrow \frac{\left(\sqrt{1+x^2}+x\right)^{19}}{360}\left[19\sqrt{1+x^2}-x\right]+C$

$\therefore m=360, n=19$

$\therefore m+n=379$

73. (b)

n cards are drawn & are found all spade, thus remaining spades $=13-$ x remaining total cards $=52-x$

Now given that P (lost card is spade) $=\frac{11}{50}$

i.e. $\frac{{}^{13-n}C_1}{{}^{52-n}C_1}=\frac{11}{50}$

$50(13-\mathrm{n})=11(52-\mathrm{n})$

$39\mathrm{n}=78$

$\mathrm{n}=2$

74. (64)

Let $\mathrm{m}=6\mathrm{a}, \mathrm{n}=6\mathrm{~b}$

$\mathrm{m}<\mathrm{n} \Rightarrow \mathrm{a}<\mathrm{b}$

where a& b are co-prime numbers also since $m \& n$ are 2 digit nos, so

$10 \leq \mathrm{m} \leq 99 \& 10 \leq \mathrm{n} \leq 99$

i.e. $2 \leq \mathrm{a} \leq 16 \& 2 \leq \mathrm{b} \leq 16$

($\because$ a is integer)

Now

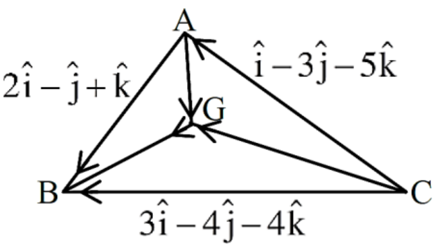
$2 \leq \mathrm{a}<\mathrm{b} \leq 16 \&$ a& b are co-prime

$\therefore$ if

$a=2, b=3,5,7,9,11,13,15$
$a=3, b=4,5,7,8,10,11,13,14,16$
$a=4, b=5,7,9,11,13,15$
$a=5, b=6,7,8,9,11,12,13,14,16$
$a=6, b=7,11,13$
$a=7, b=8,9,10,11,12,13,15,16$
$a=8, b=9,11,13,15$
$a=9, b=10,11,13,14,16$
$a=10, b=11,13$
$a=11, b=12,13,14,15,16$
$a=12, b=13$
$a=13, b=14,15,16$
$a=14, b=15$
$a=15, b=16$

64 ordered pairs

75. (164)



By given data

$\overrightarrow{AB}+\overrightarrow{AC}=\overrightarrow{CB}$

Let pv of $\vec{A}$ are $\vec{O}$ then

$\overrightarrow{AB}=\vec{B}-\vec{A}$

i.e. pv of $\vec{B}-=2\hat{\imath}-\hat{\jmath}+\hat{k}$

$\overrightarrow{CA}=\vec{A}-\vec{C}$

i.e. pv of $\vec{C} = -(\hat{\imath} - 3\hat{\jmath} - 5\hat{k})$

Now pv of centroid

$$(\vec{G}) - = \frac{\vec{A} + \vec{B} + \vec{C}}{3} = \frac{\vec{0} + (2,-1,1) + (-1,3,5)}{3}$$

$$\vec{G} = \frac{1}{3}(\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$$

$$\text{Now } \overrightarrow{AG} = \frac{1}{3}(\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$$

$$\Rightarrow |\overrightarrow{AG}|^2 = \frac{1}{9} \times 41$$

$$\overrightarrow{BG} = \left(\frac{1}{3} - 2\right)\hat{\imath} + \left(\frac{2}{3} + 1\right)\hat{\jmath} + (2-1)\hat{k}$$

$$\Rightarrow |\overrightarrow{BG}|^2 = \frac{59}{9}$$

$$\overrightarrow{CG} = \left(\frac{1}{3} + 1\right)\hat{\imath} + \left(\frac{2}{3} - 3\right)\hat{\jmath} + (2-5)\hat{k}$$

$$\Rightarrow |\overrightarrow{CG}|^2 = \frac{146}{9}$$

Now

$$6\left[|\overrightarrow{AG}|^2 + |\overrightarrow{BG}|^2 + |\overrightarrow{CG}|^2\right] = 6 \times \left[\frac{41}{9} + \frac{59}{9} + \frac{146}{9}\right]$$

$$= 6 \times \frac{246}{9} = 164$$